INTERNATIONAL SIXTH EDITION

SEDRA/SMITH

INSTRUCTOR'S SOLUTIONS MANUAL FOR MICROELECTRONIC CIRCUITS

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This Instructor's Solutions Manual contains complete solutions for the 1000+ end-of-chapter problems created specifically for the International Sixth Edition of Sedra/Smith's *Microelectronic Circuits*.

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Preface

This manual contains complete solutions for all exercises and end-of-chapter problems included in the book *Microelectronic Circuits, International Sixth Edition*, by Adel S. Sedra and Kenneth C. Smith.

We are grateful to Mandana Amiri, Shahriar Mirabbasi, Roberto Rosales, Alok Berry, Norman Cox, John Wilson, Clark Kinnaird, Roger King, Marc Cahay, Kathleen Muhonen, Angela Rasmussen, Mike Green, John Davis, Dan Moore, and Bob Krueger, who assisted in the preparation of this manual. We also acknowledge the contribution of Ralph Duncan and Brian Silveira to previous editions of this manual.

Communications concerning detected errors should be sent to the attention of the Engineering Editor, mail to Oxford University Press, 198 Madison Avenue, New York, New York, USA 10016 or e-mail to higher.education.us@oup.com. Needless to say, they would be greatly appreciated.

A website for the book is available at www.oup.com/sedra-xse

Ex: 1.1 When output terminals are open circuited

For circuit a. $v_{OC} = v_s(t)$

For circuit b. $v_{OC} = i_s(t) \times R_s$

When output terminals are short-circuited

For circuit a. $i_{sc} = \dfrac{v_S(t)}{R_S}$

For circuit b. $i_{sc} = i_S(t)$

For equivalency

$R_S i_S(t) = v_S(t)$

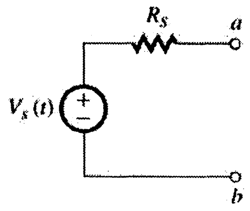


Figure 1.1a

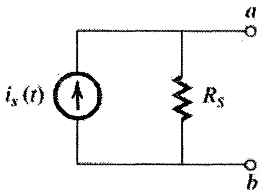
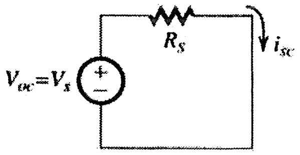


Figure 1.1b

Ex: 1.2

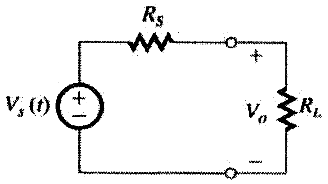


$V_{OC} = 10$ mV

$i_{SC} = 10$ μA

$R = \dfrac{V}{i} = \dfrac{10 \text{ mV}}{10\ \mu\text{A}} = 1\ \text{k}\Omega$

Ex: 1.3 Using voltage divider



$v_o(t) = v_S(t) \times \dfrac{R_L}{R_S + R_L}$

Given $v_S(t) = 10$ mV and $R_S = 1\ \text{k}\Omega$

If $R_L = 100\ \text{k}\Omega$

$v_O = 10 \text{ mV} \times \dfrac{100}{100 + 1} = 9.9 \text{ mV}$

If $R_L = 10\ \text{k}\Omega$

$v_O = 10 \text{ mV} \times \dfrac{10}{10 + 1} \simeq 9.1 \text{ mV}$

If $R_L = 1\ \text{k}\Omega$

$v_O = 10 \text{ mV} \times \dfrac{1}{1 + 1} = 5 \text{ mV}$

If $R_L = 100\ \Omega$

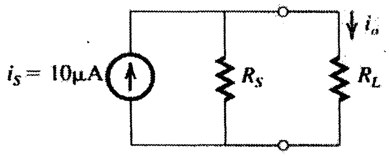
$v_O = 10 \text{ mV} \times \dfrac{100}{100 + 1 \text{ K}} \simeq 0.91 \text{ V}$

80% of source voltage $= 10 \text{ mV} \times \dfrac{80}{100} = 8 \text{ mV}$

If R_L gives 8 mV when $R_S = 1\ \text{k}\Omega$, then

$8 = 10 \times \dfrac{R_L}{1 + R_L} \Rightarrow R_L = 4\ \text{k}\Omega$

Ex: 1.4 Using current divider



$i_O = i_S \times \dfrac{R_S}{R_S + R_L}$

Given $i_S = 10\ \mu\text{A}$, $R_S = 100\ \text{k}\Omega$

For

$R_L = 1\ \text{k}\Omega$, $i_O = 10\ \mu\text{A} \times \dfrac{100}{100 + 1} = 9.9\ \mu\text{A}$

For $R_L = 10\ \text{k}\Omega$, $i_O = 10\ \mu\text{A} \times \dfrac{100}{100 + 10}$

$\simeq 9.1\ \mu\text{A}$

For

$R_L = 100\ \text{k}\Omega$, $i_O = 10\ \mu\text{A} \times \dfrac{100}{100 + 100} = 5\ \mu\text{A}$

For

$R_L = 1\ \text{M}\Omega$, $i_O = 10\ \mu\text{A} \times \dfrac{100 \text{ K}}{100 \text{ K} + 1 \text{ M}}$

$\simeq 0.9\ \mu\text{A}$

80% of source current $= 10 \times \dfrac{80}{100} = 8\ \mu\text{A}$

If a load R_L gives 80% of the source current, then

$8\ \mu\text{A} = 10\ \mu\text{A} \times \dfrac{100}{100 + R_L}$

$\Rightarrow R_L = 25\ \text{k}\Omega$

Ex: 1.5 $f = \frac{1}{T} = \frac{1}{10^{-3}} = 1000\text{ Hz}$

$\omega = 2\pi f = 2\pi \times 10^3\text{ rad/s}$

Ex: 1.6 (a) $T = \frac{1}{f} = \frac{1}{60}\text{s} = 16.7\text{ ms}$

(b) $T = \frac{1}{f} = \frac{1}{10^{-3}} = 1000\text{ s}$

(c) $T = \frac{1}{f} = \frac{1}{10^6}\text{s} = 1\ \mu\text{s}$

Ex: 1.7 If 6 MHz is allocated for each channel, then 470 MHz to 806 MHz will accomodate

$\frac{806 - 470}{6} = 56$ channels

Since it starts with channel 14, it will go from channel 14 to channel 69

Ex: 1.8 $P = \frac{1}{T}\int_0^T \frac{v^2}{R}\text{dt}$

$= \frac{1}{T} \times \frac{V^2}{R} \times T = \frac{V^2}{R}$

Alteratively,

$P = P_1 + P_3 + P_5 + \dots$

$= \left(\frac{4V}{\sqrt{2}\pi}\right)^2 \frac{1}{R} + \left(\frac{4V}{3\sqrt{2}\pi}\right)^2 \frac{1}{R} + \left(\frac{4V}{5\sqrt{2}\pi}\right)^2 \frac{1}{R} + \dots$

$= \frac{V^2}{R} \times \frac{8}{\pi^2} \times \left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \dots\right)$

It can be shown by direct calculation that the infinite series in the parentheses has a sum that approaches $\pi^2/8$; thus P becomes V^2/R as found from direct calculation.

Fraction of energy in fundamental

$= 8/\pi^2 = 0.81$

Fraction of energy in first five harmonics

$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25}\right) = 0.93$

Fraction of energy in first seven harmonics

$= \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49}\right) = 0.95$

Fraction of energy in first nine harmonics

$5\frac{8}{\pi^2}\left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81}\right) = 0.96$

Note that 90% of the energy of the square wave is in the first three harmonics; that is, in the fundamental and the third harmonic.

Ex: 1.9 (a) D can represent 15 distinct values between 0 and +15 V. Thus,

$v_A = 0\text{ V} \Rightarrow D = 0000$

$v_A = 1\text{ V} \Rightarrow D = 0001$

$v_A = 2\text{ V} \Rightarrow D = 0010$

$v_A = 15\text{ V} \Rightarrow D = 1111$

(b) (i) +1 V (ii) +2 V (iii) +4 V (iv) +8 V

(c) The closest discrete value represented by D is 5 V; thus $D = 0101$. The error is -0.2 V or $-0.2/5.2 \times 100 = -4\%$

Ex: 1.10 Voltage gain $= 20\log 100 = 40\text{ dB}$

Current gain $= 20\log 1000 = 60\text{ dB}$

Power gain $= 10\log A_p = 10\log(A_v A_i)$

$= 10\log 10^5 = 50\text{ dB}$

Ex: 1.11 $P_{dc} = 15 \times 8 = 120\text{ mW}$

$P_L = \frac{(6/\sqrt{2})^2}{1} = 18\text{ mW}$

$P_{\text{dissipated}} = 120 - 18 = 102\text{ mW}$

$\eta = \frac{P_L}{P_{dc}} \times 100 = \frac{18}{120} \times 100 = 15\%$

Ex: 1.12

$v_o = 1 \times \frac{10}{10^6 + 10} \simeq 10^{-5}\text{ V} = 10\ \mu\text{V}$

$P_L = v_O^2/R_L = \frac{(10 \times 10^{-6})^2}{10} = 10^{-11}\text{ W}$

With the buffer amplifier:

$v_O = 1 \times \frac{R_i}{R_i + R_S} \times A_{vo} \times \frac{R_L}{R_L + R_o}$

$= 1 \times \frac{1}{1+1} \times 1 \times \frac{10}{10+10} = 0.25\text{ V}$

$P_l = \frac{v_O^2}{R_L} = \frac{0.25^2}{10} = 6.25\text{ mW}$

Voltage gain $= \frac{v_O}{v_S} = \frac{0.25\text{ V}}{1\text{ V}} = 0.25\text{ V/V}$

$= -12\text{ dB}$

Power gain $(A_p) = \frac{P_L}{P_i}$

where $P_L = 6.25$ mW and $P_i = v_i i_1$.

$v_i = 0.5\text{ V}$ and

$i_i = \frac{1\text{ V}}{1\text{ M}\Omega + 1\text{ M}\Omega} = 0.5\ \mu\text{A}$

Thus,

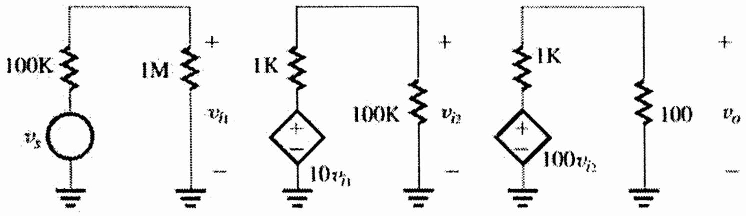
$P_i = 0.5 \times 0.5 = 0.25\ \mu\text{W}$

and,

$A_p = \frac{6.25 \times 10^{-3}}{0.25 \times 10^{-6}} = 25 \times 10^3$

$10\log A_p = 44\text{ dB}$

This figure belongs to Exercise 1.15



Ex: 1.13 Open-circuit (no load) output voltage $=$ $A_{v_o} v_i$

Output voltage with load connected

$$= A_{v_o} v_i \frac{R_L}{R_L + R_O}$$

$$0.8 = \frac{1}{R_O + 1} \Rightarrow R_O = 0.25 \text{ k}\Omega = 250\ \Omega$$

Ex: 1.14 $A_{v_o} = 40 \text{ dB} = 100 \text{ V/V}$

$$P_L = \frac{v_O^2}{R_L} = \left(A_{vo} v_i \frac{R_L}{R_L + R_O}\right)^2 / R_L$$

$$= v_i^2 \times \left(100 \times \frac{1}{1+1}\right)^2 / 1000 = 2.5 v_i^2$$

$$P_i = \frac{v_i^2}{R_i} = \frac{v_i^2}{10{,}000}$$

$$A_p = \frac{P_L}{P_i} = \frac{2.5 v_i^2}{10^{-4} v_i^2} = 2.5 \times 10^4 \text{ W/W}$$

$10 \log A_p = 44$ dB

Ex: 1.15 Without stage 3 (see figure above)

$$\frac{v_0}{v_S} = \left(\frac{1 \text{ M}}{100 \text{ K} + 1 \text{ M}}\right)(10)\left(\frac{100 \text{ K}}{100 \text{ K} + 1 \text{ K}}\right)$$

$$\times (100)\left(\frac{100}{100 + 1 \text{ K}}\right)$$

$$\frac{v_0}{v_S} = (0.909)(10)(0.9901)(100)(0.0909) = 81.8 \text{ V}$$

Ex: 1.16 Given $v_s = 1$ mV

$$\frac{v_{i1}}{v_s} = 0.909 \text{ So}$$

$$v_{i1} = 0.909\ v_S = 0.909 \times 1 = 0.909 \text{ mV}$$

$$\frac{v_{i2}}{v_s} = \frac{v_{i2}}{V_{i1}} \times \frac{v_{i1}}{V_S} = 9.9 \times 0.909 = 9 \text{ V/V}$$

For $v_s = 1$ mV

$$v_{i2} = 9 \times v_S = 9 \times 1 = 9 \text{ mV}$$

$$\frac{v_{i3}}{v_s} = \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_S} = 90.9 \times 9.9 \times 0.909$$

$= 818$ V/V

For $v_s = 1$ mV

$$v_{i3} = 818\ v_S = 818 \times 1 = 818 \text{ mV}$$

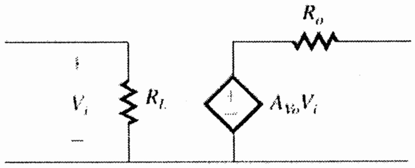
$$\frac{v_{iL}}{v_S} = \frac{v_{iL}}{v_{i3}} \times \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_S}$$

$= 0.909 \times 90.9 \times 9.9 \times 0.909 \simeq 744$

For $V_s = 1$ mV

$$V_{iL} = 744 \times 1 \text{ mV} = 744 \text{ mV}$$

Ex: 1.17 Using voltage amplifier model, it can be represented as



$R_i = 1$ MΩ

$R_o = 10\ \Omega$

$A_{vo} = A_{v1} \times A_{v2} = 9.9 \times 90.9 = 900$ V/V

The overall voltage gain

$$\frac{V_O}{V_S} = \frac{R_i}{R_i + R_S} \times A_{vo} \times \frac{R_L}{R_L + R_o}$$

For $R_L = 10\ \Omega$

Overall voltage gain

$$= \frac{1 \text{ M}}{1 \text{ M} + 100 \text{ K}} \times 900 \times \frac{10}{10 + 10} = 409 \text{ V/V}$$

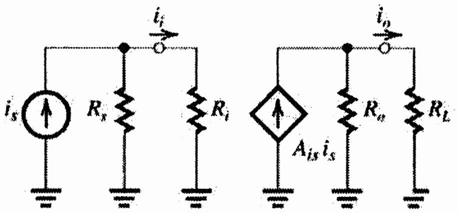
For $R_L = 1000\ \Omega$

Overall voltage gain

$$= \frac{1 \text{ M}}{1 \text{ M} + 100 \text{ K}} \times 900 \times \frac{1000}{1000 + 10} = 810 \text{ V/V}$$

$\therefore$ Range of voltage gain is from 409 to 810 V/V

Ex: 1.18


i_i
i_o
i_s
R_s
R_i
$A_{is}i_s$
R_o
R_L

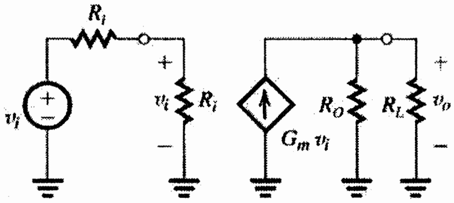
$$i_i = i_S\frac{R_S}{R_S + R_i}$$

$$i_O = A_{is}i_i\frac{R_O}{R_O + R_L} = A_{is}i_S\frac{R_S}{R_S + R_i}\frac{R_O}{R_O + R_L}$$

Thus,

$$\frac{i_O}{i_S} = A_{is}\frac{R_s}{R_S + R_i}\frac{R_O}{R_O + R_L}$$

Ex: 1.19


R_s
v_i
+
−
v_i
R_i
R_O
R_L
v_o
$G_m v_i$

$$v_i = v_S\frac{R_i}{R_i + R_s}$$

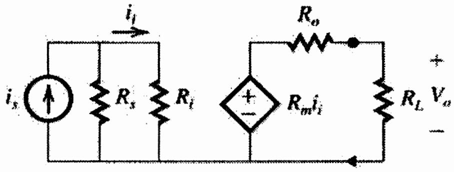
$$v_S = G_m v_i(R_O \| R_L)$$

$$= G_m v_S\frac{R_i}{R_i + R_S}(R_O \| R_L)$$

Thus,

$$\frac{v_O}{v_S} = G_m\frac{R_i}{R_i + R_S}(R_O \| R_L)$$

Ex: 1.20 Using transresistance circuit model the circuit will be


i_i
R_o
i_s
R_s
R_i
$R_m i_i$
R_L
+
V_o
−

$$\frac{i_i}{i_S} = \frac{R_S}{R_i + R_S}$$

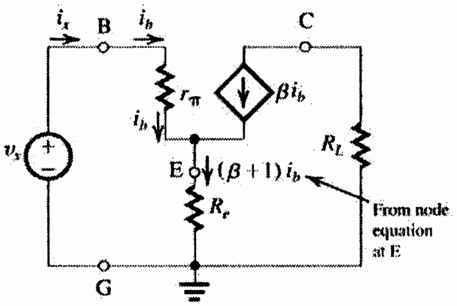
$$V_O = R_m i_i \times \frac{R_L}{R_L + R_O}$$

$$\frac{V_O}{i_i} = R_m\frac{R_L}{R_L + R_O}$$

Now $\frac{V_O}{i_S} = \frac{V_O}{i_i}\times\frac{i_i}{i_S} = R_m\frac{R_L}{R_L + R_O}\times\frac{R_S}{R_i + R_S}$

$$= R_m\frac{R_S}{R_S + R_i}\times\frac{R_L}{R_L + R_O}$$

Ex: 1.21


i_x
B
i_b
C
r_π
βi_b
i_b
v_x
R_L
E
$(\beta + 1)i_b$
R_e
G
From node equation at E

$$v_b = i_b r_\pi + (\beta + 1)i_b R_e$$

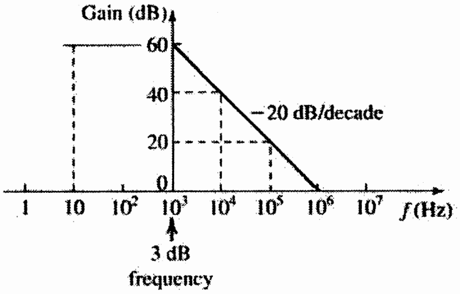
$$= i_b[r_\pi + (\beta + 1)R_e]$$

But $v_b = v_x$ and $i_b = i_x$, thus

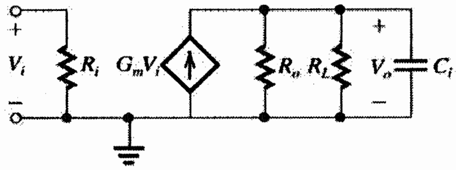
$$R_{in} \equiv \frac{v_x}{i_x} = \frac{v_b}{i_b} = r_\pi + (\beta + 1)R_e$$

Ex: 1.22

f	Gain
10 Hz	60 dB
10 kHz	40 dB
100 kHz	20 dB
1 MHz	0 dB



Ex: 1.23


+
V_i
−
R_i
G_mV_i
R_o
R_L
+
V_o
−
C_l

$$V_O = G_m V_i[R_O \| R_L \| C_L]$$

$$= \frac{G_m V_i}{\frac{1}{R_O} + \frac{1}{R_L} + sC_L}$$

Thus, $\frac{V_O}{V_i} = \frac{G_m}{\frac{1}{R_O} + \frac{1}{R_L}}\,\frac{1}{1 + \frac{sC_L}{\frac{1}{R_O} + \frac{1}{R_L}}}$

which is of the STC LP type.

DC gain $= \frac{G_m}{\frac{1}{R_O} + \frac{1}{R_L}} \geq 100$

$$\frac{1}{R_O} + \frac{1}{R_L} \leq \frac{G_m}{100} = \frac{10}{100} = 0.1 \text{ mA/V}$$

$$\frac{1}{R_L} \leq 0.1 - \frac{1}{50} = 0.08 \text{ mA/V}$$

$$R_L \geq \frac{1}{0.08} \text{ k}\Omega = 12.5 \text{ k}\Omega$$

$$\omega_O = \frac{1}{C_L}\left(\frac{1}{R_O} + \frac{1}{R_L}\right) \geq 2\pi \times 100 \text{ kHz}$$

$$C_L \leq \frac{\left(\frac{1}{50 \times 10^3} + \frac{1}{12.5 \times 10^3}\right)}{2\pi \times 10^5} = 159.2 \text{ pF}$$

Ex: 1.24 Refer to Fig. E1.23

$$\frac{V_2}{V_S} = \frac{R_i}{R_s + \frac{1}{sC} + R_i} = \frac{R_i}{R_S + R_i}\frac{s}{s + \frac{1}{C(R_S + R_i)}}$$

which is a HP STC function.

$$f_{3\text{dB}} = \frac{1}{2\pi C(R_S + R_i)} \leq 100 \text{ Hz}$$

$$C \geq \frac{1}{2\pi(1 + 9)10^3 \times 100} = 0.16 \ \mu\text{F}$$

Ex: 1.25

T = 50 K

$n_i = BT^{3/2}e^{-Eg/(2KT)}$

$= 7.3 \times 10^{15}(50)^{3/2}\, e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 50)}$

$\simeq 9.6 \times 10^{-39}/\text{cm}^3$

T = 350 K

$n_i = BT^{3/2}e^{-Eg/(2KT)}$

$= 7.3 \times 10^{15}(350)^{3/2}\, e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 350)}$

$= 4.15 \times 10^{11}/\text{cm}^3$

Ex: 1.26

$N_D = 10^{17}/\text{cm}^3$

From Exercise 3.1 n_i at

$\text{T} = 350\ \text{K} = 4.15 \times 10^{11}/\text{cm}^3$

$n_n = N_D = 10^{17}/\text{cm}^3$

$p_n \cong \frac{ni^2}{N_D}$

$= \frac{(4.15 \times 10^{11})^2}{10^{17}}$

$= 1.72 \times 10^6/\text{cm}^3$

Ex: 1.27

At 300 K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$

$p_p = N_A$

Want electron concentration

$= n_p = \frac{1.5 \times 10^{10}}{10^6} = 1.5 \times 10^4/\text{cm}^3$

$\therefore N_A = p_p = \frac{ni^2}{n_p}$

$= \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^4}$

$= 1.5 \times 10^{16}/\text{cm}^3$

Ex: 1.28

a. υ_n-drift $= -\mu_n \text{E}$

Here negative sign indicates that electrons move in a direction opposite to E

We use

υ_n-drift $= -\mu_n \text{E}$

$= 1350 \times \frac{1}{2 \times 10^{-4}} \quad \because 1\ \mu\text{m} = 10^{-4}\ \text{cm}$

$= 6.75 \times 10^6\ \text{cm/s} = 6.75 \times 10^4\ \text{m/s}$

b. Time taken to cross 2 μm

length $= \frac{2 \times 10^6}{6.75 \times 10^4} \simeq 30$ ps

c. In n-si drift current density J_n in

$J_n = qn\mu_n \text{E}$

$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \frac{1\ \text{V}}{2 \times 10^{-4}}$

$= 1.08 \times 10^4\ \text{A/cm}^2$

d. Drift current $I_n = Aqn\, \upsilon_n$-drift

$= Aqn\mu_n E$

$= 0.25 \times 10^{-8} \times 1.08 \times 10^4$

$= 27\ \mu\text{A}$

Note $0.25\ \mu\text{m}^2 = 0.25 \times 10^{-8}\ \text{cm}^2$

Ex: 1.29 $J_n = q\, D_n \frac{\text{dn}(x)}{\text{d}x}$

From Figure E1.29

$n_O = 10^{17}/\text{cm}^3 = 10^5/(\mu\text{m})^3$

$D_n = 35\ \text{cm}^3/\text{s} = 35 \times (10^4)^2(\mu\text{m})^2/\text{s}$

$= 35 \times 10^8(\mu\text{m})^2/\text{s}$

$\frac{\text{dn}}{\text{d}x} = \frac{10^5 - 0}{1} = 10^5 \mu\text{m}^{-2}$

$J_n = qD_n \frac{\text{dn}(x)}{\text{d}x}$

$= 1.6 \times 10^{-19} \times 35 \times 10^8 \times 10^5$

$= 56 \times 10^{-6}\ \text{A}/(\mu\text{m})^2$

$= 56\ \mu\ \text{A}/(\mu\text{m})^2$

For $I_n = 1\ \text{mA} = J_n \times A$

$\Rightarrow A = \frac{1\ \text{mA}}{J_n} = \frac{10^3\ \mu A}{56\ \mu\text{A}/(\mu\text{m})^2} \simeq 18\ \mu\text{m}^2$

Ex: 1.30

Using equation 1.45

$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_\text{T}$

$D_n = \mu_n V_\text{T} = 1350 \times 25.9 \times 10^{-3}$

$\cong 35\ \text{cm}^2/\text{s}$

$D_p = \mu_p V_\text{T} = 480 \times 25.9 \times 10^{-3}$

$\cong 12.4\ \text{cm}^2/\text{s}$

Ex: 1.31

Equation 3.50

$\text{W} = \sqrt{\frac{2\epsilon_S}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_O}$

$$= \sqrt{\frac{2\epsilon_S}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_O}$$

$$W^2 = \frac{2\epsilon_S}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_O$$

$$V_O = \frac{1}{2}\left(\frac{q}{\epsilon_S}\right) = \left(\frac{N_A N_D}{N_A + N_D}\right)W^2$$

Ex: 1.32

In a p⁺ n diode $N_A >> N_D$

Equation 1.50 $W = \sqrt{\frac{2\epsilon_S}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_O}$

We can neglect the term $\frac{1}{N_A}$ as Compared to $\frac{1}{N_D}$

since $N_A >> N_D$

$$\simeq \sqrt{\frac{2\epsilon_S}{qN_D} \cdot V_O}$$

Equation 1.51 $X_n = W\frac{N_A}{N_A + N_D}$

$$\simeq W\frac{N_D}{N_D}$$

$$= W$$

Equation 1.52 $X_p = W\frac{N_A}{N_A + N_D}$

since $N_A >> N_D$

$$\simeq W\frac{N_D}{N_A} = W/\left(\frac{N_A}{N_D}\right)$$

Equation 1.53 $Q_J = Aq\left(\frac{N_A N_D}{N_A + N_D}\right)$

$W \simeq Aq\frac{N_A N_D}{N_A} \cdot W$ since $N_A >> N_D$

$$\simeq AqN_D W$$

Equation 1.54 $Q_J = A\sqrt{2\epsilon_S q\left(\frac{N_A N_D}{N_A + N_D}\right)V_O}$

$\simeq A\sqrt{2\epsilon_S q\left(\frac{N_A N_D}{N_A}\right)V_O}$ since $N_A >> N_D$

$$= A\sqrt{2\epsilon_S q N_D V_O}$$

Ex: 1.33

In example 1.29 $N_A = 10^{18}/\text{cm}^3$ and

$N_D = 10^{16}/\text{cm}^3$

In the n-region of this pn juction diode

$$n_n = N_D = 10^{16}/\text{cm}^3$$

$$p_n = \frac{n_i^2}{n_n} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4/\text{cm}^3$$

As one can see from above equation, to increase minority carrier-concentration (p_n) by a factor of 2, one must lower N_D ($= n_n$) by a factor of 2.

Ex: 1.34

Equation 1.38 $I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$

since $\frac{D_p}{L_p}$ and $\frac{D_n}{L_n}$ here approximately

similar values, if $N_A >> N_D$ then the term $\frac{D_n}{L_n N_A}$

can be neglected as compared to $\frac{D_p}{L_p N_D}$

$$\therefore I_S \cong Aqn_i^2\frac{D_p}{L_p N_D}$$

Ex: 1.35

$$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$$

$$= 10^{-4} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{1})^2$$

$$\times \left(\frac{10}{5 \times 10^{-4} \times \frac{10^{16}}{2}} + \frac{10}{10 \times 10^{-4} \times 10^{18}}\right)$$

$$= 1.45 \times 10^{-14}\ \text{A}$$

$$I = I_S(e^{V/V_T} - 1)$$

$$\simeq I_S e^{V/V_T} = 1.45 \times 10^{-14} e^{0.605/(25.9 \times 10^{-3})}$$

$$= 0.2\ \text{mA}$$

Ex: 1.36

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_O - V_F)}$$

$$= \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}}\left(\frac{1}{10^{18}} + \frac{1}{10^{16}}\right)(0.814 - 0.605)}$$

$$= 1.66 \times 10^{-5}\ \text{cm} = 0.166\ \mu\text{m}$$

Ex: 1.37

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_O + V_R)}$$

$$= \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}}\left(\frac{1}{10^{18}} + \frac{1}{10^{16}}\right)(0.814 + 2)}$$

$$= 6.08 \times 10^{-5}\ \text{cm} = 0.608\ \mu\text{m}$$

Using equation 1.53

$$Q_J = \text{Aq}\left(\frac{N_A N_D}{N_A + N_D}\right)\text{W}$$

$$= 10^{-4} \times 1.6 \times 10^{-19}\left(\frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}}\right) \times 6.08 \times 10^{-5} \text{ cm}$$

$$= 9.63\ pC$$

Reverse Current $I = I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$

$$= 10^{-14} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2$$
$$\times\left(\frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{18}}\right)$$
$$= 7.3 \times 10^{-15} \text{ A}$$

Ex: 1.38

Equation 1.72

$$C_{jo} = A\sqrt{\left(\frac{\epsilon_S q}{2}\right)\left(\frac{N_AN_D}{N_A + N_D}\right)\left(\frac{1}{V_O}\right)}$$
$$= 10^{-4}\sqrt{\left(\frac{1.04 \times 10^{-12} \times 1.6 \times 10^{-19}}{2}\right)}$$
$$\sqrt{\left(\frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}}\right)\left(\frac{1}{0.814}\right)}$$
$$= 3.2\ pF$$

Equation 1.71

$$C_j = \frac{C_{jO}}{\sqrt{1 + \frac{V_R}{V_O}}}$$
$$= \frac{3.2 \times 10^{-12}}{\sqrt{1 + \frac{2}{0.814}}}$$
$$= 1.72\ pF$$

Ex: 1.39

$$C_d = \frac{dQ}{dV} = \frac{d}{dV}(\tau_T I)$$
$$= \frac{d}{dv}[\tau_T \times I_S(e^{V/V_T} - 1)]$$
$$= \tau_T I_S \frac{d}{dV}(e^{V/V_T} - 1)$$
$$= \tau_T I_S \frac{1}{V_T} e^{V/V_T}$$
$$= \frac{\tau_T}{V_T} \times I_S e^{V/V_T}$$
$$= \left(\frac{\tau_T}{V_T}\right) I$$

Ex: 1.40

Equation 1.74

$$\tau_p = \frac{L_p^2}{D_p}$$
$$= \frac{(5 \times 10^{-4})^2}{5}$$
$$= 25\ ns$$

Equation 1.81

$$C_d = \left(\frac{\tau_T}{V_T}\right) I$$

In example 1.30 $N_A = 10^{18}/\text{cm}^3$,

$N_D = 10^{16}/\text{cm}^3$

Assuming $N_A >> N_D$

$\tau_T \simeq \tau_p = 25\ ns$

$$\therefore C_d = \left(\frac{25 \times 10^{-9}}{25.9 \times 10^{-3}}\right) 0.1 \times 10^{-3}$$
$$= 96.5\ pF$$

Ex: 2.1

The minimum number of terminals required by a single op amp is five: two input terminals, one output terminal, one terminal for positive power supply and one terminal for negative power supply.

The minimum number of terminals required by a quad op amp is 14: each op amp requires two input terminals and one output terminal (accounting for 12 terminals for the four op amps). In addition, the four op amp can all share one terminal for positive power supply and one terminal for negative power supply.

Ex: 2.2

Equation are $v_3 = \mathrm{A}(v_2 - v_1)$;

$$v_{\mathrm{id}} = v_2 - v_1,\ v_{\mathrm{icm}} = \frac{1}{2}(v_1 + v_2)$$

a)

$$v_1 = v_2 - \frac{v_3}{\mathrm{A}} = 0 - \frac{2}{10^3} = -0.02\ \mathrm{V} = -2\ \mathrm{mV}$$

$$v_{\mathrm{id}} = v_2 - v_1 = 0 - (-0.002) = +0.002\ \mathrm{V}$$
$$= 2\ \mathrm{mV}$$

$$v_{\mathrm{icm}} = \frac{1}{2}(-2\mathrm{mV} + 0) = -1\ \mathrm{mV}$$

b) $-10 = 10^3(5 - v_1) \Rightarrow v_1 = 5.01\ \mathrm{V}$

$$v_{\mathrm{id}} = v_2 - v_1 = 5 - 5.01 = 0.01\ \mathrm{V} = 10\ \mathrm{mV}$$

$$v_{\mathrm{icm}} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(5.01 + 5) = 5.005\ \mathrm{V}$$
$$\simeq 5\ \mathrm{V}$$

c)

$$v_3 = \mathrm{A}(v_2 - v_1) = 10^3(0.998 - 1.002) = -4\ \mathrm{V}$$

$$v_{\mathrm{id}} = v_2 - v_1 = 0.998 - 1.002 = -4\ \mathrm{mV}$$

$$v_{\mathrm{icm}} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(1.002 + 0.998) = 1\ \mathrm{V}$$

d)

$$-3.6 = 10^3[v_2 - (-3.6)] = 10^3(v_2 + 3.6)$$
$$\Rightarrow \sqrt{2} = -3.6036\ \mathrm{V}$$
$$v_{\mathrm{id}} = v_2 - v_1 = -3.6036 - (-3.6)$$
$$= -0.0036\ \mathrm{V} = -3.6\ \mathrm{mV}$$
$$v_{\mathrm{icm}} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}[-3.6 + (-3.6)]$$
$$= -3.6\ \mathrm{V}$$

Ex: 2.3

From Figure E2.3 we have: $V_3 = \mu V_d$ and

$$V_d = (G_m V_2 - G_m V_1)R = G_m R(V_2 - V_1)$$

Therefore:

$$V_3 = \mu G_m R(V_2 - V_1)$$

That is the open-loop gain of the op amp is $A = \mu G_m R$. For $G_m = 10$ mA/V and $\mu = 100$ we have:

$A = 100 \times 10 \times 10 = 10^4$ V/V Or equivalently 80 dB

Ex: 2.4

The gain and input resistance of the inverting amplifier circuit shown in Figure 2.5 are $-\frac{R_2}{R_1}$ and R_1 respectively. Therefore, we have:

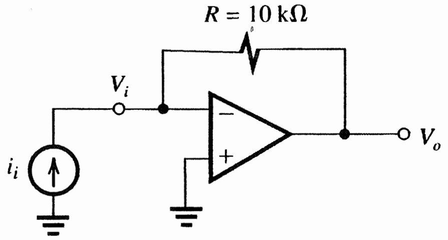
$R_1 = 100\ \mathrm{k\Omega}$ and

$$-\frac{R_2}{R_1} = -10 \Rightarrow R_2 = 10\ R_1$$

Thus:

$$R_2 = 10 \times 100\ \mathrm{k\Omega} = 1\ \mathrm{M\Omega}$$

Ex: 2.5



From Table 1.1 we have:

$$R_m = \left.\frac{V_O}{i_i}\right|_{i_o = 0}$$, i.e., output is open circuit

The negative input terminal of the op amp, i.e., V_i is a virtual ground, thus $V_i = 0$

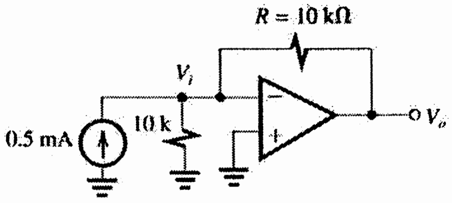
$$V_O = V_i - Ri_i = 0 - Ri_i = -Ri_i$$

$$R_m = \left.\frac{V_O}{i_i}\right|_{i_o = 0} = -\frac{Ri_i}{i_i} = -R \Rightarrow R_m = -R$$
$$= -10\ \mathrm{k\Omega}$$

$R_i = \frac{V_i}{V_i}$ and V_i is a virtual ground ($V_i = 0$),

thus $R_i = \frac{0}{i_i} = 0 \Rightarrow R_i = 0\ \Omega$

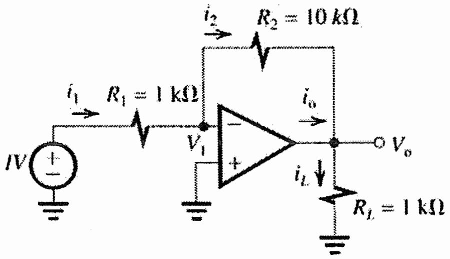
Since we are assuming that the op amp in this transresistance amplifier is ideal, the op amp has zero output resistance and therefore the output resistance of this transresistance amplifier is also zero. That is $R_O = 0\ \Omega$.



Connecting the signal source shown in Figure E2.5 to the input of this amplifier we have:
V_i is a virtual ground that is $V_i = 0$, thus the current flowing through the 10 kΩ resistor connected between V_i and ground is zero. Therefore

$$V_O = V_i - R \times 0.5\text{ mA} = 0 - 10\text{ K} \times 0.5\text{ mA}$$
$$= -5\text{ V}$$

Ex: 2.6



V_1 is a virtual ground, thus $V_1 = 0$ V

$$i_1 = \frac{1\text{ V} - V_1}{R_1} = \frac{1-0}{1\text{ k}\Omega} = 1\text{ mA}$$

Assuming an ideal op amp, the current flowing into the negative input terminal of the op amp is zero. Therefore, $i_2 = i_1 \Rightarrow i_2 = 1$ mA

$$V_O = V_1 - i_2R_2 = 0 - 1\text{ mA} \times 10\text{ k}\Omega$$
$$= -10\text{ V}$$

$$i_L = \frac{V_O}{R_l} = \frac{-10\text{ V}}{1\text{ k}\Omega} = -10\text{ mA}$$

$$i_O = i_L - i_2 = -10\text{ mA} - 1\text{ mA} = -11\text{ mA}$$

$$\text{Voltage gain} = \frac{V_O}{1\text{ V}} = \frac{-10\text{ V}}{1\text{ V}} = -10\text{ V/V}$$

or 20 dB

$$\text{Current gain} = \frac{i_L}{i_1} = \frac{-10\text{ mA}}{1\text{ mA}} = -10\text{ A/A}$$

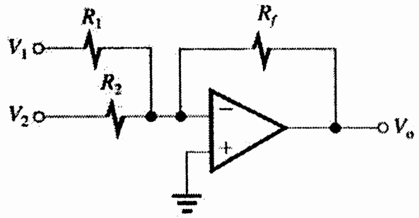
or 20 dB

$$\text{Power gain} = \frac{P_L}{P_i} = \frac{-10(-10\text{ mA})}{1\text{ V} \times 1\text{ mA}} = 100\text{ W/W}$$

or 20 dB

Note that power gain in dB is $10\log_{10}\left|\frac{P_L}{P_i}\right|$.

Ex: 2.7



For the circuit shown above we have:

$$V_O = \left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2\right)$$

Since it is required that $V_O = -(V_1 + 5V_2)$.
We want to have:

$$\frac{R_F}{R_1} = 1 \text{ and } \frac{R_F}{R_2} = 5$$

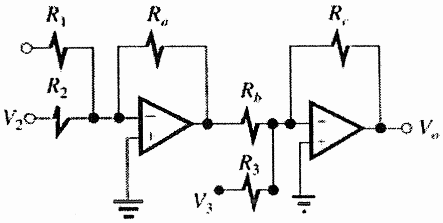
It is also desired that for a maximum output voltage of 10 V the current in the feedback resistor does not exceed 1 mA.
Therefore

$$\frac{10\text{ V}}{R_f} \le 1\text{ mA} \Rightarrow R_f \ge \frac{10\text{ V}}{1\text{ mA}} \Rightarrow R_f \ge 10\text{ k}\Omega$$

Let us choose R_f to be 10 kΩ, then

$$R_1 = R_f = 10\text{ k}\Omega \text{ and } R_2 = \frac{R_f}{5} = 2\text{ k}\Omega$$

Ex: 2.8



$$V_O = \left(\frac{R_a}{R_1}\right)\left(\frac{R_C}{R_b}\right)V_1 + \left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right)V_2 - \left(\frac{R_C}{R_3}\right)V_3$$

We want to design the circuit such that

$$V_O = 2V_1 + V_2 - 4V_3$$

Thus we need to have

$$\left(\frac{R_a}{R_1}\right)\left(\frac{R_C}{R_b}\right) = 2,\ \left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1 \text{ and } \frac{R_c}{R_3} = 4$$

From the above three equations, we have to Find six unknown resistors, therefore, we can arbitrarily choose three of these resistors. Let us choose:
Then we have

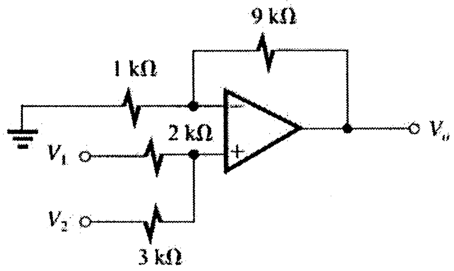
$$R_3 = \frac{R_C}{4} = \frac{10}{4} = 2.5\ \text{k}\Omega$$

$$\left(\frac{R_a}{R_1}\right)\left(\frac{R_C}{R_b}\right) = 2, \Rightarrow \frac{10}{R_1} \times \frac{10}{10} = 2 \Rightarrow R_1 = 5\ \text{k}\Omega$$

$$\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) = 1 \Rightarrow \frac{10}{R_2} \times \frac{10}{10} = 1 \Rightarrow R_2 = 10\ \text{k}\Omega$$

Ex: 2.9

Using the super position principle, to find the contribution of v_1 to the output voltage v_0, we set $V_2 = 0$



The V_+ (the voltage at the positive input of the op amp is: $V_+ = \frac{3}{2+3}V_1 = 0.6V_1$

Thus

$$V_O = \left(1 + \frac{9\ \text{k}\Omega}{1\ \text{k}\Omega}\right)V_+ = 10 \times 0.6V_1 = 6\ V_1$$

To find the contribution of V_2 to the output voltage V_0 we set $V_1 = 0$.

Then $V_+ = \frac{2}{2+3}V_2 = 0.4\ V_2$

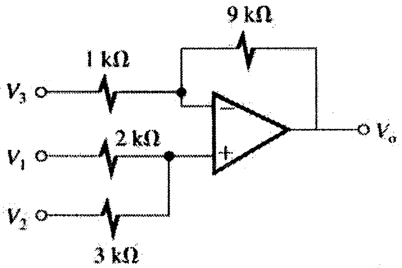
Hence

$$V_O = \left(1 + \frac{9\ \text{k}\Omega}{1\ \text{k}\Omega}\right)V_+ = 10 \times 0.4V_2 = 4V_2$$

Combining the contributions of v1 and v2

To V_O we have $V_O = 6V_1 + 4V_2$

Ex: 2.10



Using the super position principle, to find the contribution of V_1 to V_0 we set $V_2 = V_3 = 0$ Then we have (refer to the solution of exercise 2.9):

$V_O = 6V_1$

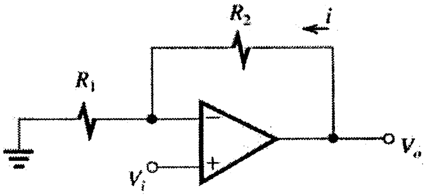
To find the contribution of V_2 to V_0 we set $V_1 = V_3 = 0$, then: $V_O = 4V_2$

To find the contribution of V_3 to V_O we set $V_1 = V_2 = 0$, then

$$V_O = -\frac{9\ \text{k}\Omega}{1\ \text{k}\Omega}V_3 = -9V_3$$

Combining the contributions of V_1,V_2 and V_3 to V_0 we have: $V_O = 6V_1 + 4V_2 - 9V_3$

Ex: 2.11



$$\frac{V_O}{V_i} = 1 + \frac{R_2}{R_1} = 2 \Rightarrow \frac{R_2}{R_1} = 1 \Rightarrow R_1 = R_2$$

If $V_O = 10\ \text{V}$ then it is desired that

$i = 10\ \mu\text{A}$.

Thus,

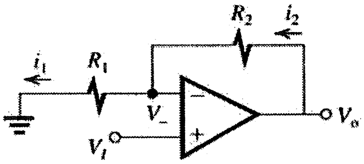
$$i = \frac{10\ \text{V}}{R_1 + R_2} = 10\ \mu\text{A} \Rightarrow R_1 + R_2 = \frac{10\ \text{V}}{10\ \mu\text{A}}$$

$R_1 + R_2 = 1\ \text{M}\Omega$ and

$R_1 = R_2 \Rightarrow R_1 = R_2 = 0.5\ \text{M}\Omega$

Ex: 2.12

a)



$$V_O = A(V_I - V_-) \Rightarrow V_- = V_I - \frac{V_O}{A}$$

$$i_2 = i_1 \Rightarrow \frac{V_O - V_-}{R_2} = \frac{V_-}{R_1} \Rightarrow \frac{V_O}{R_2} = \left(\frac{1}{R_2} + \frac{1}{R_1}\right)V_-$$

$$V_o = \left(1 + \frac{R_L}{R_1}\right)V_- = \left(1 + \frac{R_L}{R_1}\right)\left(V_I - \frac{V_o}{A}\right) \Rightarrow$$

$$V_O + \frac{1 + R_2/R_1}{A}V_O = (1 + R_2/R_1)V_I$$

$$\frac{V_O}{V_I} = \frac{1 + R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}} \Rightarrow G = \frac{1 + R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}}$$

(b) For $R_1 = 1$ kΩ and $R_2 = 9$ kΩ the ideal value for the closed-loop gain is $1 + \frac{9}{1}$, that is 10. The actual closed-loop gain is $G = \frac{10}{1 + \frac{10}{A}}$

If $A = 10^3$ then $G = 9.901$ and

$$\epsilon = \frac{G - 10}{10} \times 100 = -0.99\% \simeq -1\%$$

For $V_I = 1\text{V}$, $V_O = G \times V_I = 9.901$ V and

$$V_O = A(V_+ - V_-) \Rightarrow V_+ - V_- = \frac{V_O}{A} = \frac{9.901}{1000} \simeq 9.9 \text{ mV}$$

If $A = 10^4$ then $G = 9.99$ and $\epsilon = -0.1\%$

For $V_I = 1$ V, $V_O = G \times V_I = 9.99$ V, therefore,

$$V_+ - V_- = \frac{V_O}{A} = \frac{9.99}{10^4} = 0.999 \text{ mV} \simeq 1 \text{ mV}$$

If $A = 10^5$ then $G = 9.999$ and $\epsilon = -0.01\%$

For $V_I = 1$ V, $V_O = G \times V_I = 9.999$ thus,

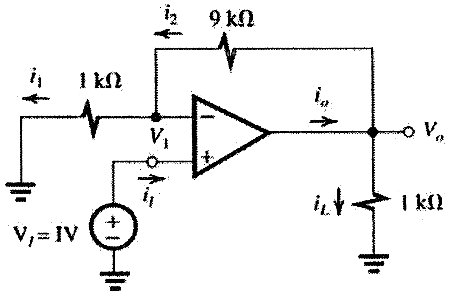
$$V_+ - V_- = \frac{V_O}{A} = \frac{9.999}{10^5} = 0.09999 \text{ mV} \simeq 0.1 \text{ mV}$$

Ex: 2.13

$i_I = 0\text{A}$, $V_1 = V_I = 1$ V,

$$i_1 = \frac{V_1}{1 \text{ k}\Omega} = \frac{1 \text{ V}}{1 \text{ k}\Omega} = 1 \text{ mA}$$

$i_2 = i_1 = 1$ mA,



$$V_O = V_1 + i_2 \times 9 \text{ k}\Omega = 1 + 1 \times 9 = 10 \text{ V}$$

$$i_L = \frac{V_O}{1 \text{ k}\Omega} = \frac{10 \text{ V}}{1 \text{ k}\Omega} = 10 \text{ mA},$$

$$i_O = i_L + i_2 = 11 \text{ mA}$$

$$\frac{V_O}{V_I} = \frac{10 \text{ V}}{1 \text{ V}} = 10\frac{\text{V}}{\text{V}} \text{ or } 20 \text{ dB}$$

$$\frac{i_L}{i_I} = \frac{10 \text{ mA}}{0} = \infty$$

$$\frac{P_L}{P_I} = \frac{V_O \times i_L}{V_I \times I_I} = \frac{10 \times 10}{1 \times 10} = \infty$$

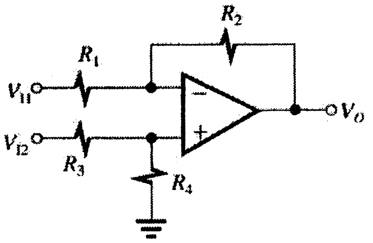
Ex: 2.14

(a) load voltage

$$= \frac{1 \text{ k}\Omega}{1 \text{ k}\Omega + 1 \text{ M}\Omega} \times 1 \text{ V} \simeq 1 \text{ mV}$$

(b) load voltage = 1V

Ex: 2.15



(a) $R_1 = R_3 = 2$ kΩ, $R_2 = R_4 = 200$ kΩ

Since $R_4/R_3 = R_2/R_1$ we have:

$$A_d = \frac{V_o}{V_{I2} - V_{I1}} = \frac{\text{R}_2}{\text{R}_1} = \frac{200}{2} = 100\text{V/V}$$

(b) $R_{id} = 2R_1 = 2 \times 2 \text{ k}\Omega = 4 \text{ k}\Omega$

Since we are assuming the op amp is ideal $R_o = 0\ \Omega$

$$\text{(c) } A_{cm} = \frac{V_o}{V_{ICM}} = \left(\frac{R_4}{R_4 + R_3}\right)\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

$$= \left(\frac{1}{1 + \frac{R_3}{R_4}}\right)\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

$$= \frac{\frac{R_4}{R_3} - \frac{R_2}{R_1}}{\frac{R_4}{R_3} + 1}$$

The worst case common-mode gain A_{cm} happens when $|A_{cm}|$ has its maximum value.

If the resistors have 1% tolerance, we have

$$\frac{R_{4\text{nom}}(1 - 0.01)}{R_{3\text{nom}}(1 + 0.01)} \le \frac{R_4}{R_3} \le \frac{R_{4\text{nom}}(1 + 0.01)}{R_{3\text{nom}}(1 - 0.01)}$$

where $R_{3\text{nom}}$ and $R_{4\text{nom}}$ are nominal values for R_3 and R_4 respectively. We have :

$R_{3\text{nom}} = 2$ kΩ and $R_{4\text{nom}} = 200$ kΩ, thus,

$$\frac{200 \times 0.99}{2 \times 1.01} \le \frac{R_4}{R_3} \le \frac{200 \times 1.01}{2 \times 0.99}$$

$$98.02 \le \frac{R_4}{R_3} \le 102.02$$

Simlarly, we can show that

$$98.02 \le \frac{R_2}{R_1} \le 102.02$$

Hence, $-102.02 \le -\frac{R_2}{R_1} \le -98.02$

Therefore,

$$-4 \le \frac{R_4}{R_3} - \frac{R_2}{R_1} \le 4 \Rightarrow \left|\frac{R_4}{R_3} - \frac{R_2}{R_1}\right| \le 4$$

In the worst case

$$\frac{\left|\frac{R_4}{R_3} - \frac{R_2}{R_1}\right|}{1 + \frac{R_4}{R_3}} \le \frac{4}{1 + 98.02} \Rightarrow |A_{cm}| \le 0.04$$

Note that the worst case A_{cm} happens when

$$\frac{R_4}{R_3} = 98.02 \text{ and } \frac{R_2}{R_1} = 102.02$$

The differential gain A_d of the amplifier is $A_d = \frac{R_2}{R_1}$, therefore, the corresponding value of CMRR for the worst case A_{cm} is :

$$\text{CMRR} = 20 \log \frac{|A_d|}{|A_{cm}|} = 20 \log \frac{102.02}{0.04} \Rightarrow$$

$$\text{CMRR} = 20 \log(2550.5) \simeq 68 \text{ dB}$$

Ex: 2.16

We choose $R_3 = R_1$ and $R_4 = R_2$ Then for the circuit to behave as a difference amplifier with a gain of 10 and an input resistance of 20 kΩ we require

$$A_d = \frac{R_2}{R_1} = 10 \text{ and}$$

$$R_{id} = 2R_1 = 20 \text{ k}\Omega \Rightarrow R_1 = 10 \text{ k}\Omega \text{ and}$$

$$R_2 = A_d R_1 = 10 \times 10 \text{ k}\Omega = 100 \text{ k}\Omega$$

Therefore, $R_1 = R_3 = 10 \text{ k}\Omega$ and

$$R_2 = R_4 = 100 \text{ k}\Omega$$

Ex: 2.17

Given $V_{Icm} = +5$ V

$V_{Id} = 10 \sin\omega t$ mV

$2R_1 = 1 \text{ k}\Omega,\ R_2 = 0.5 \text{ M}\Omega$

$R_3 = R_4 = 10 \text{ k}\Omega$

$$v_{I1} = v_{Icm} - \frac{1}{2} v_{Id} = 5 - \frac{1}{2} \times 0.01 \sin\omega t$$

$$= 5 - 0.005 \sin\omega t \text{ V}$$

$$v_{I2} = v_{Icm} + \frac{1}{2} v_{ID}$$

$$= 5 + 0.005 \sin\omega t \text{ V}$$

$$v_-(\text{Op AmpA}_1) = V_{I1} = 5 - 0.005 \sin\omega t \text{ V}$$

$$v_-(\text{Op AmpA}_2) = V_{I2} = 5 + 0.005 \sin\omega t \text{ V}$$

$$v_{Id} = v_{I2} - v_{I1} = 0.01 \sin\omega t$$

v_{01} -The to voltage at the output of op amp A.

$$v_{01} = V_{I1} - R_2 \times \frac{V_{Id}}{2R_1}$$

$$= 5 - 0.005 \sin\omega t - 500 \text{ k} \times \frac{0.01 \sin\omega t}{1 \text{ k}\Omega}$$

$$= (5 - 5.005 \sin\omega t) \text{ V}$$

v_{02} - The voltage at the output of op amp A2

$$V_{02} = v_{I2} + R_2 \times \frac{v_{Id}}{2R_1}$$

$$= (5 + 5.005 \sin\omega t) \text{V}$$

$$v_+(\text{Op Amp A}_3) = v_{02} \times \frac{R_4}{R_3 + R_4} = v_{02} \frac{10}{10 + 10}$$

$$\because R_3 = R_4 = 10 \text{ k}\Omega$$

$$= \frac{1}{2} v_{02} = \frac{1}{2}(5 + 5.005 \sin\omega t)$$

$$= (2.5 + 2.5025 \sin\omega t) \text{V}$$

$$v_-(\text{Op Amp A}_3) = V_+(\text{Op Amp A}_3)$$

$$= (2.5 + 2.5025 \sin\omega t) \text{ V}$$

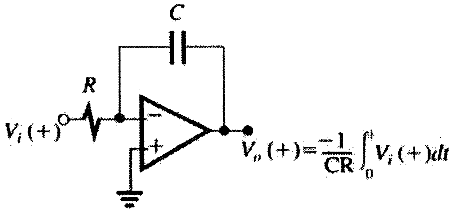
$$v_O = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) v_{Id}$$

$$\frac{10 \text{ k}}{10 \text{ k}}\left(1 + \frac{0.5 \text{ M}\Omega}{0.5 \text{ M}\Omega}\right) \times 0.01 \sin\omega t$$

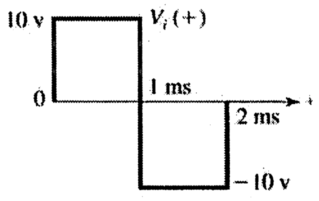
$$= 1(1 + 1000) \times 0.01 \sin\omega t$$

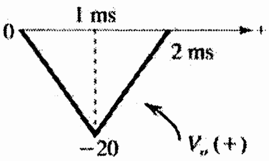
$$10.01 \sin\omega t \text{ V}$$

Ex: 2.18



The waveforms for one period of the input and the output signals are shown below:





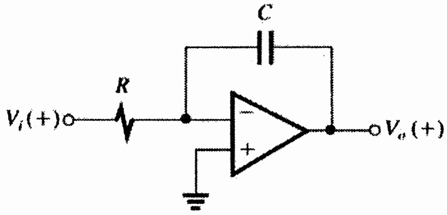
We have

$$-20 = \frac{-1}{CR}\int_{0}^{1\text{ ms}} 10\,dt$$

$$\Rightarrow -20 = \frac{-1}{CR} \times 10 \times 1\text{ ms}$$

$$CR = \frac{10}{20} \times 1\text{ ms} \times 0.5\text{ ms}$$

Ex: 2.19



The input resistance of this inverting integrator is R_1, therefore, $R = 10\text{ k}\Omega$

Since the desired integration time constant is 10^{-3} s, we have: $CR = 10^{-3}\text{ s} \Rightarrow$

$$C = \frac{10^{-3}\text{ s}}{10\text{ k}\Omega} = 0.1\ \mu\text{F}$$

From equation (2.50) the transfer function of this integrator is:

$$\frac{V_O(jw)}{V_i(jw)} = -\frac{1}{jwCR}$$

For $w = 10$ rad / s the integrator transfer function has magnitude

$$\left|\frac{V_O}{V_i}\right| = \frac{1}{10 \times 10^{-3}} = 100\text{ V/V} \text{ and phase}$$

$$\phi = 90°$$

For $w = 1$rad / s the integrator transfer function has magnitude

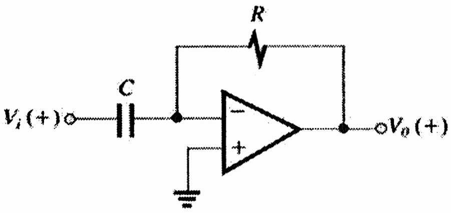
$$\left|\frac{V_O}{V_i}\right| = \frac{1}{10 \times 10^{-3}} = 1000\text{ V/V} \text{ and phase}$$

$$\phi = 90°$$

Using equation (2.53) the frequency at which the integrator gain magnitude is unity is

$$w_{\text{int}} = \frac{1}{CR} = \frac{1}{10^{-3}} = 1000\text{ rad / s}$$

Ex: 2.20



$C = 0.01\ \mu\text{F}$ Is the input capacitance of this differentiator. We want $CR = 10^{-2}$ s (the time constant of the differentiator), thus,

$$R = \frac{10^{-2}}{0.01\ \mu\text{F}} = 1\text{ M}\Omega$$

From equation (2.57), we know that the transfer function of the differentiator is of the form

$$\frac{V_O(jw)}{V_i(jw)} = -jwCR$$

Thus, for $w = 10$ rad / s the differentiator transfer function has magnitude

$$\left|\frac{V_O}{V_i}\right| = 10 \times 10^{-2} = 0.1\text{ V/V} \text{ and phase}$$

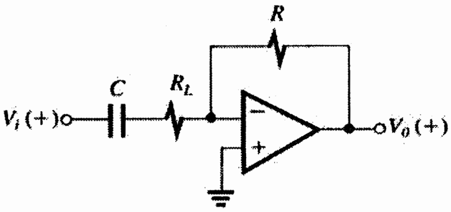
$$\phi = -90°$$

For $w = 10^3$ rad / s the differentiator transfer function has magnitude

$$\left|\frac{V_O}{V_i}\right| = 10^3 \times 10^{-2} = 10\text{ V/V} \text{ and phase}$$

$$\phi = -90°$$

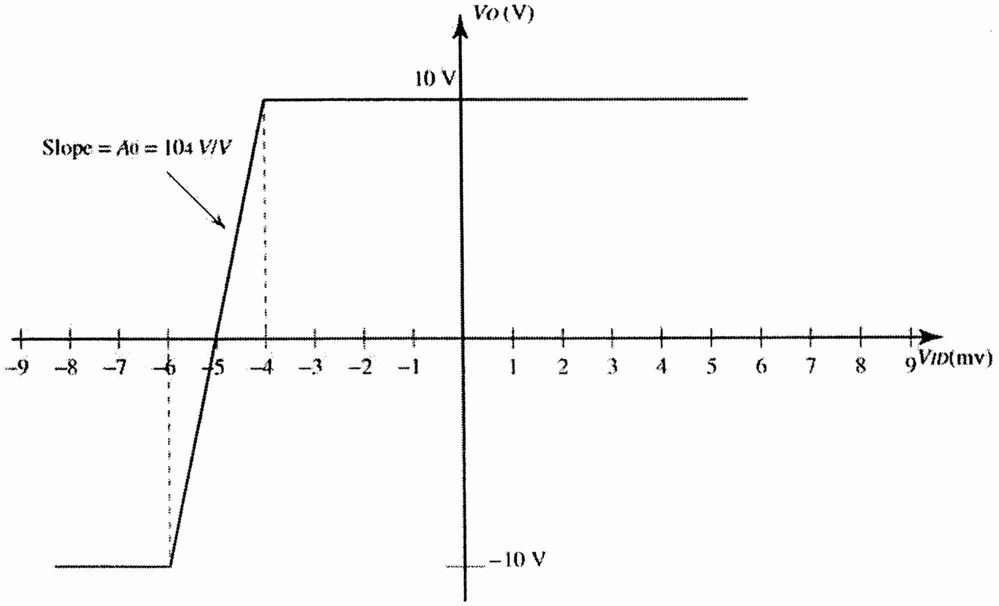
If we add a resistor in series with the capacitor to limit the high frequency gain of the differentiator to 100, the circuit would be:



At high frequencies the capacitor C acts like a short circuit. Therefore, the high-frequency gain of this circuit is: $\frac{R}{R_L}$ To limit the magnitude of this high-frequency gain to 100, we should have:

$$\frac{R}{R_L} = 100 \Rightarrow R_L = \frac{R}{100} = \frac{1\text{ M}\Omega}{100} = 10\text{ k}\Omega$$

Ex: 2.21



$V_O = V_3$

$V_{ID} = V_2 - V_1$

$V_{ID} = V_+ - V_{OS} - V_-$

when $V_+ = V_- = 0$ then

$V_{ID} = 0 - 5\text{ mV} = -5\text{ mV}$. This input offset voltage causes an offset in the voltage transfer characteristic. Rather than passing through the origin, it is now shifted to the left by V_{OS}

Ex: 2.22

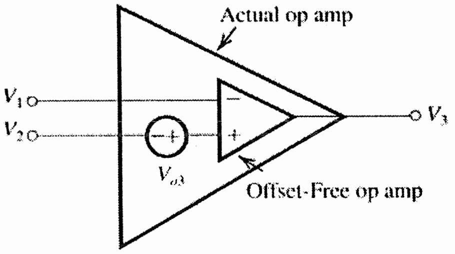
From equation (2.41) we have:

$$f_M = \frac{SR}{2\pi V_{O\max}} = 15.915\text{ kHz} \simeq 15.9\text{ kHz}$$

Using equation (2.42), for an input sinusoid with frequency $f = 5 f_M$, the maximum possible amplitude that can be accommodated at the output without incurring SR distortion is:

$$V_O = V_{O\max}\left(\frac{f_M}{5\ f_M}\right) = 10 \times \frac{1}{5} = 2\text{ V(peak)}$$

Ex: 2.23



$V_O = V_3$

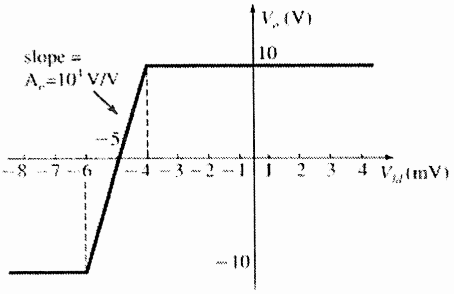
$V_{Id} = V_2 - V_1$

$V_{Id} = V_+ - V_{OS} - V_-$

In order to have zero differential input for the offset-free op amp (i.e., $V_+ - V_- = 0$) we need

$V_{Id} = V_+ - V_- - V_{OS} = 0 - 5\text{ mV} = -5\text{ mV}$

Thus, the transfer characteristic V_O versus V_{Id} is:



Ex: 2.24

From equation(2,44) we have:

$V_O = I_{B1}R_2 \simeq I_B R_2$

$= 100\text{ nA} \times 1\text{ M}\Omega = 0.1\text{ V}$

From equation (2.46) the value of resistor R_3 (placed in series with positive input to minimize the output offset voltage) is:

$$R_3 = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{10\text{ k}\Omega \times 1\text{ M}\Omega}{10\text{ k}\Omega + 1\text{ M}\Omega}$$

$= 9.9\text{ k}\Omega$

$R_3 = 9.9\text{ k}\Omega \simeq 10\text{ k}\Omega$

With this value of R_3 the new value of the output dc voltage (using equation (2.47))is:

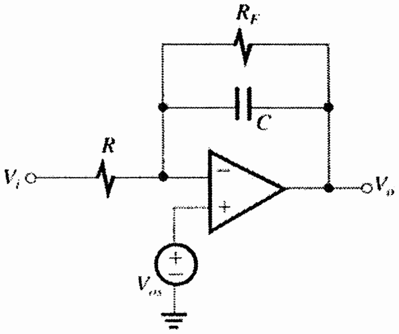
$V_O = I_{OS}R_2 = 10\text{ nA} \times 10\text{ k}\Omega \simeq 0.01\text{ V}$

Ex: 2.25

Using equation (2.54) we have:

$$V_O = V_{OS} + \frac{V_{OS}}{CR}t \Rightarrow 12 = 2\text{ mV} + \frac{2\text{ mV}}{1\text{ ms}}t$$

$$\Rightarrow t = \frac{12\text{ V} - 2\text{ mV}}{2\text{ mV}} \times 1\text{ ms} \simeq 6\text{D} \Rightarrow t = 6\text{D}$$



With the feedback resistor R_F to have at least $+10$ V of output signal swing available, we have to make sure that the output voltage due to V_{OS} has a magnitude of at most 2 V. From equation (2.43), we know that the output dc voltage due to V_{OS} is

$$V_O = V_{OS}\left(1 + \frac{R_F}{R}\right) \Rightarrow 2\text{V} = 2\text{ mV}\left(1 + \frac{R_F}{10\text{ k}\Omega}\right)$$

$$1 + \frac{R_F}{10\text{ k}\Omega} = 1000 \Rightarrow R_F \simeq 10\text{ M}\Omega$$

The corner frequency of the resulting STC network is $w = \frac{1}{CR_F}$

We know $RC = 1$ ms and

$R = 10\text{ k}\Omega \Rightarrow C = 0.1\ \mu\text{F}$

Thus $w = \frac{1}{0.1\ \mu\text{F} \times 10\text{ M}\Omega} = 1$ rad / s

$$f = \frac{w}{2\pi} = \frac{1}{2\pi} = 0.16\text{ Hz}$$

Ex: 2.26

From equation (2.28) we have:

$w_t = A_O w_b \Rightarrow f_t = A_O f_b \Rightarrow f_b = \frac{f_t}{A_O}$, and

we know

$20 \log A_O = 106$ and $f_t = 3$ MHz, therefore $f_b \simeq 15$ Hz

By definition the open-loop gain (in dB) at f_b is:

$A_O(\text{in dB}) - 3 = 106 - 3 = 103$ dB

To find the open-loop gain at frequency f we can use equation (2.31) (especially when $f >> f_b$ which is the case in this exercise) and write:

Open-loop gain at $f \simeq 20 \log\left(\frac{f_t}{f}\right)$

Therefore:

Open-loop gain at 300 Hz =

$20 \log \frac{3\text{ MHz}}{300} = 80$ dB

Open-loop gain at 3 kHz =

$20 \log \frac{3\text{ MHz}}{3\text{ kHz}} = 60$ dB

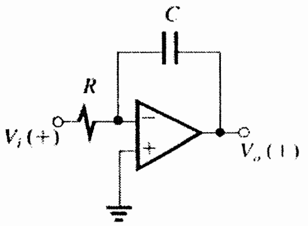
Open-loop gain at 12 kHz =

$20 \log \frac{3\text{ MHz}}{12\text{ kHz}} = 48$ dB

Open-loop gain at 60 kHz =

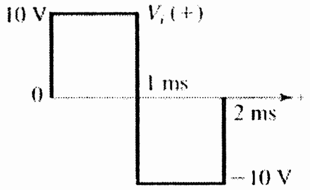
$20 \log \frac{3\text{ MHz}}{60\text{ kHz}} = 34$ dB

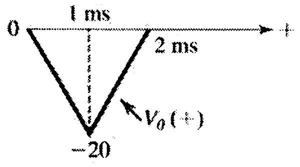
Ex: 2.27



$$\frac{-1}{CR}\int_0^t V_i(t)\,dt$$

The waveforms for one period of the input and the output signals are shown below:





We have

$$-20 = \frac{-1}{CR}\int_0^{1\text{ ms}} 10\ dt$$

$$\Rightarrow -20 = \frac{-1}{CR} \times 10 \times 1\text{ ms}$$

$$CR = \frac{10}{20} \times 1\text{ ms} = 0.5\text{ ms}$$

Ex: 2.28

Since dc gain of the op amp is much larger than the dc gain of the designed non-inverting amplifier, we can use equation(2.35).

Therefore:

$$f_{3db} = \frac{f_t}{1+\frac{R_2}{R_1}} \text{ and } 1+\frac{R_2}{R_1} = 100 \text{ and}$$

$$f_t = 2\text{ MHz}$$

$$\text{Hence } f_{3db} = \frac{2\text{ MHz}}{100} = 20\text{ kHz}$$

Ex: 2.29

For the input voltage step of magnitude V the output waveform will still be given by the exponential waveform of equation(2.40)

If $w_t V \le SR$

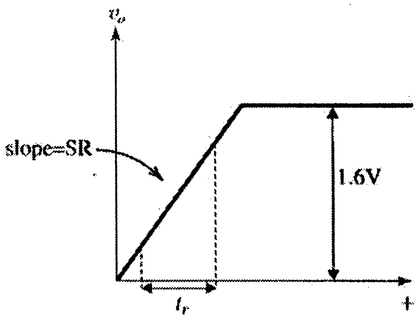
That is $V \le \frac{SR}{w_t} \Rightarrow V \le \frac{SR}{2\pi f_t}$

$V \le 0.16$ V, thus, the largest possible input voltage step is 0.16 V.

From Appendix F we know that the 10% to 90% rise time of the output waveform of the form of equation (2.40) is $t_r \simeq 2.2\frac{1}{w_t}$

Thus, $t_r \simeq 0.35$ μs

If an input step of amplitude 1.6 V (10 times as large compared to the previous case) is applied, the the output is slew-rate limited and is linearly rising with a slope equal to the slew-rate, as shown in the following figure.



$$t_r = \frac{0.9 \times 1.6 - 0.1 \times 1.6}{1\text{ V/μS}}$$

$$\Rightarrow t_r = 1.28\text{ μs}$$

Ex: 2.30

From equation (2.41) we have:

$$f_M = \frac{SR}{2\pi V_{O\max}} = 15.915\text{ kHz} \simeq 15.9\text{ kHz}$$

Using equation (2.42), for an input sinusoid with frequency $f = 5\ f_M$, the maximum possible amplitude that can be accommodated at the output without incurring SR distortion is:

$$V_O = V_{O\max}\left(\frac{f_M}{5\ f_M}\right) = 10 \times \frac{1}{5} = 2\text{ V (peak)}$$

Ex: 3.1

Refer to Fig3.3(a). for $V_I \geq 0$, the diode conducts and presents a zero voltage drop. Thus $V_O = V_I$ For $V_I < 0$, the diode is cut-off, zero current flows through R and $V_O = 0$, The results is the transfer characteristic in Fig E3.1.

Ex: 3.2

see Figure3.3aand3.3b

During the positive half of the sinusoid, the diode is forward biased, so it conducts resulting in $v_O = 0$ During the negative half of the input signal v_I, the diode is reverse biased. The diode does not conduct resulting in no current flowing in the circuit. So $v_O = 0$ and $v_D = v_I - v_O = v_I$ This results in the waveform shown in Figure E3.2

Ex: 3.3

$$\hat{i}_D = \frac{\hat{v}_I}{R} = \frac{10\text{ V}}{1\text{ k}\Omega} = 10\text{ mA}$$

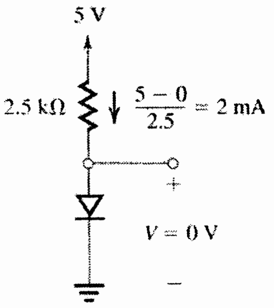
dc component of $v_O = \frac{1}{\pi}\hat{v}_O$

$$= \frac{1}{\pi}\hat{v}_I = \frac{10}{\pi}$$

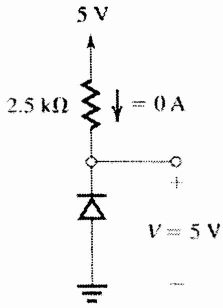
$$= 3.18\text{ V}$$

Ex: 3.4

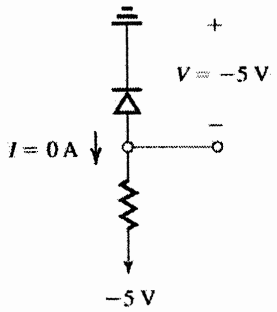
(a)


5 V
2.5 kΩ
$\frac{5-0}{2.5} = 2$ mA
+
$V = 0$ V
−

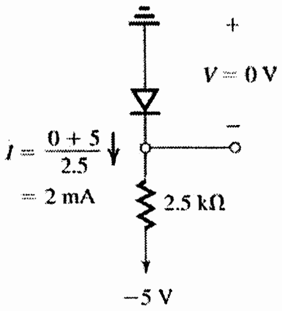
(b)


5 V
2.5 kΩ
= 0 A
+
$V = 5$ V
−

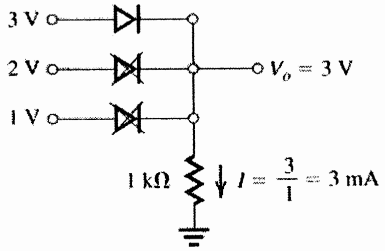
(c)



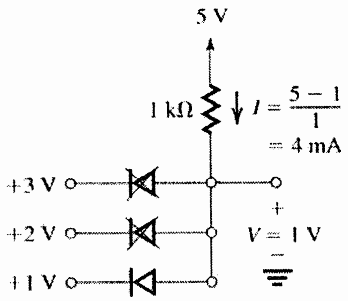
(d)


+
$V = 0$ V
−
$I = \frac{0+5}{2.5}$
= 2 mA
2.5 kΩ
−5 V

(e)


3 V
2 V
1 V
$V_o = 3$ V
1 kΩ
$I = \frac{3}{1} = 3$ mA

(f)


5 V
1 kΩ
$I = \frac{5-1}{1}$
= 4 mA
+3 V
+2 V
+1 V
+
$V = 1$ V
−

Ex: 3.5

$$V_{\text{avg}} = \frac{10}{\pi}$$

$$50 + R = \frac{\frac{10}{\pi} - 0}{1\text{ mA}} = \frac{10}{\pi}\text{ k}\Omega$$

$$\therefore R = 3.133\text{ k}\Omega$$

For an output voltage of 2.4 V, the voltage drop across each diode $= \frac{2.4}{3} = 0.8$ V

Now I, the current through each diode is

$$I = I_S e^{V/V_T} = 6.91 \times 10^{-16} e^{0.8/\left(25 \times 10^{-3}\right)}$$

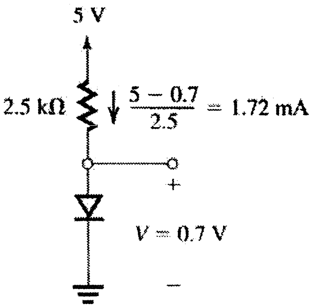
$$= 54.6 \text{ mA}$$

$$R = \frac{10 - 2.4}{54.6 \times 10^{-3}}$$

$$= 139\ \Omega$$

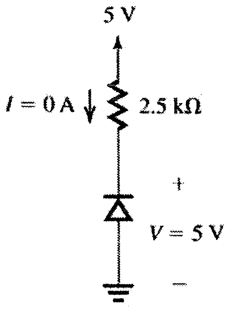
Ex: 3.12

(a)



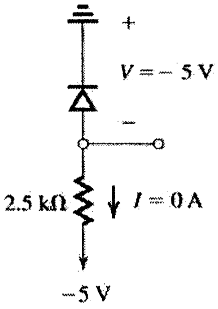
5 V
2.5 kΩ
$\frac{5 - 0.7}{2.5} = 1.72$ mA
+
$V = 0.7$ V
−

(b)



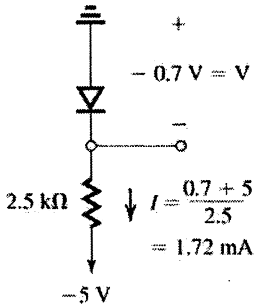
5 V
$I = 0$ A
2.5 kΩ
+
$V = 5$ V
−

(c)



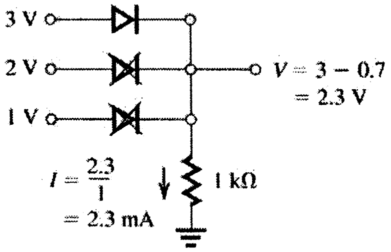
+
$V = -5$ V
−
2.5 kΩ
$I = 0$ A
−5 V

(d)



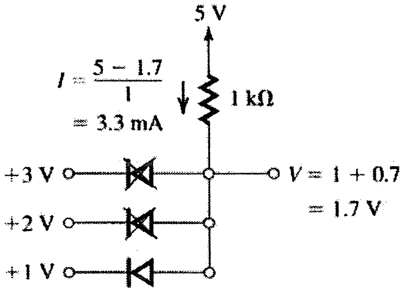
+
-0.7 V $= V$
−
2.5 kΩ
$I = \frac{0.7 + 5}{2.5}$
$= 1.72$ mA
−5 V

(e)



3 V
2 V
1 V
$V = 3 - 0.7$
$= 2.3$ V
$I = \frac{2.3}{1}$
$= 2.3$ mA
1 kΩ

(f)



5 V
$I = \frac{5 - 1.7}{1}$
$= 3.3$ mA
1 kΩ
+3 V
+2 V
+1 V
$V = 1 + 0.7$
$= 1.7$ V

Ex: 3.13

$$r_d = \frac{V_T}{I_D}$$

$$I_D = 0.1 \text{ mA} \quad r_d = \frac{25 \times 10^{-3}}{0.1 \times 10^{-3}} = 250\ \Omega$$

$$I_D = 1 \text{ mA} \quad r_d = \frac{25 \times 10^{-3}}{1 \times 10^{-3}} = 25\ \Omega$$

$$I_D = 10 \text{ mA} \quad r_d = \frac{25 \times 10^{-3}}{10 \times 10^{-3}} = 2.5\ \Omega$$

Ex: 3.14

For small signal model, using equation 3.15

$$i_D = I_D + \frac{I_D}{V_T} \cdot v_d$$

$$\Delta i_D = \frac{I_D}{V_T} \cdot \Delta v_d \qquad (1)$$

For exponential model

$i_D = I_S e^{V/V_T}$

$\frac{i_{D2}}{i_{D1}} = e^{(V_2 - V_1)/V_T} = e^{\Delta V/V_T}$

$\Delta i_D = i_{D2} - i_{D1} = i_{D1} e^{\Delta V/V_T} - i_{D1}$

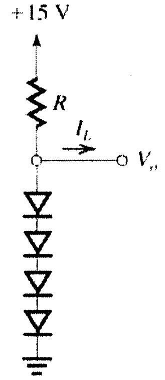
$= i_{D1}(e^{\Delta V/V_T} - 1) \qquad (2)$

In this problem $i_{D1} = I_D = 1$ mA

Using equations (1) and (2) results and using $V_T = 25$ mV

	ΔV(mV)	Δi_D(mA) small signal	Δi_D(mA) expo. model
a	−10	−0.4	−0.33
b	−5	−0.2	−0.18
c	+5	+0.2	+0.22
d	+10	+0.4	+0.49

Ex: 3.15



a. In this problem $\frac{\Delta V_O}{\Delta i_L} = \frac{20\text{ mV}}{1\text{ mA}} = 20\ \Omega$

∴ Total small signal resistance of the four diodes $= 20\ \Omega$

∴ For each diode $r_d = \frac{20}{4} = 5\ \Omega$

But $r_d = \frac{V_T}{I_D} \Rightarrow 5 = \frac{25\text{ mV}}{I_D}$

$\therefore I_D = 5$ mA

and $R = \frac{15 - 3}{5\text{ mA}} = 2.4\ \text{k}\Omega$

b. For $V_O = 3$ V, voltage drop across each diode $= \frac{3}{4} = 0.75$ V

$i_D = I_S e^{V/V_T}$

$I_S = \frac{i_D}{e^{V/V_T}} = \frac{5}{e^{0.75/25 \times 10^{-3}}} = 4.7 \times 10^{-16}$A

c. If $i_D = 5 - i_L = 5 - 1 = 4$ mA

Across each diode the voltage drop is

$V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$

$= 25 \times 10^{-3} \times \ln\left(\frac{4 \times 10^{-3}}{4.7 \times 10^{-16}}\right)$

$= 0.7443$ V

Voltage drop across 4 diodes

$= 4 \times 0.7443 = 2.977$ V

so change in $V_O = 3 - 2.977 = 23$ mV

Ex: 3.16

For a zener diode

$V_O = V_{ZO} + I_Z r_Z$

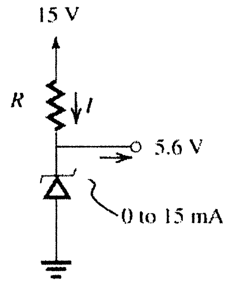
$10 = V_{ZO} + 0.01 \times 50$

$V_{ZO} = 9.5$ V

For $I_Z = 5$ mA

$V_O = 9.5 + 0.005 \times 50 = 9.75$ V

Ex: 3.17



The minimum zener current should be $5 \times I_{ZK} = 5 \times 1 = 5$ mA.

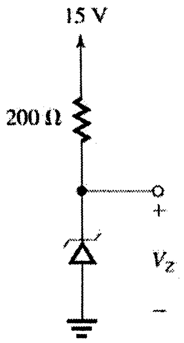
Since the load current can be as large as 15 mA, we should select R so that with $I_L = 15$ mA, a zener current of 5 mA is available. Thus the current should be 20 mA Leading to

$R = \frac{15 - 5.6}{20\text{ mA}} = 470\ \Omega$

Maximum power dissipated in the diode occurs when $I_L = 0$ is

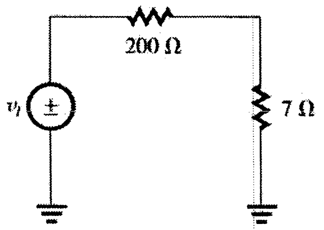
$P_{\max} = 20 \times 10^{-3} \times 5.6 = 112$ mV

Ex: 3.18



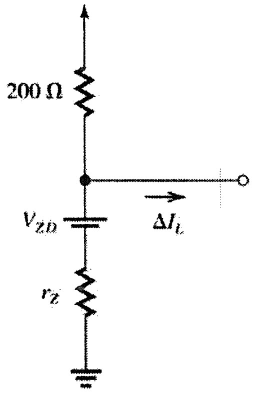
At no load $V_Z = 5.1$ V

FOR LINE REGULATION



Line Regulation $= \dfrac{v_O}{v_i} = \dfrac{7}{200 + 7} = 33.8\ \dfrac{\text{mV}}{\text{V}}$

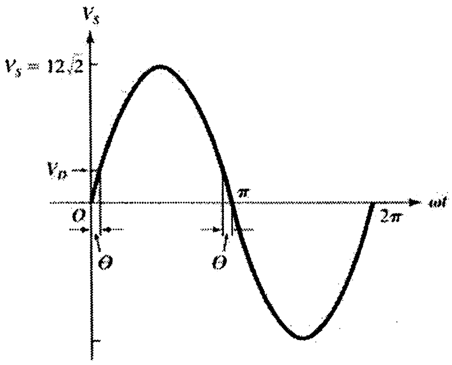
For Load Regualation:



$$\frac{\Delta V_O}{\Delta I_l} = \frac{-\Delta I_l r_Z}{1\text{ mA}}$$

$$= -7\ \frac{\text{mV}}{\text{mA}}$$

Ex: 3.19



a. The diode starts conduction at

$v_S = V_D = 0.7$ V

$v_S = V_S \sin\omega t$, here $V_S = 12\sqrt{2}$

At $\omega t = \theta$

$v_S = V_S \sin\theta = V_D = 0.7$ V

$12\sqrt{2}\sin\theta = 0.7$

$$\theta = \sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right) \simeq 2.4°$$

Conduction starts at θ and stops at $180 - \theta$.

$\therefore$ Total conduction angle $= 180 - 2\theta$

$= 175.2°$

$$\text{b. } v_{O,\text{avg}} = \frac{1}{2\pi}\int_{0}^{(\pi-\theta)} (V_S \sin\phi - V_D)d\phi$$

$$= \frac{1}{2\pi}[-V_S\cos\phi - V_D\phi]_{\phi=\theta}^{\phi=\pi-\theta}$$

$$= \frac{1}{2\pi}[V_S\cos\theta - V_S\cos(\pi-\theta) - V_D(\pi - 2\theta)]$$

But $\cos\theta \simeq 1$, $\cos(\pi - \theta) \simeq -1$ and

$\pi - 2\theta \simeq \pi$

$$v_{O,\text{avg}} = \frac{2V_S}{2\pi} - \frac{V_D}{2}$$

$$= \frac{V_S}{\pi} - \frac{V_D}{2}$$

For $V_S = 12\sqrt{2}$ and $V_D = 0.7$ V

$$v_{O,\text{avg}} = \frac{12\sqrt{2}}{\pi} - \frac{0.7}{2} = 5.05\text{ V}$$

c. The peak diode current occers at the peak diode voltage

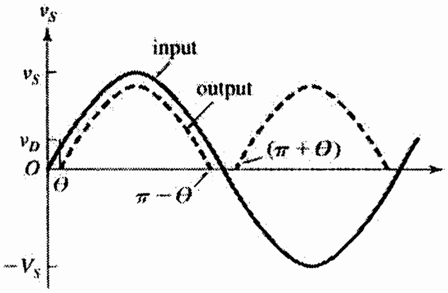
$$\therefore \hat{i}_D = \frac{V_S - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100}$$

$$= 163 \text{ mA}$$

$$\text{PIV} = +V_S = 12\sqrt{2}$$

$$\simeq 17 \text{ V}$$

Ex: 3.20



a. As shown in the diagram the output is zero between $(\pi - \theta)$ to $(\pi + \theta)$

$$= 2\theta$$

Here θ is the angle at which the input signal reaches V_D

$$\therefore V_S \sin\theta = V_D$$

$$\theta = \sin^{-1}\left(\frac{V_D}{V_S}\right)$$

$$2\theta = 2\sin^{-1}\left(\frac{V_D}{V_S}\right)$$

b. Average value of the output signal is given by

$$V_{O,\text{avg}} = \frac{1}{2\pi}\left[2 \times \int_{\phi}^{(\pi - \phi)} (V_S \sin\phi - V_D)d\phi\right]$$

$$= \frac{1}{\pi}[-V_S\cos\phi - V_D\phi]_{\phi = \theta}^{\pi - \theta}$$

$$= 2\frac{V_S}{\pi} - V_D$$

c. Peak current occers when $\phi = \frac{\pi}{2}$

Peak Current

$$= \frac{V_S \sin(\pi/2) - V_D}{R} = \frac{V_S - V_D}{R}$$

If v_S is 12 V(rms)

then $V_S = \sqrt{2} \times 12 = 12\sqrt{2}$

$$\text{Peak current} = \frac{12\sqrt{2} - 0.7}{100} \simeq 163 \text{ mA}$$

Non zero output occurs for angle $= 2(\pi - 2\theta)$

The fraction of the cycle for which $v_O > 0$ is

$$= \frac{2(\pi - 2\theta)}{2\pi} \times 100$$

$$= \frac{2\left[\pi - 2\sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right)\right]}{2\pi} \times 100$$

$$\simeq 97.4\%$$

Average output voltage V_O is

$$V_O = 2\frac{V_S}{\pi} - V_D = \frac{2 \times 12\sqrt{2}}{\pi} - 0.7 = 10.1 \text{ V}$$

Peak diode current $\hat{i}_D$ is

$$\hat{i}_D = \frac{V_S - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100}$$

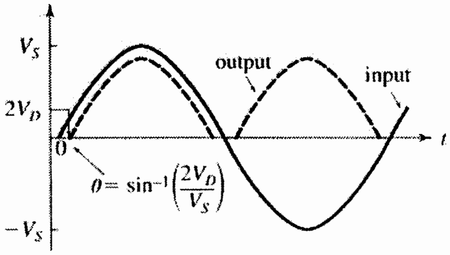
$$= 163 \text{ mA}$$

$$\text{PIV} = V_S - V_D + V_S$$

$$= 12\sqrt{2} - 0.7 + 12\sqrt{2}$$

$$= 33.2 \text{ V}$$

Ex: 3.21



$$V_{O,\text{avg}} = \frac{1}{2\pi}\int (V_S \sin\phi - 2V_D)\, d\phi$$

$$= \frac{2}{2\pi}[-V_S\cos\phi - 2V_D\phi]_{\phi = \theta}^{\pi - \theta}$$

$$= \frac{1}{\pi}[2V_S - 2V_D(\pi - 2\theta)]$$

But $\cos\theta \approx 1$

$\cos(\pi - \theta) \approx -1$

$\pi - 2\theta \approx \pi$

$$\Rightarrow V_{O,\text{avg}} = \frac{2V_S}{\pi} - 2V_D$$

$$= \frac{2 \times 12\sqrt{2}}{\pi} = -1.4 = 9.4 \text{ V}$$

(b) Peak diode current $= \dfrac{\text{Peak Voltage}}{R}$

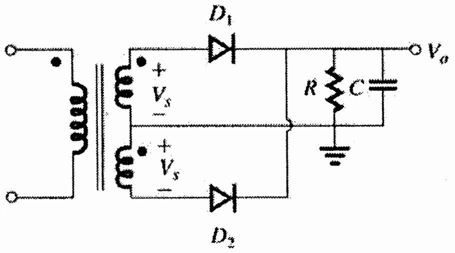
$$= \frac{V_S - 2V_D}{R} = \frac{12\sqrt{2} - 1.4}{100}$$

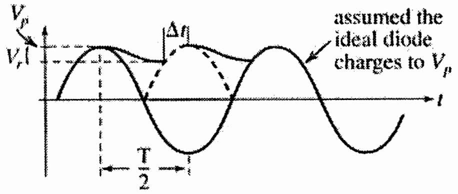
$$= 156 \text{ mA}$$

$$\text{PIV} = V_S - V_D = 12\sqrt{2} - 0.7 = 16.3 \text{ V}$$

Ex: 3.22

Full wave peak Rectifier:





The ripple voltage is the amount of discharge that occurs when the diodes are not conducting. The output voltage is given by:

$$v_O = V_p e^{-t/RC}$$

$V_p - V_r = V_p e^{-\frac{T/2}{RC}}$ ← discharge is only half the period.

$$V_r = V_p\left(1 - e^{-\frac{T/2}{RC}}\right)$$

$$e^{-\frac{T/2}{RC}} \simeq 1 - \frac{T/2}{RC}$$

for $CR >> T/2$

$$\simeq V_p\left(1 - 1 + \frac{T/2}{RC}\right)$$

$$= \frac{V_p}{2fRC} \qquad \text{(a)}$$

To find the average current, note that the charge supplied during conduction is equivalent to the charge lost during discharge.

$Q_{\text{SUPPLIED}} = Q_{\text{LOST}}$

$i_{\text{cap}}\Delta t = CV_r$ SUB (a)

$$(i_{D,\text{av}} - I_L)\Delta t = \cancel{C}\frac{V_p}{2fR\cancel{C}} = \frac{V_p}{2fR}$$

$$= \frac{V_p\pi}{\omega R}$$

$$i_{D,\text{av}} = \frac{V_p\pi}{\omega\Delta tR} + I_L$$

where $\omega\Delta t$ is the conduction angle.

Note the conduction angle is the same expression as for the half wave rectifier and is given in EQ 3.30

$$\omega\Delta t \simeq \sqrt{\frac{2V_r}{V_p}} \qquad \text{(b)}$$

Substituting for $\omega\Delta t$ we get:

$$\Rightarrow i_{D,\text{av}} = \frac{\pi V_p}{\sqrt{\frac{2V_r}{V_p}} \cdot R} + I_L$$

Since the output is approximately held at V_p,

$\frac{V_p}{R} \approx I_L$. Thus:

$$\Rightarrow i_{D,\text{av}} \cong \pi I_L\sqrt{\frac{V_p}{2V_r}} + I_L$$

$$= I_L\left[1 + \pi\sqrt{\frac{V_p}{2V_r}}\right] \text{ Q.E.D}$$

If $t = 0$ is at the peak, the maximum diode current occurs at the onset of conduction or at $t = \omega\Delta t$.

During conduction, the diode current is given by:

$$i_D = i_C + i_L$$

$$i_{D,\max} = C\frac{dv_S}{dt} + i_L$$

assuming i_L is const. $i_L \approx \frac{V_p}{R} = I_L$

$$= C\frac{d}{dt}(V_p\cos\omega t) + I_L$$

$$= -C\sin\omega t \times \omega V_p + I_L$$

$$= -C\sin(-\omega\Delta t) \times \omega V_p + I_L$$

for a small conduction angle

$\sin(-\omega\Delta t) \approx -\omega\Delta t$. Thus:

$$\Rightarrow i_{D,\max} = C\omega\Delta t \times \omega V_p + I_L$$

Sub (b) to get:

$$i_{D,\max} = C\sqrt{\frac{V_r^2}{V_p}}\omega V_p + I_L$$

SUB $\omega = 2\pi f$ sub (a) for f

$$= 2\pi\frac{V_p}{2V_rRC}$$

$$\Rightarrow i_{D,\max} = \cancel{C}\sqrt{\frac{2V_r}{V_p}}\frac{2\pi Vp^2}{2V_rR\cancel{C}} + I_L$$

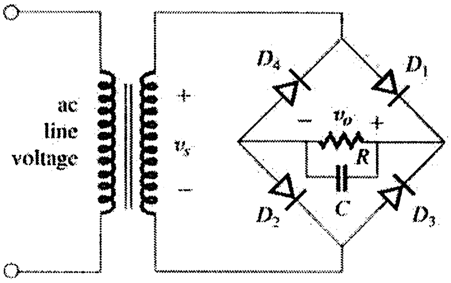
$$= \pi\frac{V_p}{V_r}I_L\sqrt{\frac{2V_r}{V_p}} + I_L$$

$$= I_L\left[1 + \frac{\pi V_p}{V_r}\sqrt{\frac{2V_r}{V_p}}\right]$$

$$= I_L\left[1 + \pi\sqrt{\frac{2V_p}{V_r}}\right]$$

$$= I_L\left[1 + 2\pi\sqrt{\frac{V_p}{2V_r}}\right] \text{ Q.E.D.}$$

Ex: 3.23



The output voltage, v_O, can be expressed as

$$v_O = (V_p - 2V_{DO})e^{-t/RC}$$

At the end of the discharge interval

$$v_O = (V_p - 2V_{DO} - V_r)$$

The discharge occurs almost over half of the time period $\simeq T/2$

For time constant $RC >> \frac{T}{2}$

$$e^{-t/RC} \simeq 1 - \frac{T}{2} \times \frac{1}{RC}$$

$$\therefore V_p - 2V_{DO} - V_r = (V_p - 2V_{DO})\left(1 - \frac{T}{2} \times \frac{1}{RC}\right)$$

$$\Rightarrow V_r = (V_p - 2V_{DO}) \times \frac{T}{2RC}$$

Here $V_p = 12\sqrt{2}$ and $V_r = 1$ V

$V_{DO} = 0.8$ V

$$T = \frac{1}{f} = \frac{1}{60}\text{ s}$$

$$1 = (12\sqrt{2} - 2 \times 0.8) \times \frac{1}{2 \times 60 \times 100 \times C}$$

$$C = \frac{(12\sqrt{2} - 1.6)}{2 \times 60 \times 100} = 1281\ \mu\text{F}$$

Without considering the ripple voltage the dc output voltage

$$= 12\sqrt{2} - 2 \times 0.8 = 15.4\text{ V}$$

If ripple voltage is included the output voltage is

$$= 12\sqrt{2} - 2 \times 0.8 - \frac{V_r}{2} = 14.9\text{V}$$

Diode current without taking ripple voltage into consideration $= \dfrac{12\sqrt{2} - 2 \times 0.8}{100\ \Omega} \simeq 0.15$ A

The conduction angle $\omega\Delta t$ can be obtained using equation 4.30

$$\omega\Delta t = \sqrt{\frac{2V_r}{V_p}} = \sqrt{\frac{2 \times 1}{12\sqrt{2} - 2 \times 0.8}} = 0.36$$

rad = 20.7°

The average and peak diode currents can be calculated using equations β . 34 and β . 35

$$i_{D\text{ ave}} = I_L\left(1 + \pi\sqrt{\frac{V_p}{2V_r}}\right) \text{ Here } I_L = \frac{14.9\text{ V}}{100\ \Omega},$$

and $V_p = 12\sqrt{2} - 2 \times 0.8$, $V_r = 1$ V

$i_{base} = 1.45$ A

$$I_{D\text{ peak}} = 1\left(1 + 2\pi\sqrt{\frac{V_p}{2V_r}}\right)$$

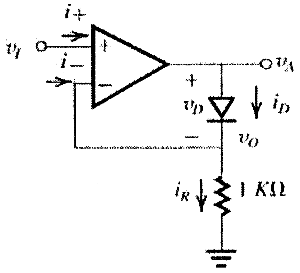
$= 2.74$ A

PIV of the diodes

$$= V_S - V_{DO} = 12\sqrt{2} - 0.8 = 16.2\text{ V}$$

To keep the safety margin, select a diode capable of a peak current of 3.5 to 4A and having a PIV rating of 20 V.

Ex: 3.24



The diode has 0.7 V drop at 1 mA current.

$$i_D = I_S e^{v_D/V_T}$$

$$\frac{i_D}{1\text{ mA}} = e^{(v_D - 0.7)/V_T}$$

$$\Rightarrow v_D = V_T \ln\left(\frac{i_D}{1\text{ mA}}\right) + 0.7\text{ V}$$

For $v_I = 10$ mV, $v_O = v_I = 10$ mV

It is ideal op amp, so $i_+ = i_- = 0$

$$\therefore i_D = i_R = \frac{10\text{ mV}}{1\text{ k}\Omega} = 10\ \mu\text{A}$$

$$v_D = 25 \times 10^{-3}\ln\left(\frac{10\text{ mA}}{1\text{ mA}}\right) + 0.7 = 0.58\text{ V}$$

$V_A = v_D + 10$ mV

$= 0.58 + 0.01$

$= 0.59$ V

For $v_I = 1$ V

$v_O = v_I = 1$ V

$$i_D = \frac{v_O}{1\text{ k}\Omega} = \frac{1}{1\text{ k}\Omega} = 1\text{ mA}$$

$v_D = 0.7$ V

$V_A = 0.7\text{ V} + 1\text{ k}\Omega \times 1\text{ mA}$

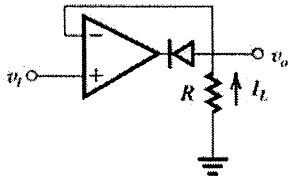
$= 1.7$ V

For $v_I = -1$ V, the diode is cutoff

$\therefore v_O = 0$ V

$V_A = -12$ V because it is ideal amplifier.

Ex: 3.25



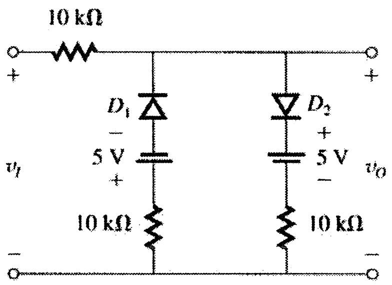
$v_I > 0$ ~ diode is cutoff

$v_O = 0$ V

$v_I < 0$ ~ diode conducts and opamp sinks load current.

$v_O = v_I$

Ex: 3.26



Both diodes are cut-off

for $-5 \leq v_I \leq +5$

and $v_o = v_I$

For $v_I \leq -5$ V

Diode D_1 conducts and

$$v_O = -5 + \frac{1}{2}(+v_I + 5)$$

$$= \left(-2.5 - \frac{v_I}{2}\right)\text{ V}$$

For $v_I \geq 5$ V

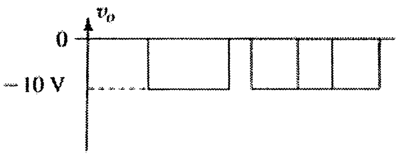
Diode D_2 conducts and

$$v_O = +5 + \frac{1}{2}(v_I - 5)$$

$$= \left(2.5 + \frac{v_I}{2}\right)\text{ V}$$

Ex: 3.27

Reversing the diode results in the peak output voltage being clamped at 0 V:



Here the dc component of $v_O = V_O = -5$ V

Ex: 4.1

$$\therefore I_C = I_s e^{v_{BE}/V_T}$$

$$v_{BE2} - v_{BE1} = V_T \ln\left[\frac{I_{C2}}{I_{C1}}\right]$$

$$v_{BE2} = 700 + 25\ln\left[\frac{0.1}{1}\right]$$

$$= 642 \text{ mV}$$

$$v_{BE2} = 700 + 25\ln\left[\frac{10}{1}\right]$$

$$= 758 \text{ mV}$$

Ex: 4.2

$$\therefore \alpha = \frac{\beta}{\beta + 1}$$

$$\frac{50}{50+1} < \alpha < \frac{150}{150+1}$$

$$0.98 < \alpha < 0.993$$

Ex: 4.3

$$I_C = I_E - I_B$$

$$= 1.460 \text{ mA} - 0.01446 \text{ mA}$$

$$= 1.446 \text{ mA}$$

$$\alpha = \frac{I_C}{I_E} = \frac{1.446}{1.460} = 0.99$$

$$\beta = \frac{I_C}{I_B} = \frac{1.446}{0.01446} = 100$$

$$I_C = I_S e^{v_{BE}/V_T}$$

$$I_S = \frac{I_C}{e^{v_{BE}/V_T}} = \frac{1.446}{e^{700/25}}$$

$$= \frac{1.446}{e^{28}}\text{A} = 10^{-15}\text{A}$$

Ex: 4.4

$$\beta = \frac{\alpha}{1-\alpha} \text{ and } I_C = 10 \text{ mA}$$

$$\text{For } \alpha = 0.99 \; \beta = \frac{0.99}{1-0.99} = 99$$

$$I_B = \frac{I_C}{\beta} = \frac{10}{99} = 0.1 \text{ mA}$$

$$\text{For } \alpha = 0.98 \; \beta = \frac{0.98}{1-0.98} = 49$$

$$I_B = \frac{I_C}{\beta} = \frac{10}{49} = 0.2 \text{ mA}$$

Ex: 4.5

Given:

$$I_S = 10^{-16} \text{ A}, \; \beta = 100, \; I_C = 1 \text{ mA}$$

$$I_{SE} = I_{SC} + I_{SB} = I_{SC}\left[1 + \frac{1}{\beta}\right]$$

$$I_{SB} = \frac{I_{SC}}{\beta} = \frac{10^{-16}}{100} = 10^{-18} \text{ A}$$

$$I_{SE} = I_{SC}\left[1 + \frac{1}{\beta}\right] = 10^{-16} \times \frac{101}{100}$$

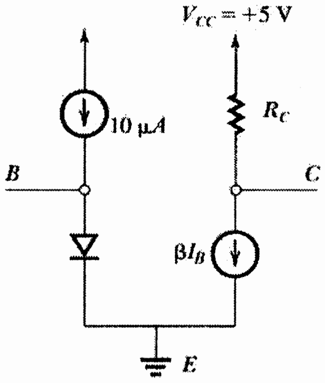
$$= 1.01 \times 10^{-16} \text{ A}$$

$$V_{BE} = V_T \ln\left[\frac{I_C}{I_S}\right] = 25\ln\left[\frac{1 \text{ mA}}{10^{-16}}\right]$$

$$= 25 \times 29.9336$$

$$= 748 \text{ mV}$$

Ex: 4.6



$$v_{BE} = 690 \text{ mV}$$

$$I_C = 1 \text{ mA}$$

For active range $V_C \geq V_B$

$$R_C(\max) = \frac{V_{CC} - 0.690}{I_C}$$

$$= \frac{5 - 0.69}{1}$$

$$= 4.31 \text{ k}\Omega$$

Ex: 4.7

$$I_S = 10^{-15} \text{ A}$$

$$\text{Area}_C = 100 \times \text{Area}_E$$

$$I_{SC} = 100 \times I_S = 10^{-13} \text{ A}$$

Ex: 4.8

$$i_C = I_S e^{V_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T}$$

for $i_C = 0$

$$I_S e^{V_{BE}/V_T} = I_{SC} e^{V_{BC}/V_T}$$

$$\frac{I_{SC}}{I_S} = \frac{e^{V_{BE}/V_T}}{e^{V_{BC}/V_T}}$$

$$= e^{(V_{BE} - V_{BC})/V_T}$$

$$\therefore V_{CE} = V_{BE} - V_{BC} = V_T \ln\left[\frac{I_{SC}}{I_S}\right]$$

For collector Area = 100 × Emitter Area

$$V_{CE} = 25\ln\left[\frac{100}{1}\right] = 115 \text{ mV}$$

Ex: 4.9

$$i_C = I_S e^{V_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T}$$

$$i_b = \frac{I_S}{\beta} e^{V_{BE}/V_T} + I_{SC} e^{V_{BC}/V_T}$$

$$\beta_{\text{forced}} = \left.\frac{i_C}{i_B}\right|_{\text{sat}} < \beta$$

$$= \beta \frac{I_S e^{V_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T}}{I_S e^{V_{BE}/V_T} + \beta I_{SC} e^{V_{BC}/V_T}}$$

$$= \beta \frac{I_S e^{(V_{BE} - V_{BC})/V_T} - I_{SC}}{I_S e^{(V_{BE} - V_{BC})/V_T} + \beta I_{SC}}$$

$$= \beta \frac{e^{V_{CE\text{sat}}/V_T} - I_{SC}/I_S}{e^{V_{CE\text{sat}}/V_T} + \beta I_{SC}/I_S}$$

$$\beta_{\text{forced}} = 100 \frac{e^{200/25} - 100}{e^{200/25} + 100 \times 100}$$

$$= 100 \times 0.2219 = 22.2$$

Ex: 4.10

$$I_E = \frac{I_S}{\alpha} e^{V_{BE}/V_T}$$

$$2 \text{ mA} = \frac{51}{50} 10^{-14} e^{V_{BE}/V_T}$$

$$V_{BE} = 25\ln\left[\frac{2}{10^3} \times \frac{50}{51} \times 10^{14}\right]$$

$$= 650 \text{ mV}$$

$$I_C = \frac{\beta}{\beta + 1} I_E = \frac{50}{51} \times 2$$

$$= 1.96 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{1.96}{50} \Rightarrow 39.2\ \mu\text{A}$$

Ex: 4.11

$$I_C = I_S e^{V_{BE}/V_T} = 1.5 \text{ A}$$

$$\therefore V_{BE} = V_T \ln[1.5 \times 10^{-11}]$$

$$= 25 \times 25.734$$

$$= 643 \text{ mV}$$

Ex: 4.12

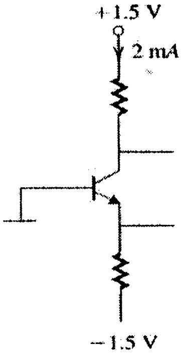


Fig 6.12

$\beta = 100$, $V_{BE} = 0.8$ V at $I_C = 1$ mA

$$V_{BE2} - V_{BE1} = V_T \ln[I_{C2}/I_{C1}]$$

$$= 25 \times 0.693 = 0.01733$$

$$\therefore V_{BE2} = 0.817 \text{ V}$$

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{1.5 - 0.5}{2} \text{ k}\Omega$$

$$= 500\ \Omega$$

$$R_E = \frac{V_{EE} - V_{BE}}{I_C} \frac{\beta}{(\beta + 1)}$$

$$= \frac{1.5 - 0.817}{2} \times \frac{100}{101} \text{ k}\Omega$$

$$= 338\ \Omega$$

Ex: 4.13

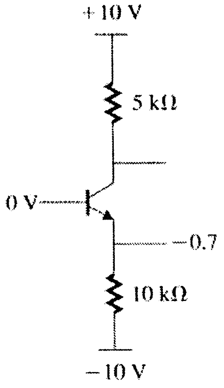


Fig 6.13

$\beta = 50$, $V_{BE} = 0.7$ V

$$V_E = V_B - 0.7 \text{ V}$$

$= 0 - 0.7 = -0.7$ V

$$I_E = \frac{-0.7 + 10}{10\text{ K}}$$

$= 0.93$ mA

$$I_C = \frac{50}{51} I_E = 0.91 \text{ mA}$$

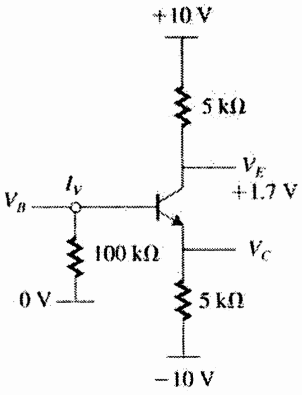
$$V_C = 10 - 0.91 \times 5$$

$= 5.45$ V

$$I_B = \frac{I_C}{\beta} = \frac{0.91}{50}$$

$= 0.0182\ \mu$A

Ex: 4.14



$$I_E = \frac{V_{EE} - V_E}{R_E}$$

$$= \frac{10 - 1.7}{5}$$

$= 1.66$ mA

$$I_B = \frac{V_B - 0}{100\text{ k}\Omega}$$

$= 0.01$ mA

$$I_C = I_E - I_B$$

$= 1.65$ mA

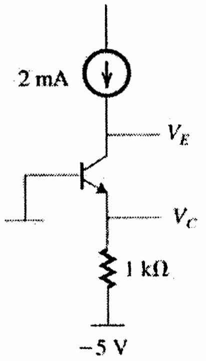
$$\alpha = \frac{I_C}{I_E} = \frac{1.65}{1.66} = 0.99$$

$$\beta = \frac{I_C}{I_B} = \frac{1.65}{0.01} = 165$$

$$V_C = V_{CC} + I_C R_C$$

$= -10 + 1.65 \times 5 = -1.75$ V

Ex: 4.15



V_{BE} decreases approx 2 mV/°C rise
for 30°C rise

$$\Delta V_{BE} = -2 \times 30$$

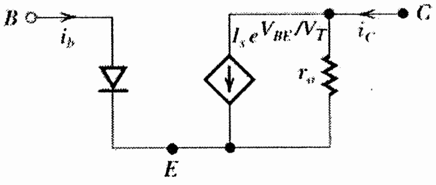
$= -60$ mV

$$\Delta V_E = -60 \text{ mV}$$

Since I_E is constant
I_C is constant

$\therefore\ \Delta V_C = 0$ V

Ex: 4.16



$$A + C, \quad i_C = I_S e^{V_{BE}/V_T} + \frac{v_{CE}}{r_O}$$

$$\text{and } r_O = \frac{V_A}{I}$$

$$\therefore\ i_C = I_S e^{V_{BE}/V_T} + \frac{V_{CE} \cdot I_S e^{V_{BE}/V_T}}{V_A}$$

$$= I_S e^{V_{BE}/V_T} \left[1 + \frac{v_{CE}}{V_A}\right]$$

QED

$$i_C = I_S e^{V_{BE}/V_T} + \frac{v_{CE}}{r_O} \text{ in Fig (a)}$$

$$i_B = \frac{I_S}{\beta} e^{V_{BE}/V_T}$$

$$\therefore i_C = \beta i_b + \frac{v_{CE}}{r_O} \text{ as in Fig (b)}$$

QED

Ex: 4.17

$$r_O = \frac{V_A}{I_C}\ \frac{(\text{V})}{(\text{A})}$$

I_C(mA)	0.1	1.0	10
r_O	$\frac{100}{0.0001}$	$\frac{100}{0.001}$	$\frac{100}{0.010}$
r_O	$10^6\ \Omega$	$10^5\ \Omega$	$10^4\ \Omega$
r_O	1 MΩ	100 kΩ	10 kΩ

Ex: 4.18

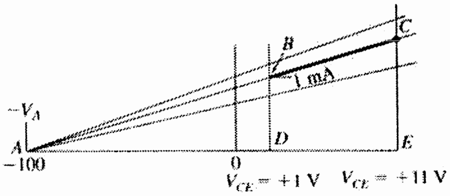


Fig 4.18

By similar triangles ABD v ACE

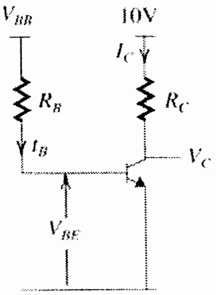
$$\frac{1}{r_O} = \frac{\Delta I_C}{\Delta V_{CE}}$$

$$\frac{BD}{CE} = \frac{AD}{AE} \text{ or } \frac{1\text{ mA}}{x} = \frac{100+1}{100+11}$$

$$\Rightarrow x = \frac{1 \times 111}{101} = 1.099\text{ mA}$$

$$\therefore \approx 1.1\text{ mA}$$

Ex: 4.19



$R_C = 10\text{ k}\Omega$, $\beta = 50$

(a) active $V_C = 5$ V

$$I_C = \frac{V_{CC} - V_C}{R_C}$$

$$= \frac{10-5}{10}$$

$= 0.5$ mA

$V_{BB} = V_{BE} + I_B R_B$

$= 0.7 + 10 \times 0.01$

$= 0.8$ V

(b) edge of saturation $v_{CE} = 0.3$ V

$$I_C = \frac{10-0.3}{10\text{ K}} = \frac{9.7}{10} = 0.97\text{ mA}$$

$I_C = I_C/\beta = 0.97/50 = 0.0194$ mA

$V_{BB} = 0.7 + 0.0194 \times 10 = 0.894$ V

(c) saturated $V_{CE} = 0.2$ V

$I_C = (10 - 0.2)/10 = 0.98$ mA

$I_B = I_C/\beta F = 0.98/10 = 0.098$ mA

$V_B = 0.7 + 0.098 \times 10 = 1.68$ V

Ex: 4.20

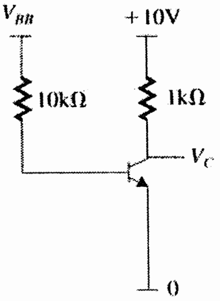


Fig 6.20

For $V_{BB} = 0$

$I_B = 0$

Transistor is OFF

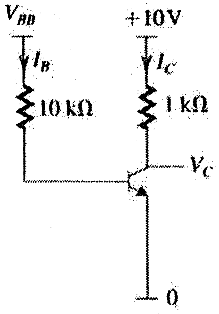
$\therefore I_C = 0$

$V_C = V_{CC} - I_C R_C$

$= +10 - 0$

$= +10$ V

Ex: 4.21


V_{BB}
+10V
I_B
I_C
10 kΩ
1 kΩ
V_C
0

For $V_{BB} = 1.7$ V

$$I_B = \frac{1.7 - 0.7}{10} = 0.1 \text{ mA}$$

$I_C = \beta I_B$

$= 50 \times 0.1 = 5$ mA

$V_C = 10 - 5 \times 1$ kΩ

$= +5 \text{ V} > V_{BE}$ (so Active)

(a) edge of saturation $V_{CE} = 0.3$ V

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{10 - 0.3}{5} = 1.94 \text{ k}\Omega$$

(b) deep saturation $v_{CE} = 0.2$ V

$I_B = 0.1$ mA (unchanged)

$I_C = \beta_{\text{Forced}} I_B = 10 \times 0.1 = 1$ mA

$$R_C = \frac{10 - 0.2}{1 \text{ mA}} = 9.8 \text{ k}\Omega$$

Ex: 4.22

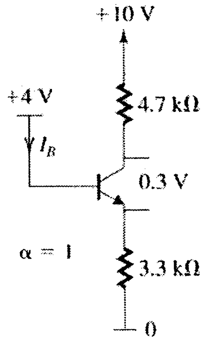

+10 V
+4 V
I_B
4.7 kΩ
0.3 V
α = 1
3.3 kΩ
0

Fig 4.22

At edge of saturation $v_{CE} = 0.3$ V

$V_{CC} = I_C R_C + 0.3 + I_E R_E$

$\simeq I_E[R_C + R_E] + 0.3$

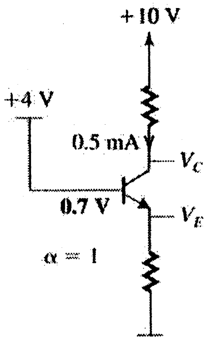
$$I_E = \frac{10 - 0.2}{4.7 + 3.3} = 1.225 \text{ mA}$$

$V_{BB} = I_E R_E + 0.7$

$= 1.225 \times 3.3 + 0.7$

$= 4.7$ V

Ex: 4.23


+10 V
+4 V
0.5 mA
V_C
0.7 V
V_E
α = 1

$V_E = V_B - 0.7$

$= 4 - 0.7 = 3.3$ V

$$R_E = \frac{3.3 \text{ V}}{0.5 \text{ mA}} = 6.6 \text{ k}\Omega$$

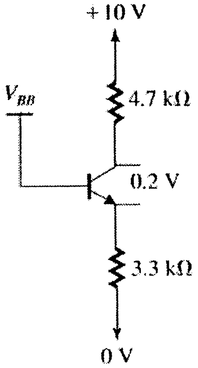
$V_C = V_B + 2$ V

$= 4 + 2 = +6$ V

$$R_C = \frac{V_{CC} - V_C}{I}$$

$$= \frac{10 - 6}{0.5} = 8 \text{ k}\Omega$$

Ex: 4.24


+10 V
V_{BB}
4.7 kΩ
0.2 V
3.3 kΩ
0 V

$\beta_{\text{Forced}} = 5$

Then $I_C = 5\, I_B$

$I_E = 6\, I_B$

$$I_B = \frac{10 - 0.2}{5 \times 4.7 + 6 \times 3.3}$$

$= 0.226$ mA

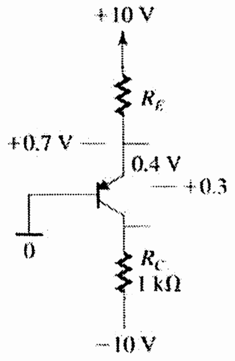
$V_E = 6\, I_B \times 3.3$

$= 4.48$ V

$V_{BB} = V_E + 0.7$

$= 5.18$ V

Ex: 4.25


+10 V
R_E
+0.7 V
0.4 V
+0.3
0
R_C
1 kΩ
−10 V

$V_B = 0$

$V_E = +0.7$ V

$$I_E = \frac{10 - 0.7}{2}$$

$= 4.65$ mA

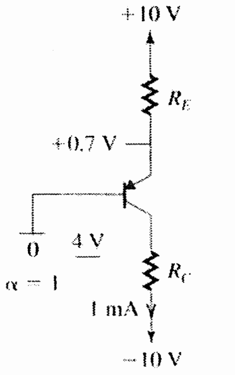
$I_C = 0.99\, I_E$

$$R_C = \frac{10 - 0.4 + 0.7}{0.99 \times 4.65}$$

$= 2.2$ kΩ

$[V_C(\text{max}) = 0 + 0.7 - 0.4 = +0.3 \text{ V}]$

Ex: 4.26


+10 V
R_E
+0.7 V
0
4 V
$\alpha = 1$
R_C
1 mA
−10 V

$I_E = I_C$

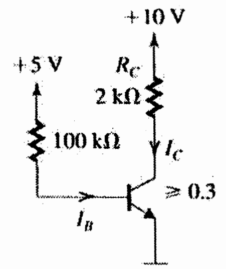
$$R_E = \frac{10 - 0.7}{1} = 9.3 \text{ k}\Omega$$

$V_C = V_B - 4$

$= -4$V

$$R_C = \frac{10 - 4}{1} = 6 \text{ k}\Omega$$

Ex: 4.27


+10 V
+5 V
R_C
2 kΩ
100 kΩ
I_C
≥ 0.3
I_B

$50 \le \beta \le 150$

In active range

$$I_B = \frac{5 - 0.7}{100 \text{ K}} = 0.043 \text{ mA}$$

V_C lowest for largest β

$I_C = \beta\, I_B = 150 \times 0.043$ A

$$R_C = \frac{V_{CC} - 0.3}{150 \times 0.043}$$

$= 1.5$ kΩ

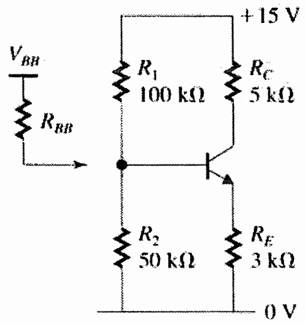
For $\beta = 50$

$V_C = 10 - 50 \times 0.043 = 6.78$ V

For $\beta = 150$

$V_C = 0.3$ V

Ex 4.28


+15 V
V_{BB}
R_{BB}
R_1
100 kΩ
R_C
5 kΩ
R_2
50 kΩ
R_E
3 kΩ
0 V

$\beta = 50$

$$V_{BB} = \frac{15 \times 50}{150} = 5 \text{ V}$$

$R_{BB} = 50 \,\|\, 100$

$= 100/3$ kΩ

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + [R_{BB}/(\beta + 1)]}$$

$$= \frac{4.3}{3 + \frac{100}{3} \cdot \frac{1}{51}}$$

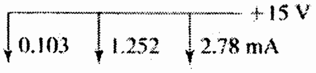
$= 1.18$ mA

$I_C = I_E \frac{50}{51} = 1.15$ mA

% change $= \frac{1.28 - 1.15}{1.28}$

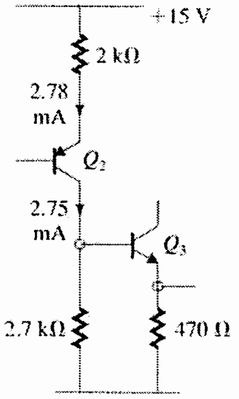
$\Rightarrow -9.8\%$

Ex: 4.29


+15 V
0.103
1.252
2.78 mA

Total current drawn
$= 0.103 + 1.252 + 2.75$ mA
$= 4.135$ mA
Power Consumed $= V \times I$
$= 15 \times 4.135$
$= 62$ mW

Ex: 4.30


+15 V
2 kΩ
2.78 mA
Q2
2.75 mA
Q3
2.7 kΩ
470 Ω

$\beta = 100$
I_{E2} unchanged
I_{C2} unchanged
$V_{E3} = V_{C2} - 0.7$ V

$$IE_3 = \frac{VC_2 - 0.7}{0.470}$$

$= 101\, I_{B3}$

$$= 101\left[2.75 - \frac{V_{C2}}{2.7}\right]$$

Hence

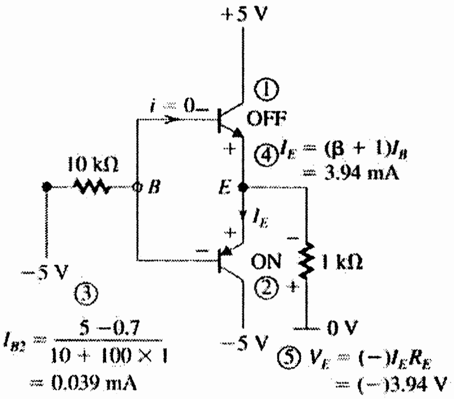
$$V_{C2}\left[\frac{1}{(\beta+1)0.47} + \frac{1}{2.7\text{ k}}\right] = 2.75 + \frac{0.7}{(\beta+1)0.47}$$

$\Rightarrow V_{C2} = 7.06$ V

$V_{E3} = V_{C2} - 0.7 = 6.36$ V

$$I_{C3} = \frac{V_{E3}}{0.47} \times \frac{100}{101} = 13.4\text{ mA}$$

Ex: 4.31

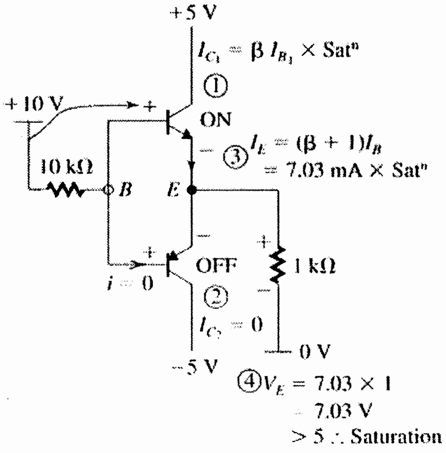

+5 V
①
OFF
i = 0
④ $I_E = (\beta + 1)I_B$
$= 3.94$ mA
10 kΩ
B
E
I_E
ON
1 kΩ
②
−5 V
③
−5 V
0 V
⑤ $V_E = (-)I_E R_E$
$= (-)3.94$ V

$$I_{B2} = \frac{5 - 0.7}{10 + 100 \times 1}$$

$= 0.039$ mA

Ans: $V_E = -3.94$ V

Ex: 4.32


+5 V
$I_{C_1} = \beta I_{B_1} \times$ Sat^n
①
ON
+10 V
③ $I_E = (\beta + 1)I_B$
$= 7.03$ mA × Sat^n
10 kΩ
B
E
OFF
1 kΩ
i = 0
②
$I_{C_2} = 0$
0 V
−5 V
④ $V_E = 7.03 \times 1$
$= 7.03$ V
> 5 ∴ Saturation

⑤ $V_B = V_E + 0.7$

$= 5.5$ V

$$I_B = \frac{10 - 5.5}{10}$$

$= 0.45$ mA

$$V_E \Rightarrow \frac{V_{CC} - 0.2}{1}$$

$= 4.8$ V

$I_E \Rightarrow 4.8$ mA

⑥ $I_{C(\text{sat})} = I_E - I_B$

$= 4.8 - 0.45$

$= 4.35$ mA

$$V_E \Rightarrow \frac{V_{CC} - 0.2}{1}$$

$= 4.8$ V

$I_E \Rightarrow 4.8$ mA

⑦ $\beta_{\text{forced}} = \frac{I_C}{I_B}$

$= \frac{4.35}{0.45} = 9.6 << 30$

Ex: 4.33

$$A_V = -\frac{V_{CC} - V_{CE}}{V_T} = \frac{10 - V_{CE}}{0.025} = -320 \text{ V/V}$$

$$\Rightarrow V_{CE} = 10 - 8 = 2.0 \text{ V}$$

$$R_C = \frac{10 - V_{CE}}{1 \text{ mA}} = \frac{10 - 2}{1} = 8 \text{ k}\Omega$$

$$V_{CE} \text{ Swing} = 2.0 - 0.3 = 1.7 \text{ V}$$

$$A_V = \frac{\Delta V_{CE}}{\Delta V_{BE}} = -320 = \frac{1.7}{\Delta V_{BE}}$$

$$\Rightarrow |\Delta V_{BE}| = \frac{1.7}{320} = 5.3 \text{ mV}$$

Ex: 4.34

Given: $g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{i_C = I_C}$

But $I_C = I_S e^{V_{BE}/V_T}$

thus $\frac{\partial I_C}{\partial V_{BE}} = \frac{I_S e^{V_{BE}/V_T}}{V_T}$

$$= \frac{I_C}{V_T}$$

Ex: 4.35

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{25 \text{ mV}} = 20 \text{ mA/V}$$

Ex: 4.36

$I_C = 0.5$ mA (constant)

$\beta = 50$	$\beta = 200$
$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{25 \text{ mV}}$	
$= 20$ mA / V	$= 20$ mA/V
$I_B = \frac{I_C}{\beta} = \frac{0.5}{50}$	$= \frac{0.5}{200}$
$= 10$ μA	$= 2.5$ mA
$r_\pi = \frac{\beta}{g_m} = \frac{50}{20}$	$= \frac{200}{20}$
$= 2.5$ kΩ	$= 10$ kΩ

Ex: 4.37

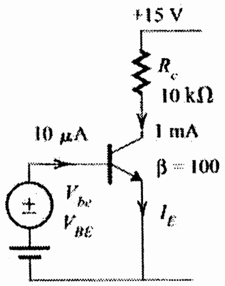
$\beta = 100 \quad I_C = 1$ mA

$$g_m = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA / V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$r_e = \frac{r_\pi}{\beta + 1} \approx 25 \ \Omega$$

Ex: 4.38



$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA / V}$$

$$A_V = \frac{v_{ce}}{v_{be}} = -g_m R_C$$

$$= -40 \times 10$$

$$= -400 \text{ V/V}$$

$$V_C = V_{CC} - I_C R_C$$

$$= 15 - 1 \times 10 = 5 \text{ V}$$

$$N_C(t) = V_C + N_C(t)$$

$$= (V_{CC} - I_C R_C) + A_V v_{be}(t)$$

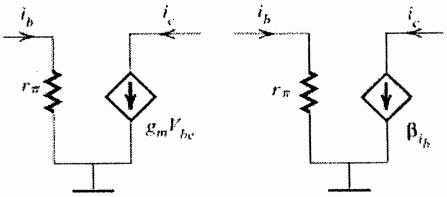
$$= (15 - 10) - 400 \times 0.005 \sin\omega t$$

$$= 5 - 2 \sin \omega t \ (t)$$

$$i_b(t) = I_B + v_{be}(t)$$

$$= 10 + 2 \sin \omega t \ (\mu\text{A})$$

Ex: 4.39



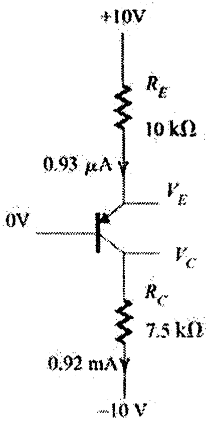
Note: $g_m = \frac{I_C}{V_T}$ and $r_\pi = \frac{\beta}{g_m}$

given $i_C = \beta i_b = (g_m r_\pi) i_b$

$$= r_\pi g_m \left(\frac{v_{be}}{r_\pi}\right)$$

$$= g_m v_{be}$$

Ex: 4 . 40


+10V
R_E
10 kΩ
0.93 μA
V_E
0V
V_C
R_C
7.5 kΩ
0.92 mA
−10 V

Change R_C to 7.5 kΩ

$V_C = -10 + 0.92 \times 7.5$

$= -3.1$ V

$$A_V = \frac{v_c}{v_e} = g_m R_C$$

$= 36.8 \times 7.5$

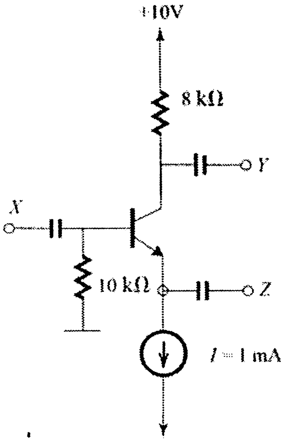
$= 276$ V/V

$\hat{v}_O = A_V v_i$

$= 276 \times \hat{1}0$ mV

$= 2.76$ V

Ex: 4 . 41


+10V
8 kΩ
Y
X
10 kΩ
Z
I = 1 mA

$I_E = 1$ mA

$$I_C = \frac{100}{101} = 0.99 \text{ mA}$$

$$I_B = \frac{1}{101} = 0.0099 \text{ mA}$$

$V_C = 10 - 8 \times 0.99 = 2.18$ V

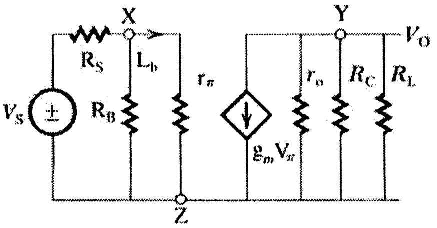
$V_B = -10 \times 0.0099 = -0.099 \approx -0.1$ V

$V_E = -0.1 - 0.7 = -0.8$ V

$$g_m = \frac{I_C}{V_T} = \frac{0.99}{0.025} = 39.6 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{39.6} = 2.53 \text{ k}\Omega$$

$$r_O = \frac{V_A}{I_C} = \frac{100}{0.99} = 101 \approx 100 \text{ k}\Omega$$


X
Y
V_O
R_S
L_b
r_π
V_S
±
R_B
r_o
R_C
R_L
$g_m V_\pi$
Z

$$\frac{v_O}{v_S} = \frac{r_\pi \| R_B}{R_S + r_\pi \| R_B} \cdot (-)g_m(r_O \| R_C \| R_L)$$

$$= \frac{2}{2+2}(-)\ 39.6\ (3.85) = -76.2 \text{ V/ V}$$

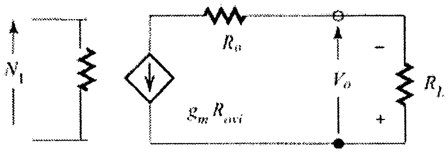
$$\frac{r_O \| R_C \| R_L}{R_C \| R_L} = \frac{3.85}{4.00}$$

thus effect of r_O is $\approx 3.9\%$

Ex (2) 4 . 41

$A_{VO} = -g_m(r_O \| R_C)$

$R_O = (r_O \| R_C)$


N_1
R_o
V_o
−
+
R_L
$g_m R_{ovi}$

$$A_V = -\frac{g_m R_O V_1}{V_1} \times \frac{R_L}{R_O + R_L}$$

$$= -g_m \times \frac{R_O R_L}{R_O + R_L}$$

$$= -g_m \times \frac{(r_O \| R_C)R_L}{(r_O \| R_C) + R_L}$$

$$= -g_m(r_O \| R_C \| R_L)$$

Ex: 4 . 42

For $I_C = 1$ mA

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025} = 40 \text{ mA / V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$r_O = \frac{V_A}{I_C} = \frac{100}{1} = 100 \text{ k}\Omega$$

$R_{IN} = r_\pi = 2.5$ kΩ

$A_{VO} = -g_m(r_O \| R_C) = -40(5 \| 100)$

$= -97.6$ V/V

(cont.)

$$\frac{v_1}{v_S} = \frac{R_{IN}}{R_S + R_{IN}} = \frac{2.5}{5 + 2.5} = \frac{1}{3}$$

$$\frac{v_O}{v_S} = \frac{v_1}{v_S} \cdot \frac{v_O}{v_1} = -\frac{1}{3} \times 97.6 = -32.5 \text{ V/V}$$

$$|v_O| = A_{VO} v_\pi = 32.5 \times \frac{15}{1000} = 0.49 \text{ V}$$

For $I_C = 0.5$ mA and $R_C = 10$ kΩ

$$g_m = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

$$r_\pi = \frac{100}{20} = 5.0 \text{ k}\Omega$$

$$r_O = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$R_{IN} = r_\pi = 5 \text{ k}\Omega$$

$$A_{VO} = -20(200 \text{ k} \| 10 \text{ k})$$
$$= -190.5 \text{ V/V}$$

$$R_O = R_C \| r_O$$
$$= 10 \text{ k} \| 200 \text{ k}$$
$$= 9.5 \text{ k}$$

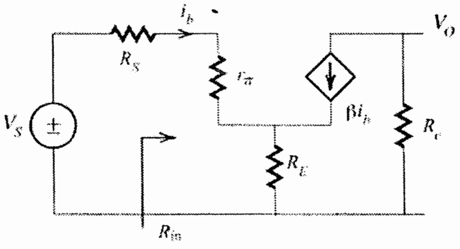
$$A_V = \frac{-190.5 \times 5}{5 + 9.5}$$
$$= -65.6 \text{ V/V}$$

$$G_V = \frac{v_O}{v_S} = \frac{1}{2} \times 65.6 = -32.8 \text{ V/V}$$

$$\frac{v_1}{v_S} = \frac{5}{5 + 5} = \frac{1}{2} \Rightarrow \hat{v}_{sig} = 10 \text{ mV}$$

$$|\hat{v}_O| = 32.8 \times 10 \text{ m} = 0.33 \text{ V}$$

Ex: 4.43



$$R_{IN} = \frac{v_b}{i_b} = r_\pi + (\beta + 1)R_E$$

$$\frac{V_S}{V_\pi} = \frac{R_S + r_\pi + (\beta + 1)R_E}{r_\pi}$$

$$= 1 + \frac{R_S}{r_\pi} + \frac{R_E}{r_\pi / (\beta + 1)}$$

$$= 1 + \frac{R_S}{r_\pi} + \frac{R_E}{r_e} \text{ QED}$$

$$\frac{v_S}{v_\pi} = \frac{100}{10} = 1 + 10 \text{ K} + \frac{R_E}{5 / 101}$$

$$\Rightarrow R_E = \frac{7 \times 5}{101} = \frac{5}{01} \approx 0.35 \text{ k}\Omega = 350 \ \Omega$$

$$R_{IN} = 5 \text{ k} + (\beta + 1)0.35 \approx 40.4 \text{ k}\Omega$$

$$G_V = \frac{\beta(r_O \| R_C \| R_L)}{R_{sig} + R_{IN}} = \frac{100 \times 10}{10 + 40.4}$$
$$= -19.8 \text{ V/V}$$

Ex: 4.44

$$g_m = \frac{I_C}{V_T} = \frac{1}{0.025} = 40 \text{ ms}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$r_e = \frac{r_\pi}{\beta + 1} \approx 25 \ \Omega$$

$$R_{IN} = r_e = 25 \ \Omega$$

$$A_{VO} = g_m(r_O \| R_C) \approx g_m R_C$$
$$= 40 \times 5 = 200 \text{ V/V}$$

$$A_V = A_{VO} \times \frac{5}{5 + 5} = 100 \text{ V/V}$$

$$G_V = \frac{R_{IN}}{R_S + R_{IN}} \cdot A_V = \frac{25}{5000 + 25} \times 100$$
$$= 0.5 \text{ V/V}$$

Ex: 4.45

$R_S = 50 \ \Omega$

$$R_{IN} = r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{I_E} = 50$$

$$\Rightarrow I_E = 25 / 50 = 0.5 \text{ mA}$$

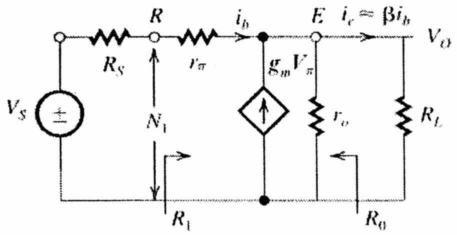
$$A_{VO} = +g_m R_C = 20 \times 5 = 100 \text{ V/V}$$

$$G_V = \frac{1}{2} \times A_V = 40 \text{ V/V}$$

$$\Rightarrow A_V = 80 = g_m R_C^*$$

$$\therefore R_C^* = 80 / 20 = 4 \text{ k}\Omega$$

Ex: 4.46



P Insert 30

$$i_b = \frac{v_1 - v_O}{r_\pi} \text{ and } v_O \approx +g_m V_\pi R_L$$

$$i_b = \frac{v_i - (g_m V\pi + i_b)R_L}{r_\pi}$$

$$\Rightarrow R_1 = \frac{v_1}{i_b} = r_\pi + (\beta + 1)R_L$$

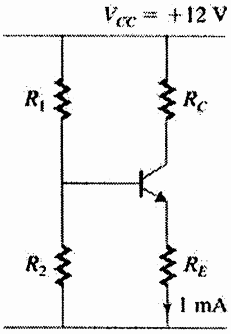
$= 0.5 + 101 \times 1 = 101.5\ \text{k}\Omega$

$G_{VO} = 1[R_L \text{ moved}, r_O = \infty]$

$$R_O = \frac{r\pi + R_S}{\beta + 1} = \frac{0.5 + 10}{101} \Rightarrow 104\ \Omega$$

$$G_V = \frac{v_O}{v_S} = \frac{R_L}{R_O + R_S} = \frac{1}{0.104 + 1} = 0.91\ \text{V/V}$$

Ex: 4.47



Design 1
$\beta = 100$
$R_E = 3\ \text{k}\Omega$

$$R_{BB} = \frac{80 \times 40}{80 + 40} = 26.7\ \text{k}\Omega$$

$$V_{BB} = \frac{12 \times 40}{80 + 40} = 4\ \text{V}$$

$$I_E = \frac{4 - 0.7}{\frac{26.7}{101} + 3} = 1.01\ \text{mA}$$

$\beta = 50$

$$I_E = \frac{3.3}{\frac{26.7}{151} + 3} = 1.04\ \text{mA}$$

$$\%\ \text{change} = \frac{1.04 - 0.937}{1} \times 100$$

$= 10.3\ \%$

Design 2
$\beta = 100$
$R_E = 3.3\ \text{k}\Omega$

$$R_{BB} = \frac{8 \times 40}{8 + 4} = 2.67\ \text{k}\Omega$$

$$V_{BB} = \frac{12 \times 4}{8 + 4} = 4\ \text{V}$$

$$I_{EE} = \frac{4 - 0.7}{\frac{26.7}{101} + 3.3} = 0.99\ \text{mA}$$

$\beta = 150$

$$I_E = \frac{3.3}{\frac{2.6}{51} + 3.3} = 0.984\ \text{mA}$$

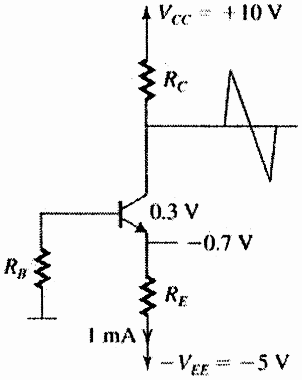
$\beta = 150$

$$I_E = \frac{3.3}{\frac{2.6}{151} + 3.3} = 0.995\ \text{mA}$$

$$\% = \frac{0.995 - 0.984}{1} \times 100$$

$= 1.1\ \%$

Ex: 4.48



$$A_V = \frac{I_C R_C}{V_T}\quad I_E \text{ given} \quad \therefore \text{ maximize } R_C$$

$V_C = I_C R_C + 2 + 0.3 + I_E R_E$

$$I_E = \frac{V_{EE} - 0.7}{R_E + R_B/(\beta + 1)} = \frac{4.3}{R_E + R_B/(\beta + 1)} = 1\ \text{mA}$$

$\Rightarrow R_E + R_B/(\beta + 1) = 4.3\ \text{k}\Omega$

For independence from β, set $R_B = 0$ (OK for C_B)

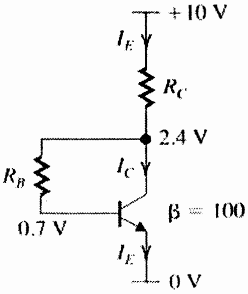
$\Rightarrow R_E = 4.3\ \text{k}\Omega$

$V_C(\text{min}) = V_E + 0.3\ \text{V} = -0.3\ \text{V}$

$VCQ = V_C(\text{min}) + 2\text{V} = +10\ \text{V}$

$$R_C = \frac{V_{CC} - V_{CQ}}{I_C} = \frac{10 -}{0.9} = 8.48\ \text{k}\Omega$$

Ex: 4.49



$V_C = V_E + 0.4 + 2$

$= +2.4\ \text{V}$

$$R_C = \frac{V_{CC} - V_C}{1} = \frac{10 - 2.4}{1} = \frac{7.6}{1} = 7.6\ \text{k}\Omega$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{1}{101}\ \text{mA}$$

(cont.)

$$R_B = \frac{V_C - V_B}{I_B} = 101(2.4 - 0.7) = 171.7\ \text{k}\Omega$$

Using 5% resistors:

$R_C = 7.5\ \text{k}\Omega$ $R_b = 180\ \text{k}\Omega$

$$0.7 + I_B R_B + I_E R_C - V_{CC} = 0$$

$$I_B = \frac{10 - 0.7}{180 + 7.5(\beta + 1)}$$

$$I_B = 9.92\ \mu$$

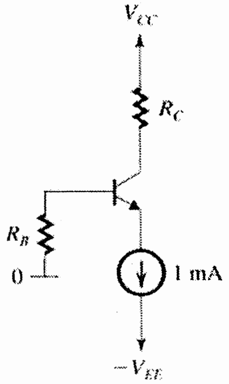
$$I_E = 101 \times I_B$$

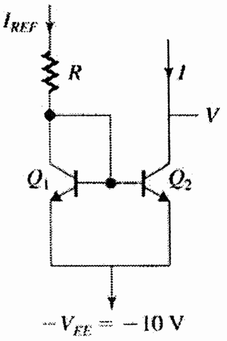
$$= 1.002\ \text{mA}$$

$$V_C = 10 - 7.5(1.002)$$

$$= 2.5\ \text{V}$$

Ex 4.50





$\beta = 100$ $R_B = 100\ \text{k}\Omega$

$R_c = 7.5\ \text{k}\Omega$

$$I_B = I_E/(\beta + 1) \approx I_E/100$$

$$= 0.01\ \text{mA}$$

$$V_B = 0 - I_B R_B = -1\ \text{V}$$

$$V_E = V_B - 0.7\ \text{V} = -1.7\ \text{V}$$

$$V_C = V_{CC} - \frac{\beta}{\beta + 1} \times 1 \times 7.5 = +2.57\ \text{V}$$

$$R = \frac{V_{CC} - 0.7 + V_{EE}}{I_{\text{REF}}} = \frac{19.3}{1} = 19.3\ \text{k}\Omega$$

Ex: 4.51

Refer to Fig E4.51

$$I_C = \alpha(1\ \text{m}) \approx 1\ \text{mA}$$

$$I_B = 0.01\ \text{mA}$$

$$V_C = 10 - 8\ \text{k}(1\ \text{m}) = +2\ \text{V}$$

$$V_B = 100\ \text{k} \times (-0.01\ \text{m}) = -1\ \text{V}$$

$$V_E = -1 - 0.7 = -1.7\ \text{V}$$

$\beta = 100$: upper limit $\Rightarrow V_{CC} - V_C = 8$ V

lower value $\Rightarrow V_B - V_C' = 0.4$ V (where

$V_C' = -1.4$ V)

Swing: $-1.4 - 2 = -3.4$ V

$\beta = 50$: upper still 8 V, lower

$\Rightarrow I_B = 0.0196$ mA so $V_B = -1.96$ V

$-1.96\ \text{V} - 0.4 - 2 = -4.4\ \text{V}$

$\beta = 200$: upper still 8V,

lower $\Rightarrow I_B = 0.005$ mA so $V_B = -0.5$ V

$-0.5 - 0.4 - 2 = -2.9\ \text{V}$

$$V_A = 100\ \text{V}$$

$$r_O = \frac{100}{1\ \text{m}} = 100\ \text{k}\Omega$$

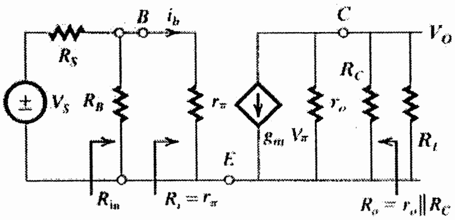
$$g_m = \frac{I_C}{V_T} = \frac{1\ \text{m}}{25\ \text{m}} = 40\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\ \text{m}} = 2.5\ \text{k}\Omega$$

$$r_e = \frac{r_\pi}{(\beta + 1)} \cong 25\ \Omega$$

Ex: 4.52

Example 4.50



$$R_i = \frac{v_b}{i_b} = r_\pi = 2.5\ \text{k}\Omega$$

$$R_{\text{IN}} = R_B \| r_\pi = 100 \| 2.5 = 2.44\ \text{k}\Omega$$

$$A_{VO} = -g_m R_C = -40 \times 8 = -320\ \text{V/V}$$

$$R_O = R_C = 8\ \text{k}\Omega$$

$$A_{VO} = -g_m(r_O \| R_C \| R_L) = -40 \times 3.5$$

$$= -119\ \text{V/V}$$

$$R_O = r_O \| R_C = 100 \| 8 = 7.4\ \text{k}\Omega$$

$$A_{VO} = -g_m(r_O \| R_C)$$

$$= -40 \times 7.4 = -296\ \text{V/V}$$

$$G_V = \frac{R_{\text{IN}}}{R_S + R_{\text{IN}}} A_V \frac{R_C}{R_O + R_L}$$

$$= \frac{2.44}{5 + 2.44}(-216)\frac{5}{7.4 + 5}$$

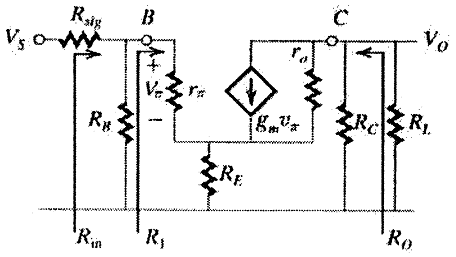
$$= -39.1\ \text{V/V}$$

$$\hat{v}_S = \frac{R_S + R_{\text{IN}}}{R_{\text{IN}}} \hat{V} = 15\ \text{mV}$$

$$|\hat{v}_O| = G_V |\hat{v}_S|$$

$$= 39.1 \times 15 = 586\ \text{mV}$$

Ex: 4.53


R_{sig}
B
C
V_S
V_O
r_o
+
V_π
r_π
−
$g_m v_\pi$
R_B
R_C
R_L
R_E
R_{in}
R_1
R_O

Example 4.50

$g_m = 40$ mA/V $\quad r_\pi = 2.5$ kΩ

$r_o = 100$ kΩ $\quad V_A = 100$ V $\quad R_B = 100$ kΩ

$R_{sig} = 5$ kΩ $\quad R_C = 8$ kΩ $\quad R_L = 5$ kΩ

$R_1 = r_\pi + (\beta + 1)R_E$

$R_{IN} = R_B \| R_1 = 4 \times R_{sig} = 20$ kΩ

$$\therefore R_E = \frac{25 - 2.5}{101} = 0.22 \text{ k}\Omega = 223\ \Omega$$

$$A_{VO} = \frac{-g_m R_C}{1 + g_m R_E} \approx \frac{-40(8)}{1 + 40(223)} = -32 \text{ V/V}$$

$R_{out} \cong 8$ kΩ

$$A_V = \frac{-R_C \| R_L}{r_e + R_E} = \frac{-8\text{ K} \| 5\text{ K}}{25 + 223} = -12.4 \text{ V/V}$$

$$G_V = \frac{-\beta(R_C \| R_L)}{R_{sig} + (\beta + 1)(r_e + R_E)} = -9.9 \text{ V/V}$$

$$\text{OR } G_V = \frac{R_{IN}}{R_{sig} + R_{IN}} \times A_V = \frac{20\text{ K}}{25\text{ K}} \times -12.4$$

Note: without R_E: $A_V = -g_m(R_C \| R_L)$

$= -123$ V/V

with R_E:

$$\frac{V_{sig}}{5 \text{ mV}} = \frac{5 + 20}{20} = \frac{5}{4} \Rightarrow V_{sig} = 6.25 \text{ mV}$$

w/o R_E:

$$\frac{V_{sig}}{5 \text{ mV}} = \frac{5 + (2.5 \| 100)}{(2.5 \| 100)} \approx \frac{3}{1} \Rightarrow V_S = 15 \text{ mV}$$

$|V_O| = |V_1| \times A_V = 512.4 = 62$ mV

Ex: 4.54

$g_m = 40$ mA/V $\quad V_A = 100$

$r_o = 100$ kΩ $\quad \beta = 100$

$r_\pi = 2.5$ kΩ $\quad \alpha = 0.99$

$r_e = 25\ \Omega$ $\quad I_E = 1$ mA

$R_{IN} = r_e = 25\ \Omega$

$$A_{VO} = +g_m(R_C \| r_O)$$

$= 40 \times 10^{-3} \times (8\text{ k} \| 100\text{ k})$

$= -296$ V/V

$R_{out} = R_C \| r_O = 7.4$ kΩ

$A_V = +g_m(R_C \| R_L \| r_O)$

$= 40 \times 3 = 120$ V/V

$$\frac{r_i}{r_{sig}} = \frac{r_e}{R_{sig} + r_e} = \frac{25}{5000 + 25}$$

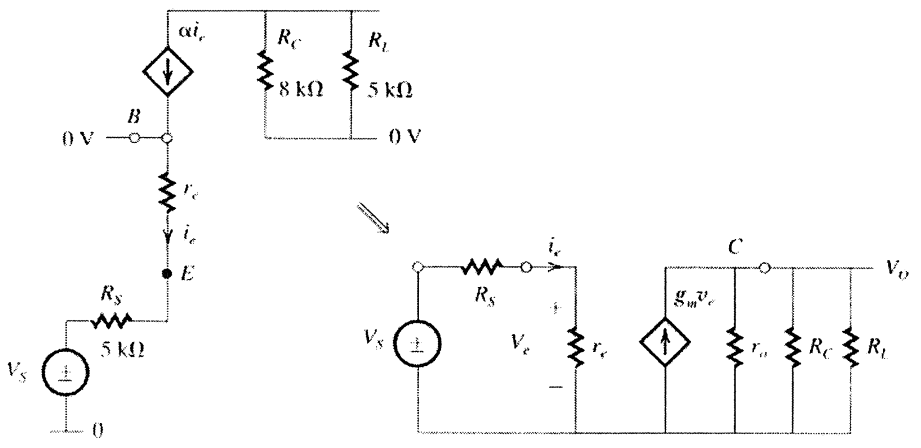
$= 0.005$ V/V

$$G_V = \frac{\alpha(R_C \| R_L)}{R_{sig} + r_e}$$

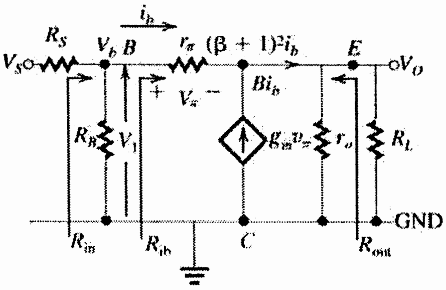
$= 0.6$ V/V

$$R_{sig} = \frac{\alpha(R_C \| R_L)}{G_V}$$

$R_{sig} = 54\ \Omega$


αi_e
R_C
8 kΩ
R_L
5 kΩ
B
0 V
0 V
r_e
i_e
E
R_S
5 kΩ
V_S
0
i_e
C
V_O
R_S
V_S
+
V_e
r_e
−
$g_m v_e$
r_o
R_C
R_L

Ex: 4.55



$I_c = 5 \text{ mA}$

$$r_\pi = \frac{\beta V_T}{I_C}$$

$$r_\pi = \frac{100(25 \text{ m})}{5 \text{ m}} = 500\ \Omega$$

$$r_O = \frac{V_A}{I_C} = \frac{100}{5 \text{ m}} = 20 \text{ k}\Omega$$

$$i_b = \frac{V_b - V_O}{r_\pi} \text{ and } V_O = (\beta + 1) i_b (r_O \| R_L)$$

$$\therefore\ i_b = \frac{V_b}{r_\pi} - \frac{i_b(\beta + 1)(r_O \| R_L)}{r_\pi}$$

$$\therefore\ R_{ib} = \frac{V_b}{i_b} = r_\pi + (\beta + 1)(r_O \| R_L)$$

$$= 0.5 + (101)(20 \| 1)$$

$$= 96.7 \text{ k}\Omega$$

$$R_{\text{IN}} = R_B \| R_{ib} = 40 \| 96.7 = 28.3 \text{ k}\Omega$$

$$G_V = \frac{V_O}{V_S} = \frac{V_1}{V_S} \times \frac{V_O}{V_1}$$

$$= -\frac{R_{\text{IN}}}{R_S + R_{\text{IN}}} \times \frac{(\beta + 1)(r_O \| R_L)}{(R_S \| R_B) + (\beta + 1)(r_e + (r_O \|) R_L)}$$

$$= 0.796 \text{ V/V}$$

$$G_{VO} = \frac{40}{10 + 40} \times \frac{20 \text{ K}}{\frac{(10 \text{ K} \| 40 \text{ K})}{101}(5\text{E} + 5 + 20 \text{ K})}$$

$$G_{VO} = 0.8 \text{ V/V}$$

$$R_{\text{out}} = r_O \| (r_e + [R_S \| R_B]) / (\beta + 1)$$

$$= 20 \| [0.05 + 0.079] \text{ k}\Omega$$

$$= 84\ \Omega$$

$$\hat{V}_O = \frac{\hat{V}_\pi \times (r_O \| r_e)}{r_e} = \frac{0.01 \times 0.95}{0.005} = 1.9 \text{ V}$$

For R_l(kΩ)	0.5	1.0	2.0
G_v (V/V)	0.68	0.735	0.765

Ex 4.56 blank

Ex: 5.1

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5\ \mathrm{pF/m}}{4\ \mathrm{nm}} = 8.625\ \mathrm{fF}/(\mu\mathrm{m})^2$$

$$\mu_n = 450\ \mathrm{cm^2/VS}$$

$$k_n' = \mu_n C_{ox} = 388\ \mu\mathrm{A/V^2}$$

$$V_{OV} = (v_{GS} - V_t) = 0.5\ \mathrm{V}.$$

$$g_{DS} = \frac{1}{1\ \mathrm{k\Omega}} = k_n'\frac{W}{L}V_{OV} \Rightarrow \frac{W}{L} = 5.15$$

$L = 0.18\ \mu\mathrm{m}$, so $W = 0.93\ \mu\mathrm{m}$

Ex: 5.2

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{34.5\ \mathrm{pF/m}}{4\ \mathrm{nm}} = 2.30\ \mathrm{fF}/\mu\mathrm{m}^2$$

$$\mu_n = 550\ \mathrm{cm^2/VS}$$

$$k_n' = \mu_n C_{ox} = 127\ \mu\mathrm{A/V^2}$$

$$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^{\ 2} = 0.2\ \mathrm{mA},\ \frac{W}{L} = 20$$

$$\therefore V_{OV} = 0.40\ \mathrm{V}.$$

$V_{DS,\,\min} = V_{OV} = 0.40\ \mathrm{V}$, for saturation

Ex: 5.3 $I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^{\ 2}$ in saturation

Change in I_D is:

(a) double L, 0.5

(b) double W, 2

(c) double V_{OV}, $2^2 = 4$

(d) double V_{DS}, no change (ignoring length modulation)

(e) changes (a) - (d), 4

case (c) would cause leaving saturation if $V_{DS} < 2V_{OV}$

Ex: 5.4 In saturation $v_{DS} \geq V_{OV}$, so $2V_{OV}$

$$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^{\ 2}\text{, so } 4\ I_D.$$

Ex: 5.5 $V_{OV} = 0.5\ \mathrm{V}$

$$g_{DS} = k_n'\frac{W}{L}V_{OV} = \frac{1}{1\ \mathrm{k\Omega}}$$

$$\therefore k_n = k_n'\frac{W}{L} = 2\ \mathrm{mA/V^2}$$

For $v_{DS} = 0.5\ \mathrm{V} \cdot = V_{OV}$

$$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^{\ 2} = 0.25\ \mathrm{mA}$$

for all $v_{DS} \geq V_{OV} = 0.5\ \mathrm{V}$.

Ex: 5.6

$$V_A = V_A' L = 50 \times 0.8 = 40\ \mathrm{V},$$

$$\lambda = \frac{1}{V_A} = 0.025\ \mathrm{V^{-1}}$$

$$V_{DS} = 1\ \mathrm{V} > V_{OV} = 0.5\ \mathrm{V}$$

$$\Rightarrow \text{Saturation: } I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2(1 + \lambda V_{DS})$$

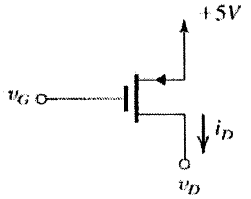
$$I_D = \frac{1}{2} \times 200 \times \frac{16}{0.8} \times 0.5^2(1 + 0.025 \times .1)$$

$$= 0.51\ \mathrm{mA}$$

$$r_o = \frac{V_A}{I_D} = \frac{40}{0.51} = 78.4\ \mathrm{k\Omega} \simeq 80\ \mathrm{k\Omega}$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_O} \Rightarrow \Delta I_O = \frac{2\ \mathrm{V}}{80\ \mathrm{K}} = 0.025\ \mathrm{mA}$$

Ex: 5.7



$$V_{tp} = -1\ \mathrm{V}.$$

$$k_p' = 60\ \mu\mathrm{A/V^2}$$

$$\frac{W}{L} = 10 \Rightarrow k_p = 600\ \mu\mathrm{A/V^2}$$

(a) Conduction occurs for $v_{GS} \leq V_{tp} = -1\ \mathrm{V}$.

or $v_G \leq V_{tp} + V_s = +4\ \mathrm{V}$.

(b) Triode region occurs for $v_{GD} \leq V_{tp}$

or $v_G - v_D \leq -1$

or $v_D \geq v_G + 1$

(c) Conversely, for saturation

$v_D \leq v_G + 1$

(d) Given $\lambda \cong 0$

$$I_D = \frac{1}{2}k_p'\frac{W}{L}|V_{OV}|^2 = 75\ \mu\mathrm{A}$$

$$\therefore |V_{OV}| = 0.5\ \mathrm{V} = -v_{GS} + V_{tp}$$

$$= -v_G + v_S + V_{tp} = 4 - v_G$$

$\therefore v_G = \; +3.5 \text{ V}.$

$v_D \leq v_G + 1 = 4.5 \text{ V}.$

(e) For $\lambda = -0.02 \text{ V}^{-1}$ and $|V_{OV}| = 0.5 \text{ V}.$

$I_D = 75 \; \mu\text{A}$ and $r_o = \dfrac{1}{|2|I_D} = 667 \text{ k}\Omega$

(f) At $V_D = 3 \text{ V}.$

$$I_D = \frac{1}{2}k'_n\frac{W}{L}|V_{OV}|^2(1 + |\lambda||v_{DS}|)$$

$$= 75 \; \mu\text{A}(1.04) = 78 \; \mu\text{A}$$

At $V_D = OV,$

$I_D = 75 \; \mu\text{A}(1.10) = 82.5 \; \mu\text{A}$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{3\text{V}.}{4.5 \; \mu\text{A}} = 667 \text{ k}\Omega$$

Ex: 5.8

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2 \Rightarrow 0.3 = \frac{1}{2} \times \frac{60}{1000}$$

$$\times \frac{120}{3}V_{OV}^2 \Rightarrow$$

$V_{OV} = 0.5 \text{ V} \Rightarrow V_{GS} = V_{GV} + V_t = 0.5 + 1$

$= 1.5 \text{ V}$

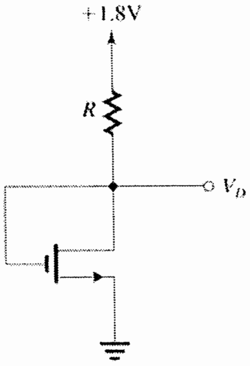
$$V_S = -1.5 \text{ V} \Rightarrow R_S = \frac{V_S - V_{SS}}{I_D}$$

$$= \frac{-1.5 - (-2.5)}{0.3}$$

$R_S = 3.33 \text{ k}\Omega$

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{2.5 - 0.4}{0.3} = 7 \text{ k}\Omega$$

Ex: 5.9



$V_{tn} = 0.5 \text{ V}.$

$\mu_n C_{ox} = 0.4 \text{ mA} / V^2$

$$\frac{W}{L} = \frac{0.72 \; \mu\text{m}}{0.18 \; \mu\text{m}} = 4.0$$

$\lambda = 0$

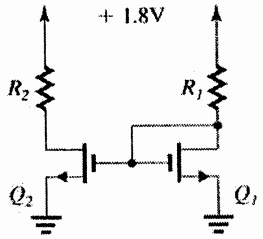
saturation mode $(v_{GD} = 0 < V_{tn})$

$V_D = 0.8 \text{ V}. = 1.8 - I_D R_D$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_D - V_{tn})^2 = 72 \; \mu\text{A}$$

$$\therefore R = \frac{1.8 - 0.8}{72 \; \mu\text{A}} = 13.9 \text{ k}\Omega$$

Ex: 5.10



From Exe. 5.9, $V_{GS} = 0.8 \text{ V}, \; V_{tn} = 0.5 \text{ V},$

$V_{OV} = 0.3 \text{ V}.$

$I_D = 72 \; \mu\text{A}$ (saturation)

At the triode/saturation boundary

$V_D = V_{OV} = 0.3 \text{ V}$

$$\therefore R_2 = \frac{1.8 \text{ V} - 0.3 \text{ V}}{72 \; \mu\text{A}} = 20.8 \text{ k}\Omega$$

Ex: 5.11

$R_D = 12.4 \times 2 = 24.8 \text{ k}\Omega$

$V_{GS} = 5 \text{ V}$, Assume triode region:

$$\left.\begin{aligned} I_D &= k'_n\frac{W}{L}[(V_{GS} - V_t)V_{DS} - \tfrac{1}{2}V_{DS}^2] \\ I_D &= \frac{V_{DD} - V_{DS}}{R} \end{aligned}\right\} \Rightarrow$$

$$\frac{5 - V_{DS}}{24.8} = 1 \times \left((5 - 1)V_{DS} - \frac{V_{DS}^2}{2}\right)$$

$$\Rightarrow V_{DS}^2 - 8.08V_{DS} + 0.4 = 0$$

$\Rightarrow V_{DS} = 0.05 \text{ V} < V_{OV} \Rightarrow$ triode region

$$I_D = \frac{5 - 0.05}{24.8} = 0.2 \text{ mA}$$

Ex: 5.12

As indicated in Example 3 . 5

$V_D \geq V_G - V_t$ for the transistor to be in saturation region.

$V_{D\,\text{min}} = V_G - V_t = 5 - 1 = 4\ \text{V}$

$I_D = 0.5\ \text{mA} \Rightarrow R_{D\,\text{max}} = \dfrac{V_{DD} - V_{D\,\text{min}}}{I_D}$

$= \dfrac{10 - 4}{0.5} = 12\ \text{k}\Omega$

Ex: 5.13

$I_D = 0.32\ \text{mA} = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2 = \frac{1}{2} \times 1 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.8\ \text{V}$

$V_{GS} = 0.8 + 1 = 1.8\ \text{V}$

$V_G = V_S + V_{GS} = 1.6 + 1.8 = 3.4\ \text{V}$

$R_{G2} = \dfrac{V_G}{I} = \dfrac{3.4}{1\ \mu} = 3.4\ \text{M}\Omega,$

$R_{G1} = \dfrac{5 - 3.4}{1\ \mu} = 1.6\ \text{M}\Omega$

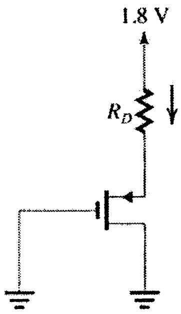
$R_S = \dfrac{V_S}{0.32} = 5\ \text{k}\Omega$

$V_{DS} \geq V_{OV} \Rightarrow V_D \geq V_{OV} + V_S \Rightarrow V_D \geq 0.8 + 1.6 = 2.4\ \text{V}$

Assume

$V_D = 3.4\ \text{V}$, then $R_D = \dfrac{5 - 3.4}{0.32} = 5\ \text{k}\Omega$

Ex: 5.14



$V_{tp} = -0.4\ \text{V}.$

$k_p' = 0.1\ \text{mA} / V^2$

$\dfrac{W}{L} = \dfrac{10\ \mu\text{m}}{0.18\ \mu\text{m}} \Rightarrow k_p = 5.56\ \text{mA} / V^2$

$V_{GS} = -0.6 + V_{tp} = -1.0\ \text{V} = -1.8 + I_D R$

$I_D R = 0.8\ \text{V}$. for $V_{OV} = -0.6\ \text{V}$

$I_D = \frac{1}{2}k_p V_{OV}^2 = 0.1\ \text{mA}$

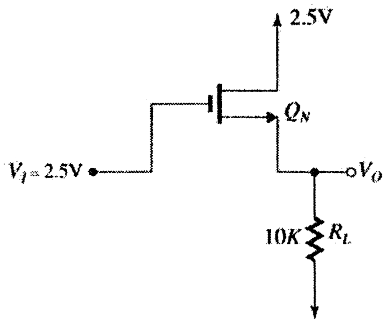
$\therefore R = 800\ \Omega$

Ex: 5.15

$v_I = 0$: Since the circuit is perfectly symmetrical $V_o = 0$ and therefore $V_{GS} = 0$ which implies the transistors are turned off and $I_{DN} = I_{DP} = 0$.

$V_I = 2.5\ \text{V}$: If we assume that the NMOS is turned on, then v_o would be less than 2.5 V and this implies that PMOS is off ($V_{GSP} > 0$)

$I_{DN} = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2$

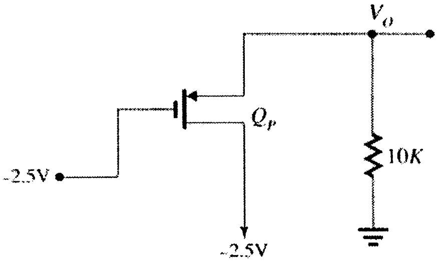


$I_{DN} = \frac{1}{2} \times 1(2.5 - V_S - 1)^2$

$I_{DN} = 0.5(1.5 - V_S)^2$

Also: $V_S = R_L I_{DN} = 10 I_{DN}$

$I_{DN} = 0.5(1.5 - 10 I_{DN})^2$



$\Rightarrow 100 I_{DN}^2 - 32 I_{DN} + 2.25 = 0 \Rightarrow I_{DN} = 0.104\ \text{mA}$

$I_{DP} = 0,\ V_O = 10 \times 0.104 = 1.04\ \text{V}$

$V_I = -2.5\ \text{V}$: Again if we assume that Q_P is turned on, then $V_o > -2.5\ \text{V}$ and $V_{GS1} < 0$ which implies the NMOS Q_N is turned off.

$I_{DN} = 0$

$I_{DP} = \frac{1}{2}k_n'\frac{W}{L}(V_{SG} - |V_t|)^2 = \frac{1}{2} \times 1$

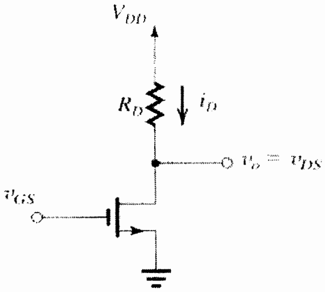
$\times (v_S + 2.5 - 1)^2$

$V_S = -10I_{DP} \Rightarrow 2I_{DP} = (-10I_{DP} + 1.5)^2$

$\Rightarrow I_{DP} = 0.104 \text{ mA} \Rightarrow V_o = -10 \times 0.104$

$= -1.04$ V

Ex: 5.16



$V_{DD} = 1.8$ V

$R_D = 17.5$ kΩ

$V_t = 0.4$ V.

$k_n = 4 \text{ mA}/V^2$

$\lambda = 0$

(A) Cutoff/Saturation Boundary

$v_{GS} = V_t = 0.4$ V., $v_o = v_{DS} = 1.8$ V.

(B) Saturation/Triode Boundary

$v_{GD} = v_{GS} - v_o = V_t = 0.4$V.

$$\Rightarrow v_{GS} - \left[V_{DD} - \frac{1}{2}k_n(v_{GS} - V_t)^2 R_D\right] = 0.4$$

$v_{GS} - [1.8 - 35(v_{GS}^2 - 0.8\ v_{GS} + 0.16)] = 0.4$

$35v_{GS}^2 - 27v_{GS} + 3.4 = 0$

$v_{GS} = 0.613$ V., 0.1585

$I_D = 90.7$ μA

$v_O = 0.213$ V.

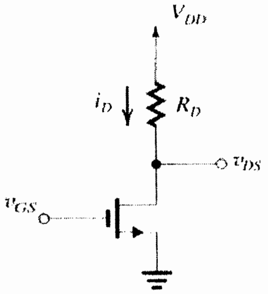
(C) For $v_{GS}|_C = V_{DD} = 1.8$ V., triode.

$V_{OV} = 1.4$ V.

$r_{DS} = (k_n V_{ov})^{-1} = 179\ \Omega$

$$V_O|_C = V_{DS}|_C \cong V_{DD}\frac{r_{DS}}{R_D + r_{DS}} = 18 \text{ mV}.$$

Ex: 5.17



$V_t = 0.4$ V.

$V_{DD} = 1.8$ V.

$V_{GS} = 0.6$ V.

$k_n' = 0.4 \text{ mA}/V^2$

$\frac{W}{L} = 10$

$R_D = 17.5$ kΩ

(a) $V_{OV} = 0.2$ V.,

$g_m = k_n' \frac{W}{L} V_{OV} = 800$ μA / V

for $A_V = -g_m R_D = -10$, make

$R_D = 12.5$ kΩ

$v_{GS} = 0.6$ V., $I_D = 0.08$ mA.

$V_{DS} = 0.8$ V.

(b) keep $R_D = 17.5$ kΩ

$-g_m R_D = -10 \Rightarrow g_m = 571$ μA / V

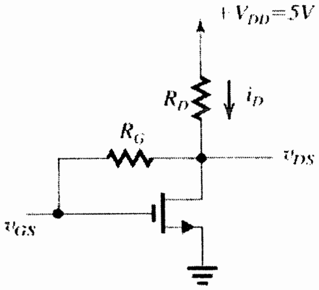
$= k_n' \frac{W}{L} V_{OV}$

$\therefore V_{OV} = 0.143$ V.

$v_{GS} = 0.54$ V, $I_D = 0.04$ mA.

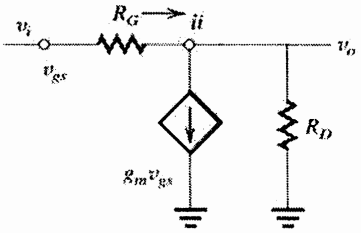
$v_{DS} = 1.1$ V.

Ex: 5.18



$V_t = 0.7$ V.

$k_n = 1$ mA / V^2



Design for $A_v = \frac{v_O}{v_i} = -25, \ R_{in} = 500 \text{ k}\Omega$

$\therefore g_m R_D = 25 = k_n V_{OV} R_D$

$R_{in} = \frac{v_i}{ii} = \frac{v_i}{v_i - v_o} R_G$

$\Rightarrow R_G = 26 R_{in} = 13 \text{ M}\Omega$

$I_D R_D = \left(\frac{1}{2} k_n V_{OV}^2\right) R_D$

$= \frac{1}{2} g_m R_D V_{OV} = 12.5 \ V_{OV}$

and

$V_{OV} = V_{DD} - V_t - I_D R_D = 4.3 - 12.5 \ V_{OV}$

$\therefore V_{OV} = 0.319$ V.

$g_m = 319 \ \mu\text{A} / V$

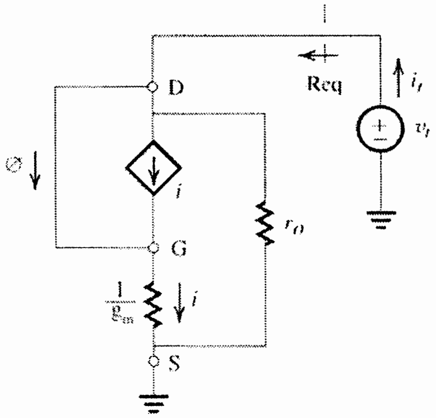
$R_D = 78.5 \text{ k}\Omega$

$V_{DS} = V_{OV} + V_t$

$\hat{v}_{GD} = 0 + 26 \hat{v}_i \leq V_t$

$\therefore \left|\hat{v}_i\right| < \frac{V_t}{26} = 27$ mV.

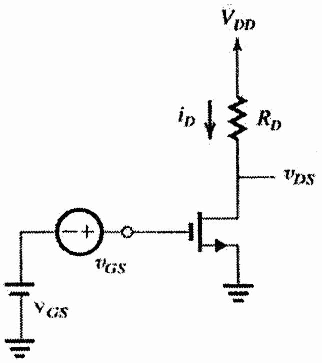
Ex: 5.19



$i_t = \frac{v_t}{r_o} + i = \frac{v_t}{r_o} + g_m v_t$

$\therefore \text{Req} = \frac{v_t}{i_t} r_o \parallel \frac{1}{g_m}$

Ex: 5.20



$V_{DD} = 5$ V.

$V_{GS} = 2$ V.

$V_t = 1$ V.

$\lambda = 0$

$k_n' = 20 \ \mu\text{A} / V^2$

$R_D = 10 \text{ k}\Omega$

$\frac{W}{L} = 20$

(a) $V_{GS} = 2V \Rightarrow V_{OV} = 1$ V.

$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 = 200 \ \mu\text{A}$

$V_{DS} = V_{DD} - I_D R_D = +3$ V

(b) $g_m = k_n' \frac{W}{L} V_{OV} = 400 \ \mu\text{A} / \text{V}$

(c) $A_V = \frac{v_{ds}}{v_{gs}} = -g_m R_D = -4$

(d) $v_{gs} = 0.2 \sin \omega t \ V.$

$v_{ds} = -0.8 \sin \omega t \ V.$

$v_{DS} = V_{DS} + v_{ds} \Rightarrow 2.2 \leq v_{DS} \leq 3.8$ V.

(e) Using (5.43)

$i_D = \frac{1}{2} k_n (V_{GS} - V_t)^2$

$+ k_n (V_{GS} - V_t) v_{gs} + \frac{1}{2} k_n v_{gs}^2$

$i_D = 200 \ \mu\text{A} + (80 \ \mu\text{A}) \sin \omega t$

$+ (8 \ \mu\text{A}) \sin^2 \omega t$

$= [200 + 80 \sin\omega t + (4 - 4 \cos\omega t)]\mu A$

I_D shifts by 4 μA

$$2HD = \frac{\hat{i}_{2\omega}}{\hat{i}_{\omega}} = \frac{4\ \mu A}{80\ \mu A} = 0.05\ (5\%)$$

Ex: 5.21

$a)\ g_m = \frac{2I_D}{V_{OV}}\ I_D = \frac{1}{2} \times k_n' \frac{W}{L} V_{OV}^2 = \frac{1}{2} \times 60$

$\times 40 \times (1.5 - 1)^2$

$I_D = 300\ \mu A = 0.3\ mA,\ V_{OV} = 0.5\ V$

$g_m = \frac{2 \times 0.3}{0.5} = 1.2\ mA/V,$

$r_o = \frac{V_A}{I_D} = \frac{15}{0.3} = 50\ k\Omega$

$I_D = 0.5\ mA \Rightarrow g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$

$= \sqrt{2 \times 60 \times 40 \times 0.5 \times 10^3}$

$g_m = 1.55\ mA/V$

$r_o = \frac{V_A}{I_D} = \frac{15}{0.5} = 30\ k\Omega$

Ex: 5.22

$I_D = 0.1\ mA,\ g_m = 1\ mA/V,\ k_n' = 50\mu A/V^2$

$g_m = \frac{2I_D}{V_{OV}} \Rightarrow V_{OV} = \frac{2 \times 0.1}{1} = 0.2\ V$

$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{k_n' V_{OV}^2}$

$= \frac{2 \times 0.1}{\frac{50}{1000} \times 0.2^2} = 100$

Ex: 5.23

$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV}$ Same bias conditions, so same V_{OV} and also same L and g_m for both PMOS and NMOS.

$\mu_n C_{ox} W_n = \mu_p C_{ox} W_p \Rightarrow \frac{\mu_p}{\mu_n} = 0.4 = \frac{W_n}{W_p}$

$\Rightarrow \frac{W_p}{W_n} = 2.5$

Ex: 5.24

$I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{SG} - |V_t|)^2$

$= \frac{1}{2} \times 60 \times \frac{16}{0.8} \times (1.6 - 1)^2$

$I_D = 216\ \mu A$

$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 216}{1.6 - 1} = 720\ \mu A/V$

$= 0.72\ mA/V$

$\lambda = 0.04 \Rightarrow V_A' = \frac{1}{\lambda} = \frac{1}{0.04} = 25\ V/\mu m$

$r_o = \frac{V_A' \times L}{I_D} = \frac{25 \times 0.8}{0.216} = 92.6\ k\Omega$

Ex: 5.25

$g_m r_o = \frac{2I_D}{V_{OV}} \times \frac{V_A}{I_D} = \frac{2V_A}{V_{OV}} = A_o$

$V_A' \times L = V_A$

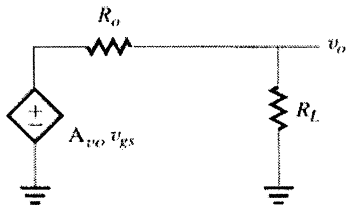
$L = 0.8\ \mu m \Rightarrow A_o = \frac{2 \times 12.5 \times 0.8}{0.2}$

$= 100\ V/V$

Ex: 5.26

(5.70) $A_{vO} = -g_m(R_D \| r_O)$

(5.72) $R_O = R_D \| r_O$



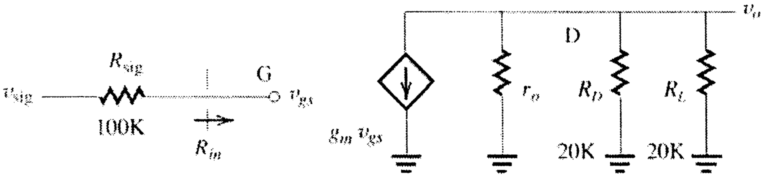
$A_v = \frac{v_O}{v_{GS}} = A_{vO} \frac{R_L}{R_O + R_L}$

$= -g_m R_O \frac{R_L}{R_O + R_L}$

$\therefore A_v = -g_m(R_O \| R_L) = -g_m(R_D \| r_O \| R_L)$

same as (5.75)

Ex: 5.27



$I_D = 0.25$ mA, $V_{OV} = 0.25$ V,

$V_A = 50$ V

$$r_O = \frac{V_A}{I_D} = 200\ \text{k}\Omega$$

$$g_m = \frac{2I_D}{V_{OV}} = 2\ \text{mS}$$

$R_{\text{in}} = \infty$

$$A_{vo} = -g_m(R_D \| r_O) \simeq -g_m R_D = -4$$

$$R_O = R_D \| r_O \simeq R_D = 20\ \text{k}\Omega$$

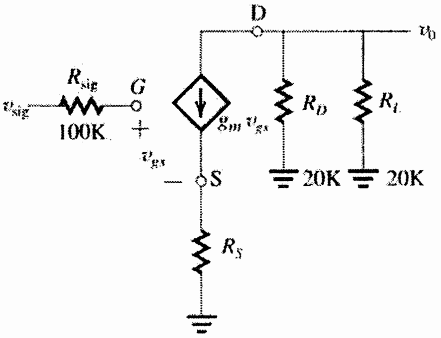
$$A_v = G_V = -g_m(R_D \| r_O \| R_L) \simeq$$

$$-g_m(R_D \| R_L) = -20\ \text{V/V}$$

for $\hat{v}_{gs} = (10\%)\, 2\text{V}_{\text{OV}} = 0.05$ V.

$$\hat{v}_O = |A_V \hat{v}_{gs}| = 1\ \text{V}.$$

Ex: 5.28


D
v_0
R_{sig}
G
v_{sig}
100K
+
v_{gs}
$g_m v_{gs}$
R_D
R_L
—
S
20K
20K
R_S

Assuming $V_A \to \infty$

From Ex 5.27

$g_m = 2$ mS

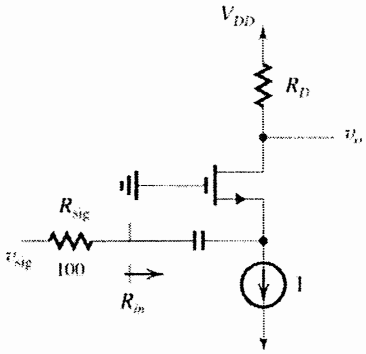
$$\frac{v_{gs}}{V_{Dis}} = \frac{1}{1 + g_m R_S} = \frac{50\ \text{mV}}{200\ \text{mV}} \Rightarrow g_m R_S = 3$$

$$\therefore\ R_S = 1.5\ \text{k}\Omega$$

$$G_V = A_V = \frac{-g_m(R_D \| R_L)}{1 + g_m R_S} = \frac{-20}{4} = -5$$

$$\hat{v}_O = |G_V \hat{v}_{\text{sig}}| = 1\text{V}.$$

Ex: 5.29


V_{DD}
R_D
v_o
R_{sig}
v_{sig}
100
R_{in}
I

$$R_{\text{in}} = \frac{1}{g_m} = R_{\text{sig}} = 100\ \Omega$$

$$\Rightarrow g_m = 10\ \text{mA / V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2I_D}{0.2\text{V}} \Rightarrow I_D = 1\ \text{mA}$$

$$G_V = \frac{v_O}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{sig}} + R_{\text{in}}} g_m R_D$$

$$= \left(\frac{1}{2}\right)(10\ \text{mA / V})(2\ \text{k}\Omega)$$

$$= +10$$

Ex: 5.30

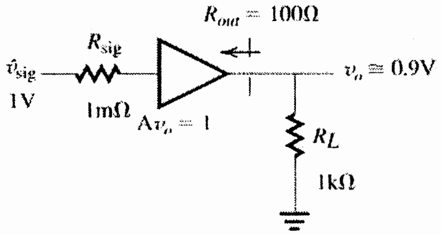
CD amplifier

$$R_{\text{out}} = \frac{1}{g_m} = 100\ \Omega \Rightarrow g_m = 10\ \text{mA / V}$$

$$g_m = \frac{2I_D}{\text{V}_{\text{OV}}} = \frac{2I_D}{0.25\text{V}} \Rightarrow I_D = 1.25\ \text{mA}$$

$$\hat{v}_O = \hat{v}_{\text{sig}} \frac{g_m R_L}{1 + g_m R_L} = 0.91\ \text{V}.$$

$$\hat{v}_{gs} = \hat{v}_{\text{sig}} \frac{1}{1 + g_m R_L} = 91\ \text{mV}.$$


$R_{out} = 100\Omega$
R_{sig}
$\hat{v}_{\text{sig}}$
1V
1mΩ
$Av_o = 1$
$v_o \cong 0.9\text{V}$
RL
1kΩ

Ex: 5.31

CD (source follower)

$$R_{\text{out}} = 200\ \Omega = \frac{1}{g_m} \Rightarrow g_m = 5\ \text{mA / V}$$

$$g_m = k_n' \frac{W}{L} \text{V}_{\text{OV}} = (0.4\ \text{mA / V}^2)$$

$$\left(\frac{W}{L}\right)(0.25\ \text{V}) \Rightarrow \frac{W}{L} = 50$$

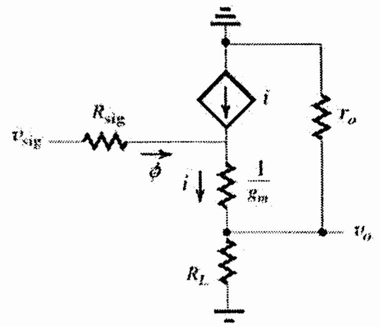
$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 = 0.625\ \text{mA}$$

$$G_V = \frac{g_m R_L}{1 + g_m R_L}$$

for 1 $K < R_L < 10$ K

$$0.83 < G_V < 0.98$$

Ex: 5.32



$$A_{vo} = A_v\big|_{R_{L \to \infty}} = \frac{g_m r_O}{1 + g_m r_O}$$

$$A_{vo} = \frac{r_O}{r_O + \frac{1}{g_m}}$$

$$A_{vo} = \frac{\left(\frac{V_A}{I_D}\right)}{\left(\frac{V_A}{I_D}\right) + \left(\frac{V_{OV}}{2I_D}\right)}$$

$$= \frac{2V_A}{2V_A + V_{OV}} = \left[1 + \frac{V_{OV}}{2V_A}\right]^{-1}$$

for $V_A = 20$ V.

$$A_{vo} = 0.99 = \frac{1}{1 + \frac{V_{OV}}{4_{OV.}}} \Rightarrow V_{OV} = 0.40 \text{ V.}$$

Ex: 5.33

$$I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2 \Rightarrow 0.5 \text{ mA}$$

$$= \frac{1}{2} \times 1(V_{GS} - 1)^2$$

$V_{GS} = 2$ V.

IF $V_t = 1.5$ V then:

$$I_D = \frac{1}{2} \times 1 \times (2 - 1.5)^2 = 0.125 \text{ mA}$$

$$\Rightarrow \frac{\Delta I_D}{I_o} = \frac{0.5 - 0.125}{0.5} = 0.75 = 75\%$$

Ex: 5.34

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 2}{0.5} = 6 \text{ k}\Omega$$

$$\rightarrow R_D = 6.2 \text{ k}\Omega$$

$$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2 \Rightarrow 0.5 = \frac{1}{2} \times 1 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 1 \text{ V}$$

$$\Rightarrow V_{GS} = V_{OV} + V_t = 1 + 1 = 2 \text{ V}$$

$$\Rightarrow V_S = -2 \text{ V}$$

$$R_S = \frac{V_S - V_{SS}}{I_D} = \frac{-2-(-5)}{0.5} = 6 \text{ k}\Omega$$

$$\rightarrow R_S = 6.2 \text{ k}\Omega$$

If we choose $R_D = R_S = 6.2$ kΩ then I_D will slightly change:

$$I_D = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2 \text{. Also}$$

$$V_{GS} = -V_S = 5 - R_S I_D$$

$$2I_D = (4 - 6.2I_D)^2$$

$$\Rightarrow 38.44 I_D^2 - 51.6 I_D^2 + 16 = 0$$

$$\Rightarrow I_D = 0.49 \text{ mA, } 0.86 \text{ mA}$$

$I_D = 0.86$ results in $V_S > 0$ or $V_S > V_G$ which is not acceptable, therefore $I_D = 0.49$ mA

$V_S = -5 + 6.2 \times 0.49 = -1.96$ V

$V_D = 5 - 6.2 \times 0.49 = +1.96$ V

R_G should be selected in the range of 1 MΩ to 10 MΩ to have low current.

Ex: 5.35

$$I_D = 0.5 \text{ mA} = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2 \Rightarrow V_{OV}^2$$

$$= \frac{0.5 \times 2}{1} = 1 \Rightarrow$$

$$V_{OV} = 1 \text{ V} \Rightarrow V_{GS} = 1 + 1 = 2 \text{ V}$$

$$= V_D \Rightarrow R_D = \frac{5-2}{0.5} = 6 \text{ k}\Omega$$

$\Rightarrow R_D = 6.2$ kΩ standard value. For this R_D we have to recalculate I_D:

$$I_D = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2$$

$$= \frac{1}{2}(V_{DD} - R_D I_D - 1)^2$$

$$(V_{GS} = V_D = V_{DD} - R_D I_D)$$

$$I_D = \frac{1}{2}(4 - 6.2I_D)^2 \Rightarrow I_D \cong 0.49 \text{ mA}$$

$$V_D = 5 - 6.2 \times 0.49 = 1.96 \text{ V}$$

Ex: 5.36

Using Eq. 3 . 53

$$I = I_{REF}\frac{(W/L)_2}{(W/L)_1} \Rightarrow I_{REF} = 0.5 \times \frac{1}{5}$$

$$\Rightarrow I_{REF} = 0.1 \text{ mA}$$

$$I_{REF} = 0.1 = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 V_{OV}^2 \Rightarrow V_{OV}^2$$

$$= \frac{0.1 \times 2}{0.8} = 0.25 \Rightarrow V_{OV} = 0.5 \text{ V}$$

$$V_{GS} = V_{OV} + V_t = 1.5 \text{ V}$$

$$\Rightarrow V_G = -5 + 1.5 = -3.5 \text{ V}$$

$$R = \frac{V_{GS} - V_G}{I_{REF}} = \frac{5 - (-3.5)}{0.1} = 85\ \text{k}\Omega$$

$V_{DS2} \geq V_{OV} \Rightarrow V_{DS\text{min}} = V_{OV} = 0.5\ \text{V}$

$\Rightarrow V_{D\text{min}} = -4.5\ \text{V}$

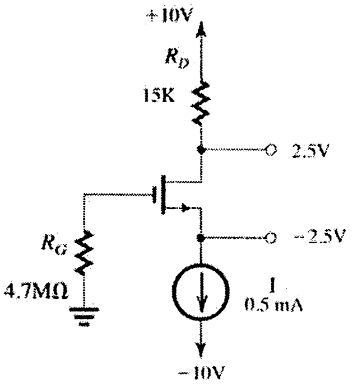
Ex: 5.37

$V_t = 1.5\ \text{V}$

$k_n' \frac{W}{L} = 1\ \text{mA/V}^2$

$V_A = 75\text{V}.$

$I_D = 0.5\text{mA} = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow \text{V}_{\text{OV}} = 1.0\ \text{V}.$


+10V
R_D
15K
2.5V
R_G
4.7MΩ
−2.5V
I
0.5 mA
−10V

$V_{GS} = V_t + \text{V}_{\text{OV}} = 2.5\ \text{V}$

$V_G = 0$

$V_S = -2.5\ \text{V}.$

$V_D = V_{DD} - I_D R_D = +2.5\ \text{V}.$

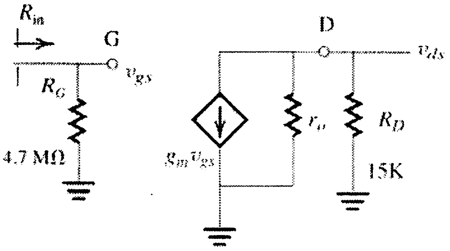
$g_m = k_n' \frac{W}{L} \text{V}_{\text{OV}} = 1\ \text{mA/V}$

$r_O = \frac{V_A}{I_D} = 150\ \text{k}\Omega$

$V_{GD} - \hat{v}_{gd} = V_t$

$-\hat{v}_{gd} \cong \hat{v}_d = V_t - V_{GD} = 4.0\ \text{V}.$

Ex: 5.38


R_{in}
G
v_{gs}
R_G
4.7 MΩ
D
v_{ds}
$g_m v_{gs}$
r_o
R_D
15K

$g_m = \sqrt{2 k_n' \frac{W}{L} I_D} = 1\ \text{mA/V}$

$r_O = \frac{V_A}{I_D} = 150\ \text{k}\Omega$

For $r_O \to \infty$

$R_{\text{in}} = R_G = 4.7\ \text{M}\Omega$

$A_{vO} = -g_m R_D = -15$

$R_{\text{out}} = R_D = 15\ \text{k}\Omega$

For $r_O = 150\ \text{k}\Omega$, $R_L = 15\ \text{k}\Omega$

$R_{\text{in}} = 4.7\ \text{M}\Omega$

$A_{vO} = -g_m (R_D \| r_O) = -13.6$

$R_{\text{out}} = R_D \| r_O = 13.6\ \text{k}\Omega$

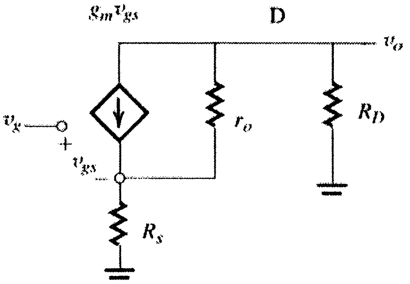
$$G_V = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} A_{VO} \frac{R_L}{R_L + R_{\text{out}}} = -7.0$$

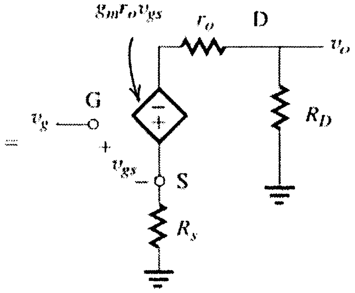
$v_{DS} = v_O = V_{DS} + v_{ds}$

$= 2.5\ \text{V} + G_V (0.4\ \text{V}_{p-p})$

v_O is a 2.8 V_{p-p} sinusoid superimposed upon a 2.5V d_C voltage.

Ex: 5.39


$g_m v_{gs}$
D
v_o
v_g
+
v_{gs}
−
r_o
R_D
R_s


$g_m r_o v_{gs}$
r_o
D
v_o
G
=
v_g
+
v_{gs}
−
S
R_D
R_s

$$(1)\ v_O = v_{gs} \frac{-g_m r_O R_D}{R_D + r_O + R_S}$$

$$(2)\ v_{gs} = v_g \frac{R_D + r_O + R_S}{R_D + r_O + R_S(1 + g_m r_O)}$$

$$(3)\ v_O = V_g \frac{-g_m r_O R_D}{R_D + r_O + R_S(1 + g_m r_O)}$$

we want

$$\left.\frac{v_O}{v_g}\right|_{R_S} = \frac{1}{3}\left.\frac{v_O}{v_g}\right|_{R_S = 0}$$

using (3)

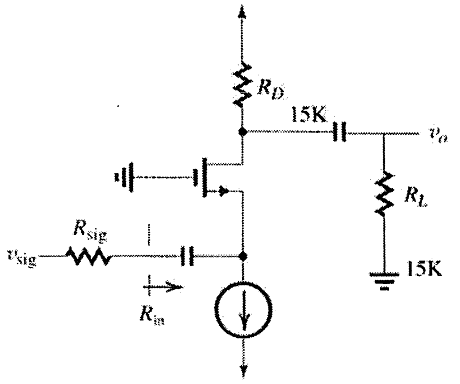
$$\frac{-g_m r_O R_D}{(R_D + r_O) + R_S(1 + g_m r_O)} = \frac{1}{3}\frac{-g_m r_O R_D}{R_D + r_O}$$

$$R_S = \frac{2(R_D + r_O)}{1 + g_m r_O} = 2.185\ \text{k}\Omega$$

based on $R_D = 15$ K, $r_O = 150$ K,

$g_m = 1$ mS

Ex: 5.40



R_D
15K
v_o
R_L
15K
R_{sig}
v_{sig}
R_{in}

$g_m = 1$ mA / V

For $R_{\text{sig}} = 50\ \Omega$

$$k_{\text{in}} = \frac{1}{g_m} = 1\ \text{k}\Omega$$

$k_{\text{out}} = R_D = 15\ \text{k}\Omega$

$A_{vo} = + g_m R_D = + 15$

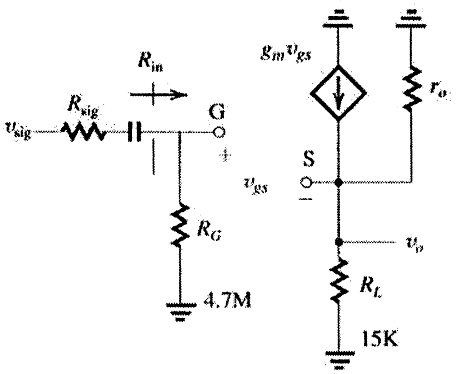
$A_V = g_m(R_D \| R_L) = + 7.5$

$$G_V = \frac{R_{\text{in}}}{R_{\text{sig}} + R_{\text{in}}} A_V = 7.1$$

For other R_{sig}

R_{sig}	G_V
1 kΩ	3.75
10 kΩ	0.68
100 kΩ	0.07

Ex: 5.41



R_{in}
R_{sig}
v_{sig}
G
+
v_{gs}
S
−
$g_m v_{gs}$
r_o
R_G
4.7M
v_o
R_L
15K

$g_m = 1$ mA / V

$r_O = 150\ \text{k}\Omega$

$R_{\text{in}} = R_G$

$$A_{vo} = \frac{g_m r_O}{1 + g_m r_O}$$

$$A_V = \frac{g_m(r_O \| R_L)}{1 + g_m(r_O \| R_L)}$$

$$R_{\text{out}} = \frac{1}{g_m} \| r_O$$

(a)

	$r_o \rightarrow \infty$	$r_o = 150$ kΩ
R_{in}	4.7 MΩ	4.7 MΩ
A_{vo}	1.0	0.993
A_v	0.938	0.932
R_{out}	1 kΩ	0.993 kΩ

(b) $G_v = \dfrac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} A_v = 0.768$

Ex 5.42

See the next page

Exercise 5–11

using eq. (5.107

$$V_t = V_{to} + r\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}]$$

$$V_t = 0.8 + 0.4[\sqrt{0.7 + 3} - \sqrt{.7}]$$

$V_t = 1.23\text{ V}$

Ex: 5.43

$V_{GS} = +1\text{ V}, V_t = -2\text{ V}$

$V_{GS} - V_t = 3\text{ V}$

TO OPERATE IN SATURATION REGION:

$$V_{DS\,\text{min}} = V_{GS} - V_t = 3\text{ V}$$

$$i_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 2 \times 3^2 = 9\text{ mA}$$

Ex: 6A.1 (a) The minimum value of I_D occurs when

$V_{ov} = 0.2$ V and $\frac{W}{L} = 0.1$, that is

$$I_{D\min} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2 \simeq 0.8\ \mu\text{A}$$

The maximum value of I_D occurs when

$V_{ov} = 0.4$ V and $\frac{W}{L} = 100$, that is

$$I_{D\max} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2 \simeq 3.1\ \text{mA}$$

(b) For a similar range of current in an npn transistor, we have

$$\frac{I_{c\max}}{I_{c\min}} = \frac{3.1\ \text{mA}}{0.8\ \mu\text{A}} = \frac{I_S e^{V_{BE\max}/V_T}}{I_S e^{V_{BE\min}/V_T}}$$

$$\Rightarrow e^{(V_{BE\max} - V_{BE\min})/V_T} = e^{\Delta V_{BE}/V_T}$$

$$= \frac{3.1\ \text{mA}}{0.8\ \mu\text{A}}$$

$\Delta V_{BE} = V_T \ln\left(\frac{3.1\ \text{mA}}{0.8\ \mu\text{A}}\right)$ and $V_T = (25)$ mV

$$\Rightarrow \Delta V_{BE} = 207\ \text{mV}$$

Ex: 6A.2 For an NMOS Fabricated in the 0.5 µm process, with $\frac{W}{L} = 10$, we want to find the transconductance and the intrinsic gain obtained for the following drain currents: ($L = 0.5$ µm)

$I_D = (10)\ \mu\text{A}$, $g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D}$.

$$\mu_n C_{ox} = (190)\ \frac{\mu\text{A}}{\text{V}^2}$$

$$g_m = \sqrt{2 \times 190 \times 10 \times 10} \simeq 0.2\ \frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{10\ \mu\text{A}} = 1\ \text{M}\Omega$$

intrinsic gain

$$= g_m r_o = 0.2\ \frac{\text{mA}}{\text{V}} \times 1\ \text{M}\Omega = 200\ \frac{\text{V}}{\text{V}}$$

For $I_D = 100$ µA we have:

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D} = \sqrt{2 \times 190 \times 10 \times 100}$$

$$g_m = 0.62\ \frac{\text{mA}}{\text{V}} \simeq 0.6\ \frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{100\ \mu\text{A}} = 100\ \text{k}\Omega$$

$$g_m r_o = 0.62\ \frac{\text{mA}}{\text{V}} \times 100\ \text{k}\Omega = 62\ \text{V/V}$$

For $I_D = 1$ mA

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D}$$

$$= \sqrt{2 \times 190 \times 10 \times 1} \simeq 2\ \frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{1\ \text{mA}} = 10\ \text{k}\Omega$$

$$g_m r_o = 2\ \frac{\text{mA}}{\text{V}} \times 10\ \text{k}\Omega = 20\ \text{V/V}$$

Ex: 6.A.3 For an NMOS fabricated in the 0.5 µm CMOS technology specified in Table 7.A.1 with $L = 0.5$ µm, $W = 5$ µm, and $V_{OV} = 0.3$ V

We have

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2 = \frac{1}{2}\,190\ \frac{\mu\text{A}}{\text{V}^2} \times \frac{5}{0.5} \times 0.3^2$$

$$I_D = 85.5\ \mu\text{A}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 85.5\ \mu\text{A}}{0.3\ \text{V}} = 0.57\ \text{mA/V}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{85.5\ \mu\text{A}} \simeq 117\ \text{k}\Omega$$

$$A_o = g_m r_o = 66.7\ \text{V/V}$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + C_{OV}$$

$$= \frac{2}{3} \times 5 \times 0.5 \times 3.8 + 0.4 \times 5$$

$$C_{gs} = 8.3\ \text{fF},\ C_{gd} = C_{OV_x}W = 0.4 \times 5 = 2\ \text{fF}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{0.57\ \frac{\text{mA}}{\text{V}}}{2\pi(8.3 + 2)}$$

$$f_T = 8.8\ \text{GHz}$$

Ex: 6.1

For this problem, use eq. (6.11

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)} \cdot \sqrt{I_D}$$

For $I_D = 10\ \mu\text{A}$,

$$g_m = \sqrt{2(387\mu\text{A/V}^2)(10)(10\ \mu)} = 0.28\ \text{mA/V}$$

using eq. (6.15

$$A_O = V_A \sqrt{\frac{2\mu_n C_{ox}(WL)}{\sqrt{I_D}}}$$

$$\frac{5\text{V}/\mu\text{m}\sqrt{2(387\mu\text{A/V}^2)(10)(.36)^2}}{\sqrt{10\ \mu\text{A}}}$$

$$A_O = 50\ \text{V/V}$$

Since g_m varies with $\sqrt{I_D}$ and A_O with $\frac{1}{\sqrt{I_D}}$

For

$$I_D = 100\ \mu\text{A} \Rightarrow g_m = 0.28\ \text{m}\left(\frac{100}{10}\right)^{\frac{1}{2}} = .89\ \text{mA/V}$$

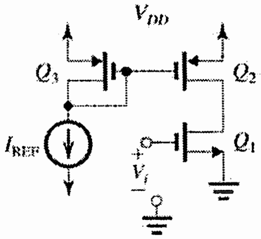
$$A_O = 50\left(\frac{10}{100}\right)^{\frac{1}{2}} = 158\ \text{V/V}$$

For $I_D = 1$ mA :

$$g_m = .28\ \text{mA/V}\left(\frac{1}{.010}\right)^{\frac{1}{2}} = 2.8\ \text{mA/V}$$

$$A_O = 50\left(\frac{.010}{1}\right)^{\frac{1}{2}} = 5\ \text{V/V}$$

Ex: 6 . 2



Since all transistors have the same

$$\frac{W}{L} = \frac{7.2\ \mu\text{m}}{0.36\ \mu\text{m}},$$

we have

$$I_{REF} = I_{D3} = I_{D2} = I_{D1} = 100\ \mu\text{A}$$

$$g_{m1} = \sqrt{2\ \mu_n C_{ox}\left(\frac{W}{L}\right)_1}\ \sqrt{I_{D1}}$$

$$= \sqrt{2(\ 387\ \mu\text{A}/\text{V}^2)\left(\frac{7.2}{0.36}\right)(100\ \mu\text{A})}$$

$$= 1.24\ \text{mA/V}$$

$$r_{o1} = \frac{V_{An}L_1}{I_{D1}} = \frac{5\ \text{V}/\mu\text{m}\ (0.36\ \mu\text{m})}{0.1\ \text{mA}} = 18\ \text{k}\Omega$$

$$r_{o2} = \frac{|V_{Ap}|L_2}{I_{D2}} = \frac{6\ \text{V}/\mu\text{m}\ (0.36\ \mu\text{m})}{0.1\ \text{mA}} = 21.6\ \text{k}\Omega$$

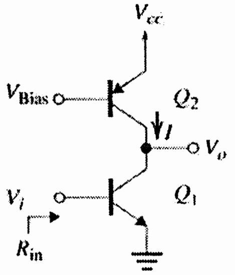
Voltage Gain is

$A_v = -\,g_m\,(r_{o1} \| r_{o2})$

$A_v = -\,(1.24\ \text{mA/V})\ (18\ \text{k}\Omega \| 21.6\ \text{k}\Omega)$

$= -\ 12.2\ \text{V/V}$

Ex: 6 . 3



$I_C = I = 100\ \mu\text{A}$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.1\ \mu\text{A}}{25\ \text{mV}} = 4\ \text{mA/V}$$

$$R_{\text{in}} = r_{\pi 1} = \frac{\beta_1}{g_{m1}} = \frac{100}{4\ \text{mA/V}} = 25\ \text{k}\Omega$$

$$r_{o1} = \frac{V_A}{I} = \frac{50\ \text{V}}{0.1\ \text{mA}} = 500\ \text{k}\Omega$$

$$r_{o2} = \frac{|V_A|}{I} = \frac{50\ \text{V}}{0.1\ \text{mA}} = 500\ \text{k}\Omega$$

$A_o = g_{m1}\,r_{o1} = (4\ \text{mA/V})\ (500\ \text{k}\Omega) = 2000\ \text{V/V}$

$A_v = -g_{m1}\,(r_{o1} \| r_{o2}) = -(4\ \text{mA/V})$

$(500\ \text{k}\Omega \| 500\ \text{k}\Omega) = -1000\ \text{V/V}$

Ex: 6 . 4 If L is halved: $L = \dfrac{0.55\ \mu\text{m}}{2}$, and

$$|V_A| = |V_A'| \cdot \text{L},$$

$$|V_A| = 5\ \text{V}/\mu\text{m}\frac{(0.55\ \mu\text{m})}{2} = 1.375\ \text{V}$$

$$R_o = \frac{|V_A|}{|V_{OV}|/2} \cdot \frac{|V_A|}{I_D} = \frac{2\ (1.375\ \text{V})^2}{(0.3\ \text{V})\ (100\ \mu\text{A})}$$

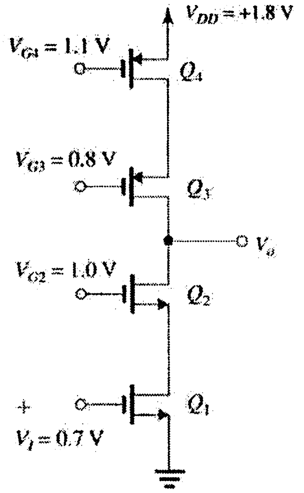
$$= 126\ \text{k}\Omega$$

Since $I_D = \dfrac{1}{2}\ (\mu_p C_{ox})\left(\dfrac{W}{L}\right)|V_{OV}|^2\left(1 + \dfrac{V_{SD}}{|V_A|}\right)$

$$\frac{W}{L} = \frac{2\ (100\ \mu\text{A})}{90\ \mu\text{A/V}^2(0.3\ \text{V})^2\left(1 + \dfrac{0.3\ \text{V}}{1.375\ \text{V}}\right)}$$

$$\frac{W}{L} = 20.3$$

Ex: 6 . 5



If all transistors are identical and the gate voltages are fixed, $|V_{OV}| = 0.7 - 0.5 = 0.2$ V

$V_{D1} = V_{S2} = V_{G2} - V_{tn} - V_{OV}$

$= 1.0 - 0.5 - 0.2 = 0.3$ V

the lowest V_{DS2} can go is $|V_{OV}| = 0.2$ V

$\therefore V_{o_{\min}} = V_{DS1} + V_{DS2} = 0.3 + 0.2 = 0.5$ V

Similarly, $V_{SG4} = V_{SG3} = 0.7$ V

$V_{D4} = V_{S3} = V_{G3} + |V_t| + |V_{OV}|$

$= 0.8 + 0.5 + 0.2 = 1.5$ V

V_{SD3} can go as low as $|V_{OV}|$, so

$V_{O\max} = V_{D4} - V_{SD3_{\min}} = 1.5 - 0.2 = 1.3$ V

Ex:6 . 6 $g_{m1} = g_{m2} = g_{m3} = g_{m4} = g_m$

$$= \frac{I_D}{\frac{|V_{OV}|}{2}} = \frac{0.2\text{ mA}}{0.2\text{V}/2} = 2\text{ mA/V}$$

$r_{o1} = r_{o2} = r_{o3} = r_{o4} = r_o$

$$= \frac{|V_A|}{I_D} = \frac{2\text{ V}}{0.2\text{ mA}} = 10\text{ k}\Omega$$

$R_{on} = (g_{m2}r_{o2})r_{o1} = (2\text{ mA/V})(10\text{ k}\Omega)^2$

$= 200\text{ k}\Omega$

$R_{op} = (g_{m3}r_{o3})(r_{o4}) = (2\text{ mA/V})(10\text{ k}\Omega)^2$

$= 200\text{ k}\Omega$

$R_o = R_{on} \| R_{op} = 100\text{ k}\Omega$

$Av = -\frac{1}{2}(g_m r_o)^2 = -\frac{1}{2}[(2\text{ mA/V})(10\text{ k}\Omega)]^2$

$Av = -200$ V/V

Ex:6 . 7 $g_m = \dfrac{I_D}{\frac{V_{OV}}{2}} = \dfrac{0.25\text{ mA}}{0.25\text{ V}/2}$

$= 2$ mA/V

$r_o = \dfrac{V_A}{I_D} = \dfrac{5\text{ V}}{0.25\text{ mA}} = 20\text{ k}\Omega$

(a) From Fig. 6 . 13

$$R_{\text{in}} = \frac{1}{g_m} + \frac{R_L}{(g_m r_o)}$$

$R_L = \infty$:

$$R_{\text{in}} = \frac{1}{2\text{ mA/V}} + \frac{R_L}{(2\text{ mA/V})(20\text{ k}\Omega)}$$

$$= 500\ \Omega + \frac{\infty}{40} \to \infty$$

$R_L = 1\text{ M}\Omega$:

$$R_{\text{in}} = 500\ \Omega + \frac{1\text{ M}\Omega}{40} = 25.5\text{ k}\Omega$$

$R_L = 100\text{ k}\Omega$:

$$R_{\text{in}} = 500\ \Omega + \frac{100\text{ k}\Omega}{40} = 3\text{ k}\Omega$$

$R_L = 20\text{ k}\Omega$:

$$R_{\text{in}} = 500\ \Omega + \frac{20\text{ k}\Omega}{40} = 1\text{ k}\Omega$$

$R_L = 0$:

$$R_{\text{in}} = 500\ \Omega + \frac{0}{40} = 0.5\text{ k}\Omega$$

(b) From Fig. 6 . 13

$R_O = r_o + R_S + (g_m r_o)R_S$

$R_S = 0$:

$R_O = 20\text{ k}\Omega + 0 + (40)(0) = 20\text{ k}\Omega$

$R_S = 1\text{ k}\Omega$:

$R_O = 20\text{ k}\Omega + 1\text{ k}\Omega + (40)(1\text{ k}\Omega) = 61\text{ k}\Omega$

$R_S = 10\text{ k}\Omega$:

$R_O = 20\text{ k}\Omega + 10\text{ k}\Omega + (40)(10\text{ k}\Omega) = 430\text{ k}\Omega$

$R_S = 20\text{ k}\Omega$:

$R_O = 20\text{ k}\Omega + 20\text{ k}\Omega + (40)(20\text{ k}\Omega) = 840\text{ k}\Omega$

$R_S = 100\text{ k}\Omega$:

$R_O = 20\text{ k}\Omega + 100\text{ k}\Omega + (40)(100\text{ k}\Omega) = 4.12\text{ M}\Omega$

Ex:6 . 8 $g_{m1} = g_{m2} = g_m$

$$= \frac{I_D}{\frac{V_{OV}}{2}} = \frac{100\ \mu\text{A}}{0.2\text{ V}/2} = 1\text{ mA/V}$$

$r_{o1} = r_{o2} = r_o$

$$= \frac{V_A}{I_D} = \frac{2\ \text{V}}{0.1\ \text{mA}} = 20\ \text{k}\Omega$$

so, $(g_m r_o) = 1\ \text{mA/V}(20\ \text{k}\Omega) = 20$

(a) For $R_L = 20\ \text{k}\Omega$,

$$R_{in2} = \frac{R_L + r_{o2}}{1 + g_{m2} r_{o2}} = \frac{20\ \text{k}\Omega + 20\ \text{k}\Omega}{1 + 20} = 1.9\ \text{k}\Omega$$

$$\therefore A_{V1} = -g_{m1}(r_{o1} \parallel R_{in2})$$

$$= -1\ \text{mA/V}(20\ \text{K} \parallel 1.9\ \text{K}) = -1.74\ \text{V/V}$$

or

If we use the approximation of eq. (6.35

$$R_{in2} \simeq \frac{R_L}{g_{m2} r_{o2}} + \frac{1}{g_{m2}} = \frac{20\ \text{k}\Omega}{20} + \frac{1}{1\ \text{mA/V}}$$

$$= 2\ \text{k}\Omega$$

then,

$$A_{V1} = -1\ \text{mA/V}(20\ \text{k}\Omega \parallel 2\ \text{k}\Omega) = -1.82\ \text{V/V}$$

Either method is correct.

continuing, from eq. (6.31

$$A_V = -gm_1[(gm_2 r_{o2} r_{o1}) \parallel R_L]$$

$$A_V = -1\ \text{mA/V}\{[(20)(20\ \text{k}\Omega)] \parallel 20\ \text{k}\Omega\}$$

$$= -19.04\ \text{V/V}$$

$$A_{V2} = \frac{Av}{Av_1} = \frac{-19.04}{-1.82} = 10.5\ \text{V/V}$$

(b) Now, for $R_L = 400\ \text{k}\Omega$,

$$R_{in2} \simeq \frac{R_L}{g_{m2} r_{o2}} + \frac{1}{g_{m2}} = \frac{400\ \text{k}\Omega}{20} + \frac{1}{1\ \text{mA/V}}$$

$$= 21\ \text{k}\Omega$$

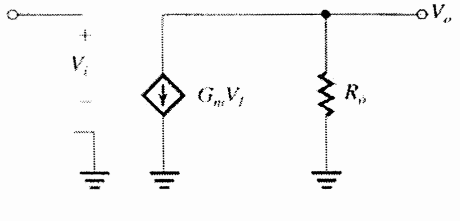
$$A_{V1} = -1\ \text{mA/V}(20\ \text{k}\Omega \parallel 21\ \text{k}\Omega) = -10.2\ \text{V/V}$$

$$A_V = -1\ \text{mA/V}\{[(20)(20\ \text{k}\Omega)] \parallel 400\ \text{k}\Omega\}$$

$$= -200\ \text{V/V}$$

$$A_{V2} = \frac{A_V}{A_{V1}} = \frac{-200}{-10.2} = 19.6\ \text{V/V}$$

Ex: 6.9 The circuit of Fig. 6.14 can be modeled as



Where $G_m = \dfrac{g_m}{1 + g_m Rs}$

and $R_o \simeq (1 + g_m Rs) r_o$

The open-circuit (no load) Voltage gain is

$$Av_o = -G_m Ro = \frac{g_m}{1 + g_m Rs} \cdot (1 + g_m Rs) r_o$$

$$= -g_m Ro$$

so, the gain remains the same

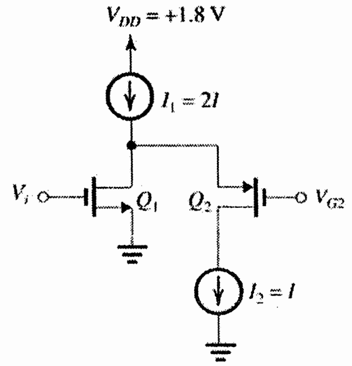
If R_L is connected to the output,

$$Av = \frac{-g_m}{1 + g_m Rs}[(1 + g_m Rs) r_o] \parallel R_L$$

$$= \frac{-g_m}{1 + g_m Rs} \cdot \frac{(1 + g_m Rs) r_o R_L}{(1 + g_m Rs) r_o + R_L}$$

$$= -(g_m r_o) \frac{R_L}{R_L + (1 + g_m Rs) r_o}$$

Ex: 6.10



(a) $I_{D1} = I$ and $I_{D2} = I$

Since $Vov_1 = Vov_2 = 0.2$ V we have

$$\frac{I_{D2}}{I_{D1}} = \frac{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 V_{ov_2}^2}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{ov_1}^2} = \frac{I}{I} = 1$$

and

$$\frac{k_p'\left(\frac{W}{L}\right)_2}{k_n'\left(\frac{W}{L}\right)_1} = 1 \Rightarrow \left(\frac{W}{L}\right)_2 = \frac{k_n'}{k_p'}\left(\frac{W}{L}\right)_1 = \frac{k_n'}{\frac{k_n'}{4}}\left(\frac{W}{L}\right)_1$$

or

$$\left(\frac{W}{L}\right)_2 = 4\left(\frac{W}{L}\right)_1$$

(b) The minimum voltage allowed across current source I_1 would be $|Vov| = 0.2$ V if made with a single transistor. If a 0.1 V_{pp} signal swing is to be allowed at the drain of Q_1, the highest dc bias voltage would be

$$V_{DD} - |Vov| - \frac{0.1\ V_{PP}}{2} = 1.8 - 0.2 - \frac{1}{2}(0.1)$$

$$= 1.55\ \text{V}$$

(c) $V_{SG2} = |Vov| + |Vtp| = 0.2 + 0.5 = 0.7$ V

V_{GS2} can be set at $1.55 - 0.7 = 0.85$ V

(d) Since current source I_2 is implemented with a cascoded current source similar to Fig.6.10, the minimum voltage required across it for proper operation is $2V_{OV} = 2(0.2\text{ V}) = 0.4\text{ V}$

(e) From parts (c) and (d), the allowable range of signal swing at the output is from 0.4 V to 1.55 V − V_{OV} or 1.35 V.

so, $0.4\text{ V} \le V_O \le 1.35\text{ V}$

Ex:6.11 Referring to fig. 6.19,

$R_{op} = (g_{m3}r_{O3})(r_{O4} \,\|\, r_{\pi 3})$ and

$R_{on} = (g_{m2}r_{O2})(r_{O1} \,\|\, r_{\pi 2})$

If Q_1 and Q_4 can be selected and biased so that r_{O1} and r_{O4} are very high and have insignificant effect ($r_O >> r_\pi$) then,

$R_{on} = (g_{m2}r_{O2})r_{\pi 2}$

$R_{op} = (g_{m3}r_{O3})r_{\pi 3}$

Since $g_m r_\pi = \beta$.

$R_{on} = \beta_2 r_{O2}$

$R_{op} = \beta_3 r_{O3}$

Since $Av = -g_{m1}(R_{on} \,\|\, R_{op})$,

$|Av_{max}| = g_{m1}(\beta_2 r_{O2} \,\|\, \beta_3 r_{O3})$

Ex:6.12 For the npn trasistors,

$$g_{m1} = g_{m2} = \frac{|I_C|}{|V_T|} = \frac{0.2\text{ mA}}{25\text{ mV}} = 8\text{ mA/V}$$

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = \frac{100}{8\text{ mA/V}} = 12.5\text{ k}\Omega$$

$$r_{O1} = r_{O2} = \frac{|V_A|}{|I_C|} = \frac{5\text{ V}}{0.2\text{ mA}} = 25\text{ k}\Omega$$

From Fig. 6.19,

$R_{on} = (g_{m2}r_{O2})(r_{O1} \,\|\, r_{\pi 2})$

$= (8\text{ mA/V})(25\text{ k}\Omega)(25\text{ k}\Omega \,\|\, 12.5\text{ k}\Omega)$

$R_{on} = 1.67\text{ M}\Omega$

For the pnp transistors,

$$g_{m3} = g_{m4} = \frac{|I_C|}{V_T} = \frac{0.2\text{ mA}}{25\text{ mV}} = 8\text{ mA/V}$$

$$r_{\pi 3} = r_{\pi 4} = \frac{\beta}{g_m} = \frac{50}{8\text{ mA/V}} = 6.25\text{ k}\Omega$$

$$r_{O3} = r_{O4} = \frac{|V_A|}{|I_C|} = \frac{4\text{ V}}{0.2\text{ mA}} = 20\text{ k}\Omega$$

$R_{op} = (g_{m3}r_{O3})(r_{O4} \,\|\, r_{\pi 3})$

$= (8\text{ mA/V})(20\text{ k}\Omega)(20\text{ k}\Omega \,\|\, 6.25\text{ k}\Omega)$

$R_{op} = 762\text{ k}\Omega$

$A_v = -g_{m1}(R_{on} \,\|\, R_{op})$

$= -(8\text{ mA/V})(1.67\text{ M}\Omega \,\|\, 0.762\text{ M}\Omega)$

$A_v = -4.186\text{ V/V}$

Av_{max} occurs when Q_1 and Q_4 are selected and bias so that r_{O1} and r_{O4} are >> r_π

Then, $R_{on} = (g_{m2}r_{O2})r_{\pi 2} = \beta_2 r_{O2}$

$R_{on} = 100(25\text{ k}\Omega) = 2.5\text{ M}\Omega$

$R_{op} = (g_{m3}r_{O3})r_{\pi 3} = \beta_3 r_{O3}$

$R_{op} = 50(20\text{ k}\Omega) = 1\text{ M}\Omega$

Finally,

$A_{vmax} = -(8\text{ mA/V})(2.5\text{ M}\Omega \,\|\, 1.0\text{ M}\Omega)$

$A_{vmax} = -5714\text{ V/V}$

Ex:6.13 $g_m = \dfrac{|I_C|}{V_T} = \dfrac{1\text{ mA}}{25\text{ mV}} = 40\text{ mA/V}$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\text{ mA/V}} = 2.5\text{ k}\Omega$$

$$r_O = \frac{V_A}{I_C} = \frac{10\text{ V}}{1\text{ mA}} = 10\text{ k}\Omega$$

Referring to Fig. 6.20,

$R_o \simeq r_O[1 + g_m(R_e \,\|\, r_\pi)]$

$$R_o \simeq 10\text{ k}\Omega\left[1 + 40\frac{\text{mA}}{\text{V}}(0.5\text{ k}\Omega \,\|\, 2.5\text{ k}\Omega)\right]$$

$R_o \simeq 176.7\text{ k}\Omega$

without R_e (that is, $R_e = 0$).

$R_o = r_O = 10\text{ k}\Omega$

Ex:6.14 Fig.6.21(a)

$$r_{O1} = r_{O2} = r_O = \frac{V_A}{I} = \frac{5\text{ V}}{0.1\text{ mA}} = 50\text{ k}\Omega$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}(W/L)I_D}$$

$$g_{m1} = \sqrt{2(200\ \mu\text{A}/\text{V}^2)(25)(100\ \mu\text{A})}$$

$$g_{m1} = 1\frac{\text{mA}}{V},\ G_m = g_{m1} = 1\text{ mA/V}$$

$$g_{m2} = \frac{|I_C|}{V_T} = \frac{0.1\text{ mA}}{25\text{ mV}} = 4\text{ mA/V}$$

$$r\pi_2 = \frac{\beta}{g_{m2}} = \frac{100}{4\text{ mA/V}} = 25\text{ k}\Omega$$

Assuming an ideal current source.

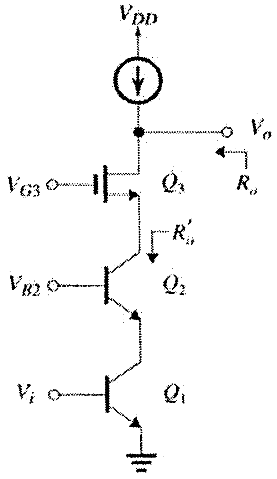
$R_o = (g_{m2}r_{O2})(r_{O1} \,\|\, r_{\pi 2})$

$R_o = (4\text{ mA/V})(50\text{ k}\Omega)(50\text{ k}\Omega \,\|\, 25\text{ k}\Omega) = 3.33\text{ M}\Omega$

$A_{vO} = -G_m R_O = -(1\text{ mA/V})(3.33\text{ M})$

$= -3.33 \times 10^3\text{ V/V}$

Fig. 7.21 (b)



From part (a).

$g_{m3} = 1$ mA/V

$g_{m1} = g_{m2} = 4$ mA/V

$r_{O3} = r_{O2} = r_{O1} = r_O = 50\ \text{k}\Omega$

$r_{\pi 1} = r_{\pi 2} = r_\pi = 25\ \text{k}\Omega$

$G_m \simeq g_{m1} = 4$ mA/V

From Fig. 6 . 19 $R_O' = (g_{m2}r_{O2})(r_{O1} \| r_{\pi 2})$

$R_o = (g_{m3}r_{O3})R_o'$ so,

$R_o = (1\ \text{mA/V})(50\ \text{k}\Omega)(4\ \text{mA/V})(50\ \text{k}\Omega)$
$(50\ \text{k}\Omega \| 25\ \text{k}\Omega)$

$R_o = 167\ \text{M}\Omega$

$A_{vO} = -g_{m1}R_o = -4\ \text{mA/V}(167\text{M}\Omega)$
$= -668 \times 10^3$ V/V

Ex:6 . 15 In the current source of Example 6 . 15 we have $I_O = 100\ \mu\text{A}$ and we want to reduce the change in output current, ΔI_O, corresponding to a 1 V change in output voltage, ΔV_O, to 1% of I_O.

That is $\Delta I_O = \dfrac{\Delta V_O}{ro_2} = 0.01 I_O \Rightarrow \dfrac{1\ \text{V}}{ro_2}$

$= 0.01 \times 100\ \mu\text{A}$

$$ro_2 = \frac{1\ \text{V}}{1\ \mu\text{A}} = 1\ \text{M}\Omega$$

$$ro_2 = \frac{V_A' \times L}{I_O} \Rightarrow 1\ \text{M}\Omega = \frac{20 \times L}{100\ \mu\text{A}}$$

$$\Rightarrow L = \frac{100\ \text{V}}{20\ \text{V}/\mu\text{m}} = 5\ \mu\text{m}$$

To keep V_{OV} of the matched transistors the same as that of Example 6 . 15 $\dfrac{W}{L}$ of the transistor should remain the same. Therefore

$$\frac{W}{5\ \mu\text{m}} = \frac{10\ \mu\text{m}}{1\ \mu\text{m}} \Rightarrow W = 50\ \mu\text{m}$$

So the dimensions of the matched transistors Q_1 and Q_2 should be changed to:

$W = 50\ \mu\text{m}$ and $L = 5\ \mu\text{m}$

Ex:6 . 16 For the circuit Figure4 . 7 we have:

$$I_2 = I_{\text{REF}}\frac{(W/L)_2}{(W/L)_1},\ I_3 = I_{\text{REF}}\frac{(W/L)_3}{(W/L)_1}$$

and $I_5 = I_4\dfrac{(W/L)_5}{(W/L)_4}$

Since all channel lengths are equal

$L_1 = L_2 = \ldots = L_5 = 1\ \mu\text{m}$

and

$I_{\text{REF}} = 10\ \mu\text{A},\ I_2 = 60\ \mu\text{A},\ I_3 = 20\ \mu\text{A},$

$I_4 = I_3 = 20\ \mu\text{A}$ and $I_5 = 80\ \mu\text{A},$

we have:

$$I_2 = I_{\text{REF}}\frac{W_2}{W_1} \Rightarrow \frac{W_2}{W_1} = \frac{I_2}{I_{\text{REF}}} = \frac{60}{10} = 6$$

$$I_3 = I_{\text{REF}}\frac{W_3}{W_1} \Rightarrow \frac{W_3}{W_1} = \frac{I_3}{I_{\text{REF}}} = \frac{20}{10} = 2$$

$$I_5 = I_4\frac{W_5}{W_4} \Rightarrow \frac{W_5}{W_4} = \frac{I_5}{I_4} = \frac{80}{10} = 4$$

In order to allow the voltage at the drain of Q_2 to go down to within 0.2 V of the negative supply voltage we need $V_{OV2} = 0.2$ V

$$I_2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 V^2{}_{OV_2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2 V^2{}_{OV_2}$$

$$60\ \mu\text{A} = \frac{1}{2}200\ \frac{\mu\text{A}}{\text{V}^2}\left(\frac{W}{L}\right)_2 (0.2)^2 \Rightarrow$$

$$\left(\frac{W}{L}\right)_2 = \frac{120}{200 \times (0.2)^2} = 15 \Rightarrow W_2 = 15 \times L_2$$

$$W_2 = 15\ \mu\text{m}.\ \frac{W_2}{W_1} = 6 \Rightarrow W_1 = \frac{W_2}{6} = 2.5\ \mu\text{m}$$

$$\frac{W_3}{W_1} = 2 \Rightarrow W_3 = 2 \times W_1 = 5\ \mu\text{m}$$

In order to allow the voltage at the drain of Q_5 to go up to within 0.2 V of positive supply we need

$$V_{OV_5}\ I_5 = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_5 \Rightarrow V^2{}_{OV2}$$

$$80\ \mu\text{A} = \frac{1}{2}80\frac{\mu\text{A}}{\text{V}^2}\left(\frac{W}{L}\right)_5 (0.2)^2 \Rightarrow$$

$$\left(\frac{W}{L}\right)_5 = \frac{2 \times 80}{80 \times (0.2)^2} = 50 \Rightarrow W_5 = 50\ L_5$$

$W_5 = 50\ \mu\text{m}$

$$\frac{W_5}{W_4} = 4 \Rightarrow W_4 = \frac{50\ \mu\text{m}}{4} = 12.5\ \mu\text{m}$$

Thus:

$W_1 = 2.5\ \mu m$, $W_2 = 15\ \mu m$, $W_3 = 5\ \mu m$

$W_4 = 12.5\ \mu m$ and $W_5 = 50\ \mu m$

Ex:6.17 From equation 6.72 we have:

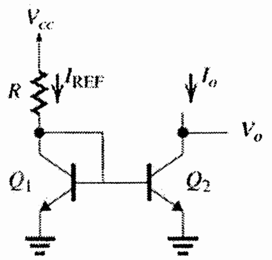
$$I_o = I_{REF}\left(\frac{m}{1+\frac{m+1}{\beta}}\right)\left(1+\frac{V_o - V_{BE}}{V_{A2}}\right)$$

$$I_o = 1\ mA\left(\frac{1}{1+\frac{1+1}{100}}\right)\left(1+\frac{5-0.7}{100}\right) = 1.02\ \text{mA}$$

$I_o = 1.02$ mA

$$R_O = r_{O2} = \frac{V_A}{I_O} = \frac{100\ V}{1.02\ \text{mA}} = 98\ \text{k}\Omega \simeq 100\ \text{k}\Omega$$

Ex:6.18



From equation 6.74 we have:

$$I_O = \frac{I_{REF}}{1+(2/\beta)}\left(1+\frac{V_O - V_{BE}}{V_A}\right) \Rightarrow$$

$$0.5\ \text{mA} = \frac{I_{REF}}{1+(2/100)}\left(1+\frac{2-0.7}{50}\right) \Rightarrow$$

$$I_{REF} = 0.5\ \text{mA}\frac{1.02}{1.026\ \text{mA}} = 0.497\ \text{mA}$$

$$I_{REF} = \frac{V_{CC}-V_{BE}}{R} \Rightarrow R = \frac{V_{CC}-V_{BE}}{I_{REF}}$$

$$R = \frac{5-0.7}{0.497\ \text{mA}} = \frac{4.3}{0.497} 8.65\ \text{k}\Omega$$

$V_{O\min} = V_{CESAT} = 0.3$ V

For $V_o = 5$ V, From equation 6.74 we have:

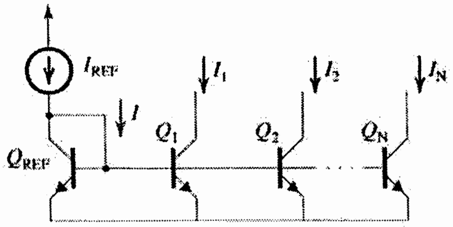
$$I_o = \frac{I_{REF}}{1+(2/\beta)}\left(1+\frac{V_o-V_{BE}}{V_A}\right)$$

$$I_o = \frac{0.497}{1+(2/100)}\left(1+\frac{5-0.7}{50}\right) = 0.53\ \text{mA}$$

Ex: 6.19

See next page:

Exercise 6–8



Ignoring the effect of finite output resistances, we have

$$I_1 = I_2 = \dots = I_N = I_{CQ_{REF}}$$

$$I_{CQ_{REF}} + I = I_{REF}(*)$$

$$I = I_{BQ_{REF}} + I_{B1} + \dots + I_{BN}$$

$$I = \frac{I_{CQ_{REF}}}{\beta} = \frac{I_{C1}}{\beta} + \dots + \frac{I_{CN}}{\beta}$$

$$I = I_{CQ_{REF}}\left(\frac{1}{\beta} + \frac{1}{\beta} + \dots + \frac{1}{\beta}\right)$$

$$I = I_{CQ_{REF}}\frac{N+1}{\beta}$$

From (*) we have:

$$I_{CQ_{REF}} + I = I_{REF} \Rightarrow$$

$$I_{REF} = I_{CQ_{REF}} + I_{CQ_{REF}}\frac{N+1}{\beta}$$

$$\Rightarrow I_{CQ_{REF}} = \frac{I_{REF}}{1+\frac{N+1}{\beta}}$$

Thus:

$$I_1 = I_2 = \dots = I_N = \frac{I_{REF}}{1+\frac{N+1}{\beta}}$$

For an error not exceeding 10% we need:

$$\frac{I_{REF}}{1+\frac{N+1}{\beta}} \geq I_{REF}(1-0.1)$$

$$\frac{I_{REF}}{1+\frac{N+1}{\beta}} \geq 0.9 I_{REF} \Rightarrow \frac{1}{1+\frac{N+1}{\beta}} \geq 0.9$$

$$\Rightarrow 1 + \frac{N+1}{\beta} \leq \frac{1}{0.9} \Rightarrow 1 + \frac{N+1}{\beta} \leq 1.11$$

$$\frac{N+1}{\beta} \leq 0.11 \Rightarrow N+1 \leq 0.11\,\beta \Rightarrow$$

$$N+1 \leq 11 \Rightarrow N \leq 10$$

The maximum number of outputs for an error not exceeding to be lessthan 10% then we need $N<10$. In this case the maximum number of outputs for and error of less than 10% is $N = 9$.

Ex:6 . 20 Refering to Fig. 6 . 32

$I_{D1} = I_{D2} = I_{D3} = I_{D4} = I_{REF} = 100\ \mu A$

Since $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V^2{}_{OV}$

$$V_{OV} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{\frac{2(100\ \mu A)}{(387\ \mu A/V^2)\left(\frac{3.6}{0.36}\right)}}$$

$= 0.23\ V$

The minimum output voltage is

$V_{tn} + 2V_{ov} = 0.5\ V + 2(0.23\ V) = 096\ V$

To obtain the output resistance, R_O, we need g_{m3}.

$$g_{m3} = \frac{I_{D3}}{V_{OV}/2} = \frac{2(0.1\ mA)}{0.23\ V} = 0.87\ mA/V$$

$$r_{O2} = r_{O3} = \frac{V_A{}'(L)}{I_D} = \frac{(5V/\mu m)(0.36\ \mu m)}{0.1\ mA}$$

$= 18\ k\Omega$. From eq. 6 . 77

$R_o \approx g_{m3} r_{o3} r_{o2} = (0.87\ mA/V)(18\ k\Omega)^2$

$= 282\ k\Omega$

Ex: For the Wilson mirror from the equation (6 . 80 we have :

$$\frac{I_o}{I_{REF}} = \frac{1}{1 + \frac{2}{\beta(\beta+2)}} = 0.9998$$

Thus $\frac{|I_o - I_{REF}|}{I_{REF}} \times 100 = 0.02\%$

whereas for the simple mirror from equation 6 . 69 we have :

$$\frac{I_o}{I_{REF}} = \frac{1}{1 + \frac{2}{\beta}} = 0.98$$

Hence $\frac{|I_o - I_{REF}|}{I_{REF}} \times 100 = 2\%$

For the Wilson current mirror we have

$R_o = \frac{\beta r_o}{2} = \frac{100 \times 100\ k\Omega}{2} = 5\ M\Omega$ and for

the simple mirror $R_o = r_o = 100\ k\Omega$

Ex:6 . 22 For the two current sources designed in Example 6 . 6

we have :

$g_m = \frac{I_C}{V_T} = \frac{10\ \mu A}{25\ mV} = 0.4\ \frac{mA}{V}$ and

$r_O = \frac{V_A}{I_C} = \frac{100\ V}{10\ \mu A} = 10\ M\Omega,\ r_\pi = \frac{\beta}{g_m} = 250\ k\Omega$

For the current source in Fig 6 . 37a we have

$R_o = r_{o2} = r_o = 10\ M\Omega$

For the current source in Fig. 6 . 37b from equation 6 . 98 we have:

$R_o \simeq [1 + g_m(R_E \| r_\pi)]r_o$

In Example 6 . 6 $R_E = R_3 = 11.5\ k\Omega$,

therefore,

$$R_o \simeq \left[1 + 0.4\ \frac{mA}{V}(11.5\ k\Omega \|\ 250\ k\Omega)\right]10\ M\Omega$$

$\Rightarrow R_o = 54\ M\Omega$

Ex: 7 . 1

Referring to Fig 7 . 3

If R_D is doubled to 5 K,

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2}R_D$$

$$= 1.5 - \frac{0.4\text{ mA}}{2}(5\text{ K}) = 0.5\text{ V}$$

$$V_{CM_{max}} = V_t + V_D = 0.5 + 0.5 = +1.0\text{ V}$$

Since the currents I_{D1}, and I_{D2} are still 0.2 mA each,

$$V_{GS} = 0.82\text{ V}$$

So, $V_{CM_{min}} = V_{SS} + V_{CS} + V_{GS}$

$$= -1.5\text{ V} + 0.4\text{ V} + 0.82\text{ V} = -0.28\text{ V}$$

So, the common-mode range is

−0.28 V to 1.0 V

Ex: 7 . 2

(a) The value of v_{id} that causes Q_1 to conduct the entire current is $\sqrt{2}\,V_{OV}$

$$\rightarrow \sqrt{2} \times 0.316 = 0.45\text{ V}$$

then, $V_{D1} = V_{DD} - I \times R_D$

$$= 1.5 - 0.4 \times 2.5 = 0.5\text{ V}$$

$$V_{D2} = V_{DD} = +1.5\text{ V}$$

(b) For Q_2 to conduct the entire current:

$$v_{id} = -\sqrt{2}\,V_{OV} = -0.45\text{ V}$$

then,

$$V_{D1} = V_{DD} = +1.5\text{ V}$$

$$V_{D2} = 1.5 - 0.4 \times 2.5 = 0.5\text{ V}$$

(c) Thus the differential output range is:

$V_{D2} - V_{D1}$: from $1.5 - 0.5 = +1\text{ V}$

to $0.5 - 1.5 = -1\text{ V}$

Ex: 7 . 3

Refer to answer table for Exercise7 . 3 where values were obtained in the following way:

$$V_{OV} = \sqrt{I/KW/L} \rightarrow \frac{W}{L} = \frac{I}{KV_{OV}^2}$$

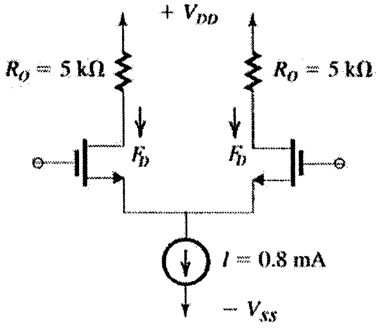
$$g_m = \frac{I}{V_{OV}}$$

$$\left(\frac{v_{id}/2}{V_{OV}}\right)^2 = 0.1 \rightarrow v_{id} = 2\,V_{OV}\sqrt{0.1}$$

Ex: 7 . 4

$$I_D = \frac{I}{2} = \frac{0.8\text{ mA}}{2} = 0.4\text{ mA}$$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{OV})^2 \text{ So that}$$



$$V_{OV} = \sqrt{\frac{2I_D}{k_n'\left(\frac{W}{L}\right)}} = \sqrt{\frac{2(0.4\text{ mA})}{0.2(\text{mA}/\text{V}^2)(100)}} = 0.2\text{ V}$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{0.4\text{ mA}(2)}{0.2\text{ V}} = 4\text{ mA}/\text{V}$$

$$r_O = \frac{V_A}{I_D} = \frac{20\text{ V}}{0.4\text{ mA}} = 50\text{ k}\Omega$$

$$A_d = g_m(R_D \| r_O)$$

$$A_D = (4\text{ mA}/\text{V})(5\text{ K} \| 50\text{ K}) = 18.2\text{ V}/\text{V}$$

Ex: 7 . 5

With $I = 200\ \mu\text{A}$. for all transistors,

$$I_D = \frac{I}{2} = \frac{200\ \mu\text{A}}{2} = 100\ \mu\text{A}$$

$$L = 2(0.18\ \mu\text{m}) = 0.36\ \mu\text{m}$$

$$r_{O_1} = r_{O2} = r_{O3} = r_{O4} = \frac{|V_A|L}{I_D}$$

$$= \frac{(10\text{ V}/\mu\text{m})\,(0.36\ \mu\text{m})}{0.1\text{ mA}} = 36\text{ k}\Omega$$

Since $I_{D1} = I_{D2} = \frac{1}{2}\mu_A C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2I_D}{\mu_A C_{ox}(V_{OV})^2}$$

$$\frac{2(100\ \mu\text{A})}{(400\ \mu\text{A}/\text{V}^2)(0.2\text{ V})^2} = 12.5$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{2I_D}{\mu_A C_{ox}|V_{OV}|^2}$$

$$\frac{2(100\ \mu\text{A})}{(100\ \mu\text{A}/\text{V}^2)(0.2)^2} = 50$$

$$g_m = \frac{I_D}{|V_{OV}|/2} = \frac{(100\ \mu\text{A})(2)}{0.2\text{ V}} = 1\text{ mA}/\text{V},$$

so,

$$A_D = g_{m1}(r_{O1} \| r_{O3}) = 1(\text{ mA}/\text{V})(36\text{ K} \| 36\text{ K})$$

$$= 18\text{ V}/\text{V}$$

Ex 7.6

$L = 2(0.18\ \mu\text{m}) = 0.36\ \mu\text{m}$

All $r_O = \dfrac{|V_A'| \cdot L}{|I_D|}$

The drain current for all transistors is

$$I_D = \frac{I}{2} = \frac{200\ \mu\text{A}}{2} = 100\ \mu\text{A}$$

$$r_O = \frac{(10\ \text{V}/\mu\text{m})(0.36\ \mu\text{m})}{0.1\ \text{mA}} = 36\ \text{k}\Omega$$

Refering to Fig7.12(a).

Since $I_D = \frac{1}{2}\mu_A C_{ox}\left(\frac{W}{L}\right)(V_{OV})^2$ for all NMOS transistors

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4$$

$$= \frac{2\,I_D}{\mu_n C_{ox}(V_{OV})^2} = \frac{2(100\ \mu\text{A})}{400\ \mu\text{A}/\text{V}^2(0.2\ \text{V})^2} = 12.5$$

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8$$

$$= \frac{2\,I_D}{\mu_p C_{ox}(V_{OV})^2} = \frac{2(100\ \mu\text{A})}{100\ \mu\text{A}/\text{V}^2(0.2\ \text{V})^2} = 50$$

For all transistors.

$$g_m = \frac{|I_D|}{|V_{OV}|/2} = \frac{(0.1\ \text{mA})(2)}{(0.2\ \text{V})} = 1\ \text{mA}/\text{V}$$

From Fig.7.12(b),

$$R_{on} = (g_{m3}\, r_{O3})r_{O1} = (1\ \text{mA}/\text{V})(36\ \text{k})^2$$

$$= 1.296\ \text{M}\Omega$$

$$R_{op} = (g_{m5}\, r_{O5})r_{O7} = (1\ \text{mA}/\text{V})(36\ \text{k})^2$$

$$= 1.296\ \text{M}\Omega$$

Using eq.(7.38

$$A_d = g_{m1}\,(R_{on} \| R_{op})$$

$$= (1\ \text{mA}/\text{V})1.296(\ \text{M}\Omega \|\ 1.296\ \text{M}\Omega)$$

$$= 648\ \text{V/V}$$

Ex: 7.7

The transconductance for each transistor is

$$g_m = \sqrt{2\,\mu_n C_{ox}(W/L)I_D}$$

$$I_D = \frac{I}{2} = \frac{0.8\ \text{mA}}{2} = 0.4\ \text{mA}$$

from eq(7.35 the differential gain for matched R_D values is $A_d = \dfrac{V_{O2} - V_{O1}}{V_{id}} = g_m R_D$

If we ignore the 1% here,

$$A_d = g_m R_D = (4\ \text{mA}/\text{V})(5\ \text{K}) = 20\ \text{V}/\text{V}$$

From eq. (7.49

$$A_{CM} = \frac{V_{od}}{V_{icm}} = \pm\frac{\Delta R_D}{2\,R_{SS}} = \frac{(0.01)(5\ \text{K})}{2(25\ \text{K})} = 0.001\ \text{V/V}$$

$$CMRR(dB) = 20\log 10\frac{|A_d|}{|A_{CM}|} = 20\log 10\left(\frac{20}{0.001}\right)$$

$$= 86\ \text{dB}$$

Ex: 7.8

From Exercise 7.7

$W/L = 100, \mu_n C_{ox}(0.2\ \text{mA}/\text{V}^2),$

$$I_D = \frac{I}{2} = \frac{0.8\ \text{mA}}{2} = 0.4\ \text{mA}$$

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D}$$

$$= \sqrt{2(0.2\ \text{mA}/\text{V}^2)(100)(0.4\ \text{mA})}$$

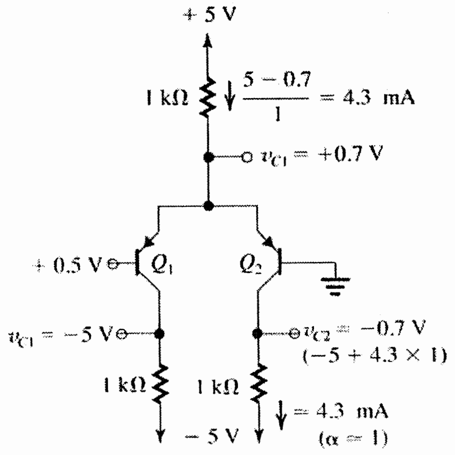
$$g_m = 4\ \text{mA}/\text{V}$$

using eq. (7.64 and the fact that $R_{SS} = 25\ \text{k}\Omega$

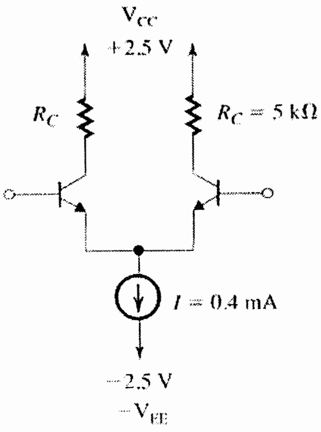
$$CMRR = \frac{(2\,g_m\,R_{SS})}{\left(\frac{\Delta g_m}{g_m}\right)} = \frac{2(4\ \text{mA}/\text{V})(25\ \text{K})}{0.01} = 20,000$$

$$CMRR(\text{dB}) = 20\log_{10}(20{,}000) = 86\ \text{dB}$$

Ex: 7.9



Ex: 7.10



$$I_{C1} = I_{C2} = I_{E1} = I_{E2} = \frac{I}{2} = \frac{0.4\text{ mA}}{2}$$

$= 0.2$ mA

From eq. 7.66

$V_{CM\,\max} \simeq V_C + 0.4\text{ V}$

$= V_{CC} - I_C R_C + 0.4\text{ V}$

$= 2.5 - 0.2\text{ mA}(5\text{ K}) + 0.4\text{ V} = +1.9\text{ V}$

From eq. 7.67

$V_{CM\,\min} = -V_{EE} + V_{CS} + V_{BE}$

$V_{CM\,\min} = -2.5\text{ V} + 0.3\text{ V} + 0.7\text{ V} = -1.5\text{ V}$

Input range is -1.5 V to $+1.9$ V

Ex: 7.11

Substituting $i_{E1} + i_{E2} = I$ in Eqn.7.70 yields

$$i_{E1} = \frac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$$

$$0.99\,I = \frac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$$

$$v_{B1} - v_{B2} = -V_T \ln\left(\frac{1}{0.99} - 1\right)$$

$= -25\ln(1/99)$

$= 25\ln(99) = 115$ mV

Ex: 7.12

(a) The DC current in each transister is 0.5 mA. Thus V_{BE} for each will be

$$V_{BE} = 0.7 + 0.025\ln\left(\frac{0.5}{1}\right)$$

$= 0.683$ V

$\Rightarrow vE = 5 - 0.683 = +4.317$ V

(b) $g_m = \dfrac{IC}{VT} = \dfrac{0.5}{0.025} = 20\ \dfrac{\text{mA}}{\text{V}}$

(c) $i_{C1} = 0.5 + g_{m1}\Delta v_{BE1}$

$= 0.5 + 20 \times 0.005\text{Sin}(2\pi \times 1000t)$

$= 0.5 + 0.1\text{Sin}(2\pi \times 1000t)$, mA

$i_{C2} = 0.5 - 0.1\text{Sin}(2\pi \times 1000t)$, mA

(d)

$v_{C1} = (V_{CC} - I_C R_C) - 0.1 \times Rc\text{Sin}(2\pi \times 1000\text{t})$

$= (15 - 0.5 \times 10) - 0.1 \times 10\text{Sin}(2\pi 1000t)$

$= 10 - 1\text{Sin}(2\pi \times 1000t)$, V

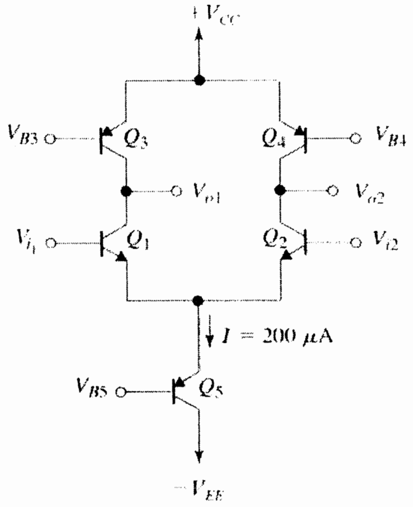
$v_{C2} = 10 + 1\text{Sin}(2\pi \times 1000t)$, V

(e) $v_{C2} - v_{C1} = 2 \cdot \text{Sin}(2\pi \times 1000t)$, V

(f) Voltage gain $= \dfrac{v_{C2} - v_{C1}}{v_{B1} - v_{B2}}$

$$= \frac{2\text{V}_{\text{Peak}}}{0.1\text{V}_{\text{Peak}}} = 200\text{ V/V}$$

Ex: 7.13



$I = 200\ \mu$A

Since $\beta \gg 1$,

$$I_{C1} \simeq I_{C2} \simeq \frac{I}{2} = \frac{200\ \mu\text{A}}{2} = 100\ \mu\text{A}$$

$$g_{m1} = g_{m2} = g_m = \frac{|I_C|}{V_T} = \frac{100\ \mu\text{A}}{25\text{ mV}} = 4\text{ mA/V}$$

$$R_{C1} = R_{C2} = R_C = r_O = \frac{|V_A|}{I_C}$$

$$= \frac{10\text{ V}}{100\ \mu\text{A}} = 100\text{ k}\Omega$$

$$R_{EE} = \frac{V_A}{I} = \frac{10\text{ V}}{200\ \mu\text{A}} = 50\text{ k}\Omega$$

$$r_{e_1} = r_{e_2} = r_e = \frac{V_T}{I_E} = \frac{25\text{ mV}}{0.1\text{ mA}} = 0.25\text{ k}\Omega$$

Since $R_O >> r_e$.

$|A_d| \simeq$

$$\frac{R_C \| r_O}{r_e} = \frac{100\text{ K} \| 100\text{ K}}{0.25\text{ K}} = 200\text{ V/V}$$

$$R_{id} = 2r_\pi \quad r_\pi = \frac{\beta}{g_m} = \frac{100}{4\text{ mA/V}} = 25\text{ k}\Omega$$

$R_{id} = 2(25\text{ K}) = 50\text{ K}$

If the total load resistance is assumed to be mismatched by 1%.

$$|A_{cm}| = \frac{\Delta R_C}{2R_{EE}} = \frac{(0.01)(100\text{ K})}{2(50\text{ K})} = 0.01$$

$$\text{CMRR(dB)} = 20\log_{10}\left|\frac{A_d}{A_{cm}}\right| = 20\log_{10}\left|\frac{200}{0.01}\right|$$

$= 86$ dB

Note: If only the load transistors are mismatched, and since $\alpha \simeq 1$,

$$V_{O1} = -\frac{R_C \| V_O}{2R_{EE} + r_e} V_{icm} = -\frac{(100\text{ K} \| 100\text{ K})V_{icm}}{2(50\text{ K}) + 0.25\text{ K}}$$

$$= -0.499\ V_{icm}$$

$$V_{O2} \simeq \frac{-[(R_C + \Delta R_C) \| r_O]V_{icm}}{2R_{EE} + r_e}$$

$$= -\frac{[(1.01)100\text{ K}] \| 100\text{ K}}{2(50\text{ K}) + 0.25\text{ K}} \cdot V_{icm} = 0.501 V_{icm}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{200}{0.501 - 0.499}$$

$$= 100,000 \rightarrow 100\text{ dB}$$

Using eq. 7.103

$$R_{icm} = \beta R_{EE} \cdot \frac{1 + \dfrac{R_C}{\beta r_O}}{1 + \dfrac{R_C + 2R_{EE}}{r_O}} = (100)(50\text{ K})$$

$$\cdot \frac{1 + \dfrac{100\text{ K}}{100(100\text{ K})}}{1 + \dfrac{100\text{ K} + 2(50\text{ K})}{100\text{ K}}}$$

$$R_{icm} \simeq 1.68\text{ M}\Omega$$

Ex: 7.14

From Exercise 7.4

$V_{ov} = 0.2$ V

Using Equ. 7.108 we obtain V_{os} due to $\Delta R_D / R_D$ as:

$$V_{OS} = \left(\frac{V_{OV}}{2}\right) \cdot \left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{0.2}{2} \times 0.02 = 0.002\text{ V} \quad \text{i.e } 2\text{ mV}$$

To obtain V_{os} due to $\dfrac{\Delta W / L}{W / L}$

Use Eqn. (7.113

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\Delta W / L}{W / L}\right)$$

$$\rightarrow V_{OS} = \left(\frac{0.2}{2}\right) \times 0.02 = 0.002$$

$$\rightarrow 2\text{ mV}$$

The offset voltage arising from ΔVt is obtained from Eqn. (7.116

$V_{os} = \Delta Vt = 2$ mV

Finally, from Eqn. 7.117 the total input offset is:

$$V_{OS} =$$

$$\left[\left(\frac{V_{OV}}{2}\frac{\Delta R_D}{R_D}\right)^2 + \left(\frac{V_{OV}}{2}\frac{\Delta W / L}{W / L}\right)^2 + (\Delta V_t)^2\right]^{1/2}$$

$$= \sqrt{(2 \times 10^{-3})^2 + (2 \times 10^{-3})^2 + (2 \times 10^{-3})^2}$$

$$= \sqrt{3 \times (2 \times 10^{-3})^2}$$

$$= 3.46\text{ mV}$$

Ex: 7.15

From Eqn. 7.127

$$V_{OS} = V_T \sqrt{\left(\frac{\Delta R_C}{R_C}\right)^2 + \left(\frac{\Delta I_S}{I_S}\right)^2}$$

$$= 25\sqrt{(0.02)^2 + (0.1)^2}$$

$$= 2.5\text{ mV}$$

$$I_B = \frac{100}{2(\beta + 1)} = \frac{100}{2 \times 101} \cong 0.5\ \mu\text{A}$$

$$I_{OS} = I_B\left(\frac{\Delta\beta}{\beta}\right)$$

$$= 0.5 \times 0.1\ \mu\text{A} = 50\text{ nA}$$

Ex: 7.16

$$(W / L)_n \times \mu_n C_{ox} \quad 0.2\text{ m} \times 100 = 20\text{ m}\frac{\text{A}}{\text{V}}$$

$$(W / L)p \times \mu p C_{ox} = 0.1\text{ m} \times 200 = 20\text{ m}\frac{\text{A}}{\text{V}}$$

Since all transistors have the same drain current $(I / 2)$ and the name product $W / L \times \mu C_{ox}$ then all transconductances g_m are identical.

$$|V_{OV}| = \sqrt{\frac{I_D}{20\text{ mA/V}}} = \sqrt{\frac{0.8\text{ mA}}{20\text{ mA/V}}} = 0.2\text{ V}$$

thus,

$$g_m = \frac{I_D}{V_{OV}} = \frac{(0.8\text{ mA/2})}{0.2\text{ V}} = 4\text{ m}\frac{\text{A}}{\text{V}}$$

From Eqn. (7.138

$$G_m = g_m = 4\text{ mA / V}$$

$$R_O = r_{O2} \| r_{O4}$$

$$r_{O2} = \frac{V_{An}}{I_{D2}} = \frac{20}{(0.8\text{ m/2})} = 50\text{ k}\Omega$$

$$r_{O4} = \frac{V_{Ap}}{I_{D4}} = \frac{20}{(0.8\text{ m/2})} = 50\text{ k}\Omega$$

thus,

$$R_O = 50 \| 50 = 25\text{ k}\Omega$$

From Eqn. (7.141

$$A_d = G_m R_O = 4\frac{\text{mA}}{\text{V}} \times 25\text{ k}\Omega = 100\frac{\text{V}}{\text{V}}$$

From Eqn. (7.148a

$$A_{cm} \simeq \frac{1}{2g_{m3}R_{SS}} = \frac{1}{2 \times 4 \times 25} = 0.005\text{ V/V}$$

$$CMRR = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.005} = 20,000$$

$$\rightarrow 86\text{ dB}$$

Ex: 7.17

From Eqn. (7.156 $G_m = g_m$

$$g_m \simeq \frac{I / 2}{V_T} = \frac{(0.8\text{ mA/2})}{25\text{ mV}} = 16\frac{\text{mA}}{\text{V}}$$

From Eqn. (7.159

$$R_O = r_{O2} \| r_{O4}$$

$$= \frac{V_A}{I_{C2}} \| \frac{V_A}{I_{C4}} \simeq \frac{1}{2}\frac{V_A}{I/2}$$

$$= \frac{100\text{ V}}{0.8\text{ mA}} = 125\text{ k}\Omega$$

$$A_d = G_m \times R_O = 16 \times 125 = 2000\ \frac{\text{V}}{\text{V}}$$

From Eqn. (7.162

$R_{id} = 2\times r_\pi$

$$\simeq 2 \times \frac{V_T}{(I/2)}\beta_n = \frac{2 \times 25\text{ m} \times 160}{(0.8\text{ m}/2)}$$

$= 20\text{ k}\Omega$

For a simple current mirror the output resistance (thus R_{EE}) is r_o

$$\rightarrow R_{EE} = \frac{V_A}{I} = \frac{100\text{ V}}{0.8\text{ mA}} = 125\text{ k}\Omega$$

From Eqn. (7.167

$$A_{cm} = \frac{-r_{O4}}{\beta_3 R_{EE}}$$

$$A_{cm} = \frac{-2 \times 125\text{ K}}{160 \times 125\text{ K}}$$

$$A_{cm} = -.0125\frac{\text{V}}{\text{V}}$$

$$C_{MRR} = \left|\frac{2000}{.0125}\right|$$

$$C_{MRR} = 160,000$$

$$20\log_{10}(160,000) = 104\text{ dB}$$

Ex: 7.18

$$G_m = gm_{1,2} = \frac{I/2}{V_T} = \frac{1\text{ m}/2}{25\text{ m}} = 20\text{ m}\frac{\text{A}}{\text{V}}$$

$$r_{O4} = r_{O5} = \frac{V_A}{I/2} = \frac{100\text{ V}}{0.5\text{ mA}} = 200\text{ k}\Omega$$

$$\rightarrow R_{O4} = \beta_4 r_{O4} = 50 \times 200\text{ K} = 10\text{ M}\Omega$$

$$R_{O5} = \beta_5\frac{r_{O5}}{2} = 100 \times \frac{200\text{ K}}{2} = 10\text{ M}\Omega$$

From Eqn. (7.174

$$R_O = \left[\beta_4 r_{O4} \| \beta_5\frac{r_{O5}}{2}\right]$$

$= (10 \| 10)\text{ M}\Omega = 5\text{ M}\Omega$

$A_d = g_m \times R_o = 20 \times 5000 = 10^5$ V/V

i.e. 100 dB

Ex: 7.19

Refer to Fig (7.41

(a) Using Eqn. (7.178

$$I_6 = \frac{(W/L)_6}{(W/L)_4}(I/2)$$

$$\Rightarrow 100 = \frac{(W/L)_6}{100} \times 50$$

thus, $(W/L)6 = 200$

Using Eqn. (7.179

$$I_7 = \frac{(W/L)_7}{(W/L)_5}(I)$$

$$\Rightarrow 100 = \frac{(W/L)_7}{200} \times 100$$

thus, $(W/L)_7 = 200$

(b) For Q_1,

$$I = \frac{1}{2}\mu_P C_{ox}\left(\frac{W}{L}\right)_1 V_{OV_1}^2$$

$$\Rightarrow V_{OV1} = \sqrt{\frac{50}{\frac{1}{2} \times 30 \times 200}} = 0.129\text{ V}$$

Similarly for Q_2, $V_{OV2} = 0.129$ V

For Q_6,

$$100 = \frac{1}{2} \times 90 \times 200\ V_{OV_6}^2$$

$$\Rightarrow V_{OV6} = 0.105\text{ V}$$

(c) $g_m = \dfrac{2I_D}{V_{OV}}$

	I_D	V_{OV}	g_m
Q_1	50 μA	0.129 V	0.775 mA.V
Q_2	50 μA	0.129 V	0.775 mA/V
Q_6	100 μA	0.105 V	1.90 mA/V

(d) $r_{o2} = 10/0.05 = 200\text{ k}\Omega$

$r_{o4} = 10/0.05 = 200\text{ k}\Omega$

$r_{o6} = 10/0.1 = 100\text{ k}\Omega$

$r_{o7} = 10/0.1 = 100\text{ k}\Omega$

(e) Eqn. (7.176

$$A_1 = -g_{m1}(r_{O2} \| r_{O4})$$

$$= -0.775\ (200 \| 200) = -77.5\ \frac{\text{V}}{\text{V}}$$

Eqn. (7.177

$$A_2 = -g_{m6}(r_{O6} \| r_{O7})$$

$= -95$ V/V

Overall voltage gain is:

$A_1 \times A_2 = 77.5 \times 95 = 7363$ V/V

Ex: 7.20

Referring to Fig.7.42 all I_D values are the same.

so, $V_{GS13} = V_{GS12} + I_B R_B$

Using the equation developed in the text.

$$R_B = \frac{2}{\sqrt{2\mu_n C_{OX}\left(\frac{W}{L}\right)_{12} I_B}} \cdot \left(\sqrt{\frac{\left(\frac{W}{L}\right)_{12}}{\left(\frac{W}{L}\right)_{13}}} - 1\right)$$

$$R_B = \frac{2}{\sqrt{2\ (90\ \mu\text{A}/\text{V}^2)(80)(10\ \mu\text{A})}} \cdot \left(\sqrt{\frac{80}{20}} - 1\right)$$

$= 5.27\text{ k}\Omega$

$$g_{m12} = \frac{2}{R_B}\left(\sqrt{\frac{\left(\frac{W}{L}\right)_{12}}{\left(\frac{W}{L}\right)_{13}}}-1\right)$$

$$g_{m12} = \frac{2}{5.27\text{ K}}\cdot\left(\sqrt{\frac{80}{20}}-1\right) = 0.38\text{ mA/V}$$

Ex 7.21

$I_D = 90\ \mu A$

$\mu_n C_{ox} = 160\ \mu A/V^2$

$\mu_p C_{ox} = 40\ \mu A/V^2$

For Q_8 and Q_9 : $W/L = 40/0.8$

(as given in Example 7.5)

$$|V_{OV}| = \sqrt{\frac{2I_D}{\mu p C_{ox}(W/L)}}$$

$$\rightarrow |V_{OV}|_{8,9} = \sqrt{\frac{2\times 90\ \mu}{40\ \mu \times \frac{40}{0.8}}} = 0.3\text{ V}$$

then,

$$g_{m8,9} = \frac{2I_D}{|V_{OV}|} = \frac{2\times 90\ \mu A}{0.3\text{ V}}$$

$= 0.6$ mA/V

Since g_m of Q_{10}, Q_{11} and Q_{13} are identical to g_m of Q_8 and Q_9 then:

$V_{ov13} = 0.3$ V

Thus, for Q_{13}

$$(0.3)^2 = \frac{2\times 90\ \mu}{160\ \mu\ (W/L)_{13}}$$

$$\rightarrow (W/L)_{13} = 12.5$$

i.e. (10 / 0.8)

Since Q_{12} is 4 times as wide as Q_{13}, then

$$\left(\frac{W}{L}\right)_{12} = \frac{4\times 10}{0.8} = \frac{40}{0.8}$$

$$R_B = \frac{2}{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{12} I_B}}\cdot\left(\sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}}-1\right)$$

$$= \frac{2}{\sqrt{2\times 160\ \mu \times \frac{40}{0.8}\times 90\ \mu}}\cdot\left(\sqrt{\frac{40/0.8}{12.5}}-1\right)$$

$$\rightarrow R_B = 1.67\text{ k}\Omega$$

The voltage drop on R_B is:

1.67 kΩ × 90 μA = 150 mV

$$V_{OV\,12} = \sqrt{\frac{2\times 90\ \mu}{160\ \mu\times\frac{40}{0.8}}} = 0.15\text{ V}$$

$V_{ov12} = V_{GS12} - V_{tn}$

$V_{GS12} = 0.15 + 0.7 = 0.85$ V

thus, $V_{G12,13} = V_{GS12} + I_B R_B - V_{SS}$

$= 0.85 + 0.15 - 2.5$

$= -1.5$ V

$$V_{OV\,11} = |V_{OV8}| = 0.3\text{ V}$$

$$\rightarrow V_{GS\,11} = 0.3 + 0.7 = 1\text{ V}$$

$$V_{G\,11} = -1.5 + 1 = -0.5\text{ V}$$

Finally,

$V_{G8} = V_{DD} - V_{SG8} = +2.5 + (-0.3 - 0.8)$

$= +1.4$ V

Ex: 7.22

$R_{in} = 20.2$ kΩ

$A_{vo} = 8513$ V/V

$R_o = 152\ \Omega$

With $R_s = 10$ kΩ and $R_L = 1$ kΩ

$$A_V = \frac{20.2}{20.2+10}\times 8513\times\frac{1}{(1+0.152)}$$

$= 4943$ V/V

Ex 7.23

$$\frac{i_{e8}}{i_{b8}} = \beta_8 + 1 = 101$$

$$\frac{i_{b8}}{i_{c7}} = \frac{R_5}{R_5 + R_{i4}} = \frac{15.7}{15.7+303.5} = 0.0492$$

$$\frac{i_{c7}}{i_{b7}} = \beta_7 = 100$$

$$\frac{i_{b7}}{i_{c5}} = \frac{R_3}{R_3 + R_{i3}} = \frac{3}{3+234.8} = 0.0126$$

$$\frac{i_{c5}}{i_{b5}} = \beta_5 = 100$$

$$\frac{i_{b5}}{i_{c2}} = \frac{R_1 + R_2}{R_1 + R_2 + R_{i2}} = \frac{40}{40+5.05} = 0.8879$$

$$\frac{i_{c2}}{i_1} = \beta_2 = 100$$

Thus the overall current gain is:

$$\frac{i_{e8}}{i_1} = 101\times 0.0492\times 100\times 0.0126\times 100..$$

$\times\ 0.8879 \times 100$

$= 55993$ A/A

and the overall voltage gain is

$$\frac{V_O}{V_{id}} = \frac{R_6}{R_{i1}}\cdot\frac{i_{e8}}{i_1}$$

$$= \frac{3}{20.2}\times 55593 = 8256\text{ V/V}$$

Ex: 8 . 1

$$A_M = \frac{-R_G}{R_G + R_{\text{sig}}} \times g_m(R_L \| R_D)$$

$$= -\frac{10}{10 + 0.1} \times 2 \times \frac{10\text{ K}}{2}$$

$A_M = -9.9$ V/V

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{\text{sig}})}$$

$$= \frac{1}{2\pi \times 1\ \mu \times (10 + 0.1\text{ M})} = 0.016\text{ Hz}$$

$$f_{P2} = \frac{1}{2\pi C_S / g_m} = \frac{1}{2\pi \times 1\ \mu / 2\text{ m}} = 318\text{ Hz}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_L + R_D)} = \frac{1}{2\pi 1\ \mu \times (10 + 10)}$$

$= 8$ Hz

$f_L \simeq f_{P2} = 318$ Hz

Ex: 8 . 2

$C_{C1} = C_E = C_{C2} = 1\ \mu$F

$$g_m = 40\frac{\text{mA}}{\text{V}} \rightarrow I_C = 40\text{ m} \times 25\text{ m} = 1\text{ mA}$$

$$r_\pi = 2.5\text{ k}\Omega = \frac{\beta}{g_m} \Rightarrow \beta = 100$$

$$r_e = \frac{V_T}{I_E} = \frac{25\text{ mV}}{1\text{ mA}} = 25\ \Omega$$

$$f_{P1} = \frac{1}{2\pi C_{C1}[R_B \| r_\pi + R_{\text{sig}}]}$$

$$= \frac{1}{2\pi 1\mu[100\text{ K} \| 2.5\text{ K} + 5\text{ K}]}$$

$f_{P1} = 21.4$ Hz

$$f_{P2} = \frac{1}{2\pi \cdot C_E\left[r_e + \dfrac{R_B \| R_{\text{sig}}}{\beta + 1}\right]}$$

$$f_{P2} = \frac{1}{2\pi \cdot 1\mu\left[25 + \dfrac{100\text{ K} \| 5\text{ K}}{101}\right]}$$

$f_{P2} = 2.2$ KHz

$$f_{P3} = \frac{1}{2\pi \cdot C_{C2} \cdot (R_C + R_L)}$$

$$= \frac{1}{2\pi \cdot 1\mu(8\text{ K} + 5\text{ K})}$$

$f_{P3} = 12.2$ Hz

Ex: 8 . 3

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}}{10 \times 10^{-9}}$$

$$= 3.45 \times 10^{-3}\text{F/m}^2 = 3.45\text{ fF}/\mu\text{m}^2$$

$$C_{OV} = WL_{OV}C_{ox} = 10 \times 0.05 \times 3.45$$

$= 172$ fF

$$C_{gs} = \frac{2}{3}WLC_{ox} + C_{OV}$$

$$= \frac{2}{3} \times 10 \times 1 \times 3.45 + 1.72 = 24.72\text{ fF}$$

$C_{gd} = C_{OV} = 1.72$ fF

$$C_{sb} = \frac{C_{sbo}}{\sqrt{1 + \dfrac{V_{SB}}{V_O}}} = \frac{10}{\sqrt{1 + \dfrac{1}{0.6}}} = 6.1\text{ fF}$$

$$C_{db} = \frac{C_{dbo}}{\sqrt{1 + \dfrac{V_{DB}}{V_O}}} = \frac{10}{\sqrt{1 + \dfrac{2 + 1}{0.6}}} = 4.1\text{ fF}$$

Ex: 8 . 4

Peak current occurs At $V_I = V_{th} = 5$ V

$$\text{i Peak} = \frac{1}{2}K_n'\left(\frac{W}{L}\right)_n (V_{th} - V_{tn})^2$$

$$= \frac{1}{2} \times 20 \times 20(5 - 2)^2 = 1800$$

Ex: 8 . 5

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{25\text{ mV}} = 40\frac{\text{mA}}{\text{V}}$$

$$C_{de} = \tau_F \cdot g_m = 20 \times 10^{-12} \times 40 \times 10^{-3}$$

$= 0.8$ pF

$C_{je} = 2\ C_{jeo} = 2 \times 20 = 40$ fF

$C_\pi = C_{de} + C_{je} = 0.84$ pF

$$C\mu = \frac{C_{\mu O}}{\left(1 + \dfrac{V_{CB}}{V_{OC}}\right)^{m_{CBJ}}}$$

$$= \frac{20\text{ fF}}{\left(1 + \dfrac{2}{0.5}\right)^{0.33}} = 12\text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C\pi + C\mu)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(0.84 + 0.012) \times 10^{-12}} = 7.47\text{ GHz}$$

Ex: 8 . 6

$$|h_{fe}| \simeq \frac{f_T}{f} \rightarrow 10 = \frac{f_T}{50}$$

$\Rightarrow f_T = 500$ MHz

$$f_T = \frac{g_m}{2\pi(C\pi + C\mu)}$$

$$C\pi + C\mu = \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6} = 12.7\text{ pF}$$

$C\pi = 12.7 - C\mu = 12.7 - 2 = 10.7$ pF

Ex:8 . 7 Diffusion component of $C\pi$ at I_C of 1 mA

$= 10.7 - 2 = 8.7$ pF

Since C_{de} is proportional to I_C, then:

$C_{de}\,(I_C = 0.1\text{ mA}) = 0.87\text{ pF}$
$C\pi\,(I_C = 0.1\text{ mA}) = 2.87\text{ pF}$

$$f_T(I_C = 0.1\text{ mA}) = \frac{g_m}{2\pi(C\pi + C\mu)}$$

$$= \frac{4 \times 10^{-3}}{2\pi(2.87 + 2) \times 10^{-12}}$$

$$= 130.7\text{ MHz}$$

Ex: 8 . 8

$$A_M = -\frac{R_G}{R_G + R_{\text{sig}}} g_m R_L^{'}$$

$R_L^{'} = 7.14\text{ k}\Omega,\ g_m = 1\text{ mA/V}$
$R_{\text{sig}} = 10\text{ k}\Omega$

$$A_M = \frac{-4.7\text{ M}\Omega}{(4.7 + 0.01)\text{ M}\Omega} \times 1 \times 7.14$$

$$= -7.12\text{ V/V}$$

$$f_H = \frac{1}{2\pi C_{\text{in}}(R_{\text{sig}} || R_G)} \quad C_{\text{in}} = 4.26\text{ pF}$$

$$f_H = \frac{1}{2\pi \times 4.26(10\text{ K} \,\|\, 4.7\text{ M})} = 3.7\text{ MHz}$$

Ex: 8 . 9

$C_{gs} = 1\text{ pF}$

$C_{eq} = (1 + g_m R_L^{'})C_{gd} = (1 + 1 \times 7.14)$

$C_{gd} = 8.14 C_{gd}$

$$f_T \geq 1\text{ MHz} \Rightarrow \frac{1}{2\pi C_{\text{in}}(R_{\text{sig}} || R_G)} \geq 1\text{ MHz}$$

$C_{\text{in}} = C_{gs} + C_{eq} = 1\text{ pF} + 8.14\, C_{gd(\text{PF})}$

$$\frac{1}{2\pi(1 + 8.14 C_{gd})\text{pF}(100\text{ K} \,\|\, 4.7\text{ M})} \geq 1\text{ MHz}$$

$\Rightarrow 1.63 \geq 1 + 8.14 C_{gd}$

$C_{gd} \leqslant 0.077\text{ pF}$ or $C_{gd} \leqslant 77\text{ fF}$

Ex: 8 . 10

① $A_M = -39/2 = -19.5\text{ V/V}$

$$A_M = \frac{-R_B}{R_B + R_{\text{sig}}} \cdot \frac{r_\pi \cdot g_m \cdot R_L^{'}}{r_\pi + r_X + (R_B + R_{\text{sig}})}$$

$$A_M = \frac{-100}{100 + 5} \cdot \frac{2.5 \times 40 \cdot 10^{-3} \times R_L^{'}}{2.5 + 0.05 + (100 \,\|\, 5)}$$

$$= -0.013 \times R_L^{'}$$

$\Rightarrow R_L^{'} = 1.5\text{ k}\Omega = r_o \,\|\, R_C \,\|\, R_L$

$1.5\text{ k}\Omega = (100 \,\|\, 8 \,\|\, R_L)\text{ k}\Omega$

$= 7.4\text{ k} \,\|\, R_L$

$\rightarrow R_L = 1.9\text{ k}\Omega$

② $$f_H = \frac{1}{2\pi C_{\text{in}} \cdot R_{\text{sig}}^{'}} \quad R_{\text{sig}}^{'} = 1.65\text{ k}\Omega$$

$C_{\text{in}} = C\pi + C\mu(1 + g_m R_L^{'})$

$C_{\text{in}} = 7 + 1\,(1 + 40 \times 10^{-3} + 1.5 + 10^3)$

$= 68\text{ pF}$

$$\Rightarrow f_H = \frac{1}{2\pi\, 68\text{p} \cdot 1.65\text{ K}} = 1.42\text{ MHz}$$

Ex: 8 . 11 Using equations 8 . 61 and 8 . 63 we can write the general form of the transfer function of a direct-coupled amplifier as:

$$A(s) = \frac{A_{DC}}{1 + \dfrac{s}{2\pi f_{3dB}}}$$ where A_{DC} is the DC gain of the amplifier and f_{3dB} is the upper 3dB frequency of the amplifier.

In this case we have $A_{DC} = 1000$ and $f_{3dB} = 100\text{ KHz} = 10^5\text{ Hz}$

$$\text{Therefore } A(s) = \frac{1000}{1 + \dfrac{s}{2\pi \times 10^5}}$$

Ex: 8 . 12

For this amplifier we have:

$$H(s) = \frac{A_M}{\left(1 + \dfrac{s}{\omega_{P1}}\right)\left(1 + \dfrac{s}{\omega_{P2}}\right)}$$

By definition at $\omega = \omega_H$ we have

$$|H(j\omega_H)|^2 = \frac{A_M^2}{2} \Rightarrow$$

$$\frac{A_M^2}{\left(1 + \left(\dfrac{\omega_H}{\omega_{P1}}\right)^2\right)\left(1 + \left(\dfrac{\omega_H}{\omega_{P2}}\right)^2\right)} = \frac{A_M^2}{2} \Rightarrow$$

$$\left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{\omega_{P2}}\right)^2\right] = 2$$

If $\omega_{P2} = K\omega_{P1}$ and $\omega_H = 0.9\,\omega_{P1}$, then

$$\left[1 + \left(\frac{0.9\omega_{P1}}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{0.9\omega_{P1}}{K\omega_{P1}}\right)^2\right] = 2$$

$$(1 + 0.9^2)\left(1 + \left(\frac{0.9}{\text{K}}\right)^2\right) = 2$$

$$1 + \left(\frac{0.9}{\text{K}}\right)^2 = 1.1 \Rightarrow \left(\frac{0.9}{\text{K}}\right)^2 = 0.1 \Rightarrow \text{K} = 2.78$$

If $\omega_H = 0.99\,\omega_{P1}$, then :

$$\left[1 + \left(\frac{0.99\omega_{P1}}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{0.99\omega_{P1}}{\text{K}\omega_{P1}}\right)^2\right] = 2$$

$$(1 + 0.99^2)\left(1 + \left(\frac{0.99}{\text{K}}\right)^2\right) = 2 \Rightarrow$$

$$1 + \left(\frac{0.99}{\text{K}}\right)^2 = 1.01 \Rightarrow \left(\frac{0.99}{\text{K}}\right)^2 = 0.01 \Rightarrow$$

K = 9.88

Ex: 8.13

From Exercise 8.12 we have:

$$\left[1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{\omega_{P2}}\right)^2\right] = 2 \text{ and}$$

$\omega_{P2} = K\omega_{P1}$

$$K = 1 \Rightarrow \left[1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{\omega_{P2}}\right)^2\right] = 2$$

$$1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2 = \sqrt{2} \Rightarrow \left(\frac{\omega_H}{\omega_{P1}}\right)^2 = \sqrt{2}-1$$

$\omega_H = \sqrt{\sqrt{2}-1}\ \ \omega_{P1} = 0.64\,\omega_{P1}$ (exact value)

(note that in this case the zeros are at $S = \infty$) we have :

$$\omega_H = 1/\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}} = 1/\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{K^2\omega_{P1}^2}}$$

$$\omega_H = \omega_{P1}/\sqrt{1+\frac{1}{K^2}}$$

For $K = 1 \Rightarrow \omega_H = \frac{1}{\sqrt{2}}\omega_{P1} = 0.71\,\omega_{P1}$

For the case of K = 2, the exact value of ω_H can be found from the following equation:

$$\left[1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{K\omega_{P1}}\right)^2\right] = 2$$

Assuming $\frac{\omega_H}{\omega_{P1}} = X$ we have

$$(1+X^2)\left(1+\frac{X^2}{K^2}\right) = 2 \Rightarrow$$

$$\frac{1}{K^2}X^4 + \left(1+\frac{1}{K^2}\right)X^2 + 1 = 2 \Rightarrow$$

$$X^4 + (K^2+1)X^2 - K^2 = 0 \Rightarrow$$

$$X^2 = \frac{-(K^2+1)+\sqrt{(K^2+1)^2+4K^2}}{2}$$

$$\frac{\omega_H}{\omega_{P1}} = \sqrt{\frac{-(K^2+1)+\sqrt{(K^2+1)^2+4K^2}}{2}} \quad (*)$$

For $K = 2 \Rightarrow \frac{\omega_H}{\omega_{P1}} = 0.84 \Rightarrow \omega_H = 0.84\omega_{P1}$

In this case, the approximate value of ω_H is:

$$\omega_H = \omega_{P1}/\sqrt{1+\frac{1}{K^2}} = 0.89\omega_{P1}$$

For K = 4, using equation (*), the exact value of ω_H is:

$\omega_H = 0.95\,\omega_{P1}$

In this case, the approximate value of ω_H is :

$$\omega_H = \omega_{P1}/\sqrt{1+\frac{1}{K^2}} = 0.97\omega_{P1}$$

Ex: 8.14

We have $A_M =$ −10.8 V/V and $f_H \simeq$ 128.3 KHz, therefore, the gain-bandwidth product is:

10.8 × 128.3 = 1.3856 MHz ≃ 1.39 MHz

Now we want to find the value of R_L' that will result in f_H = 180 KHz. We have:

$$\tau_{gs} + \tau_{gd} = \frac{1}{\omega_H} = \frac{1}{2\pi f_H}$$

$$\tau_{gs} + \tau_{gd} = \frac{1}{2\pi \times 180\text{ KHz}} = 884.2\text{ nsec}$$

$$\tau_{gs} = 80.8\text{ nsec} \Rightarrow \tau_{gd} = 884.2 - 80.8$$

$$\tau_{gd} = 803.4\text{ nsec}$$

$$\tau_{gd} = R_{gd}C_{gd} = (R' + R_L' + g_mR_L'R')C_{gd}$$

$$R' = R_{in} \| R_{sig} = 80.8\text{ k}\Omega,\ g_m = 4\ \frac{\text{mA}}{\text{V}},$$

$C_{gd} = 1$ pF

Thus

$$803.4\text{ nsec} = (80.8\text{ k}\Omega + R_L' + 323.2R_L') + 1\text{ pF}$$

$$\Rightarrow 324.2R_L' = \frac{803.4\text{ nsec}}{1\text{ pF}} - 80.8\text{ k}\Omega$$

$$\Rightarrow R_L' = \frac{722.6\text{ k}\Omega}{324.2} = 2.23\text{ k}\Omega$$

$$\Rightarrow R_L' = 2.23\text{ k}\Omega$$

For this value of R_L' we have

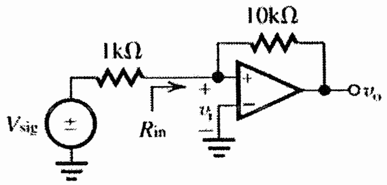
$$A_M = -\frac{R_{in}}{R_{in}+R_{sig}}(g_mR_L')$$

$$A_M = -\frac{420}{420 \times 100} \times 4 \times 2.23 = -7.2\text{ V/V}$$

Therefore, the gain-bandwidth product is:

7.2 × 180 KHz = 1.296 MHz ≃ 1.3 MHz

Ex: 8.15



Using Miller's theorem we have

$$R_{in} = \frac{10\text{ k}\Omega}{A+1},\ V_i = \frac{R_{in}}{R_{in}+1\text{ k}\Omega}V_{sig} \text{ and}$$

$V_o = -AV_i$

Assuming V_{sig} = 1 V we have

A(V/V)	$R_{in}(\Omega)$	V_i(mV)	V_o(V)	V_o/V_{sig} $\left(\frac{V}{V}\right)$
10	909	476	4.76	4.76
100	99	90	−9	−9
1000	9.99	9.9	−9.9	−9.9
10000	1	0.999	−9.99	−9.99

Ex:8 . 16 Refering to the solution of Example 8 . 10 the value of f_H determined by the exact analysis is:

$f_H = f_{P1} = 143.4$ MHz

Also,

$$A_M = -g_m \cdot R_L' = -1.25 \times 10 = -12.5 \text{ V/V}$$

Therefore the gain-bandwidth product (f_T) is:

$f_T = 143.4 \times 12.5 = 1.79$ GHz

Since f_T is less than $f_{P2} = 2.44$ GHz and $f_z =$ 40 GHz, therefore it is a good approximation of the unity gain frequency.

Ex: Referring to the solution of Example 8 . 10 if a load resistor is connected at the output halving the value of R_L', then we have

$$R_L' = \frac{r_{o1} \| r_{o2}}{2} \text{ and therefore}$$

$$|A_M| = g_m \frac{r_{o1} \| r_{o2}}{2} = 1.25 \times \frac{10}{2} = 6.25 \text{ V/V}$$

Using equation 8 . 92 and assuming $f_H \simeq f_{P1}$, we have:

$$f_H \simeq \frac{1}{2\pi \cdot [[C_{gs} + C_{gd}(1 + g_m R_L')] \cdot R_{\text{sig}}' + (C_L + C_{gd})R_L']}$$

$$f_H \simeq \frac{1}{2\pi[[20f + 5f(1 + 1.25 \times 5)] \cdot 10 \text{ K}} \quad \frac{1}{+ (25f + 5f) \times 5 \text{ K}]}$$

$f_H \simeq 223$ MHz

$f_T = |A_M| \cdot f_H = 6.25 \times 223 = 1.4$ GHz

Ex:8 . 18 Referring to the solution of Example 8 . 10

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} \text{ is } I_{D1} \text{ is increased by 4 and } V_{ov} \text{ by 2}$$

then :

$$g_m = \frac{2(4I_{D1})}{(2V_{OV1})} = 2g_{m1} = 2 \times 1.25 \frac{\text{mA}}{\text{V}}$$

$$= 2.50 \frac{\text{mA}}{\text{V}}$$

To calculate R_L'

If $R_{L1}' = r_{OQ1} \| r_{OQ2} = 10$ kΩ in example 9.10

Since $r_O = \dfrac{V_A}{I_D} \rightarrow$ increasing I_D by 4 reduces both r_{OQ1} and r_{OQ2} by 4

Thus :

$$r_{OQ1} \| r_{OQ2} = 10 \text{ K} \rightarrow \frac{r_{OQ1}}{4} \| \frac{r_{OQ2}}{4} = \frac{1}{4} (r_{OQ1} \| r_{OQ2})$$

$$R_L' = \frac{1}{4} \times 10 \text{ K} = 2.5 \text{ k}\Omega$$

$$|A_M| = g_m \cdot R_L' = 2.5 \times 2.5$$
$$= 6.25 \text{ A/A}$$

Using equation 8 . 93 and assuming $f_H \simeq f_{P1}$ we have:

$$f_H = \frac{1}{2\pi\{[C_{gs} + C_{gd}(1 + g_m R_L')]R_{\text{sig}} + (C_L + C_{gd})R_L'\}}$$

$$f_H = \frac{1}{2\pi\{[20 \text{ f} + 5 \text{ f}(1 + 6.25)]10 \text{ K} + (25 + 5) \text{ f} \times 2.5 \text{ K}}$$

$f_H = 250 \text{ MHz} \Rightarrow \quad f_{P1} \simeq f_H = 250$ MHz

$f_T \simeq |A_M| \cdot f_H = 6.25 \times 250 = 1.56$ GHz

Ex: 8 . 19

$$r_{onpn} = \frac{V_{An}}{I} = \frac{130 \text{ V}}{1 \text{ mA}} = 130 \text{ k}\Omega$$

$$r_{opnp} = \left|\frac{V_{Ap}}{I}\right| = \frac{50 \text{ V}}{1 \text{ mA}} = 50 \text{ k}\Omega$$

$$R_L' = r_{onpn} \| r_{opnp} = 130 \text{ k}\Omega \| 50 \text{ k}\Omega$$

$$R_L' = 36 \text{ k}\Omega$$

$$g_m = \frac{I}{V_T} = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \frac{\text{mA}}{\text{V}}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{40 \frac{\text{mA}}{\text{V}}} = 5 \text{ k}\Omega$$

(a) From equation 8 . 97 we have:

$$A_M = -\frac{r_\pi}{R_{\text{sig}} + r_x + r_\pi}(g_m R_L')$$

$$= -\frac{5}{36 + 0.2 + 5}(40 \times 36 \text{ k}\Omega) \simeq -175 \frac{\text{V}}{\text{V}}$$

$$A_M = -175 \frac{\text{V}}{\text{V}}$$

(b) Using Miller's theorem we have:

$$C_{\text{in}} = C_\pi + C_\mu(1 + g_m R_L')$$

$$= 16 \text{ pF} + 0.3 \text{ pF}(1 + 40 \times 36) = 448 \text{ pF}$$

$$C_{\text{in}} = 448 \text{ pF}$$

$$f_H \simeq \frac{1}{2\pi C_{\text{in}} R_{\text{sig}}'} = \frac{1}{2\pi C_{\text{in}}[r_\pi \| (R_{\text{sig}} + r_x)]}$$

$$f_H = \frac{1}{2\pi \times 448 \text{ pF} \underbrace{[5 \| (36 + 0.2)]}_{\simeq 4.3 \text{ k}\Omega}} = 82.6 \text{ kHz}$$

(c) Using the method of open-circuit time constants, from equation 8 . 100 we have:

$$\tau_H = C_\pi R_{\text{sig}}' + C_\mu[(1 + g_m R_L')R_{\text{sig}}' + R_L'] + C_L R_L'$$

We have $R_{\text{sig}}' = r_\pi \| (R_{\text{sig}} + r_x) \simeq 4.3$ kΩ

$R_L' = r_{onpn} \| r_{opnp} = 36$ kΩ

Thus:

$$\tau_H = 16 \times 4.3 + 0.3[(1 + 40 \times 36)4.3 + 36]$$
$$+5 \times 36$$

$$\tau_H = 2.12 \text{ nsec}$$

$$f_H = \frac{1}{2\pi\tau_H} \simeq 75.1 \text{ kHz}$$

(d) Using equations 8.102, 8.103 and 8.104 we have:

$$f_Z = \frac{1}{2\pi}\frac{g_m}{C_\mu} = \frac{1}{2\pi}\frac{40\ \frac{\text{mA}}{\text{V}}}{0.3 \text{ pF}} = 21.2 \text{ GHz}$$

$f_{p1} \simeq$

$$\frac{1}{2\pi}\frac{1}{[C_\pi + C_\mu(1 + g_mR_L')]R_{\text{sig}}' + (C_L + C_\mu)R_L'}$$

$$\Rightarrow f_{p1} = 75.1 \text{ kHz}$$

$f_{p2} \simeq$

$$\frac{1}{2\pi}\frac{[C_\pi + C_\mu(1 + g_mR_L')]R_{\text{sig}}' + (C_L + C_\mu)R_L'}{[C_\pi(C_L + C_\mu) + C_LC_\mu]R_{\text{sig}}'R_L}$$

$$f_{p2} = 25.2 \text{ MHz}$$

Since $f_{p1} << f_z$ and $f_{p1} << f_{p2}$, thus $f_H \simeq f_{p1} = 75.1$ KHz

(e) $f_t \simeq |A_M| f_H = 175 \times 75.1 \text{ kHz} = 13.1 \text{ MHz}$

$f_t = 13.1 \text{ MHz}$

Ex:8.20 Referring to the solution of Example 8.11 we have $f_T = |A_M| \cdot f_H$, since $|A_M|$ remains the same as that of the example, to place f_t at 2 GHz we need

$$2 \text{ GHz} = f_T = |A_M| \cdot f_H = \frac{|A_M|}{2\pi(C_L + C_{gd})R_L'}$$

$$\Rightarrow C_L = \frac{|A_M|}{2\pi R_L' \cdot f_T} - C_{gd}$$

$$= \frac{12.5}{2\pi \times 10 \text{ k}\Omega \times 2 \text{ GHz}} - 5 \times \text{fF}$$

$$\Rightarrow C_L = 94.5 \text{ fF}$$

Ex:8.21 For a CS amplifier fed with $R_{\text{sig}} = 0$ we know that:

$$f_t = \frac{g_m}{2\pi(C_L + C_{gd})}$$

and $f_z = \dfrac{g_m}{2\pi C_{gd}}$

Therefore,

$$\frac{f_z}{f_t} = \frac{g_m/(2\pi C_{gd})}{g_m/[2\pi(C_L + C_{gd})]} = \frac{C_L + C_{gd}}{C_{gd}}$$

$$\frac{f_z}{f_t} = \frac{C_L}{C_{gd}} + 1 \Rightarrow \frac{f_z}{f_t} = 1 + \frac{C_L}{C_{gd}}$$

Ex:8.22 $R_L = 500 \text{ k}\Omega$, and from Example 8.12

$g_m = 1.25 \text{ mA/V}$, $r_o = 20 \text{ k}\Omega$,

$C_{gs} = 20 \text{ fF}$, $C_{gd} = 5 \text{ fF}$ $C_L = 15 \text{ fF}$,

$R_{\text{sig}} = 10 \text{ k}\Omega$, $R_L = 20 \text{ k}\Omega$.

$$R_{\text{in}} = \frac{1}{g_m} + \frac{R_L}{g_mr_o} = \frac{1}{1.25 \text{ m}} + \frac{500 \text{ K}}{1.25 \times 20}$$

$$= 20.8 \text{ k}\Omega$$

$$G_v = \frac{R_L}{R_{\text{sig}} + R_{\text{in}}} = \frac{500}{10 + 20.8} = 16.2 \text{ V/V}$$

To obtain f_H:

$R_{gs} = R_{\text{sig}} \| R_{\text{in}} = 10 \text{ K} \| 20.8 \text{ K} = 6.75 \text{ k}\Omega$

$R_{gd} = R_L \| R_O$

$R_O = r_O + R_{\text{sig}} + (g_mr_O) \cdot R_{\text{sig}} = 280 \text{ K}$

(same as in Eq. 8.12)

$R_{gd} = 500 \text{ K} \| 280 \text{ K} = 179.5 \text{ k}\Omega$

$$\tau_H = C_{gs} \cdot R_{gs} + (C_{gd} + C_L) \cdot R_{gd}$$
$$= 20 \text{ f} \times 6.75 \text{ K} + (5 \text{ f} + 15 \text{ f}) \times 179.5 \text{ K}$$

$$\tau_H = 0.135 \text{ ns} + 3.59 \text{ ns} = 3.72 \text{ ns}$$

Thus, $f_H = \dfrac{1}{2\pi T_H} = 42.7 \text{ MHz}$

Ex:8.23 a) Low-frequency gain

A_V: $g_mr_O 40$; $R_L = r_O$

CS − Amplifier: $A_V = -g_m(R_L \| r_O)$

Since $R_L = r_O \rightarrow$

$$A_V = -\frac{1}{2}(g_mr_O) = -\frac{1}{2} \times 40 = -20 \text{ V/V}$$

CASCODE Amplifier:

$A_V = -g_m(R_O \| R_L) = -g_m(R_O \| r_O)$

where $R_O = r_{O2} + r_{O1} + (g_{m2}r_{O2})r_{O1}$

since $r_{O2} = r_{O1} = r_O$ and $g_{m2} = g_{m1} = g_m$

$R_O = 2r_O + (g_mr_O) \cdot r_O = r_O(2 + g_mr_O)$

$= 42r_O$

$\Rightarrow A_V = -g_m(42r_O \| r_O) \simeq -g_m \cdot r_O$

$= -40 \text{ V/V}$

$$\frac{A_{V\ \text{CASCODE}}}{A_{VCS}} = \frac{-40}{-20} = 2$$

b) f_H : Neglect components of τ_H that do not include R_{sig}; also $C_{gd} = 0.25\ C_{gs}$

CS - Amplifier:

$$\tau_H = C_{gs} \cdot R_{\text{sig}} + C_{gd}[(1 + g_m \cdot R_L')R_{\text{sig}} + R_L']$$
$$+ (C_L + C_{ds})R_L'$$

where: $R_L' = r_O \| r_L = \dfrac{r_O}{2}$

$$\Rightarrow \tau_H = C_{gs} \cdot R_{\text{sig}} + C_{gd}\left[\left(1 + \frac{g_mr_O}{2}\right)R_{\text{sig}}\right]$$

since $C_{gd} = 0.25\ C_{gs}$ and $g_mr_O = 40$

$$\Rightarrow \tau_H = C_{gs} \cdot R_{sig} + 0.25 \times C_{gs} \times 21 \times R_{sig}$$
$$= R_{sig} \times C_{gs} \times 6.25$$

CASCODE - Amplifier:
Using Eq8.118 and neglecting the terms that do not include R_{sig}

$$\tau_H = R_{sig}[C_{gs} + C_{gd}(1 + g_m R_{d1})]$$

$$R_{d1} = r_O \| \left(\frac{r_O + R_L}{g_m r_O}\right) = r_O \| \frac{2r_O}{g_m r_O}$$
$$= r_O \| \frac{r_O}{20} \simeq \frac{r_O}{20}$$

$$\rightarrow \tau_H = R_{sig}\left[C_{gs} + 0.25 \times C_{gs}\left(1 + \frac{g_m r_O}{20}\right)\right]$$
$$= R_{sig} \cdot C_{gs} \times 1.75$$

$$\frac{f_{H\ \mathrm{CASCODE}}}{f_{H\ \mathrm{CS}}} = \frac{\tau_{cs}}{\tau_{\mathrm{CASCODE}}}$$
$$= \frac{R_{sig} \cdot C_{gs} \times 6.25}{R_{sig} \cdot C_{gs} \times 1.75} = 3.6$$

c) $f_T = |A_V| \cdot f_H$

$$\frac{f_{T\ \mathrm{CASCODE}}}{f_{T\ CS}} = \left(\frac{|A_V|_{\mathrm{CASCODE}}}{|A_V|_{CS}}\right) \times \left(\frac{f_{H\ \mathrm{CASCODE}}}{f_{H\ CS}}\right)$$
$$= 2 \times 3.6$$
$$= 7.2$$

Ex:8.24 Referring to the solution of Exercise8.19 we have:

$$g_m = 40\ \frac{\text{mA}}{\text{V}} \text{ and } r_\pi = 5\ \text{k}\Omega$$

Note that for the cascode amplifier considered in this exercise:

$$r_{\pi 1} = r_{\pi 2} = r_\pi = 5\ \text{k}\Omega$$

$$g_{m1} = g_{m2} = g_m = 40\ \frac{\text{mA}}{\text{V}}$$

$$R_{in} = r_{\pi 1} + r_x = 5\ \text{k}\Omega + 0.2\ \text{k}\Omega = 5.2\ \text{k}\Omega$$

$$A_o = g_m \cdot r_o = 40 \times 130 = 5200\ \text{V/V}$$

$$R_{o1} = r_{o1} = r_o = 130\ \text{k}\Omega$$

$$R_{in2} \simeq r_{e2} \cdot \frac{r_{o2} + R_L}{r_{o2} + \dfrac{R_L}{\beta + 1}} = \frac{5\ \text{K}}{200 + 1}$$
$$\times \frac{130 + 50}{130 + \dfrac{50}{201}}$$

$$R_{in2} \simeq 35\ \Omega$$

$$R_o \simeq \beta_2 r_{O2} = 200 \times 130\ \text{k}\Omega = 26\ \text{M}\Omega$$

$$A_M = \frac{-r_\pi}{r_\pi + r_x + R_{sig}} \cdot g_m(\beta r_o \| R_L)$$

$$A_M \simeq -242\ \frac{\text{V}}{\text{V}}$$

To calculate f_H we use the method of open-circuit time constants. From Figure8.30 we have:

$$R'_{sig} = r_{\pi 1} \| (r_{x1} + R_{sig}) = 5\ \text{K} \| (0.2 + 36)$$
$$R'_{sig} = 4.4\ \text{k}\Omega$$
$$R_{\pi 1} = R'_{sig} = 4.4\ \text{k}\Omega$$

$$R_{O1} = r_{O1} \| R_{in2} = r_O \| \left[r_{e2}\left(\frac{r_{O2} + R_L}{r_{O2} + \dfrac{R_L}{\beta_2 + 1}}\right)\right]$$

$$R_{O1} = 130\ \text{K} \| 35\ \Omega \simeq 35\ \Omega$$
$$R_{\mu 1} = R'_{sig}(1 + g_{m1} R_{O1}) + R_{O1}$$
$$R_{\mu 1} = 10.6\ \text{k}\Omega$$
$$\tau_H = C_{\pi 1} \cdot R_{\pi 1} + C_{\mu 1} \cdot R_{\mu 1} + (C_{cs1} + C_{\pi 2})$$
$$R_{O1} + (C_L + C_{cs2} + C_{\mu 2}) \cdot (R_L \| R_O)$$
$$\tau_H = 16\text{p} \times 4.4\ \text{K} + 0.3\ \text{p} \times 10.6\ \text{K} + (0 + 16\ \text{p})$$
$$\times 35 + (5\ \text{p} + 0 + 0.3\ \text{p})(50\ \text{K} \| 26\ \text{M})$$
$$\tau_H = 339\ \text{ns}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 339\ \text{ns}} \simeq 469\ \text{KHz}$$

$$f_T \simeq |A_M| \cdot f_H = 242 \times 469\ \text{KHz}$$
$$\simeq 113.5\ \text{MHz}$$

Compared to the CE amplifier in Exercise 8.19 $|A_M|$ has increased from 175 V/V to 242 V/V, f_H has increased from 75.1 KHz to 469 KHz and f_T has increased from 13.1 MHz to 113.5 MHz

To increase f_H to 1 MHz we need:

$$\tau_H = \frac{1}{2\pi f_H} = 159\ \text{ns, thus}$$

$$16\ \text{p} \times 4.4\ \text{K} + 0.3\ \text{p} \times 10.6\ \text{K} + 16\ \text{p} \times 35$$
$$+ (C_L + 0.3\ \text{p}) \cdot (50\ \text{K} \| 26\ \text{M}) = 159\ \text{ns}$$
$$\Rightarrow C_L = 1.4\ \text{pf}$$

Ex:8.25 $R'_L = R_L \| r_o = 20\ \text{K} \| 20\ \text{K}$
$$= 10\ \text{K}$$

From Eq.8.121 we have:

$$A_M = \frac{g_m R'_L}{1 + g_m R'_L} = \frac{1.25 \times 10}{1 + 1.25 \times 10} = 0.93\ \frac{\text{V}}{\text{V}}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{1}{2\pi} \times \frac{1.25\ \text{m}}{(20\ \text{f} + 5\ \text{f})}$$
$$= 8\ \text{GHz}$$

$$f_z = \frac{1}{2\pi} \cdot \frac{g_m}{C_{gs}} = \frac{1}{2\pi} \cdot \frac{1.25\ \text{m}}{20\ \text{f}} \simeq 10\ \text{GHz}$$

$$R_{gd} = R_{sig} = 10\ \text{K}$$

$$R_{gs} = \frac{R_{sig} + R'_L}{1 + g_m \cdot R'_L} = \frac{10\ \text{K} + 10\ \text{K}}{1 + 1.25 \times 10}$$
$$= 1.48\ \text{k}\Omega$$

Ex:8.28

$$f_Z = \frac{1}{2\pi \cdot C_{SS} \cdot R_{SS}}$$

$$= \frac{1}{2\pi \cdot (0.4\ \text{p})25\ \text{K}} = 15.9\ \text{MHz}$$

Ex:8.29 For a loaded bipolar differential amplifier:

$$A_d = \frac{1}{2} g_m \cdot r_O$$

where,

$$g_m = \frac{I/2}{V_T} = \frac{0.5\ \text{mA}}{25\ \text{mV}} = 20\ \text{mA/V}$$

$$r_O = \frac{V_A}{I/2} = \frac{100\ \text{V}}{0.5\ \text{mA}} = 200\ \text{k}\Omega$$

$$\Rightarrow A_d = \frac{1}{2} \times 20\ \frac{\text{mA}}{\text{V}} \times 200\ \text{k}\Omega$$

$$= 2000\ \text{V/V}$$

The dominant pole is set by the output load capacitance

$$f_z = \frac{1}{2\pi \cdot C_L(r_{O2} \| r_{O4})}$$

$$= \frac{1}{2\pi \times 2\ \text{pF} \times (200\ \text{K} \|\ 200\ \text{K})\Omega}$$

$$= 0.796\ \text{MHz} \simeq 0.8\ \text{MHz}$$

Ex:8.30 (a)

$$A_M = -g_m \times R'_L = -g_m(R_L \|\ r_o)$$

$$A_M = -2\ \frac{\text{mA}}{\text{V}} \times (20\ \text{k}\Omega \|\ 20\ \text{k}\Omega) = -20\ \frac{\text{V}}{\text{V}}$$

To calculate τ_H using the method of open-circuit time-constants we can employ Eq (8.84

$$\tau_H = C_{gs} \cdot R_{\text{sig}} + C_{gd}[R_{\text{sig}}(1 + g_m R'_L) + R'_L] + C_L \cdot R'_L$$

$$\tau_H = 20\ \text{f} \times 20\ \text{K} + 5\ \text{f}[20\ \text{K}(1 + 20) + 10\ \text{K}] + 5\ \text{f} \times 10\ \text{K}$$

$$\tau_H = 2.6\ \text{ns} \Rightarrow f_H = \frac{1}{2\pi\tau_H} = 61.2\ \text{MHz}$$

The gain-bandwidth product is:

$$\text{GBP} = 20 \times 61.2\ \text{M} = 1.22\ \text{GHz}$$

(b) With source degeneration of $R_S = \frac{2}{g_m}$

$$R_O = r_O[1 + g_m R_S] = 3r_O$$

$$R_O = 3 \times 20\ \text{k}\Omega = 60\ \text{k}\Omega$$

$$A_M = -g_m r_O \times \frac{R_L}{R_L + R_O} = -2 \times 20 \times \frac{20}{20 + 60} = -10\ \frac{\text{V}}{\text{V}}$$

$$G_m = \frac{g_m}{1 + g_m R_S} = \frac{g_m}{1 + 2} = \frac{g_m}{3} = \frac{2}{3}\ \frac{\text{mA}}{\text{V}}$$

Using Eq (8.153 to (8.157 we have:

$$R'_L = R_L \|\ R_O = 20\ \text{K} \|\ 60\ \text{K} = 15\ \text{k}\Omega$$

$$R_{gd} = R_{\text{sig}}(1 + G_m R'_L) + R'_L = 235\ \text{k}\Omega$$

$$R_{gs} \simeq \frac{R_{\text{sig}} + R_S}{1 + g_m R_S \times \dfrac{r_O}{r_O + R_L}} = \frac{R_{\text{sig}} + R_S}{1 + 2 \times \dfrac{20}{20 + 20}}$$

$$= \frac{R_{\text{sig}} + R_S}{2}$$

$$R_S = \frac{2}{g_m} = 1\ \text{k}\Omega \Rightarrow R_{gs} = \frac{20 + 1}{2} = 10.5\ \text{k}\Omega$$

$$\tau_H = C_{gs} R_{gs} + C_{gd} \cdot R_{gd} + C_L R'_L$$

$$= 20\ \text{f}(10.5\ \text{K} + 5\ \text{f} \times 235\ \text{K} + 5\ \text{f} \times 15\ \text{K})$$

$$\tau_H = 1.46\ \text{ns} \Rightarrow f_H = \frac{1}{2\pi\tau_H} = 109\ \text{MHz}$$

$$GBP = 10 \times 109\ \text{MHz} \simeq 1.1\ \text{GHz}$$

Ex:8.31

$$R_{\text{in}} = (\beta_1 + 1)(r_{e1} + r_{e2})$$

Since in this case $r_{e1} = r_{e2} = r_e$ and $\beta_1 = \beta_2 = \beta$ we have

$$R_{\text{in}} = r_\pi + r_\pi = 2r_\pi = \frac{2\beta V_T}{I_C} \simeq 10\ \text{k}\Omega$$

we have:

$$\frac{V_O}{V_{\text{Sig}}} = \frac{1}{2}\left(\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{Sig}}}\right) g_m R_L$$

$$\frac{V_O}{V_{\text{Sig}}} = \frac{1}{2}\left(\frac{10}{10 + 10}\right)\frac{V_T}{I_C} R_L = 50\ \frac{\text{V}}{\text{V}}$$

we have:

$$f_{P1} = \frac{1}{2\pi\left(\dfrac{C_\pi}{2} + C_\mu\right)(R_{\text{sig}} \|\ 2r_\pi)}$$

$$f_{P1} = \frac{1}{2\pi\left(\dfrac{6\ \text{pF}}{2} + 2\ \text{pF}\right)(10\ \text{K} \|\ 10\ \text{K})}$$

$$\simeq 6.4\ \text{MHz}$$

we have:

$$f_{P2} = \frac{1}{2\pi C_\mu R_L} = \frac{1}{2\pi \times 2\ \text{pF} \times 10\ \text{K}}$$

$$\simeq 8\ \text{MHz}$$

Therefore, the transfer function of this CC-CB amplifier is:

$$A(s) = \frac{A_M}{\left(1 + \dfrac{s}{2\pi f_{P1}}\right)\left(1 + \dfrac{s}{2\pi f_{P2}}\right)}$$

$$|A(s)|_{s = j\omega_H} = \frac{|A_M|}{\sqrt{2}} \text{ or } |A(s)|^2_{s = j\omega_H} = \frac{A_M^2}{2}$$

Thus:

$$\frac{A_M^2}{\left(1+\frac{(2\pi f_M)^2}{(2\pi f_{P1})^2}\right)\left(1+\frac{(2\pi f_M)^2}{(2\pi f_{P2})^2}\right)} = \frac{A_M^2}{2}$$

$$\left(1+\frac{f_M^2}{f_{P1}^2}\right)\left(1+\frac{f_M^2}{f_{P2}^2}\right) = 2$$

Solving this equation for f_H we have:

$f_H \simeq 4.6\text{ MHz}$

Using the approximate formula, we have:

$$f_H \simeq \frac{1}{\sqrt{\frac{1}{f_{P1}^2}+\frac{1}{f_{P2}^2}}} \simeq 5\text{ MHz}$$

Ex:8 . 32 From Eq 8 . 178

$\omega_t = \frac{G_{m1}}{C_C}$ from Example 7 . 4

$G_{m1} = G_{m1,2} = 0.3\text{ mA/V}$

thus, for $f_T = 10\text{ MHz}$:

$$C_C = \frac{0.3\text{ mA/V}}{2\pi \times 10 \times 10^6} = 4.8\text{ pF}$$

From Eqn. (8 . 173

$$f_Z = \frac{G_{m2}}{2\pi \cdot C_C} \quad G_{m2} = g_{m6} = 0.6\text{ mA/V}$$

$$\Rightarrow f_Z = \frac{0.6\text{ mA/V}}{2\pi \times 4.8\text{ pF}} = 20\text{ MHz}$$

From Eqn. (8 . 177

$$f_{P2} = \frac{G_{m2}}{2\pi \cdot C_2} = \frac{0.6\text{ mA/V}}{2\pi \times 2\text{ pF}} = 48\text{ MHz}$$

Ex:8 . 33**To obtain Req:**

$\text{Req} = R_2 \| r_{O2} \| r_{\pi 5}$

$R_2 = 20\text{ k}\Omega$

$$r_{O2} = \frac{V_A}{I_{C2}} \simeq \frac{100\text{ V}}{0.25\text{ mA}} = 400\text{ k}\Omega$$

$$r_{\pi 5} = (\beta+1)\frac{V_T}{I_S} = 101 \times \frac{25\text{ mV}}{1\text{ mA}}$$

$= 2525\ \Omega$

Thus,

$\text{Req} = 20\text{ K} \| 400\text{ K} \| 2525 = 2.2\text{ k}\Omega$

To obtain Ceq :

$Ceq = C_{\mu 2} + C_{\pi 5} + C_{\mu 5}(1 + g_{m5}R_{L5})$

$C_{\mu 2} = C_{\pi 5} = 2\text{ pF}$

$R_{L5} \simeq R_3 = 3\text{ k}\Omega$

$$g_{m5} = \frac{I_5}{V_T} = \frac{1\text{ mA}}{25\text{ mV}} = 40\frac{\text{mA}}{\text{V}}$$

$$C_{\pi 5} + C_{\mu 5} = \frac{g_{m5}}{2\pi f_T}$$

$$\Rightarrow C_{\pi 5} = \frac{40\text{ m}}{2\pi \times 400\text{ m}} - 2\text{ p} = 14\text{ pF}$$

thus,

$Ceq = 2\text{ pF} + 14\text{ pF} + 2\text{ pF}(1 + 40 \times 3)$

$= 258\text{ pF}$

Finally,

$$f_P = \frac{1}{2\pi \cdot Req \cdot Ceq}$$

$$= \frac{1}{2\pi \times 2.2\text{ K} \times 258\text{ p}}$$

$= 280\text{ KHz}$

Ex:9.1 (c) $A = 100$ V/V and $A_f = 10$ V/V

$$A_f = \frac{A}{1+\beta A} \Rightarrow \beta = \frac{1}{A_f} - \frac{1}{A} = \frac{1}{10} - \frac{1}{100}$$

$$= 0.09$$

$\beta = 0.09$

since: $1 + \dfrac{R_2}{R_1} = \dfrac{1}{\beta}$

$$\Rightarrow \frac{R_2}{R_1} = \frac{1}{0.09} - 1 = 10.11$$

(d) The amount of feed-back is:

$1 + A\beta = 1 + 100 \times 0.09 = 10$ which is 20 dB

(e) For $V_S = 1$ V; $V_O = A_f V_S = 10 \times 1 = 10$ V

$$V_f = \beta \cdot V_O = 0.09 \times 10 = 0.9 \text{ V}$$

$$V_i = \frac{V_O}{A} = \frac{10}{100} = 0.1 \text{ V}$$

(f) If A decreases by 20%:

$A = 0.8 \times 100 = 80$ V/V

$$A_f = \frac{80}{1 + 80 \times 0.09} = 9.7561$$

$\Delta A_f = 10 - 9.7561 \rightarrow 2.44\%$ of $A_f = 10$

Ex:9.2 (c) $A = 10^4$ V/V and $A_f = 10^3$

$$A_f = \frac{A}{1+\beta A} \Rightarrow \beta = \frac{1}{A_f} - \frac{1}{A} = \frac{1}{10^3} - \frac{1}{10^4}$$

$$= 9 \times 10^{-4}$$

$$\Rightarrow \frac{R_2}{R_1} = \frac{1}{\beta} - 1 = 110.1$$

(d) The amount of feed-back is:

$1 + A\beta = 1 + 10^4 \times 9 \times 10^{-4} = 10$ which is 20 dB

(e) For $V_S = 0.01$ V:

$$V_O = A_f \cdot V_S = 10^3 \times 0.01 = 10 \text{ V}$$

$$V_f = \beta \cdot V_O = 9 \times 10^{-4} \times 10 = 0.009 \text{ V}$$

$$V_i = \frac{V_O}{A} = \frac{10}{10^4} = 0.001 \text{ V}$$

Ex:9.3 $\dfrac{dA_f}{A_f} = 0.1\%$ and $\dfrac{dA}{A} = 10\%$

$$\frac{dA_f}{A_f} = \left(\frac{1}{1+A\beta}\right) \cdot \frac{dA}{A} \Rightarrow 0.01 = \frac{1}{1+A\beta}$$

$A_f = \dfrac{A}{1+A\beta}$ and the largest close-loop gain possible occurs when $A = 1000$ V/V

$\Rightarrow A_f = 0.01 \times 1000 = 10$ V/V

If three of these amplifiers are cascaded:

$A_{\text{TOT}} = A_{f1} \times A_{f2} \times A_{f3} = 1000$ V/V and the total variability is:

$$\frac{dA_1}{A_1} + \frac{dA_2}{A_2} + \frac{dA_3}{A_3} = 0.3\% \text{ maximum}$$

Ex:9.4 For Example 7.1

$A_O \approx 6000$, $\beta = 10^{-3}$

$$(1 + A\beta) = (1 + (6 \times 10^3) \times 10^{-3}) = 7$$

$$\therefore f_{Hf} = f_H(1 + A\beta) = 1 \times 7 = 7 \text{ kHz}$$

Ex:9.5

$$V_o = V_s \frac{A_1 A_2}{1 + A_1 A_2 \beta} + V_n \frac{A_1}{1 + A_1 A_2 \beta} = V_{sf} + V_{nf}$$

$$= \frac{(1 \times 100) \times V_S}{1 + (100 \times 1) \times 1} + \frac{1 \times V_n}{1 + (100 \times 1) \times 1}$$

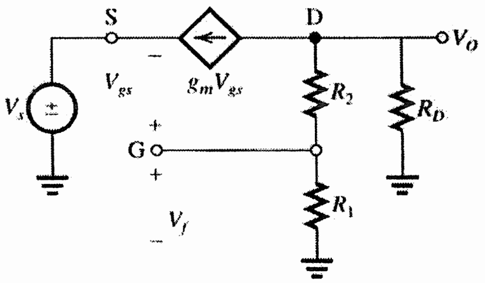
$= 0.99 + 0.0099$

Thus, $V_{sf} \approx 1$ V and $V_{nf} \approx 0.01$ V

New S/N ratio $\approx 100/1$

an improvement of $20 \log(100/1) = 40$ dB

Ex:9.6 Replacing the amplifier by its small signal model



Open-loop gain: without R_2 and R_1 ang the gate grounded:

$$V_O = -g_m V_{gs} \times R_D$$

$$V_{gs} = -V_S \Rightarrow A = g_m R_D$$

Feed-back factor:

$$\beta = \frac{V_f}{V_O} \Rightarrow V_f = \frac{R_1}{R_1 + R_2} \cdot V_O \rightarrow \beta = \frac{R_1}{R_1 + R_2}$$

Closed-loop gain A_f

$$A_f = \frac{V_O}{V_S}$$

$$V_O = -g_m V_{gs} \times \{(R_1 + R_2) \parallel R_D\}$$

$$= -g_m V_{gs} \cdot \frac{(R_2 + R_1)R_D}{R_2 + R_1 + R_D}$$

but $R_2 + R_1 >> R_D$ $\quad R_2 + R_1 + R_D \simeq R_2 + R_1$

$$\rightarrow V_O = \left(-g_m V_{gs} \cdot \frac{(R_2 + R_1)R_D}{R_2 + R_1}\right)$$

$$= -g_m V_{gs}, R_D$$

$$-V_{gs} = V_S - V_f = V_S - \frac{R_1 V_O}{R_1 + R_2}$$

$$\Rightarrow V_O = g_m R_D \left\{ V_S - \frac{R_1 V_O}{R_1 + R_2} \right\}$$

$$\Rightarrow V_O \left\{ 1 + \frac{g_m R_D R_1}{R_1 + R_2} \right\} = g_m R_D V_S$$

Thus:

$$A_f = \frac{V_O}{V_S} = \frac{g_m R_D}{1 + g_m R_D R_1 / (R_1 + R_2)}$$

if $A \cdot \beta >> 1 \quad \Rightarrow (g_m R_D) \cdot R_1 / (R_1 + R_2) >> 1$

$$A_f = \frac{g_m R_D}{g_m R_D R_1 / (R_1 + R_2)} = \frac{R_1 + R_2}{R_1}$$

$$= 1 + \frac{R_2}{R_1}$$

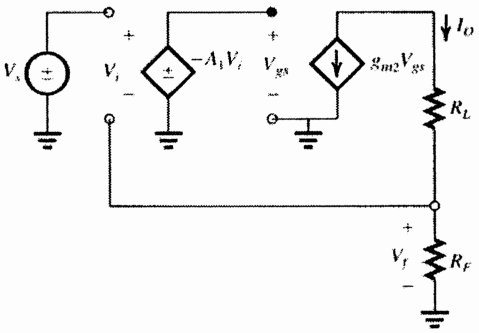
Ex:9.7 From Example 9.2 we know that:

$$A_f = \frac{-g_{m2} R_D}{1 + \dfrac{g_{m2} R_D}{1 + \dfrac{R_F}{R_M}}}$$ where $-g_{m2} R_D$ is the open-loop gain

if loop-gain $A\beta >> 1 \quad \Rightarrow \dfrac{g_{m2} R_D}{1 + \dfrac{R_F}{R_M}} >> 1$

thus: $A_f \cong \dfrac{-g_{m2} R_D}{\dfrac{g_{m2} R_D}{\left(1 + \dfrac{R_F}{R_M}\right)}} \cong -\left(1 + \dfrac{R_F}{R_M}\right)$

Ex: 9.8 The equivalent small-signal model for Fig 9.10 b) is:



Open-loop gain: $V_s \equiv V_i$

$I_o = -g_{m2} V_{gs}$, and $V_{gs} = -A_1 V_i$

$$\Rightarrow I_o = g_{m2}(A_1 V_i) \rightarrow A = \frac{I_o}{V_i} = A_1 g_{m2}$$

$$\beta = \frac{V_f}{I_O}$$

$$V_f = I_O \cdot R_f \rightarrow \frac{V_f}{I_O} = R_f \Rightarrow \beta = R_f$$

Closed-loop gain: $A_f = \dfrac{I_O}{V_S}$

$I_O = A_1 g_{m2} V_i$ and $V_i = V_S - V_F$

$I_O = A_1 g_{m2} \{ V_S - V_f \} = A_1 g_{m2} \{ V_S - I_O R_F \}$

$\Rightarrow I_O(1 + R_F A_1 g_{m2}) = A_1 g_{m2} \cdot V_S$

$A_f = \dfrac{I_O}{V_S} = \dfrac{A_1 g_{m2}}{1 + R_F A_1 g_{m2}}$ which is the same

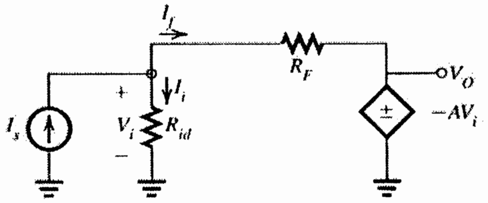
as $A_f = \dfrac{A}{1 + \beta A}$

If $A\beta >> 1 \quad \Rightarrow R_F A_1 g_{m2} >> 1$ and

$$A_f \simeq \frac{A_1 g_{m2}}{R_F A_1 g_{m2}}$$

$$\rightarrow A_f \simeq \frac{1}{R_F}$$

Ex:9.9 The equivalent small-signal circuit for Fig 9.11b)



$$V_O = -AV_i \rightarrow V_i = \frac{-V_O}{A}$$

$$I_S = I_i + I_f \qquad I_i = \frac{V_i}{R_{id}} = \frac{-V_O}{AR_{id}} \qquad (1)$$

$$I_f = \frac{V_i - V_O}{R_F} = \frac{-V_O}{AR_F} - \frac{V_O}{R_F}$$

$$= -V_O\left(\frac{1}{AR_F} + \frac{1}{R_F}\right) \qquad (2)$$

$$(1) + (2): I_S = -V_O\left\{\frac{1}{AR_{id}} + \frac{1}{AR_F} + \frac{1}{R_F}\right\}$$

$$= \frac{-V_O}{R_F}\left\{\frac{1}{AR_{id}} + \frac{1}{A} + 1\right\}$$

$$\Rightarrow \frac{V_O}{I_S} = \frac{-R_F}{\left(1 + \dfrac{1}{A} + \dfrac{R_F}{AR_{id}}\right)}$$

if $A >> 1$ and $AR_{id} >> R_F$

$$\Rightarrow 1 + \frac{1}{A} + \frac{R_F}{AR_{id}} \simeq 1 \text{ and } A_f \simeq -R_F$$

Ex: 9.10

From Example 7.1

$A_0 \approx 6000$, $\beta = 10^{-3}$

$(1 + A\beta) = (1 + (6 \times 10^3) \times 10^{-3}) = 7$

$\therefore f_{Hf} = f_H(1 + A\beta) = 1 \times 7 = 7\ \text{kHz}$

Ex: 9.11

$I_{E1} = I_{E2} = 0.5\text{mA}$

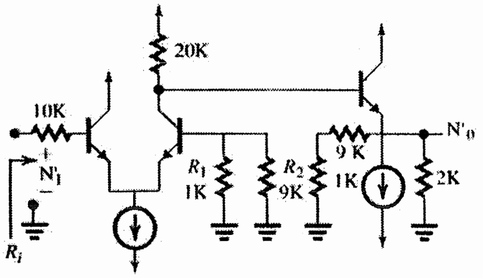
$V_{C2} = 10.7 - 0.5 \times 20 = \quad + 0.7v$

$V_o = 0.7 - V_{BE3} = 0$

$I_{E3} = 5\ \text{mA}$

$r_{e1} = r_{e2} = V_A / I = 50\Omega,\ r_{e3} = 5\Omega$

A-Circuit:



$$A = \frac{V_0'}{V_1'} = \frac{[20 \| (\beta_2 + 1)(r_{e3} + 2 \| 10)]}{r_{e1} + r_{e2} + \dfrac{10}{\beta_1 + 1} + \dfrac{(1 \| 9)}{\beta_2 + 1}} \times$$

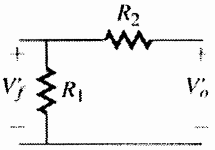
$$\frac{(2 \| 10)}{r_{e3} + (2 \| 10)} = 85.7\ \text{V/V}$$

$R_i = R_S + (\beta + 1)(r_{e1} + r_{e2}) + R_E \| R_4$

$= 10 + 101(50 + 50) + (1 \| 9) = 21\ \text{k}\Omega$

$$R_0 = 2 \| 10 \| \left[r_{e3} + \frac{20}{\beta_2 + 1}\right] = 181\ \Omega$$

B-Circuit:



$\beta = V_f' / V_o'$

$$= \frac{1}{9 + 1} = 0.1\text{V/V}$$

$$A_f = \frac{V_o}{V_S} = \frac{A}{1 + A\beta} = \frac{85.7}{1 + 85.7 \times 0.1} = 8.96\text{V/V}$$

$R_{if} = R_i(1 + A\beta) = 21 \times 9.57 = 201\ \text{k}\Omega$

$R_{iN} = R_{if} - R_S = 201 - 10 = 191\ \text{k}\Omega$

$$R_{of} = (R_{out} \| R_L) = \frac{R_o}{1 + A\beta} = \frac{181}{9.57} = 18.8\ \Omega$$

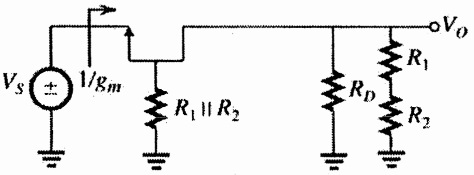
$\Rightarrow R_{out} = 19.1\ \Omega$

Ex: 9.12 The feed-back network is composed of the voltage-driver resistors R_1 and R_2

a) The loading effect of the feed-back network at the input is: $R_1 \| R_2$

b) The loading effect of the feed-back network at the output is: $R_1 + R_2$

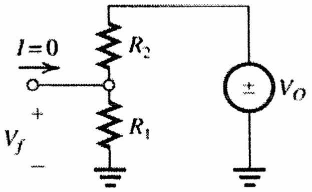
The A circuit is:



For the *CG* amplifier:

$A = g_m[R_D \| (R_1 + R_2)]$

To obtain β:



$$\beta = \frac{V_F}{V_O} = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta}$$ Substituting:

$$A_f = \frac{g_m R_D}{1 + \dfrac{R_D(1 + g_m R_1)}{R_1 + R_2}}$$

if $R_1 + R_2 >> R_D$ we obtain the same result as in Exercise 10.6

From the A circuit:

$$R_i = 1 / g_m \Rightarrow R_{in} = \frac{1}{g_m}(1 + A\beta)$$

$$R_D = R_D \| (R_1 + R_2) \Rightarrow R_{out} = \frac{(R_D \| R_1 + R_2)}{1 + A\beta}$$

(c) $R_i = R_S \parallel R_F$

Following the procedure used in Example 9.7

$$R_{if} = \frac{R_i}{1 + A\beta} \Rightarrow \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{A\beta}{R_i}$$

$$\frac{1}{R_{if}} = \frac{1}{R_i} + \frac{(R_S \parallel R_F) g_m (r_O \parallel R_F)}{(R_S \parallel R_F) \cdot R_F}$$ if we cal

$$\mu' = g_m(r_O \parallel R_F)$$

$$\Rightarrow \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{\mu'}{R_F} \text{ or } R_{if} = R_i \parallel \frac{R_F}{\mu'}$$

Substituting for $R_i = R_S \parallel R_F$:

$$R_{if} = R_S \parallel R_F \parallel \frac{R_F}{\mu'}$$

$$= R_S \parallel \frac{R_F}{(1 + \mu')}$$

Since

$$R_{if} = R_S \parallel R_{in} \rightarrow R_{in} = \frac{R_F}{1 + \mu'}$$

$$= \frac{R_F}{1 + g_m(r_O \parallel R_F)}$$

(d) $R_O = r_O \parallel R_F$

Following the procedure used in Example 9.7

$$R_{of} = \frac{R_O}{1 + A\beta} \Rightarrow \frac{1}{R_{of}} = \frac{1}{R_O} + \frac{A\beta}{R_O}$$

$$\frac{1}{R_{of}} = \frac{1}{R_O} + \frac{(R_S \parallel R_F) g_m (r_O \parallel R_F)}{R_F \cdot (r_O \parallel R_F)}$$ if we call

$$\mu' = g_m(R_S \parallel R_F)$$

$$R_{of} = R_O \parallel \frac{R_F}{\mu'}$$

Substituting for

$$R_O = r_O \parallel R_F: R_{of} = r_O \parallel R_F \parallel \frac{R_F}{\mu'}$$

$$R_{of} = r_O \parallel \frac{R_F}{1 + \mu'}$$

Since: $R_{of} = R_{out} \parallel R_L$ and $R_L = \infty$

$$\Rightarrow R_{out} = r_O \parallel \frac{R_F}{1 + g_m(R_S \parallel R_F)}$$

(e) For $g_m = 5\,\frac{\text{mA}}{\text{V}}$ $r_O = 20\text{ k}\Omega$

$R_f = 10\text{ k}\Omega$

$R_S = 1\text{ k}\Omega$

$$A = -(1\text{ K} \parallel 10\text{ K}) \cdot 5\text{ m}(20\text{ K} \parallel 10\text{ K})$$

$$= -30.3\text{ k}\Omega$$

$\beta = -1/R_F = -1/10\text{ K} = -0.1\text{ mA/V}$

$A\beta = -30.3 \times -0.1 = 3.03$

$$A_f = \frac{A}{1 + A\beta} = \frac{-3.03\text{ K}}{1 + 3.03} = -7.52\text{ k}\Omega$$

$R_i = R_S \parallel R_F = 1 \parallel 10 = 909\ \Omega$

$R_O = r_O \parallel R_F = 20 \parallel 10 = 6.67\text{ k}\Omega$

$$R_{in} = \frac{10\text{ K}}{1 + 5\text{ m}(20\text{ K} \parallel 10\text{ K})} = 291\ \Omega$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{909}{4.03} = 225.6\ \Omega$$

$$R_{of} = \frac{R_O}{1 + A\beta} = \frac{6.67\text{ K}}{4.03} = 1.66\text{ k}\Omega$$

$$R_{out} = 20\text{ K} \parallel \frac{10\text{ K}}{1 + 5\text{ m}(1\text{ K} \parallel 10\text{ K})} = 1.66\text{ k}\Omega$$

Ex: 9.16 $\mu = 100$

$R_S = \infty$ $r_{O1} = 1\text{ K}\Omega$ $R_1 = 10\text{ K}\Omega$

$R_2 = 90\text{ k}\Omega$ $g_m = 5\text{ mA/V}$

$r_O = 20\text{ k}\Omega$

Refer to Example 9.8

$R_i = 100\text{ K}$, unchanged

$$A = -\mu \cdot \frac{R_i}{R_1 \parallel R_2} = -100 \cdot \frac{100}{10 \parallel 90}$$

$$= -1.11 \times 10^3\text{ A/A}$$

$\beta = -0.1\text{ A/A}$, unchanged

$A\beta = 111$

$$A_f = \frac{-1.11 \times 10^3}{1 + 111} = -9.91\text{ A/A}$$

$$R_{in} = \frac{90\text{ K}}{100} = 900\ \Omega$$

$R_O = 900\text{ K}\Omega$, unchanged

$R_{out} = (1 + 111).\ 900\text{ K} = 100\text{ M}\Omega$

Ex: 9.17 If $R_1 = 0 \Rightarrow \beta = \dfrac{R_1}{R_1 + R_2} = -1$

All of I_O is fed-back.

$\Rightarrow$ if $A\beta >> 1 \rightarrow$ ideal

$$A_f = -\frac{1}{\beta} = 1\text{ A/A}$$

$R_i = R_S \parallel R_{id} \parallel R_1 = \infty \parallel \infty \parallel R_1 = R_1$

$R_O = r_{O2} + (R_1 \parallel 0) + g_m r_{O2}(R_1 \parallel 0) = r_{O2}$

Replacing R_2 for 0 in Eq 9.69

$$A = \frac{-\mu R_i r_{O2}}{\frac{1}{g_m} \times r_{O2}} = -\mu g_m R_i = -\mu g_m R_1$$

$$A_f = \frac{A}{1 + A\beta} = \frac{-\mu \cdot g_m \cdot R_1}{1 + \mu g_m \cdot R_1}.$$

From Eq 9 . 77

$$R_{\text{out}} = \mu \cdot \frac{R_1}{R_1} \cdot g_m r_{O2} \cdot R_1 = \mu g_m r_{O2} \cdot R_1$$

To obtain R_{in}:

$$R_{\text{if}} = \frac{R_i}{1 + A\beta} = \frac{R_1}{1 + \mu g_m R_1}$$

$$R_{\text{in}} = \frac{1}{\frac{1}{R_{\text{if}}} - \frac{1}{R_S}}$$

$$R_S = \infty \rightarrow R_{\text{in}} = \frac{R_1}{1 + \mu g_m R_1} = \frac{1}{\frac{1}{R_1} + \mu g_m}$$

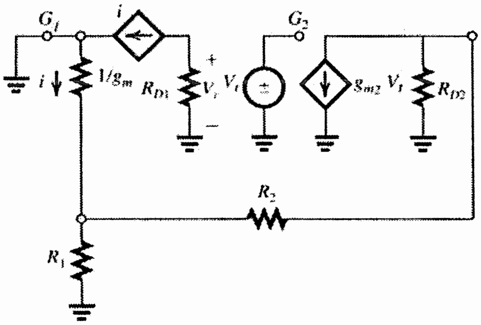
Since $\mu g_m >> \frac{1}{R_1} \Rightarrow R_{\text{in}} = 1 / \mu g_m$

Ex:9 . 18 Small-signal equivalent circuit:

$$\frac{-V_r}{V_t} = \frac{g_{m2} \cdot R_{D2}}{R_{D2} + R_2 + R_1 \parallel 1/g_{m1}} \cdot \left(\frac{R_1}{R_1 + 1/g_{m1}}\right) \cdot R_{D1}$$

$$A\beta = \frac{4\text{ m} \times 10\text{ K}}{(10\text{ K} + 9\text{ K} + 1\text{ K} \parallel 1/4\text{ m})} \cdot \left(\frac{1\text{ K}}{1\text{ K} \parallel 1/4\text{ m}}\right) \cdot 10\text{ K} = 16.66$$

Compared to 17.39 obtained in Example 9 . 4

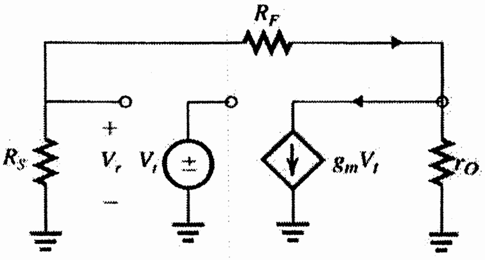


Ex 9 . 19 $V_x = \frac{-g_m r_O}{r_O + R_F + R_S} \cdot R_S \cdot V_t$

$$\frac{-V_x}{V_t} = A\beta = \frac{g_m r_O}{r_O + R_F + R_S} \cdot R_S$$

$$A\beta = \frac{5\text{ m} \times 20\text{ K} \times 1\text{ K}}{20\text{ K} + 10\text{ K} + 1\text{ K}} = 3.22$$

as compared to 3.03 obtained in Exercise 9 . 15



Ex 9 . 20 $A(j\omega) = \left(\frac{10}{1 + j\omega / 10^4}\right)^3$

Thus $\phi = -3\tan^{-1}(\omega / 10^4)$

At $\omega_{180^\circ}, \phi = 180^\circ \Rightarrow \tan^{-1}(\omega_{180^\circ}/10^4) = 60^\circ$

$(\omega_{180}/10^4) = \sqrt{3} \Rightarrow \omega_{180} = \sqrt{3} \times 10^4$ rad|s

Amplifier stable if $|A\beta| < 1$ at ω_{180°

When $|A\beta| = 1$: $\beta_{cr} = \frac{1}{|A(j\omega_{180})|}$

$$\therefore \beta_{cr} = \frac{1}{1000 / (1 + (\sqrt{3})^2)^{3/2}} = 0.008$$

Ex:9 . 21 Pole is shifted by factor $(1 + A_o\beta)$

$= 1 + 10^5 \times 0.01 = 1001$

$f_{pf} = f_p(1 + A_O\beta) = 100 \times 1001 = 100.1$kHz

For closed loop gain = 1, β = 1

$f'_{pf} = f''_p(1 + A_O\,\beta) = 10^5(1001) = 10^7$ Hz

Ex:9 . 22 From Eq. 9 . 92 Poles will coincide when

$(\omega_{P1} + \omega_{P2})^2 - 4(1 + A_o\beta)\ \ \omega_{P1}\omega_{P2} = 0$

Using

$A_o = 100, \omega_{P1} = 10^4, \omega_{P2} = 10^6$ rad|s

$(10^4 + 10^6)^2 - 4(1 + 100\,\beta) \times 10^{10} = 0$

$1 + 100\,\beta = (1.01)^2 \times 100 / 4$

$\Rightarrow \beta = 0.245$

Corresponding $Q = 0.5$

For maximally flat response $Q = 0.707$ and

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{(1 + 100\beta) \times 10^{10}}}{10^4 + 10^6} \Rightarrow \beta = 0.5$$

Corresponding gain is

$$A = \frac{A}{1 + A_O\beta} = \frac{100}{1 + 100 \times 0.5} = 1.96\text{ V/V}$$

Ex9.23 Closed loop poles are found using

$$1 + A(s)\beta = 0$$

$$1 + \frac{10^3}{(1 + S/10^4)^3}\beta = 0$$

$$(1 + S/4)^3 + 10^3\beta = 0$$

$$\frac{S^3}{10^{12}} + \frac{3S^2}{10^8} + \frac{3S^1}{10^4} + (1 + 100\beta) = 0$$

$$= S_n^3 + 3S_n + 3S_n + (1 + 100\beta) = 0 \text{ for}$$

$$S_n = \frac{S}{10^4}$$

Roots of this cubic equation are:

$$(-1 - 10\beta^{1/3}), -1 + 5\beta^{1/3} \pm j\,5\sqrt{3}\beta^{1/3}$$

Amplifier becomes unstable when complex poles are on $j\omega$ axis ie. when $\beta = \beta_{cr}$

$$10\beta_{cr}^{1/3} = \frac{1}{\cos 60^\circ} = 2 \Rightarrow \beta_{cr} = 0.008$$

Ex: 9.24 $A = \dfrac{A_o}{1 + j\dfrac{\omega}{\omega_p}} = \dfrac{A_o}{1 + j\,f/f_p}$

$$= \frac{10^5}{1 + j\,f/10}$$

$$\beta = 0.01 \quad |A\beta| = \frac{10^5 \times 0.01}{\sqrt{1 + f^2/100}} = 1$$

thus $1 + f^2/100 = 10^6 \Rightarrow f = 10^4$ Hz

At $f = 10^4$ Hz

$$\phi = -\tan^{-1}(10^4/10) \approx -90^\circ$$

making phase margin $180 - 90 = 90^\circ$

Ex:9.25 From Eqn 9.105

$|A_f(j\omega_2)| = \dfrac{1/\beta}{|1 + e^{-j\theta}|}$ and $\dfrac{1}{\beta} \approx$ low frequency gain $\theta = 180^\circ -$ Phase margin

For $PM = 30^\circ, \theta = 150^\circ$

$$|A_f(j\omega_1)|/(1/\beta) = 1.93$$

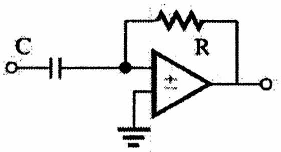
For $PM = 60^\circ, \theta = 120^\circ$

$$|A_f(j\omega_2)|/(1/\beta) = 1.0$$

For $PM = 90^\circ, \theta = 90^\circ$

$$|A_f(j\omega_{a?})|/(1/\beta) = 0.707$$

Ex:9.26 $\beta = \dfrac{1/SC}{R + 1/SC} = \dfrac{1}{1 + SCR}$

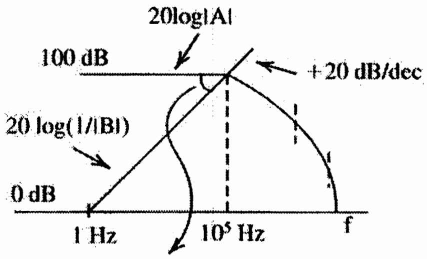


$$\left|\frac{1}{\beta}\right| = \sqrt{1 + (\omega CR)^2}$$

$$\therefore \frac{1}{2\pi CR} \leq 1 \text{ Hz}$$

$$CR \leq \frac{1}{2\pi}$$

Thus $CR \geq 0.1595$.

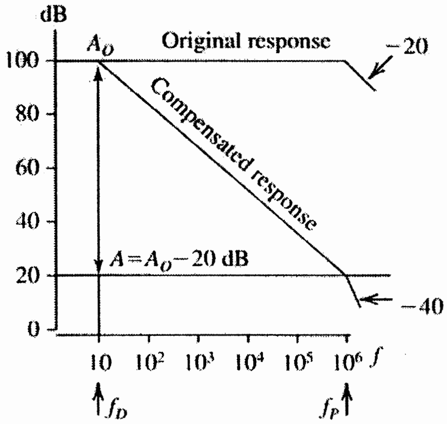


Rate of closure = 20 dB/dec

Ex:9.27 Must place new dominant pole at

$$f_D = \frac{f_P}{A} = \frac{10^6}{10^4}$$

$$\therefore f_D = 100 \text{ Hz}$$



Ex: 9.28 The pole must be moved f_{P1} to f_D where

$$f_D = \frac{\text{Frequency of 2nd pole}}{A_O \div A_F}$$

$$= \frac{10 \times 10^6}{10^4} \leftarrow (100 \text{ dB} - 20 \text{ db})$$

$$= 10^3 \text{ Hz}$$

The capacitance at the controlling node must be increased by same factor as f is lowered.

$$\therefore C_{\text{new}} = C_{\text{old}} \times 1000$$

Ex 13.1

For Q_1

$$I = \frac{V_{CC} - V_{CEsat}}{R_L} = \frac{15 - 0.2}{1\ k\Omega}$$

$I = 14.8$ mA

$$R = \frac{-V_D - (-V_{CC})}{14.8} = \frac{-0.7 - (-15)}{14.8}$$

$= 0.97\ k\Omega$

$v_{omax} = V_{CC} - V_{CEsat}$

$= 15 - 0.2$

$= 14.8$ V

$v_{omin} = -V_{CC} + V_{CEsat}$

$= -15 + 0.2$

$= -14.8$

Output signal swing is from 14.8 V to -14.8 V

Maximum emitter current $= 2I = 2 \times 14.8$

$= 29.6$ mA

Ex 13.2

At $v_o = -10$ V, the load current is -10 mA and the emitter current of Q_1 is $14.8 - 10 = 4.8$ mA.

Thus, $v_{BE1} = 0.6 + 0.025 \ln\left(\frac{14.8}{1}\right)$

$= 0.64$ V

Thus, $v_I = -10 + 0.64 = -9.36$ V

At $v_o = 0$ V, $i_L = 0$ and $i_{E1} = 14.8$ mA

Thus, $v_{BE1} = 0.6 + 0.025 \ln\frac{4.8}{1}$

$= 0.67$ V

$v_I = +0.67$ V

At $v_o = +10$ V, $i_L = 10$ mA and $i_{E1} = 24.8$ mA

Thus, $v_{BE1} = 0.6 + 0.025 \ln(24.8)$

$= 0.68$ V

$v_I = 10.68$ V

To calculate the incremental voltage gain we use

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}}$$

At $v_o = -10$ V, $i_{E1} = 4.8$ mA and

$$r_{e1} = \frac{25}{4.8} = 5.2\ \Omega$$

Thus, $\frac{v_o}{v_i} = \frac{1}{1 + 0.0052} = 0.995$ V / V

Similarly, at $v_o = 0$ V, $r_{e1} = \frac{25}{14.8} = 1.7\ \Omega$

and, $\frac{v_o}{v_i} = \frac{1}{1 + 0.0017} = 0.998$ V / V

At $v_o = +10$ V, $i_{E1} = 24.8$ mA and $r_{e1} = 1\ \Omega$

Thus, $\frac{v_o}{v_i} = \frac{1}{1 + 0.001} = 0.999$ V / V

Ex 13.3

a. $P_L = \frac{(\hat{V}_o / \sqrt{2})^2}{R_L} = \frac{(8/\sqrt{2})^2}{100} = 0.32$ W

$P_s = 2\,V'_{cc} \times I = 2 \times 10 \times 100 \times 10^{-3}$

$= 2$ W

Efficiency $\eta = \frac{P_L}{P_S} \times 100$

$= \frac{0.32}{2} \times 100$

$= 16\%$

Ex 13.4

(a) $P_L = \frac{1}{2}\frac{\hat{V}_o^2}{R_L}$

$= \frac{1}{2}\frac{(4.5)^2}{4} = 2.53$ W

(b) $P_+ = P_- = V_{CC} \times \frac{1}{\pi}\frac{\hat{V}_o}{R_L}$

$= 6 \times \frac{1}{\pi} \times \frac{4.5}{4} = 2.15$ W

(c) $\eta = \frac{P_L}{P_S} \times 100 = \frac{2.53}{2 \times 2.15} \times 100$

$= 59\ \%$

(d) Peak input currents $= \frac{1}{\beta + 1}\frac{\hat{V}_o}{R_L}$

$= \frac{1}{51} \times \frac{4.5}{4}$

$= 22.1$ mA

(e) Using Eq. 10.22

$$P_{DNmax} = P_{DPmax} = \frac{V_{CC}^2}{\pi^2 R_L}$$

$$= \frac{6^2}{\pi^2 \times 4} = 0.91\ \text{W}$$

Ex 13.5

(a) The quiescent power dissipated in each transistor $= I_Q \times V_{cc}$

Total power dissipated in the two transistors

$= 2I_Q \times V_{cc}$

$= 2 \times 2 \times 10^{-3} \times 15$

$= 60$ mW

(b) I_Q is increased to 10 mA

At $V_o = 0$, $i_N = i_P = 10$ mA

From equation 13.31

$$R_{out} = \frac{V_T}{i_P + i_N} = \frac{25}{10 + 10} = 1.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{out}} = \frac{100}{100 + 1.25}$$

$\frac{v_o}{v_i} = 0.988$ at $v_o = 0\text{ V}$

At $v_o = 10\text{ V}$,

$$i_L = \frac{10\text{ V}}{100\ \Omega} = 0.1\text{ A} = 100\text{ mA}$$

use equation 13.27 to calculate i_N

$i_N^2 - i_N i_L - I_Q^2 = 0$

$i_N^2 - 100\, i_N - 10^2 = 0$

$\Rightarrow i_N = 99.99\text{ mA}$

using equation 13.26

$$i_P = \frac{I_Q^2}{I_N} \simeq 1\text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{99.99 + 1} \simeq 0.2475\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 0.2475} \simeq 1$$

$$\%\text{ change} = \frac{1 - 0.988}{1} \times 100 = 1.2\%$$

In example 13.5 $I_Q = 2\text{ mA}$, and for $v_o = 0$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{2 + 2} = 6.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 6.25} = 0.94$$

$v_o = 10\text{ V}$

$$I_L = \frac{10\text{ V}}{100\ \Omega} = 100\text{ mA}$$

Again calculate i_N (for $I_Q = 2\text{ mA}$) using equation 13.27 $i_N = 99.96\text{ mA}$

$$i_P = \frac{I_Q^2}{I_N} = \frac{2^2}{99.96} = 0.04\text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{99.6 + 0.04} = 0.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} \simeq 1$$

$$\%\text{ Change} = \frac{1 - 0.94}{1} \times 100 = 6\%$$

For $I_Q = 10\text{ mA}$, change is 1.2%

For $I_Q = 2\text{ mA}$, change is 6%

(c) The quiesent power dissipated in each transistor $= I_Q \times V_{CC}$

Total power dissipated $= 2 \times 10 \times 10^{-3} \times 15$
$= 300\text{ mW}$

Ex 13.6

From example 13.4 $V_{CC} = 15\text{ V}$, $R_L = 100\ \Omega$

Q_N and Q_P matched and $I_S = 10^{-13}\text{ A}$ and $\beta = 50$,

$I_{\text{Bias}} = 3\text{ mA}$

For $v_o = 10\text{ V}$, $I_L = \frac{10}{100} = 0.1\text{ A}$

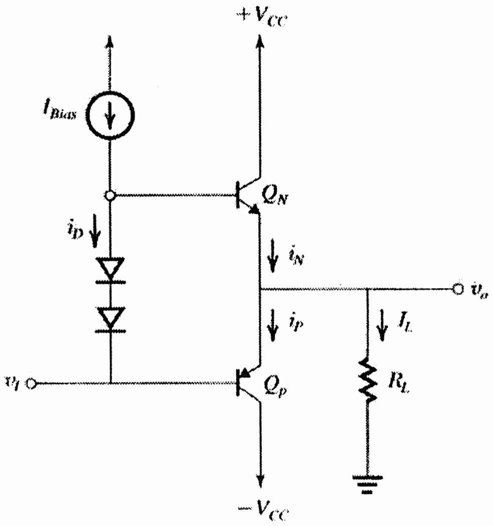
As a first approximation $i_N \simeq 0.1\text{ A}$, $i_P = 0$, $i_{BN} \simeq$

$$\frac{0.1\text{ A}}{50 + 1} \simeq 2\text{ mA}$$

$i_D = I_{\text{Bias}} - i_{BN} = 3 - 2 = 1\text{ mA}$

$$V_{BB} = 2\, V_T \ln\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)$$

This $\frac{1}{3}$ is because biasing diodes have $\frac{1}{3}$ area of the output devices.



But $V_{BB} = V_{BEN} + V_{BEP} =$ (1)

$$V_T \ln\left(\frac{i_N}{I_S}\right) + V_T \ln\left(\frac{i_N - i_L}{I_S}\right)$$

$$= V_T \ln\left[\frac{i_N(i_N - i_L)}{I_S^2}\right] \qquad (2)$$

Equating equations 1 and 2

$$2V_T \ln\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) = V_T \ln\left(\frac{i_N - i_L}{I_S^2}\right)$$

$$\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)^2 = \frac{i_N(i_N - 0.1)}{(10^{-13})^2}$$

$i_N(i_N - 0.1) = 9 \times 10^{-6}$

If i_N is in mA, then

$i_N(i_N - 100) = 9$

$i_N^2 - 100\, i_N - 9 = 0$

$\Rightarrow i_N = 100.1\text{ mA}$

$i_P = i_N - i_L = 0.1\text{ mA}$

For $v_O = -10\text{ V}$ and $i_L = \frac{-10}{100} = -0.1\text{ A}$

$= -100\text{ mA}$

As a first approximation assume $i_P \cong 100\text{ mA}$.

$i_N \simeq 0$ since $i_N = 0$, current through diodes $= 3\text{ mA}$

$$\therefore V_{BB} - 2V_T \ln\left(\frac{3\times 10^{-3}}{\frac{1}{3}\times 10^{-13}}\right) \qquad (3)$$

$$\text{But } V_{BB} = V_T \ln\left(\frac{i_N}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right)$$

$$= V_T \ln\left(\frac{i_P - i_L}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right) \quad (4)$$

Here $i_L = 0.1$ A

Equating equations 3 and 4

$$2V_T \ln\left(\frac{3\times 10^{-3}}{\frac{1}{3}\times 10^{-13}}\right) =$$

$$V_T \ln\left(\frac{i_P - 0.1}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right)$$

$$\left(\frac{3\times 10^{-3}}{\frac{1}{3}\times 10^{-13}}\right)^2 = \frac{i_P(i_P - 0.1)}{(10^{-13})^2}$$

$i_P(i_P - 0.1) = 81 \times 10^{-6}$

Expressing currents in mA

$i_P(i_P - 100) = 81$

$i_P^2 - 100\, i_P - 81 = 0$

$\Rightarrow i_P = 100.8$ mA

$i_N = i_P - i_L = 0.8$ mA

Ex 13.7

$\Delta I_C = g_m \times 2$ mV / °C × 5 °C, mA

where g_m is in mA / mV

$$g_m = \frac{10 \text{ mA}}{25 \text{ mV}} = 0.4 \text{ mA / mV}$$

Thus, $\Delta I_C = 0.4 \times 2 \times 5 = 4$ mA

Ex 13.8

Refer to Fig. 10.14

(a) To obtain a terminal voltage of 1.2 V, and since β_1 is very large, it follows, that $V_{R1} = V_{R2} = 0.6$ V.

Thus $I_{C1} = 1$ mA

$$I_R = \frac{1.2 \text{ V}}{R_1 + R_2} = \frac{1.2}{2.4} = 0.5 \text{ mA}$$

Thus, $I = I_{C1} + I_R = 1.5$ mA

(b) For $\Delta V_{BB} = +50$ mV:

$$V_{BB} = 1.25 \text{ V} \qquad I_R = \frac{1.25}{2.4} = 0.52 \text{ mA}$$

$$V_{BE} = \frac{1.25}{2} = 0.625 \text{ V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = e^{0.025/0.025}$$

$= 2.72$ mA

$I = 2.72 + 0.52 = 3.24$ mA

For $\Delta V_{BB} = +100$ mV

$$V_{BB} = 1.3 \text{ V} \; I_R = \frac{1.3}{2.4} = 0.54 \text{ mA}$$

$$V_{BE} = \frac{1.3}{2} = 0.65 \text{ V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = 1 \times e^{0.05/0.025}$$

$= 7.39$

$I = 7.39 + 0.54 = 7.93$ mA

For $\Delta V_{BB} = +200$ mV:

$$V_{BB} = 1.4 \text{ V} \qquad I_R = \frac{1.4}{2.4} = 0.58 \text{ mA}$$

$V_{BE} = 0.7$ V

$$I_{C1} = 1 \times e^{0.1/0.025} = 54.60 \text{ mA}$$

$I = 54.60 + 0.58 = 55.18$ mA

For $\Delta V_{BB} = -50$ mV

$$V_{BB} = 1.15 \text{ V} \qquad I_R = \frac{1.15}{2.4} = 0.48 \text{ mA}$$

$$V_{BE} = \frac{1.15}{2}$$

$= 0.575$

$$I_{C1} = 1 \times e^{-0.025/0.025} = 0.37 \text{ mA}$$

$I = 0.48 + 0.37 = 0.85$ mA

For $\Delta V_{BB} = -100$ mV:

$$V_{BB} = 1.1 \text{ V} \; I_R = \frac{1.1}{2.4} = 0.46 \text{ mA}$$

$V_{BE} = 0.55$ V

$$I_{C1} = 1 \times e^{-0.05/0.025} = 0.13 \text{ mA}$$

$I = 0.46 + 0.13 = 0.59$ mA

For $\Delta V_{BB} = -200$ mV:

$$V_{BB} = 1.0 \text{ V} \qquad I_R = \frac{1}{2.4} = 0.417 \text{ mA}$$

$V_{BE} = 0.5$ V

$$I_{C1} = 1 \times e^{-0.1/0.025} = 0.018 \text{ mA}$$

$I = 0.43$ mA

Ex 13.9

Using equation 13.43

$$I_Q = I_{\text{Bias}} \frac{(W/L)_n}{(W/L)_1}$$

$$1 = 0.2 \frac{(W/L)_n}{(W/L)_p}$$

$$\frac{(W/L)_n}{(W/L)_1} = 5$$

$$Q_1: I_{\text{Bias}} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 (V_{GS} - V_{tn})^2$$

$$0.2 = \frac{1}{2} \times 0.250 \left(\frac{W}{L}\right)_1 (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 40$$

Q_2: $I_{\text{Bias}} = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_2 (V_{GS} - |V_t|)^2$

$0.2 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_2 \times (0.2)^2$

$\Rightarrow \left(\frac{W}{L}\right)_2 = 100$

Q_N: $I_Q = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_N (V_{GS} - V_t)^2$

$1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_N 0.2^2$

$\Rightarrow \left(\frac{W}{L}\right)_N = 200$

Q_P: $I_Q = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_P (V_{GS} - |V_t|)^2$

$1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_P \times 0.2^2$

$\left(\frac{W}{L}\right)_P = 500$

Now $V_{iG} = V_{GS1} + V_{GS2}$

$= (V_{ov1} + V_t) + (V_{ov2} + |V_t|)$

$= (0.2 + 0.5) + (0.2 + 0.5)$

$= 1.4\text{ V}$

Ex 13.10

$I_N = i_{L\max} = 10\text{ mA}$

$\therefore 10 = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_n V_{ov}^2$

$10 = \frac{1}{2} \times 0.250 \times 200 \times V_{ov}^2$

$\Rightarrow V_{ov} = 0.63\text{ V}$

Using equation 13.46

$V_{o\max} = V_{DD} - V_{ov}|_{\text{Bias}} - V_{in} - V_{ovN}$

$= 2.5 - 0.2 - 0.5 - 0.63$

$= 1.17\text{ V}$

Ex 13.11

New values of W/L are

$\left(\frac{W}{L}\right)_p = \frac{2000}{2} = 1000$

$\left(\frac{W}{L}\right)_N = \frac{800}{2} = 400$

$I_Q = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_p V_{ov}^2$

$1 \times 10^{-3} = \frac{1}{2} \times 0.1 \times 10^{-3} \times 1000 \times V_{ov}^2$

$\Rightarrow V_{ov} = 0.14\text{ V}$

Gain Error =

$-\frac{V_{ov}}{4\mu I_Q R_L} = -\frac{0.14}{4 \times 10 \times 1 \times 10^{-3} \times 100}$

$= -0.035$

Gain Error $= -0.035 \times 100 = 3.5\%$

$g_{mn} = g_{mp} = \frac{2I_Q}{V_{ov}} = \frac{2 \times 1 \times 10^{-3}}{0.14}$

$= 14.14\text{ mA/V}$

$R_{\text{out}} = \frac{1}{\mu(g_{mp} + g_{mn})} =$

$\frac{1}{10 \times (14.14 + 14.14) \times 10^{-3}}$

$\simeq 3.5\ \Omega$

Ex 13.12

See solution on next page

Ex 13.12

Need to prove when $V_{o2} = 4\,I_Q R_L$ then $V_{GSN2} = V_{tn}$

Assume Q_N off ($V_{GSN} = V_{tn}$) so $i_{N2} = 0$ and

$i_{p2} = i_{L2}$

$$i_{p2} = i_{L2} = \frac{V_{O2}}{R_L} = 4I_Q$$

$$4I_Q = \frac{1}{2}k'_p\left(\frac{W}{L}\right)_p(V_{SGP2} - |V_{tp}|)^2$$

$$\sqrt{4\left(\cancel{\tfrac{1}{2}}\cancel{k'_p}\left(\cancel{\tfrac{W}{L}}\right)_p(V_{SGPQ} - |V_{tp}|)^2\right)}$$

$$= \sqrt{\cancel{\tfrac{1}{2}}\cancel{k'_p}\left(\cancel{\tfrac{W}{L}}\right)_p(V_{SGP2} - |V_{tp}|)^2}$$

$$2(V_{SGPQ} - |V_{tp}|) = (V_{SGP2} - |V_{tp}|)$$

$$V_{SGP2} = 2V_{SGPQ} - 2|V_{tp}| + |V_{tp}|$$

$$= 2V_{SGPQ} - |V_{tp}| \qquad (1)$$

Find V_{i2} for the gate voltage, V_{GP2}:

$$V_{GP2} = (V_{DD} - V_{SGPQ}) + \mu(V_{O2} - V_{i2})$$

$$(V_{GP2} - V_{DD}) = -V_{SGPQ} + \mu(V_{O2} - V_{i2})$$

$$[V_{GSP2}\ \text{OR}] - V_{SGP2} = -V_{SGPQ} + \mu(V_{O2} - V_{i2})$$

using (1):

$$-2V_{SGPQ} + |V_{tp}| = -V_{SGPQ} + \mu(V_{O2} - V_{i2})$$

$$\mu(V_{i2} - V_{O2}) = -V_{SGPQ} + 2V_{SGPQ} - |V_{tp}|$$

$$V_{i2} = +V_{O2} + \frac{(V_{SGPQ} - |V_{tp}|)}{\mu} = V_{O2} + \frac{V_{OVQ}}{\mu}$$

Plug this value for V_{i2} into the value for V_{GN2} and show $V_{GSN2} = V_{tN}$

$$(-\cancel{V_{SS}} + V_{GSNQ}) + \mu(V_{O2} - V_{i2}) = V_{GN2} - (-\cancel{V_{SS}})$$

$$V_{GSNQ} + \cancel{\mu}\left(\cancel{V_{O2}} - \cancel{V_{O2}} - \frac{V_{OVQ}}{\cancel{\mu}}\right) = V_{GSN2}$$

where

$$V_{OVQ} = (V_{GSNQ} - V_{tn}) = (V_{SGPQ} - |V_{tp}|)$$

$\cancel{V_{GSNQ}} - \cancel{V_{GSNQ}} + V_{tn} = V_{GSN2}$ Q.E.D.

Same proof for p transistor.

Ex: 10.1

$V_{ICM(\max)} \le V_{DD} - |V_{OV5}| - |V_{tp}| - |V_{OV1}|$

$\le +1.65 - 0.3 - 0.5 - 0.3$

$\le +0.55\text{ V}$

$V_{ICM(\min)} \ge -V_{SS} + V_{OV3} + V_{tn} - |V_{tp}|$

$\ge -1.65 + 0.3 + 0.5 - 0.5$

$\ge -1.35\text{ V}$

$V_{0(\min)} \le V_{DD} - |V_{OV7}|$

$\le +1.65 - 0.5$

$\le +1.15\text{V}$

$V_{0(\min)} \ge -V_{SS} + V_{OV6}$

$\ge -1.65 + 0.5$

$\ge -1.15\text{V}$

Ex: 10.2

$|V_A| = 30\text{V},\ I_6 = 0.5\text{ mA},$

$V_{OV1} = 0.2\text{V},\ V_{OV6} = 0.5\text{ V}$

$I = K(V_{OV})^2$

For $Q_6: 0.5 = K(0.5)^2 \Rightarrow K = 2\text{ mA / V}^2$

For $Q_2: I_2 = 2(0.2)^2 \Rightarrow I_2 = 0.8\text{ mA}$

$$g_m = \frac{I}{V_{OV}} \Rightarrow g_{m2} = \frac{0.8}{0.2} = 4\text{ mA / V}$$

$$\Rightarrow g_{m6} = \frac{0.5}{0.5} = 1\text{ mA / V}$$

$$r_o = \frac{V_A}{I} \Rightarrow r_{o2} = \frac{20}{0.8} = 25\text{ k}\Omega$$

$$\Rightarrow r_{o6} = r_{o7} = \frac{20}{0.5} = 40\text{ k}\Omega$$

$A_1 = -g_{m2}r_{o2} = -4 \times 25 = -100\text{ V / V}$

$A_2 = -g_{m6}r_{o6} = -1 \times 40 = -40\text{ V / V}$

$A = A_1A_2 = (-100)(-40) = +4000\text{ V / V}$

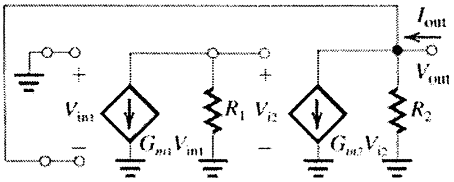
$$R_o = (r_{o6} \| r_{o7}) = \frac{40\text{k}}{2} = 20\text{ k}\Omega$$

Ex: 10.3

The small-signal equivalent circuit for the op-amp in Fig. 10.1 on page of the Text is redrawn below for a unity-gain buffer.

From Eq10.8, 10.7, 10.14, 10.15on page of the Text : $G_{m1} = g_{m1}\ G_{m2} = g_{m6}$

$R_1 = r_{o2} \| r_{o4}\ R_2 = r_{o6} \| r_{o7}$



From the above we can write :

$$I_{\text{out}} = G_{m2}V_{i2} + \frac{V_{\text{out}}}{R_2} \text{ where}$$

$V_{i2} = -G_{m1}V_{\text{in1}}R_1$ and

$V_{\text{in1}} = -V_{\text{out}} \Rightarrow V_{i2} = G_{m1}R_1V_{\text{out}}$ therefore :

$$I_{\text{out}} = g_{m6}g_{m1}R_1V_{\text{out}} + \frac{V_{\text{out}}}{R_2}$$

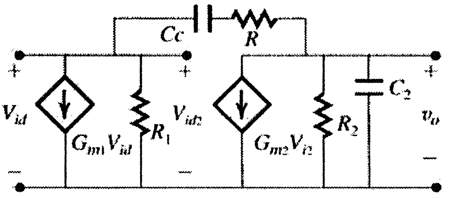
$$I_{\text{out}} = V_{out}\left(g_{m6}g_{m1}R_1 + \frac{1}{R_2}\right)\text{or}$$

$$R_o = \frac{1}{g_{m6}g_{m1}(r_{o2} \| r_{o4}) + \dfrac{1}{r_{o6} \| r_{o7}}}$$

Since $r_{o6} \| r_{o7}$ is huge, we can neglect $\dfrac{1}{r_{o6} \| r_{o7}}$ and have :

$$R_o \approx \frac{1}{g_{m6}g_{m1}(r_{o2} \| r_{o4})}$$

Ex: 10.4



$G_{m1} = 1\text{ mA / V}\ \ G_{m2} = 2\text{ mA / V}$

$C_2 = 1P$

$r_{o2} = r_{o4} = 100\text{ K}, r_{o6} = r_{o7} = 40\text{ K}$

$$\text{(a) } f_t = \frac{G_{m1}}{2\pi C_C} = \frac{1\text{ mA / V}}{2\pi C_C} = 100\text{ MHz}$$

$\Rightarrow C_C = 1.6\text{ pF}$

$A_1 = -G_{m1}R_1 = (1 \times 10^{-3})(r_{o2} \| r_{o4})$

$= (1 \times 10^{-3})(100\text{ K} / 2) = -50\text{ V / V}$

$A_2 = -G_{m2}R_2 = (2 \times 10^{-3})(r_{o6} \| r_{o7})$

$= (2 \times 10^{-3})(40(\text{ K}) / 2) = -40\text{ V / V}$

$A = A_1 \cdot A_2 = (-50)(-40) = +2000$

$f_p = f_t / A = 100 \times 10^6 / 2 \times 10^3 = 50\text{ KHz}$

(b) to move zero to $S = \infty$

$$R = \frac{1}{G_{m2}} = \frac{1}{2 \times 10^{-3}} = 500\ \Omega$$

$$f_{p2} \approx \frac{G_{m2}}{2\pi C_2} = \frac{0.2 \times 10^{-3}}{2\pi 10^{-9}} = 318 \times 10^6 \text{ HZ}$$

$$\theta = \tan^{-1}\frac{f_t}{f_P} = \tan^{-1}\frac{100 \times 10^6}{318 \times 10^6} = 17.4^\circ$$

$$\text{PM} = 90 - \theta = 72.6^\circ$$

Ex: 10.5

Find SR for $f_t = 100$ MHz

$V_{OV1} = 0.2$ V

$$SR = 2\pi f_t V_{OV} = 2\pi \times 100 \times 10^6 \times 0.2$$

$$= 125.67 \cong 126 \text{ V}/\mu\text{S}$$

$$SR = \frac{I}{C_C} \Rightarrow I = SR \times C_C$$

$$\therefore I = 126 \times 10^6 \times 1.6 \times 10^{-12}$$

$$= 200\ \mu\text{A}$$

Ex: 10.6

$$V_{ICM(\max)} \leq V_{DD} - V_{OV9} + V_{tn}$$

$$\leq +1.65 - 0.3 + 0.5 = +1.85 \text{ V}$$

$$V_{ICM(\min)} \geq -V_{SS} + V_{OV11} + V_{OV1} + V_{tn}$$

$$\geq -1.65 + 0.3 + 0.3 + 0.5 = -0.55 \text{ V}$$

$$V_{o(\max)} \leq V_{DD} - |V_{OV9}| - |V_{OV}|$$

$$\leq +1.65 - 0.3 - 0.3 = +1.05 \text{ V}$$

$$V_{o(\min)} \geq -V_{SS} + V_{OV11} + V_{OV1} + V_{tn}$$

$$\geq -1.66 + 0.3 + 0.3 + 0.5 = -0.55 \text{ V}$$

Ex: 10.7

$|V_A| = 20$ V, $V_{OV} = 0.2$V, $I = 100\ \mu$A

$$G_m = \frac{2I}{V_{OV}} = \frac{2 \times 100 \times 10^{-6}}{0.2} = 1.0 \text{ mA/V}$$

$$r_o = \frac{V_A}{I} = \frac{20 \times 10^6}{100} \Rightarrow 200 \text{ k}\Omega$$

$$R_O = [g_{m4}r_{o4}(r_{o2} \| r_{o10})] \| [g_{m6}r_{o6}r_{o5}]$$

$$= g_m r_o^2\left[\frac{1}{2}(1 \| 2)\right]$$

$$= 1.0 \times 200^2 \times 1/3 \times 10^6 = 13.33 \text{ M}\Omega$$

$$A = G_m R_o = 1.0 \times 10^{-3} \times 13.33 \times 10^6$$

$$= 13.33 \times 10^3 \text{ V/V}$$

Ex: 10.8

Given : all $V_{OV} = 0.3$ V, $|V_T| = 0.7$ V

$V_{DD} = V_{SS} = 2.5$V

(a) $V_{IC(\max)}$ for NMOS

$$V_{ICM(\max)_N} \leq V_{DD} - V_{OV} + V_T$$

$$\leq +2.5 - 0.3 + 0.7 = +2.9 \text{ V}$$

$$V_{ICM(\min)_N} \geq -V_{SS} + V_{OV} + V_{OV} + V_T$$

$$\geq -2.5 + 0.3 + 0.3 + 0.7 = -1.2 \text{ V}$$

$$\therefore -1.2V \leq (V_{ICM})_N \leq +2.9\text{V}$$

(b) By Sym.

$$-2.9 \text{ V} \leq (V_{ICM})_P \leq +1.2 \text{ V}$$

(c) $-1.2 \text{ V} \leq (V_{ICM})_{\text{BOTH}} \leq +1.2$ V

(d) $-2.9 \text{ V} \leq (V_{ICM})_{\text{overall}} \leq +2.9$ V

Ex: 10.9

$$I_1 = \frac{1}{2}\text{K}(W/L)(V_{GS1} - V_T)^2$$

$$I_2 = \frac{1}{2}\text{K}(W4/L)(V_{GS2} - V_T)^2$$

For $I_1 = I_2$:

$$(V_{GS1} - V_T)^2 = 4(V_{GS2} - V_T)^2$$

i.e., $V_{GS1} - V_T = 2(V_{GS2} - V_T)$

or $V_{GS1} = 2V_{GS2} - V_T$

Ex: 10.10

npn : $I_S = 10^{-14}$A, $\beta = 200$, $V_A = 125$ V

pnp : $I_S = 10^{-14}$A, $\beta = 50$, $V_A = 50$ V

$$I = I_S e^{V_{BE}/V_T}$$

$$\Rightarrow V_{BE} = V_T \ln\frac{I}{I_S}$$

$$V_{BE} = 25 \text{ mV} \ln\frac{10^{-3}}{10^{-14}} = 633 \text{ mV}$$

$$g_m = \beta / r_E = 200/5\text{k} = 40 \text{ mA/v}$$

$$r_\pi \approx \beta_{re} \approx 200 \times 25 \Rightarrow 5 \text{ k}\Omega$$

$$r_e = \frac{V_T}{I} = \frac{25\text{mV}}{1\text{mA}} = 25\ \Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{125\text{V}}{1\text{mA}} = 125 \text{ k}\Omega$$

Ex: 10.11

$$I = I_S e^{V_B/V_T} \Rightarrow V_{BE} = V_T \ln(I/I_S)$$

and $I_3 = I_4$, $I_1 = I_2$

From ckt : $V_{BE1} + V_{BE2} = V_{BE3} + V_{BE4}$

$$V_T \ln\left[\frac{I_1}{I_{S1}}\right] + V_T \ln\left[\frac{I_2}{I_{S2}}\right] = V_T \ln\left[\frac{I_3 I_4}{I_{SS} I_{S4}}\right]$$

$$\therefore \ln\left[\frac{I_1^2}{I_{S1} \cdot I_{S2}}\right] = \ln\left[\frac{I_3^2}{I_{S3} I_{S4}}\right]$$

$$\therefore I_3 = I_1\left[\frac{I_{S3} I_{S4}}{I_{S1} I_{S2}}\right]^{\frac{1}{2}}$$

Ex: 10.12

$V_{BE} = 0.7$ V for $I_C = 1$ mA for $Q11$

$I_C = 10\ \mu$A for $Q10$

$$V_{BE10} = 0.7 + V_t \ln\left[\frac{10\ \mu\text{A}}{1\ \text{mA}}\right] = 0.585\ \text{V}$$

Voltage across $R_4 = V_{BE10} - V_{BE11}$

$= 0.7 - 0.585 = 0.115$ V

$$\Rightarrow I_{R4} = \frac{V_{R4}}{R_4} \Rightarrow R_4 = \frac{0.115\ \text{V}}{10\ \mu\text{A}} \Rightarrow 11.5\ \text{k}\Omega$$

Ex: 10.13

$V_{ICM(max)} = V_{DD} - V_{BE8} - V_{1sat} + V_{BE}$

$= +15 - 0.6 - 0.3 + 0.6$

$= +14.7$ V

$V_{ICM(min)} = -V_{SS} + V_{BE5} + V_{BE7} + V_{3sat} + V_{BE1}$

$= -15 + 0.6 + 0.6 + 0.3 + 0.6$

$= -12.9$ V

(neglecting R_1 & R_2 drops)

Ex: 10.14

Assume Q_{18}, Q_{19} (now diode connected transistors) have normal area

Q_{14}, Q_{20} have area = 3 * normal

$$I_{14} = 0.25 I_{\text{REF}} \sqrt{\frac{I_{S14} \cdot I_{S20}}{I_{S18} \cdot I_{S19}}}$$

$$I_{14} = 180\ \mu \sqrt{\frac{3I}{I} \cdot \frac{3I}{I}}$$

$$I_{14} = 180\ \mu \times 3 = 540\ \mu\text{A}$$

Ex: 10.15

Assume $I_{C7} \approx I_{C5} \approx I_{C6} = 9.5\ \mu$A

(a) $V_{B6} = I_E(R_2 + r_{e6}) = i_e(1 + 2.63)$

$= 3.63 I_3$

(b) $I_{E7} = \dfrac{V_{B6}}{R_3} + I_{B5} + I_{B6}$

$= \dfrac{V_{B6}}{R_3} + \dfrac{2I_E}{\beta + 1} = \dfrac{3.63 I_E}{50} + \dfrac{2I_E}{201}$

$= 0.08 I_E$

(c) $I_{B7} = \dfrac{V_{E7}}{\beta + 1} \approx \dfrac{0.08 I_E}{201} \approx 0.0004 I_E$

(d) $V_{B7} = V_{B6} + I_{E7} r_{e7}$

$= 3.63 I_E + 0.08 I_E \times \dfrac{25\ \text{mV}}{9.5\ \mu\text{A}}$

$= 3.84\text{k}\Omega \times I_E$

(e) $R_{in} = \dfrac{(\beta + 1)}{\beta} \dfrac{V_{B7}}{I_E} \approx 3.84\ \text{k}\Omega$

Ex:

See Fig10.22 on page of the Text, Let $R_1 = R$, $R_2 = R + \Delta R$ Assume $\beta >> 1$ and $r_{e5} = r_{e6}$, then

$V_{B5} = V_{B6} = i(r_{e5} + R_1) = i(r_{e6} + R_2)$

$$\therefore i_{C6} = i \frac{r_{e5} + R}{r_{e5} + R + \Delta R}$$

$i_o = i_{C6} - i = i$

$$\therefore i_6 = i \frac{r_{e5} + R}{r_{e5} + R + \Delta R} - i = i \frac{\Delta R}{r_{e5} + R + \Delta R}$$

$$\Rightarrow \epsilon_m = \frac{\Delta R}{r_{e5} + R + \Delta R}$$

For $\dfrac{\Delta R}{R} = 0.02$:

$$\epsilon_m = \frac{0.02R}{R + 0.02R + r_{e5}} = \frac{0.02}{1.02 + \frac{r_{e5}}{R}}$$

Substituting $R = 1\ \text{k}\Omega$ and $r_e = 2.63\ \text{k}\Omega$ for 741 op-amp, we have

$$\epsilon_m = \frac{0.02}{1.02 + \frac{2.63}{1}} = 5.5 \times 10^{-3}$$

Ex: 10.17

From Fig.10.23 on page of the Text:

$R_o = R_{o9} \parallel R_{o10}$

$$R_{o9} = r_{o9} = \frac{V_A}{I} = \frac{50}{19 \times 10^{-6}} = 2.63\ \text{M}\Omega$$

$$R_{o10} = r_{o10}(1 + g_{m10}(r_{\pi10} \parallel R_4)) = \frac{125}{19 \times 10^{-6}}$$

$$\left(1 + \frac{19 \times 10^{-6}}{25 \times 10^{-3}}\left(\frac{200 \times 25 \times 10^{-3}}{19 \times 10^{-6}} \parallel 5 \times 10^{3}\right)\right)$$

$R_{o10} = 31.1\ \text{M}\Omega$

$R_o = 2.63 \parallel 31.1 = 2.42\ \text{M}\Omega$

Ex: 10.18

From Eq.10.93 we have: $G_{mcm} = \dfrac{\epsilon_m}{2R_o}$ From Ex.10.16and10.17in the Text, we have:

$\epsilon_m = 5.5 \times 10^{-3}$, $R_o = 2.43\ \text{M}\Omega$

Hence:

$$G_{mcm} = \frac{5.5 \times 10^{-3}}{2 \times 2.43 \times 10^{6}} = 1.13 \times 10^{-6}\ \text{mA/V}$$

From Eq.10.95we have:

$CMRR = 2g_{m1}(R_{o9} \parallel R_{o10}) / \epsilon_m$

$$CMRR = \frac{2\left(\frac{9.5 \times 10^{-6}}{25 \times 10^{-3}}\right)\left(\frac{2.63\ \mu \times 3.1.1\ \mu}{2.63\ \mu + 31.1\ \mu}\right)}{5.5 \times 10^{-3}}$$

1.68×10^5 or 104.5 dB

If the common-mode feedback is not present, as explained in the text, common-mode transconductance and common-mode gain are both reduced by a factor of β_P. Hence,

$$CMRR = \frac{1.68 \times 10^5}{50} = 3360 \text{ or}$$

$$CMRR = 70.5 \text{ dB}$$

Ex: 10.19

$\beta_{16} = \beta_{17} = 200$

$$r_{e16} = \frac{25 \text{ mV}}{16.2\ \mu\text{A}} = 1.54 \text{ k}\Omega$$

$$r_{e17} = \frac{25 \text{ mV}}{0.55 \text{ mA}} = 45.5\ \Omega$$

$R_8 = 100\ \Omega$, $R_9 = 50 \text{ k}\Omega$

Substituting into Eq. (8.77

$$R_{i2} = 201[1.54 + 50 \| (201 \times 0.0455)]$$

$$\simeq 4 \text{ M}\Omega$$

Ex: 10.20

$$i_{c17} = \frac{\beta}{\beta + 1} \cdot \frac{V_{b17}}{r_{e17} + R_8} \simeq \frac{V_{b17}}{45.5 + 100} = \frac{V_{b17}}{145.5}$$

$$V_{b17} = V_{i2}\frac{(R_9 \| R_{i17})}{(R_9 \| R_{i17}) + r_{e16}}$$

needs $R_{i17} = (\beta + 1)(r_{e17} + R_8)$

$$= 201(45.5 + 100) = 29.2 \text{ k}\Omega$$

$\therefore V_{b17} \simeq V_{i2} \times 0.92$

Ex: 10.21

$R_{o2} = R_{o13B} \| R_{o17}$

where $R_{o13B} = r_{o13B} = \dfrac{50\text{V}}{0.55 \text{ mA}} = 90.9 \text{ k}\Omega$

$$R_{o17} = r_{o17}(1 + g_{m17}(R_8 \| r_{\pi17}))$$

$$r_{o17} = \frac{125\text{V}}{0.55 \text{ mA}} = 227.3 \text{ k}\Omega$$

$$g_{m17} = \frac{0.55 \text{ mA}}{0.025 \text{ mV}} = 22 \text{ mA / V}$$

$$r_{\pi 17} = \frac{\beta}{g_m} = \frac{200}{22} = 9.09 \text{ k}\Omega$$

$R_8 = 100\ \Omega$

Thus $R_{o17} \Rightarrow 722 \text{ k}\Omega$

Hence $R_{o2} = 90.9 \| 722k \simeq 81 \text{ k}\Omega$

Ex: 10.22

$$\frac{I_1}{V_1} = \frac{1}{18.8} + 6.6 \times 0.917 = 0.05 + 6.05$$

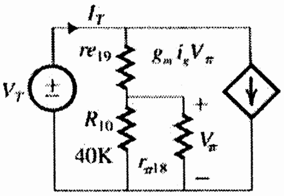
$$= 6.11$$

open-circuit voltage gain

$$(A_2)_{\text{o/c}} = -G_{m2} R_{02}$$

$$= -6.5 \times 8.1 = -526.5 \text{ V / V}$$

Ex: 10.23



$$r_{e19} = \frac{25 \text{ mV}}{16\ \mu\text{A}}$$

$$= 1.56 \text{ K}$$

$$r_{\pi 18} = \frac{200}{40 \times 0.165}$$

$$= 30.3 \text{ k}\Omega$$

$$I_T = \frac{V_T}{r_{e19} + (R_{10} \| r_{\pi 18})} + \frac{g_{m18} V_T (R_{10} \| r_{\pi 18})}{r_{e19} + (R_{10} \| r_{\pi 18})}$$

$$\therefore I_T = V_T\left[\frac{1}{18.8 \text{ K}} + \frac{6.6 \times 0.917}{18.8 \text{ K}}\right]$$

$$= V_T[0.05 + 6.05]$$

$$\Rightarrow R_T \equiv \frac{V_T}{I_T} = 163\ \Omega$$

Ex: 10.24

$$R_O \simeq r_{e14} + \frac{\left[R_{18,19} + r_{e23} + \dfrac{R_{O2}}{\beta_{23} + 1}\right]}{\beta_{14} + 1}$$

$$R_O \simeq \frac{0.025}{0.005} + \frac{\left(163 + \dfrac{25 \text{ M}}{180\ \mu} + \dfrac{81 \text{ K}}{51}\right)}{201}$$

Assuming $\beta_{23} = 50$ and $\beta_{14} = 200$ and

$I_{c13A} = 180\ \mu\text{A}$ from Table

$$R_O \simeq \frac{5 + (163 + 139 + 1588)}{201} = 5 + 94$$

$R_O \simeq 14.4\ \Omega$

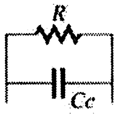
Ex: 10.25

$SR = 0.63\ \mathrm{V/\mu S}$

using eq.

$$f_n = \frac{SR}{2\pi V_{O\,max}} = \frac{0.63}{2\pi(10)\times 1\times 10^{-6}} = 10\ \mathrm{kHz}$$

Ex: 10.26



$A_o = 2.43147\times 10^5$

$G_{m1} = \frac{1}{5.26}\times 10^{-3}$

$\therefore A_o = G_{m1}R$

$\therefore R = A_o/G_{m1} \Rightarrow 1279\ \mathrm{MHz}$

Ex: 10.27

$SR = \frac{W_t}{a}$ where

$a = \frac{G_{m1}}{2I} \Rightarrow SR = \frac{2I}{G_{m1}}\times w_t$

with R_E inserted in emitters of Q_3, Q_4

$$G_{m1} = 2\times\frac{1}{4r_e + 2R_E} = \frac{1}{2r_e + R_E}$$

$$= \frac{1}{2\times\frac{0.025\ \mathrm{mV}}{I} + R_E} = \frac{I}{2r_T + IR_E}$$

for $I = 9.5\times 10^{-6}$ A

$$R_E = \frac{0.050}{9.5\times 10^{-6}} = 5.26\ \mathrm{k\Omega}$$

now $SR = \frac{2Iwt}{I}\times(2V_T + IR_E)$

$= 4[V_T + I_{RE}/2]^{wt}$ QED

new C_c: $\frac{G'_{m1}}{C_c} = \frac{G_{m1}}{2C_c}$

$\therefore$ C_c must be reduce × factor of 2

$C_{c\,new} = \frac{C_{c\,old}}{2} = \frac{30}{2} = 15\ \mathrm{pF}$

Gain A α G_{m1} $\therefore$ A also halved

$A_{new} = A_{old} - 6\ \mathrm{dB} = 101.7\ \mathrm{dB}$

$f_p = f_t/\mathrm{A} = f_t$ has been halved

$f_{p\,new} = 2\times f_{p\,old} = 8.2\ \mathrm{Hz}$

Ex: 10.28

using eq. (10.129

$I = \frac{V_T}{R_2}\ln\left(\frac{I_{S2}}{I_{S1}}\right)$ where $V_T = 25\ \mathrm{mV}$

$R_3 = R_4 = \frac{0.2}{10\ \mu} = 20,000\ \Omega$

$R_2 = \frac{V_T}{I}\ln\left(\frac{I_{S2}}{I_{S1}}\right)$

$R_2 = \frac{25\ \mathrm{m}}{10\ \mu}\ln(2) = 1,733\ \Omega$

Ex: 10.29

use $R_3 = R_4 = 20\text{ k}\Omega$

and $I = 10\ \mu\text{A}$ from Exercise 10.28

For $I_8 = 10\ \mu\text{A} = I$, then $\left(\frac{W}{L}\right)_8 = \left(\frac{W}{L}\right)_3$

For $I_9 = 20\ \mu\text{A} = 2I$, then $\left(\frac{W}{L}\right)_9 = 2\left(\frac{W}{L}\right)_3$

For $I_{10} = 5\ \mu\text{A} = \frac{I}{2}$, then $\left(\frac{W}{L}\right)_{10} = \frac{1}{2}\left(\frac{W}{L}\right)_3$

Since V_S has to equal the original $(V_{CC} - I * R_4) = V_{CC} - 0.2$ so R_8, R_9, and R_{10} can be found by

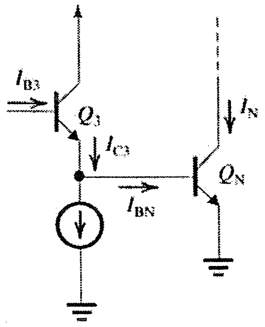
$$R_8 = \frac{0.2}{10\ \mu} = 20\text{ k}\Omega$$

$$R_9 = \frac{0.2}{20\ \mu} = 10\text{ k}\Omega$$

$$R_{10} = \frac{0.2}{5\ \mu} = 40\text{ k}\Omega$$

Ex: 10.30

(a) Find $\frac{i_N}{i_{B3}}$ for (V_{IN})



Assume

$i_{C3} \approx i_{BN}$

$i_{B3} \le \frac{i_{C3}}{\beta_N}$

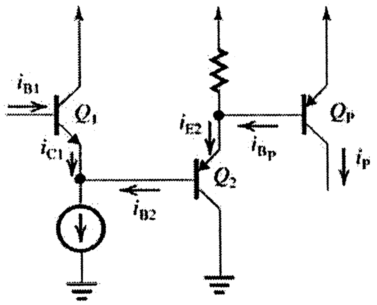
$i_{C3} \approx i_{BN} = \frac{i_N}{\beta_N}$

$$\therefore \frac{i_N}{i_{B1}} = \frac{i_N}{\left(\frac{i_N}{\beta_N}\right)\left(\frac{1}{\beta_N}\right)} = \beta_N^2$$

Find $\frac{i_P}{i_{B1}}$ for (V_{IP})

Assume $i_{\beta P} \approx i_{E2}$ and $i_{C1} \approx i_{B2}$

$$\therefore \frac{i_P}{i_{B1}} = \frac{i_P}{\left(\frac{i_P}{\beta_P}\right)\left(\frac{1}{\beta_P}\right)\left(\frac{1}{\beta_N}\right)} = \beta_P^2 \cdot \beta_N$$



(b) $i_{B3} = \frac{i_N}{\beta_N^2}$ (Assume $\beta_N \approx 40$)

$$i_{B3} = \frac{10\text{ m}}{(40)^2} = 6.25\ \mu\text{A}$$

$i_{B1} = \frac{i_P}{\beta_P^2 \beta_N}$ (Assume $\beta_P \approx 10$)

$$i_{B1} = \frac{10\text{ m}}{(10)^2 \cdot 40} = 2.5\ \mu\text{A}$$

Ex: 11.1

$A = -20 \log |T|$ [dB]

$\|T\| =$	1	0.99	0.9	0.8	0.7	0.5	0.1	0
$A \simeq$	0	0.1	1	2	3	6	20	∞

Ex: 11.2

$A_{max} = 20 \log 1.05 - 20 \log 0.95 = 0.9$ dB

$$A_{min} = 20 \log \left[\frac{1}{0.001}\right] = 40 \text{ dB}$$

Ex: 11.3

$$T(s) = k\frac{(s + j2)\ (s - j2)}{\left(s + \frac{1}{2} + j\sqrt{\frac{3}{2}}\right)\left(s + \frac{1}{2} - j\sqrt{\frac{3}{2}}\right)}$$

$$= k\frac{(s^2 + 4)}{s^2 + s + \frac{1}{4} + \frac{3}{4}}$$

$$= k\frac{(s^2 + 4)}{s^2 + s + 1}$$

$$T(o) = k\frac{4}{1} = 1$$

$$k = \frac{1}{4}$$

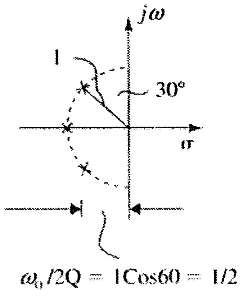
$$\therefore T(s) = \frac{1}{4}\,\frac{(s^2 + 4)}{s^2 + s + 1}$$

Ex: 11.4

$$T(s) = k\frac{s(s^2 + 4)}{(s + 0.1 + j8)\ (s + 0.1 - j8)\ (s + 0.1 + j1.2)\ (s + 0.1 - j1.2)}$$

$$= k\frac{s(s^2 + 4)}{(s^2 + 0.23 + 0.65)\ (s^2 + 0.23 + 1.45)}$$

Ex: 11.5



As shown, the pair of computex poles has $\omega_0 = 1$ and $Q = 1$

$$\omega_0 / zQ = 1 \cos 60 = \frac{1}{2}$$

$$\frac{1}{2Q} = \frac{1}{2}$$

$$Q = 1$$

$$\therefore T(s) = k\frac{1}{(s + 1)\ (s^2 + s + s + 1)}$$

since $T(0) = 1, k = 1$

$$\text{Thus: } T(s) = \frac{1}{(s + 1)\ (s^2 + s + 1)}$$

$$T(j\omega) = \frac{1}{\sqrt{1 + \omega^2}\sqrt{(1 - \omega^2)^2 + \omega^2}}$$

$$= \frac{1}{\sqrt{(1 - \omega^4)\ (1 - \omega^2) + \omega^2\ (1 + \omega^2)}}$$

$$= \frac{1}{\sqrt{1 - \omega^4 - \omega^2 + \omega^6 + \omega^2 + \omega^4}}$$

$$\frac{1}{\sqrt{1 + \omega^6}} \text{ Q.E.D}$$

$$\text{Thus: } \frac{1}{\sqrt{2}} = \frac{1}{(1 + \omega_{3dB}^6)^{\frac{1}{2}}} \Rightarrow \omega_{3dB} = 1 \text{ rad / s}$$

$$A(3) = -20 \log \frac{1}{\sqrt{1 + 3^6}} = 28.6 \text{ dB}$$

Ex: 11.6

$$\epsilon = \sqrt{10^{\frac{A_{max}}{10}} - 1} = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

$$A(\omega_s) = -20 \log|T(j\omega_s)|$$

$$= 10 \log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

Thus, $10 \log[1 + 0.5888^2 \times 1.5^{2N}] \geq 30$

$N = 10$ LHS $= 29.35$ dB

$N = 11$ LHS $= 32.87$ dB

$\therefore$ Use $N = 11$ and obtain

$A_{min} = 32.87$ dB

For A_{min} to be exactly 30 dB

$10 \log[1 + \epsilon^2 \times 1.5^{22}] = 30$

$\epsilon \Rightarrow 0.3654 \Rightarrow A_{max} + 20 \log \sqrt{1 + 0.3654^2}$

$= 0.54$ dB

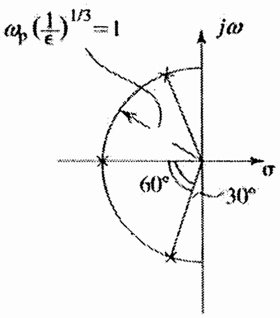
Ex: 11.7

The real pole is at $s = -1$

The complex conjugate poles are at

$$s = 2\cos 60^\circ \pm j\sin 60^\circ$$

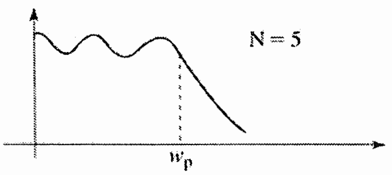
$$= -0.5 \pm j\sqrt{\frac{3}{2}}$$



$$T(s) = \frac{1}{(s+1)\left(s+0.5+j\sqrt{\frac{3}{2}}\right)\left(s+0.5-j\sqrt{\frac{3}{2}}\right)}$$

$$= \frac{1}{(s+1)(s^2+s+1)} \text{ for } DC_{gain} = 1$$

Ex: 11.8



$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\cos^2\left[N\cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right]}}$$

for $\omega < \omega_p$.

Peaks are obtained when

$$\cos^2 5\left[N\cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right] = 0$$

$$\cos^2 5\left[5\cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right] = 0$$

$$5\cos^{-1}\left(\frac{\omega}{\omega_p}\right) = (2k+1)\frac{\pi}{2}, k = 0, 1, 2$$

$$\therefore \hat{\omega} = \omega_p \mathrm{Cos}\left[\frac{(2k+1)\pi}{10}\right], k = 0, 1, 2$$

$$\hat{\omega}_1 = \omega_p \mathrm{Cos}\left(\frac{\pi}{10}\right) = 0.95\omega_p$$

$$\hat{\omega}_2 = \omega_p \mathrm{Cos}\left(\frac{3}{10}\pi\right) = 0.59\omega_p$$

$$\hat{\omega}_3 = \omega_p \mathrm{Cos}\left(\frac{5}{10}\pi\right) = 0$$

Valleys are obtained when

$$\mathrm{Cos}^2\left[N\mathrm{Cos}^{-1}\left(\frac{\omega}{\omega_p}\right)\right] = 1$$

$$5\mathrm{Cos}^{-1}\left(\frac{\omega}{\omega_p}\right) = k\pi, k = 0, 1, 2$$

$$\therefore \hat{\omega} = \omega_p \mathrm{Cos}\left(\frac{k\pi}{5}\right), k = 0, 1, 2$$

$$\hat{\omega}_1 = \omega_p \mathrm{Cos}0 = \omega_p$$

$$\hat{\omega}_2 = \omega_p \mathrm{Cos}\frac{\pi}{5} = 0.81\omega_p$$

$$\hat{\omega}_3 = \omega_p \mathrm{Cos}\frac{2\pi}{5} = 0.31\omega_p$$

Ex: 11.9

$$\epsilon = \sqrt{10^{\frac{A_{max}}{10}} - 1} = \sqrt{10^{\frac{0.5}{10}} - 1} = 0.3493$$

$$A(\omega_s) = 10\log\left[1+\epsilon^2\cosh^2\left(N\cosh^{-1}\frac{\omega_s}{\omega_p}\right)\right]$$

$$= 10\log[1+0.3493^2\cosh^2(7\cosh^{-1}2)]$$

$$= 64.9 \text{ dB}$$

For $A_{max} = 1$ dB, $\epsilon = \sqrt{10^{0.1}-1} = 0.5088$

$$A(\omega_s) = 10\log[1+0.5088^2\cosh^2(7\cosh^{-1}2)]$$

$$= 68.2 \text{ dB}$$

This is an increase of 3.3 dB

Ex: 11.10

$$\epsilon = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$$

(a) For the Chebyshev Filter:

$$A(\omega_s) = 10\log[1+0.5088^2\cosh^2(N\cosh^{-1}1.5)]$$

$$\geq 50 \text{ dB}$$

$N = 7.4 \therefore$ choose $N = 8$

Excess Attenuation =

$$10\log[1+0.5088^2\cosh^2(8\cosh^{-1}1.5)] - 50$$

$$= 55 - 50 = 5 \text{ dB}$$

(b) For a Butterworth Filter

$\epsilon = 0.5088$

$$A(\omega_s) = 10\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

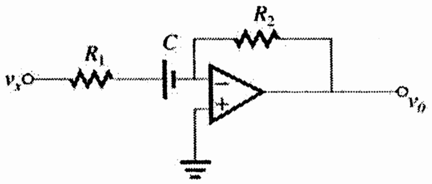
$$= 10\log[1+0.5088^2(1.5)^{2N}] \geq 50$$

$N = 15.9 \therefore$ choose $N = 16$

Excess attenuation =

$$10\log([1+0.5088^2(1.5)^{32}] - 50) = 0.5 \text{ dB}$$

Ex: 11.11



$$10^4 = \frac{1}{CR_1} \quad R_1 = 10\ \text{k}\Omega$$

$$C = 0.01\ \mu\text{F}$$

$$\text{H.F. Gain} = \frac{-R_2}{R_1} = -10$$

$$R_2 = 100\ \text{k}\Omega$$

Ex: 11.12

Refer to Fig. 11.14

$$\omega_0 = \frac{1}{CR} = 10^3\ \text{rad/s}$$

For R arbitrarily selected to be

$$10\ \text{k}\Omega\ C = \frac{1}{10^3 \times 10^4} = 0.1\ \mu\text{F}$$

The two resistors labelled R_i can also be selected to be 10 kΩ each.

Ex: 11.13

$$T(s) = \frac{\omega_0^2}{s + s\sqrt{2\omega_0 + \omega_0^2}} \text{ (for dc gain = 1)}$$

$$|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\omega_0^2\,\omega^2)}}$$

$$= \frac{\omega_0^2}{\sqrt{\omega_0^4 + \omega^4}}$$

$$= \frac{\omega_0^2}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^4}}$$

At $\omega = \omega_0$, $|T| = \frac{1}{\sqrt{2}}$ which is 3 dB below the value at dc(unity) Q.E.D.

Ex: 11.14

This $T(s) = \dfrac{10^3 s}{s^2 + 10^3 s + 10^{10}}$

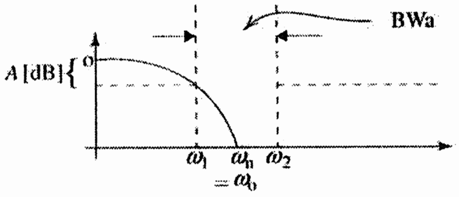
$$\omega_0 = 10^5\ \text{rad/s}$$

$$\frac{\omega_0}{Q} = 10^3\text{rad/s}$$

selected to yeild a centre. Frequency gain of 10.

Ex: 11.15

(a)



$$T(S) = \frac{s^2 + \omega_O^2}{s^2 + s\,\omega_O/Q + \omega_O^2}$$

$$|T(j\omega)| = \frac{\omega_O^2 - \omega^2}{\sqrt{(\omega_O^2 - \omega^2)^2 + \frac{\omega^2\omega_O^2}{Q^2}}}$$

$$= \frac{1}{\sqrt{1 + \frac{\omega_O^2\omega^2}{(\omega_O^2 - \omega^2)^2 Q^2}}}$$

for any two frequencies ω_1 and ω_2 at which $|T|$ is the same

$$\frac{\omega_1^2\omega_O^2}{(\omega_O^2 - \omega_1^2)^2} = \frac{\omega_2^2\,\omega_O^2}{(\omega_O^2 - \omega_2^2)^2}$$

$$\omega_1(\omega_O^2 - \omega_2^2) = \omega_2(\omega_O^2 - \omega_1^2)$$

$$\Rightarrow \omega_1\omega_2 = \omega_O^2 \qquad (1)$$

Now to obtain attenuation ≥ A dB at ω_1 and ω_2 where $\omega_2 - \omega_1 = \text{Bw}_a$

$$10 \log\left[1 + \frac{\omega_O^2\omega_1^2}{(\omega_O^2 - \omega_1^2)^2 Q^2}\right] \geq A$$

$$\frac{\omega_1\omega_O}{\omega_O^2 - \omega_1^2}\,\frac{1}{Q} \geq \sqrt{10^{A/10} - 1} \text{ SUB (1)}$$

$$\frac{\omega_1\omega_O}{\omega_1\omega_2 - \omega_1^2}\,\frac{1}{Q} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{\omega_O}{\omega_2 - \omega_1}\,\frac{1}{Q} \geq \sqrt{10^{A/10} - 1}$$

$$\frac{\omega_O}{BW_a}\,\frac{1}{Q} \geq \sqrt{10^{A/10} - 1}$$

$$\Rightarrow Q \leq \frac{\omega_O}{BW_a\sqrt{10^{A/10} - 1}} \quad \text{Q.E.D.}$$

(b) For $A = 3$ dB

$$Q = \frac{\omega_O}{BW_3\sqrt{10^{0.3} - 1}} = \frac{\omega_O}{BW_3}$$

OR $BW_3 = \omega_o/Q$ Q.E.D.

Ex: 11.16

From Fig 10.16(e)

$$\omega_{max} = \omega_O\sqrt{\frac{(\omega_n/\omega_O)^2\left(1-\frac{1}{2Q^2}\right)-1}{(\omega_n/\omega_o)^2+\frac{1}{2Q^2}-1}}$$

For $\omega_o = 1$ rad/s, $\omega_n = 1.2$ rad/s $Q = 10$

$$\text{dc gain} = |a_2|\left(\frac{\omega_n^2}{\omega_o^2}\right) = 1$$

$$|a_2| = \omega^2{}_o/\omega^2{}_n = \frac{1}{1.44}$$

$$\omega_{max} = 1\sqrt{\frac{1.44\left(1-\frac{1}{200}\right)-1}{1.44+\frac{1}{200}-1}}$$

$= 0.986$ rad/s

$$|T(j\omega_{max})| = \frac{a_2|(\omega^2{}_n-\omega^2{}_{max})|}{\sqrt{(\omega^2{}_n-\omega^2{}_{max})^2+\left(\frac{\omega_o\omega_{max}}{Q}\right)^2}}$$

$= 3.17$

$$|T(j\infty)| = Q_2 = \frac{1}{1.44} = 0.69$$

Ex: 11.17

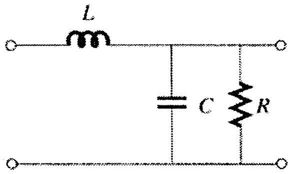
Maximally flat $\Rightarrow Q = \frac{1}{\sqrt{2}}$

$\omega_o = 2\pi \times 100 \times 10^3$

Arbitrarily selecting R = 1 kΩ

$$Q = \omega_o CR \Rightarrow C = \frac{1}{\sqrt{2}\times 2\pi 10^5 \times 10^3}$$

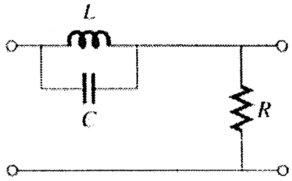
$= 1125$ pF



Also $Q = \frac{R}{\omega_o L}$

$$\therefore L = \frac{R}{\omega_o Q} = \frac{10^3}{2\pi 10^5 \times \frac{1}{\sqrt{2}}} = 2.25 \text{ mH}$$

Ex: 11.18



From Exercise 11.16 above 3dB bandwidth

$= \omega_o/Q$

$2\pi 10 = 2\pi 60/Q \Rightarrow Q = 6$

$Q = \omega_o CR$

$6 = 2\pi 60 \times C \times 10^4 \Rightarrow C = 1.6\mu F$

$$Q = \frac{R}{\omega_o L}$$

$$L = \frac{R}{\omega_o Q} = \frac{10^4}{2\pi 60 \times 6} = 4.42 \text{ H}$$

Ex: 11.19

$f_o = 10$ kHz $\Delta f_{3dB} = 500$ Hz

$$Q = \frac{f}{\Delta f_{3dB}} = \frac{10^4}{500} = 20$$

Using the data at the top of Table 11.1

$C_A = C_6 = 1.2$ nF

$$R_1 = R_2 = R_3 = R_5 = \frac{1}{\omega_o C}$$

$$= \frac{1}{2\pi 10^4 \times 1.2 \times 10^{-9}} = 13.26 \text{ k}\Omega$$

$$R_6 = Q/\omega_{oC} = \frac{20}{2\pi 10^4 \times 1.2 \times 10^{-9}} = 265 \text{ k}\Omega$$

Now using the data in Table 11.1 for the bandpass case

K = centre-frequency gain = 10

$1 + r_2/r_1 = 10$

Selecting $r_1 = 10$ kΩ then $r_2 = 90$ kΩ

Ex: 11.20

Eq (16.25) ~ $\omega_p = 2\pi 10^4$

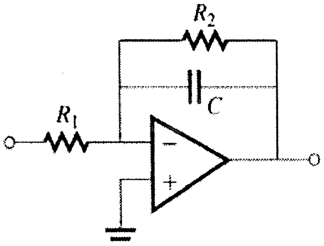
$$T(s) = \frac{\omega_p^5}{8.1408(s+0.2895\omega_p)} \times$$

$$\frac{1}{(s^2+s0.4684\omega_p+0.4293\omega_p^2)} \times$$

$$\frac{1}{(s^2+s0.1789\omega_p+0.9883\omega_p^2)}$$

The circuit consists of 3 sections in cascade:

(a) First order section



the number coefficient was set so that the dc gain = 1.

$$T(s) = \frac{-0.2895\omega_p}{s+0.2895\omega_p}$$

Let $R_1 = 10\text{ k}\Omega$

dc gain $= R_2 / R_1 = 1 = R_2 = 10\text{ k}\Omega$

as $j\omega \rightarrow \infty$

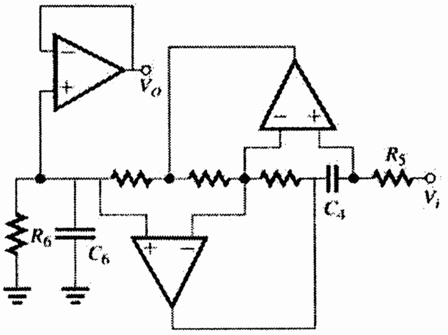
$$|T(j\omega)| \rightarrow \frac{0.2985\omega_p}{\omega} = \frac{1}{\omega CR_1}$$

$$C = \frac{1}{0.2895 \times 2\pi 10^4 \times 10^4} = 5.5\text{ nF}$$

(b) Second-Order section with transfer function:

$$T(s) = \frac{0.4295\omega_p^2}{s^2 + 0.4684\omega_p + 0.4293\omega_p^2}$$

where the numerator coefficient was selected to yield a dc gain of unity



Select $R_1 = R_2 = R_3 = R_5 = 10\text{ k}\Omega$

$$\Rightarrow C = \frac{1}{\sqrt{0.4293} \times 2\pi 10^4 \times 10^4} = 2.43\text{ nF}$$

$C_4 = C_6 = C = 2.43\text{nF}$

$$Q = \frac{\sqrt{0.4293}\omega_p}{0.4684\omega_p} = 1.4 \Rightarrow R_6 = \frac{Q}{\omega_0 C} = 14\text{ k}\Omega$$

(c) Second-Order Section with Transfer-function:

$$T(s) = \frac{0.9883\omega^2{}_p}{s^2 + s0.1789\omega_p + 0.9883\omega^2{}_p}$$

The circuit is similar to that in (b) above but with $R_1 = R_2 = R_3 = R_5 = 10\text{ k}\Omega$

$$C_4 = C_6 = \frac{1}{\omega_0 \times 10^4} = \frac{1}{\sqrt{0.9883} \times 2\pi 10^4 \times 10^4}$$

$= 1.6\text{ nF}$

$$Q = \frac{\sqrt{0.9883}}{0.1789} = 5.56$$

Thus $R_6 = Q / \omega_0 C = 55.6\text{ k}\Omega$

Placing the three sections in cascade, i.e. connecting the output of the first-order section to the input of the second-order section in (b) and the output of of section (b) to the input of (c) results in the overall transfer function in eq. 11.25

Ex: 11.21

Refer to the KHN circuit in Fig. 11.24 Choosing $C = 1\text{nF}$

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi 10^4 \times 10^{-9}} = 15.9\text{ k}\Omega$$

Using Eq. 11.62 and selecting $R_1 = 10\text{ k}\Omega$

$R_f = R_1 = 10\text{ k}\Omega$

Using Eq. 11.63 and setting $R_2 = 10\text{ k}\Omega$

$R_3 = R_2(2Q - 1) = 10(2 \times 2 - 1) = 30\text{ k}\Omega$

$$\text{High freuency gain} = K = 2 - \frac{1}{Q} = 1.5\text{ V/V}$$

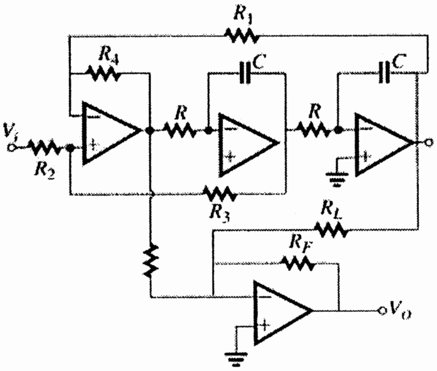
The transfer function to the output of the first integrator is

$$\frac{V_{np}}{V_i} = -\frac{1}{SCR} = \frac{V_{np}}{V_i} = \frac{sK/(CR)}{s^2 + s\frac{\omega_0}{Q} + \omega^2{}_0}$$

Thus the centre-frequency gain

$$= \frac{K\ Q}{CR\omega_0} = KQ = 1.5 \times 2 = 3\text{ V/V}$$

Ex: 11.22



$$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 + (R_F/R_L)\omega^2{}_0}{s^2 + s\omega_0/Q + \omega^2{}_0}$$

given $C = 1\text{nF}$ $R_L = 10\text{ k}\Omega$

$$R = \frac{1}{\omega_{0C}} = \frac{1}{2\pi 5 \times 10^3 \times 10^{-9}} = 31.83\text{ k}\Omega$$

$R_1 = 10\text{ k}\Omega \Rightarrow R_F = 10\text{ k}\Omega$

$R_2 = 10\text{ k}\Omega \Rightarrow R_3 = R_2(2Q - 1)$

$= 10(10 - 1) = 90\text{ k}\Omega$

$$\frac{R_H}{R_L}\omega_0{}^2 = \omega_n{}^2 \Rightarrow R_H = 10\left(\frac{8}{5}\right)^2 = 25.6\text{ k}\Omega$$

$$\text{DC gain} = K\frac{R_F}{R_L} = \left(2 - \frac{1}{Q}\right)\frac{R_F}{R_L} = 3$$

$$R_F = \frac{3 \times 10}{2 - 1/5} = 16.7\text{ k}\Omega$$

Ex: 11.23

Refer to Fig.16.25(b)

$$CR = \frac{1}{\omega_0} \Rightarrow C = \frac{1}{2\pi 10^3 \times 10^4} = 1.59 \text{ nF}$$

$$R_d = QR = 20 \times 10 = 200 \text{ k}\Omega$$

Centre frequency gain = $KQ = 1$

$$\therefore K = \frac{1}{Q} = \frac{1}{20}$$

$$R_g = R/K = 20R = 200 \text{ k}\Omega$$

Ex: 11.24

Refer to Fig11.26and Table 11.2

$$C = 10 \text{ nF}$$

$$R = \frac{1}{\omega_o C} = \frac{1}{10^4 \times 10 \times 10^{-9}} = 10 \text{ k}\Omega$$

$$QR = 5 \times 10 = 50 \text{ k}\Omega$$

$$C_1 = C \times \text{flat gain} = 10 \times 1 = 10 \text{ nF}$$

$$R_1 = \infty$$

$$R_2 = \frac{R}{\text{gain}} = R/1 = 10 \text{ k}\Omega$$

$$r = 10 \text{ k}\Omega$$

$$R_3 = \frac{Q_r}{\text{gain}} = \frac{5 \times 10}{1} = 50 \text{ k}\Omega$$

Ex: 11.25

From eq (11.76

$$CR = \frac{2Q}{\omega_o} = \frac{2 \times 1}{10^4} = 2 \times 10^4 \text{s}$$

For $C = C_1 = C_2 = 1\text{nF}$

$$R = \frac{2 \times 10^{-4}}{10^{-9}} = 200 \text{ k}\Omega$$

Thus $R_3 = 200 \text{ k}\Omega$

From eq. (11.75

$$m = 4Q^2 = 4$$

$$\text{Thus, } R_4 = \frac{R}{M} = \frac{200}{4} = 50 \text{ k}\Omega$$

Ex: 11.26

The transfer function of the feedback network is given in Fig. (11.28a .The poles are the roots of the denominator polynomial.

$$s^2 + s\left(\frac{1}{C_1R_3} + \frac{1}{C_2R_3} + \frac{1}{C_1R_4}\right) + \frac{1}{C_1C_2R_3R_4} = 0$$

For $C_1 = C_2 = 10^{-9}$ F, $R_3 = 2 \times 10^5 \ \Omega$,

$R_4 = 5 \times 10^4 \ \Omega$,

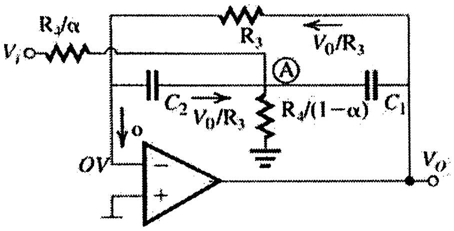
$$s^2 + s\left(\frac{2}{10^{-9} \times 2 \times 10^5} + \frac{1}{10^{-9} \times 5 \times 10^4}\right) + \frac{1}{10^{-18}10^{10}} = 0$$

$$s^2 + s(3 \times 10^4) + 10^8 = 0$$

$$s = \frac{-3 \times 10^4 \pm \sqrt{9 \times 10^8 - 4 \times 10^8}}{2}$$

$$= -0.382 \times 10^4 \text{ and } -2.618 \times 10^4 \ \frac{\text{rad}}{\text{s}}$$

Ex:11.27



$$V_A = O - \frac{V_O}{SC_2R_3}$$

ΣI at (A)

$$\frac{V_O}{R_3} + SC_1(V_O - V_A) + \frac{-V_A}{R_y/(1-\alpha)}$$

$$+ \frac{V_i - V_A}{R_y/\alpha} = 0$$

$$\frac{V_O}{R_3} + SC_1V_O + \frac{SC_1V_O}{SC_2R_3} + \frac{(1-\alpha)V_O}{SC_2R_3R_4} + \frac{\alpha V_i}{R_4}$$

$$+ \frac{\alpha V_O}{SC_2R_3R_4} = 0$$

$$\frac{V_O}{V_i} = \frac{-\alpha/R_4}{SC_1 + 1/R_3 + \frac{C_1}{C_2R_3} + \frac{1}{SC_2R_3R_4}}$$

$$= \frac{-S\alpha/(C_1R_4)}{S^2 + S\left(\frac{1}{C_1R_3} + \frac{1}{C_2R_3}\right) + \frac{1}{C_1C_2R_3R_4}}$$

This is a bandpass function whose poles are identical to the zeros of $t(s)$ in Fig. (11.28a).

For $C_1 = C_2 = 10^{-9}$ F, $R_3 = 2 \times 10^5 \ \Omega$ & $R_4 = 5 \times 10^4 \ \Omega$

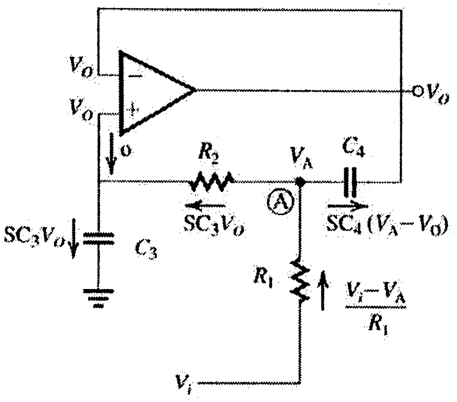
$$\frac{V_O}{V_i} = \frac{-S \times 2 \times 10^4 \times \alpha}{S^2 + S \times 10^4 + 10^8}$$

For unity centre-frequency gain

$$2 \times 10^4 \times \alpha = 10^4 \Rightarrow \alpha = 0.5$$

$$\text{Thus } \frac{R_4}{\alpha} = 100 \text{ k}\Omega; \ \frac{R_4}{1-\alpha} = 100 \text{ k}\Omega$$

Ex: 11.28



$V_A = V_o + SC_3V_oR_2$
$= V_o(1 + SC_3R_2)$
$\Sigma\, I$ at (A)

$$SC_3V_O - \frac{V_i}{R_1} + \frac{V_O}{R_1} + \frac{V_O}{R_1}SC_3R_2 + V_OSC_4(SC_3R_2)$$

$$\frac{V_i}{R_1} = V_O\left[S^2C_3C_4R_2 + \frac{SC_3R_2}{R_1} + SC_3 + \frac{1}{R_L}\right]$$

$$\frac{V_O}{V_i} = \frac{1/C_3C_4R_1R_2}{S^2 + S\frac{1}{C_4R_2}\left(1 + \frac{R_2}{R_1}\right) + \frac{1}{C_3C_4R_1R_2}}$$

$$\omega_O = \frac{1}{\sqrt{C_3C_4R_1R_2}} \text{ as in Eq. (16.77)}$$

$$Q = \frac{1}{\sqrt{C_3C_4R_1R_2}}\ \frac{C_4}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \text{ as in Eq (11.78}$$

D.C. gain = 1 Q.E.D.

Ex: 11.29

From Fig11.34(c)
$R_1 = R_2 = R = 10\text{ k}\Omega$
$CR = 2Q/\omega_o$

$$C = \frac{2Q}{\omega_O R} = \frac{2/\sqrt{2}}{2\pi(4\times10^3)10^4} = 5.63\text{ nF}$$

$$m = 4Q^2 = 4\left(\frac{1}{2}\right) = 2$$

$C_1 = 2.81$ nF

Ex: 11.30

Refer to the results in Example 11.2
(a) $\Delta R_3/R_3 = +2\%$

$$S^{\omega_o}_{R3} = -1/2 \Rightarrow \frac{\Delta\omega_O}{\omega_O} = -\frac{1}{2}\times 2 = -1\%$$

$$S^{Q}_{R3} = -\frac{1}{2} \Rightarrow \Delta Q/Q = \frac{1}{2}\times 2 = 1\%$$

(b) $\Delta R_4/R_4 = 2\%$

$$S^{\omega_o}_{R_4} = -\frac{1}{2} \Rightarrow \frac{\Delta\omega_O}{\omega_O} = -1\%$$

$$S^{Q}_{R_4} = -\frac{1}{2} \Rightarrow \frac{\Delta Q}{Q} = -\frac{1}{2}\times 2 = -1\%$$

(c) Combining the results in (a) & (b)

$$\frac{\Delta\omega_O}{\omega_O} = -1 - 1 = -2\%$$

$$\frac{\Delta Q}{Q} = 1 - 1 = 0\%$$

(d) using the results in (c) for both resistors being 2% high we have:

$$\frac{\Delta\omega_O}{\omega_O} = S^{\omega_o}_{C1}\frac{\Delta C_1}{C_1} + S^{\omega_o}_{C2}\frac{\Delta C_2}{C_2} - 2$$

$$= -\frac{1}{2}(-2) + \frac{-1}{2}(-2) - 2$$

$$= 2 - 2 = 0\%$$

$$\frac{\Delta Q}{Q} = S^{Q}_{C1}\frac{\Delta C_1}{C} + S^{Q}_{C2}\frac{\Delta C_2}{C_2} + 0$$

$$= 0(-2) + (0)(-2) + 0 = 0\%$$

Ex: 11.31

From Eq (11.96 & (11.97
$C_3 = C_4 = \omega_O T_C C$

$$= 2\pi 10^4 \times \frac{1}{200\times10^3} \times 20$$

$= 6.283$ pF

From Eq. (11.99

$$C_5 = \frac{C_4}{Q} = \frac{6.283}{20} = 0.314\text{ pF}$$

From Eq (11.100

$$\text{Centre-frequency gain} = \frac{C_6}{C_5} = 1$$

$C_6 = C_5 = 0.314$ pF

Ex: 11.32

$R_p = \omega_O L Q_O = 2\pi 10^6 \times 3.2\times10^{-6}\times 150 = 3\text{ k}\Omega$
$R = R_L \parallel r_O \parallel R_p = 2\text{ k}\Omega \Rightarrow R_L = 15\text{ k}\Omega$

Ex: 11.33

$Q = (R_1 \parallel R_{in})/\omega_O L$

$$= \frac{10^3 \parallel 10^3}{(2\pi\times455\times10^3)\times5\times10^{-6}} = 35$$

$BW = f_O/Q = 455/35 = 13$ KHz

$$C_1 + C_{in} = \frac{1}{\omega_O^2 L}$$

$$= \frac{1}{(2\pi\times455\times10^3)^2\times5\times10^{-6}}$$

$= 24.47$ nF

$C_1 = 24.47 - 0.2 = 24.27$ nF

Ex: 11.34

To just meet specifications

$$Q = \frac{f_0}{BW} = \frac{455}{10} = 45.5$$

$$\therefore \frac{R_1 \| n^2 R_{in}}{\omega_0 L} = 45.5$$

$$R_1 \| n^2 R_{in} = 45.5 \times 455 \times 10^3 \times 5 \times 10^{-6}$$

$$= 650\ \Omega$$

$$n^2 R_{in} = 1.86\ \text{k}\Omega$$

$$n = \sqrt{\frac{1.86}{1}} = 1.36$$

$$C_1 + \frac{C_{in}}{n^2} = \frac{1}{\omega_0^2\ L} = 24.47$$

$$C_1 = 24.36\ \text{nF}$$

At resonance, the voltage developed across R_1 is $I(R_1 \| n^2 R_{in})$. Thus, $V_{De} = IR/n$ &

$I_C = g_m V_{De} = g_m IR/n$, here

$$\frac{I_C}{I} = g_m R/n = \frac{40 \times 0.65}{1.36} = 19.1 \frac{\text{A}}{\text{A}}$$

Ex: 11.35

$$200 = f_0/Q\sqrt{2^{1/2} - 1} \qquad \text{Eq (16.110)}$$

$$\frac{f_0}{Q} = 310.8\ \text{kHz}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi \times 10.7 \times 10^6)^2 \times 3 \times 10^{-6}}$$

$$= 73.7\ \text{pF}$$

$$\frac{\omega_0}{Q} = \frac{1}{C_R}$$

$$R = \frac{1}{73.7 \times 10^{-12} \times 2\pi \times 310.8 \times 10^3}$$

$$= 6.95\ \text{k}\Omega$$

Ex: 11.36

$$f_{01} = f_0 + \frac{2\pi B}{2\sqrt{2}} \qquad \text{Eq(16.115)}$$

$$= 10.7\ \text{MHz} + \frac{200}{2\sqrt{2}}\ \text{kHz} = 10.77\ \text{MHz}$$

$$B_1 = B/\sqrt{2} = 200/\sqrt{2} = 141.4\ \text{KHz}$$

$$f_{02} = f_0 - \frac{2\pi B}{2\sqrt{2}} \qquad \text{Eq(16.116)}$$

$$= 10.7\ \text{MHz} - 200/2\sqrt{2} = 10.63\ \text{MHz}$$

$$B_2 = \frac{200}{\sqrt{2}} = 141.4\ \text{KHz}$$

For stage 1

$$C = \frac{1}{\omega_{01}^2 L} = \frac{1}{(2\pi\ 10.77 \times 10^6)^2 \times 3 \times 10^{-6}}$$

$$= 72.8\ \text{pF}$$

$$R = \frac{1}{CB_1} = \frac{1}{72.8 \times 10^{-12} \times 141.4 \times 2\pi\ 10^3}$$

$$= 15.5\ \text{k}\Omega$$

For stage 2

$$C = \frac{1}{\omega_{02}^2 L} = \frac{1}{(2\pi\ 10.63 \times 10^6)^2 \times 3 \times 10^{-6}}$$

$$= 74.7\ \text{pF}$$

$$R = \frac{1}{CB_2} = \frac{1}{74.7 \times 10^{-12} \times 141.4 \times 2\pi\ 10^3}$$

$$= 15.1\ \text{k}\Omega$$

Ex: 11.37

Gain of stagger-tuned amplifier at f_0 is proportional to

$$\frac{1}{\sqrt{2}} R_{\text{stage1}} \times \frac{1}{\sqrt{2}} R_{\text{stage2}}$$

$$= \frac{1}{2} \times 15.5 \times 15.1 = 117$$

Gain of synchronous-tuned amplifier at to

$$\alpha\ R_{\text{stage1}} \times R_{\text{stage2}}$$

$$= 6.95 \times 6.95$$

$$= 48.3$$

$$\therefore \text{Ratio} = \frac{117}{48.3} = 2.42$$

Ex: 12.1

Pole frequency f_o = 1 kHz

$$\text{Centre frequency gain} = \frac{1}{\text{AMPLIFIER GAIN}}$$

$$= \frac{1}{2}\text{V / V}$$

Ex: 12.2

$$L_+ = V^{R_1/R_5} + V_D\left(1 + \frac{R_4}{R_3}\right)$$

$$= 15\left(\frac{3}{9}\right) + 0.7\left(1 + \frac{3}{9}\right)$$

$$= 5 + 0.93 = +5.93\text{ V}$$

$$L_- = -V\frac{R_3}{R_2} - V_D\left(1 + \frac{R_3}{R_2}\right)$$

$$= -15 \times \frac{3}{9} - 0.7\left(1 + \frac{3}{9}\right)$$

$$= -5.93\text{ V / V}$$

$$\text{Limiter gain} = \frac{-R_f}{R_1} = \frac{-60}{30}$$

$$= -2\text{V / V}$$

Thus limiting occurs at $\frac{\pm 5.93}{2}$

$= \pm 2.97$ V

Slope in the limiting regions

$$= \frac{-R_f \parallel R_4}{R_1} = -\frac{60 \parallel 3}{30} = -0.095\ \frac{\text{V}}{\text{V}}$$

Ex: 12.3

$$\text{(a) } L(s) = \left(1 + \frac{R_2}{R_1}\right)\frac{Z_P}{Z_P + Z_S}$$

$$= \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{1 + Z_S Y_P}\right)$$

$$= \left(1 + \frac{20.3}{10}\right)\left(\frac{1}{1 + \left(R + \frac{1}{SC}\right)\left(\frac{1}{R} + SC\right)}\right)$$

$$= \frac{3.03}{3 + SCR + \frac{1}{SCR}}$$

where R = 10 kΩ and C = 16 nF

Thus

$$L(s) = \frac{3.03}{3 + S16 \times 10^{-5} + \frac{1}{S \times 16 \times 10^{-5}}}$$

The closed loop poles are found by setting $L(s) = 1$, that is, they are the values of s, satifiying

$$3 + S \times 16 \times 10^{-5} + \frac{1}{S \times 16 \times 10^{-5}} = 3.03$$

$$\Rightarrow S = \frac{10^5}{16}(0.015 \pm j)$$

(b) The frequency of oscillation is $(10^5/16)$ rad/s or 1 kHz

(c) Refer to fig. 12.5At the positine peak $\hat{V}_O$, the voltage at node b will be one diode drop (0.7 V) above the voltage V_1 which is about 1/3 of V_o; thus $V_b = 0.7 + \hat{v}_O/3$. Now if we neglect the current through D_2 in comparison with the currents through R_5 and R_6 we find that

$$\frac{\hat{V}_O - V_b}{R_5} \cong \frac{V_b - (-15)}{R_6}$$

Thus,

$$\frac{\hat{V}_O - V_b}{1} = \frac{V_b + 15}{3}$$

$$\hat{V}_O = \frac{4}{3}V_b + 5$$

$$\hat{V}_O = \frac{4}{3}\left(0.7 + \frac{\hat{V}_O}{3}\right) + 5$$

$$\Rightarrow \hat{V}_O = 10.68\text{ V}$$

from symmetry, we see that the negative peak is equal to the positive peak. Thus the output peak-to-peak voltage is 21.36 V

Ex: 12.4

a) for oscillations to start, $R_2/R_1 = 2$ thus the potentiometer should be set so that its resistance to ground is 20 kΩ

$$\text{(b) } f_o = \frac{1}{2\pi RC} = \frac{1}{2\pi 10 \times 10^3 \times 16 \times 10^{-9}}$$

$$= 1\text{ kHz}$$

Ex: 12.5

Working from the output back to the input and continuing the equations we get I

$$I = \frac{V_O}{R_f} + \frac{V_O}{SCR_fR} + \frac{V_O}{SCR_fR} + \frac{1}{SCR}$$

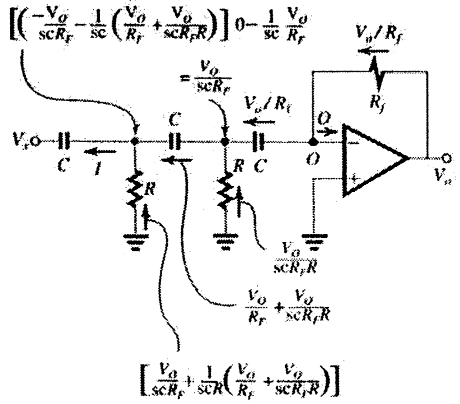
$$\left(\frac{V_O}{R_f} + \frac{V_O}{SCR_fR}\right)$$

$$V_x = \frac{-V_O}{SCR_f} - \frac{1}{SC}\left(\frac{V_O}{R_f} + \frac{V_O}{SCR_fR}\right) - \frac{I}{SC}$$

$$V_x = \frac{-V_O}{SCR_f}\left(2 + \frac{1}{SCR}\right)$$

$$\frac{-V_O}{SCR_f}\left[1 + \frac{1}{SCR} + \frac{1}{SCR} + \frac{1}{SCR}\left(1 + \frac{1}{SCR}\right)\right]$$

$$= \frac{-V_O}{SCR_f}\left(3 + \frac{4}{SCR} + \frac{1}{S^2C^2R^2}\right)$$


$\left[\left(-\frac{V_O}{scR_f}-\frac{1}{sc}\left(\frac{V_O}{R_f}+\frac{V_O}{scR_fR}\right)\right]0-\frac{1}{sc}\frac{V_O}{R_f}$
V_O/R_f
$=\frac{V_O}{scR_f}$
V_O/R_f
R_f
V_x
C
I
C
R
R
C
O
O
V_O
$\frac{V_O}{scR_fR}$
$\frac{V_O}{R_f}+\frac{V_O}{scR_fR}$
$\left[\frac{V_O}{scR_f}+\frac{1}{scR}\left(\frac{V_O}{R_f}+\frac{V_O}{scR_fR}\right)\right]$

Thus:

$$\frac{V_O}{V_x} = \frac{-SCR_f}{3 + \frac{4}{SCR} + \frac{1}{S^2C^2R^2}}$$

$$\frac{V_O}{V_x}(j\omega) = \frac{-j\omega CR_f}{4 + j\left(3\omega CR - \frac{1}{\omega CR}\right)}$$

Ex: 12.6

The circuit will oscillate at the value of ω that makes $\frac{V_O}{V_x}(j\omega)$ a real number.

It follows that ω_o is obtained from

$$3\omega_0 CR = \frac{1}{\omega_0 CR} \Rightarrow \omega_0 = \frac{1}{\sqrt{3}CR}$$

$$\text{Thus, } f_o = \frac{1}{2\pi\sqrt{3} \times 16 \times 10^{-9} \times 10 \times 10^3}$$

$$= 574.3 \text{ Hz}$$

For oscillations to begin, the magnitude of $\frac{V_O}{V_x}(j\omega)$ should equal to (or greater than) unity, that is

$$\frac{\omega_0^2 C^2 RR_f}{4} \geq 1$$

Thus the minimum value of R_f is

$$R_f = \frac{4}{\omega_0^2 C^2 R} = \frac{4\text{ R}}{\omega_0^2 C^2 R^2} = \frac{4\text{ R}}{\frac{1}{3}}$$

$$= 12 \text{ R or } 120 \text{ k}\Omega$$

Ex: 12.7

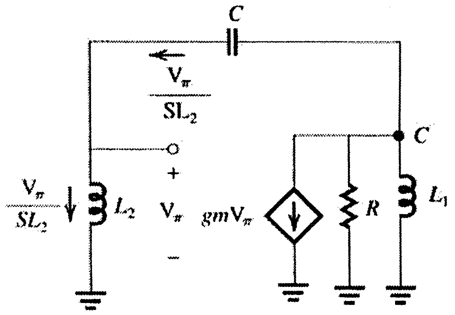
$$\omega_0 = \frac{1}{CR} \Rightarrow CR = \frac{1}{2\pi 10^3}$$

For $C = 16$ nF $R = 10$ kΩ

$\therefore$ the output is twice as large as the voltage across the resonator, the peak-to-peak amplitude is

$$\frac{4\text{ V}}{\pi} = \frac{4(2 \times 1.4)}{\pi} = 3.6 \text{ V}$$

Ex: 12.8


C
$\frac{V_\pi}{SL_2}$
C
$\frac{V_\pi}{SL_2}$
L_2
$+$
V_π
$-$
gmV_π
R
L_1

$$V_c = V_\pi + \frac{V_\pi}{SL_2} \cdot \frac{1}{SC} = V_\pi\left(1 + \frac{1}{S^2CL_2}\right)$$

Node equation at collector:

$$\frac{V_\pi}{SL_2} + g_m V_\pi + \frac{V_c}{R} + \frac{V_c}{SL_1} = 0$$

$$\frac{V_\pi}{SL_2} + g_m V_\pi + \frac{V_\pi}{R}\left(1 + \frac{1}{S^2CL_2}\right)$$

$$+ \frac{V_\pi}{S_4}\left(1 + \frac{1}{S^2CL_2}\right) = 0$$

Since $V_\pi \neq D$, (oscillations have started) it can be eliminated resulting in

$$S^3 L_1 L_2 C\left(g_m + \frac{1}{R}\right) + S^2(L_1 C + L_2 C)$$

$$+ S\frac{L_1}{R} + 1 = 0$$

Substituting $S = j\omega$

$$[1 - \omega^2 C(L_1 + L_2)] + j\omega$$

$$\left[\frac{L_1}{R} - \left(g_m + \frac{1}{R}\right) \times \omega^2 L_1 L_2 C\right] = 0$$

$$R_E = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{C(L_1 + L_2)}} \text{ Q.E.D.}$$

$$I_m = 0 \Rightarrow g_m R + 1 = \frac{1}{\omega_0^2 L_2 C}$$

$$= \frac{L_1 + L_2}{L_2}$$

$$\Rightarrow g_m R = L_1 / L_2$$

for oscillations to start

$g_m R > L_1 / L_2$ Q.E.D.

Ex: 12.9

$$R = \frac{Q}{\omega_0} \| R_L \| \Gamma_O$$

$$= \frac{100}{10^6 \times 10^{-8}} \| 2 \times 10^3 \| 100 \times 10^3$$

$$= 10 \| 2 \| 100 = 1.64 \text{ k}\Omega$$

$$\frac{C_2}{C_1} = g_m R = 40 \times 1.64 = 65.6$$

$$C_2 = 65.6 \times 0.01 = 0.66\ \mu\text{F}$$

$$L = \frac{1}{\omega_0^2 \dfrac{C_1 C_2}{C_1 + C_2}}$$

$$= \frac{1}{10^{12} \times \dfrac{0.01 \times 0.66 \times 10^{-6}}{0.01 + 0.66}} \cong 100\ \mu\text{A}$$

Ex: 12.10

from Eq (12.24

$$f_S = \frac{1}{2\pi\sqrt{LC_s}} = \frac{1}{2\pi\sqrt{0.52 \times 0.012 \times 10^{-12}}}$$

$$= 2.015\ \text{MHz}$$

from Eq (12.25

$$f_P = \frac{1}{2\pi\sqrt{L\dfrac{C_S C_P}{C_p + C_P}}}$$

$$= \frac{1}{2\pi\sqrt{0.52 \times \dfrac{0.012 \times 4 \times 10^{-12}}{0.012 + 4}}}$$

$$= 2.018\ \text{MHz}$$

$$Q = \frac{\omega_o L}{r} \cong \frac{\omega_S L}{r}$$

$$= \frac{2\pi \times 2.015 \times 10^6 \times 0.52}{120}$$

$$\cong 55,000$$

Ex: 12.11

$$V_{TH} = V_{TL} = \beta\, |L \pm|$$

$$5 = \frac{R_1}{R_1 + R_2} \times 13$$

$$\frac{R_2}{R_1} = 1.6$$

$$R_2 = 16\ \text{k}\Omega$$

Ex: 12.12

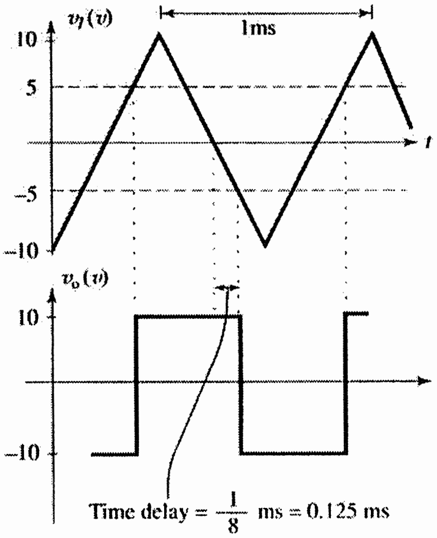
$$V_{TH} - V_{TL} = \frac{R_1}{R_2}|L|$$

$$5 = \frac{R_1}{R_2} \times 10$$

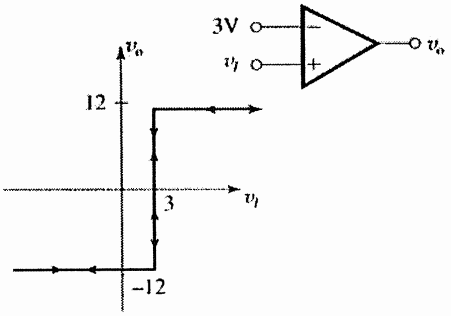
$$R_2 = 2R_1$$

Possible choice $R_1 = 10\ \text{k}\Omega \quad R_2 = 20\ \text{k}\Omega$

Ex: 12.13



Ex: 12.14



A comparator with a threshold of 3 V and output levels of ±12 V

Ex: 12.15

$$|V_T| = \frac{100}{2} = 50\ \text{mV}$$

$$50 \times 10^{-3} = 10\,\frac{R_1}{R_2}$$

$$\frac{R_2}{R_1} = \frac{10}{0.06}$$

$$R_2 = 200\,R_1$$

for $R_1 = 1\ \text{k}\Omega \quad R_2 = 200\ \text{k}\Omega$

Ex: 12.16

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{100}{100 + 1000} = 0.091\ \frac{\text{V}}{\text{V}}$$

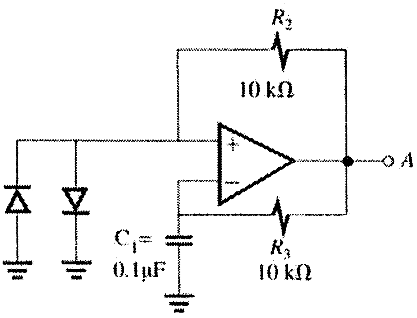
$$T = 2\tau \ln \frac{1 + \beta}{1 - \beta}$$

$$2 \times 0.01 \times 10^{-6} \times 10^{6} \times \ln\left(\frac{1.091}{1 - 0.091}\right)$$

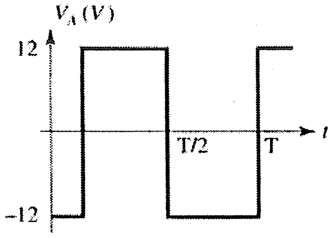
$$= 0.00365\ \text{s}$$

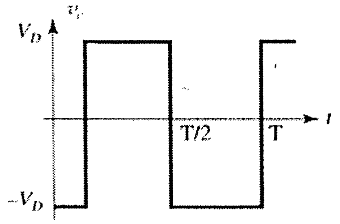
$$f_o = \frac{1}{T} = 274\ \text{Hz}$$

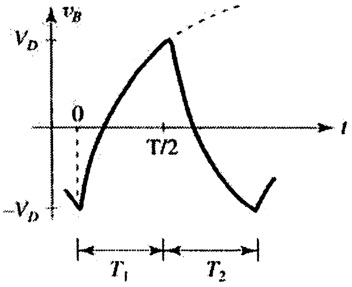
Ex: 12.17



$$T_1 = T_2 = T/2$$







During T_1

$$V_B(t) = 12 - (12 + V_D)e^{-t/5}$$

$$V_B = V_D \text{ at } t = T/2$$

$$V_D = 12 - (12 + V_D)e^{-T/25}$$

$$T = 2\tau \ln\left(\frac{12 + V_D}{12 - V_D}\right)$$

$$= 2 \times 0.1 \times 10^{-6} \times 10 \times 10^{3} \times \ln\left(\frac{12 + V_D}{12 - V_D}\right)$$

$$f = \frac{1}{T} = \frac{500}{\ln\left(\frac{12 + V_D}{12 - V_D}\right)}$$

$$f|_{25^\circ C} = \frac{500}{\ln\left(\frac{12.7}{11.3}\right)}$$

$$= 4281\ \text{Hz}$$

At 0°C, $V_D = 0.7 + .05 = 0.75$ V

$$f|_{0^\circ C} = \frac{500}{\ln\left(\frac{12.75}{11.25}\right)} = 3,995\ \text{Hz}$$

At 50°C, $V_D = 0.7 - 0.05 = 0.65$ V

$$f|_{50^\circ C} = \frac{500}{\ln\left(\frac{12.65}{11.35}\right)} = 4,611\ \text{Hz}$$

At 100°C, $V_D = 0.7 - 0.15 = 0.55$ V

$$f|_{100^\circ C} = \frac{500}{\ln\left(\frac{12.55}{11.45}\right)} = 5,451\ \text{Hz}.$$

Ex: 12.18

To obtain a triangular waveform with 10-V peak-to-peak amplitute we should have

$$V_{TH} = -V_{TL} = 5\ \text{V}$$

But $V_{TL} = -L + \frac{R_1}{R_2}$

Thus $-5 = -10 \times \frac{10}{R_2}$

$R_2 = 20$ kΩ

For 1 kHz frequency, T = 1ms
Thus,

$$T/2 = 0.5\times10^{-3} = CR\frac{V_{TH}-V_{TL}}{4}$$

$$= 0.01\times10^{-6}\times R\times10/10$$

$$R = 50\text{ k}\Omega$$

Ex: 12.19

Using Eq (12.37

$$100\times10^{-6} = 0.1\times10^{-6}\times R_3\ln\left(\frac{12.7}{10.8}\right)$$

$$R_3 = 6171\ \Omega$$

Ex: 12.20

$T = 1.1CR \Rightarrow R = T/1.1C = 9.1\text{ k}\Omega$

Ex: 12.21

$$T = 0.69C(R_A+2R_B)$$

$$\frac{1}{100\times10^3} = 0.69\times10^3\times10^{-12}(R_A+2R_B)$$

$$\Rightarrow R_A+2R_B = \frac{1}{0.69\times10^{-4}} = 14.49\text{ k}\Omega\ (1)$$

Using Eq (13.45)

$$0.75 = \frac{A+R_B}{R_A+2R_B}$$

$$R_A+R_B = 0.75\times14.44 = 10.88\text{ k}\Omega \qquad (2)$$

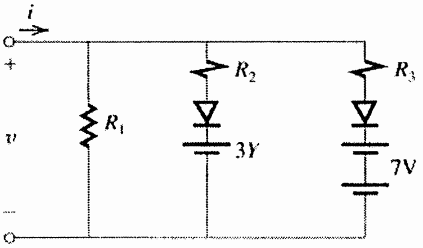
$$(1)-(2)\Rightarrow R_B = 3.61\text{ k}\Omega$$

Now, substituting into (2)

$$R_A = 7.27\text{ k}\Omega$$

Use 7.2 kΩ and 3.6 kΩ, standard 5% resistors.

Ex: 12.22



$i = 0.1\ v^2$

At $v = 2$ V, $i = 0.4$ mA

$$\text{Thus } R_1 = \frac{2}{0.4} = 5\text{ k}\Omega$$

For $3V \le V \le 7V$

$$i = \frac{v}{R_1}+\frac{v-3}{R_2}$$

To obtain a perfect match at $V = 4$ V (i.e. to obtain $i = 1.6$ mA)

$$1.6 = \frac{4}{5}+\frac{4-3}{R_2}$$

$$R_2 = 1.25\text{ k}\Omega$$

for $v \ge 7V$

$$i = \frac{v}{R_1}+\frac{v-3}{R_2}+\frac{v-7}{R_3}$$

To obtain a perfect match at $v = 8V$ we must have to select R_3 so that $i = 6.4$ mA,

$$6.4 = \frac{8}{5}+\frac{8-3}{1.25}+\frac{8.7}{R_3}$$

$$\Rightarrow R_3 = 1.25\text{ k}\Omega$$

At $v = 3$ V, the circuit provides .

$$i = \frac{3}{5} = 0.6\text{ mA while ideally}$$

$i = 0.1\times9 = 0.9$ mA . Thus the error is -0.3 mA.

* At V = 5V, the circuit provides

$$i = \frac{5}{5}+\frac{5-3}{1.25} = 2.6\text{ mA , while ideally}$$

$i = 0.1\times25 = 2.5$ mA . Thus the error is $+0.1$ mA.

* At $v = 7$ V, the circuit provides

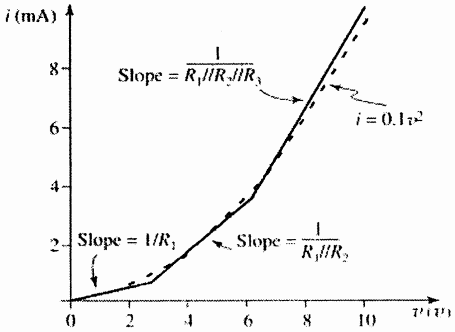
$$i = \frac{7}{5}+\frac{7-3}{1.25} = 4.6\text{ mA , while ideally. Thus}$$

the error is -0.3 mA

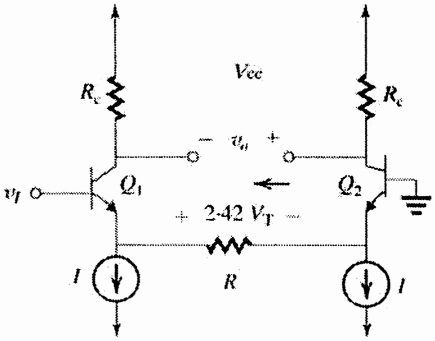
* At $v = 10V$, the circuit provides,

$$i = \frac{10}{6}+\frac{10-3}{1.25}+\frac{10-7}{1.25} = 10\text{ mA , while}$$

ideally $i = 10$ mA. Thus the error is 0 A.



Ex: 12.23



$I_{e_1} = I + (2.42\ V_T)/R$

$= I\left[1 + \dfrac{2.42\ V_T}{I_R}\right]$

$= I\left[1 + \dfrac{2.42\ V_T}{2.5\ V_T}\right]$

$= I\left(1 + \dfrac{2.42}{2.5}\right)$

$I_{e_1} \cong I\left(1 + \dfrac{2.42}{2.5}\right)$

$I_{e_2} \cong I\left(1 - \dfrac{2.42}{2.5}\right)$

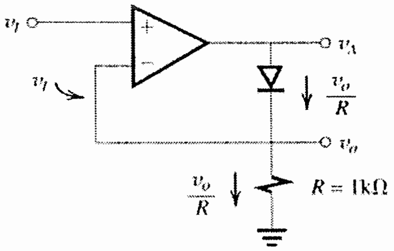
$v_O = (V_{cc} - I_{c2}R_c) - (V_{cc} - I_{c1}R_c)$

$= (I_{c1} - I_{c2})R_c$

$= IR_c \times 2 \times \dfrac{2.42}{2.5}$

$= 0.25 \times 10 \times 2 \times \dfrac{2.42}{2.5} = 4.84\ \text{V}$

Ex: 12.24



$\therefore$ The opamp is ideal $v_o = v_I$ for $v = > 0$.

$v_I = 10\ \text{mV} \quad v_O = 10\ \text{mV}$

$i_D = \dfrac{10\ \text{mV}}{R} = 10\ \mu\text{A}$

Given $\Rightarrow i_D = 1\ \text{mA} \quad 0.1\ \text{mA} \quad 10\ \mu\text{A}$

$v_D = 0.7\ \text{V} \quad 0.6\ \text{V} \quad 0.5\ \text{V}$

Thus, $v_D = 0.5$ V so

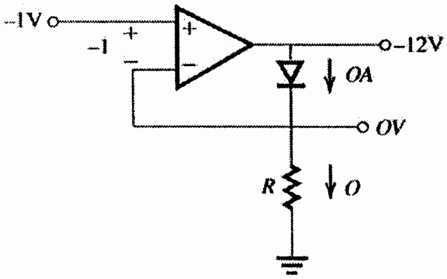
$v_A = v_O + v_D = 0.51$ V

$v_I = 1\ \text{V} \Rightarrow v_O = 1$ V

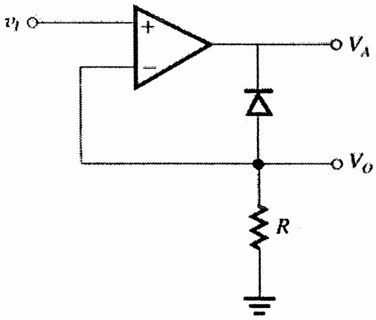
$i_D = 1$ mA, $v_D = 0.7$ V $v_A = 1.7$ V

$v_I = -1$ V – The negative feedback loop is not operative.

$v_o = 0\ \text{V}$ $v_A = -12$ V



Ex: 12.25



For the diode to conduct and close the negative feedback loop, v_o must be negative, in which case, the negative feedback causes a virtual short circuit to appear between the input terminals of the op amp and thus $v_o = v_I$ For positive v_I, the op amp saturates in the positive saturation level. The diode will be cut off and $v_D = 0$.

In summary

$v_o = 0$ for $v_I \geq 0$

$v_o = v_I$ for $v_I \leq 0$

Ex: 12.26

Refer to Fig 12.34

For $v_I = +1$ V:

D_2 will conduct and close the negative feedback loop around the op amp. $v\_ = 0$, the current through R_1 and D_2 will be 1 mA. Thus the voltage at the op amp output, $v_A = -0.7$ V which will set

D_1 off and no current will flow through R_2. Thus

$v_D = 0\text{ V}$

For $v_I = -10\text{ mV}$

D_1 will conduct through R_2 & R_1 to v_I. The negative feedback loop of the op amp will thus be closed and a virtual ground will appear at the inverting input terminal. D_2 will be cutoff. The current through R_1, R_2 and D_1 will be

$\frac{10\text{ mV}}{1\text{ k}\Omega} = 10\ \mu\text{A}$. Thus the diode, D_1, voltage will be 0.5 V.

$v_o = O + 10\ \mu\text{A} \times 10\text{ k}\Omega = +0.1\text{ V}$

$v_A = v_{D1} + v_o = 0.5 + 0.1 = 0.6\text{ V}$

For $v_I = -1\text{ V}$

This is similar to the case when $v_I = -10\text{ mV}$.

The current through R_1, R_2, D_1 will be

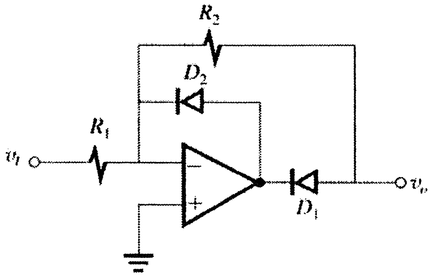
$I = \frac{1}{1\text{ k}\Omega} = 1\text{ mA}$

$\therefore\ v_{D1} = 0.7\text{ V}$

$v_O = 0 + 1\text{mA} \times 10\text{ k}\Omega = 10\text{ V}$

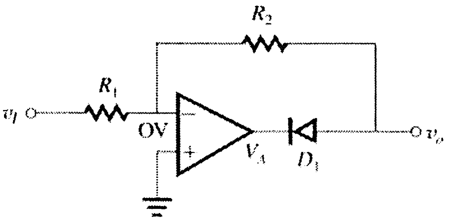
$v_A = 10 + v_{D1} = 10.7\text{ V}$

Ex: 12.27



$v_I > 0$

Current flows from v_I through R_1, R_2, D_1 into the output terminal of the opamp. v_0 goes negative and is thus off. The following circuit results:



$\frac{v_0}{v_I} = \frac{-R_2}{R_1}$

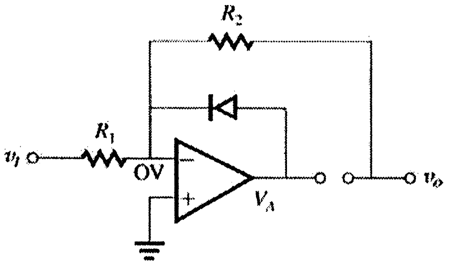
$v_I < 0$ ~ D_2 on

~ v_0 goes the & forms D_1 off

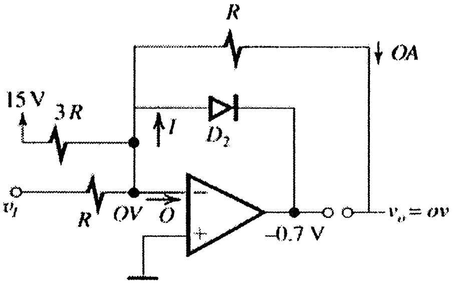
~ no current flows through

$R_2 = v_0 = 0\text{ V}$

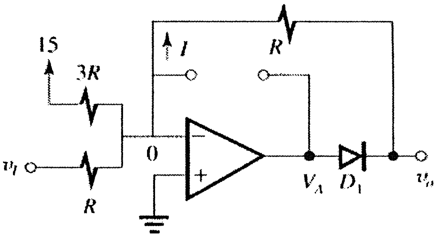
~ $V_A = 0.7\text{ V}$



Ex: 12.28



$VI > 0$ - Equivalant Circuit
~ D_2 on , D_1 off



$I = \frac{15}{3R} + \frac{v_I}{R}$

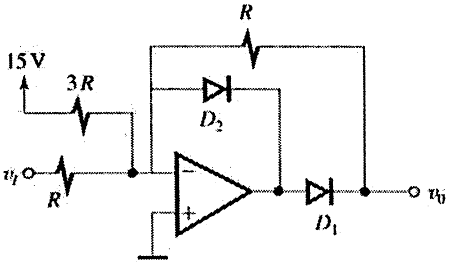
As v_I goes negative, the above circuit holds so that $v_0 = 0$. This occurs as the 15 V supply sources the current I even for small negative v_I. This situation remains the case until $I = 0$

$\therefore\ \frac{15}{3\text{R}} + \frac{v_I}{R} = 0$

$v_I = -5\text{ V}$

$v_I < -5\text{ V}$ ~ D_2 off D_1 -on

Exercise 12–8


R
15 V
3R
D_2
v_I
R
D_1
v_0

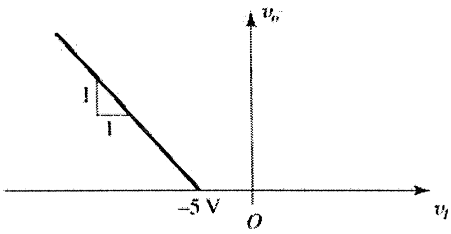
$v_0 = 0 - IR$

$= 0 - \left(\frac{15}{3R} + \frac{v_I}{R}\right)R$

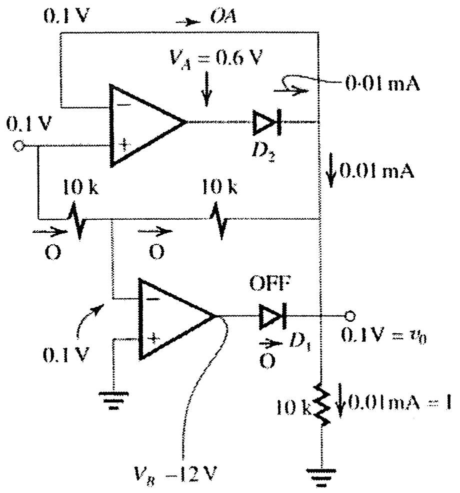
$= -v_I - 5$

note $v_A = v_0 + 0.7 = -v_I - 4.3 > 0$

$\therefore\ v_{D2} = 0 - V_A < 0 - D_2$ off!



Ex: 12.29


0.1 V
→ OA
$V_A = 0.6$ V
0·01 mA
0.1 V
D_2
0.01 mA
10 k
10 k
O
O
OFF
0.1 V
D_1
O
0.1 V = v_0
10 k
0.01mA = I
V_B –12 V

a) $v_I = 0.1$ V

NB

for all circuits, currents are given in mA, resistance in kΩ & voltages in V.

b) $v_I = 1$ V ~ similar to the circuit in (a) but with all of the undergrounded opamp input terminals at $v_I = 1$ V

$v_0 = 1$ V

$I = 1/10\ \text{k}\Omega = 0.1$ mA

$v_A = 1 + U_{D2}$

$= 1 + 0.7 + 0.1 \log\left(\frac{0.1}{1}\right)$

$= 1.6$ V

(c)

$v_I = 10$ V ~ similar to (a) & (b)

~ all input terminals (not grounded) of opamps is equal to 10 V.

$v_o = 10$ V

$I = \frac{10}{10} = 1$ mA ~ diode voltages = 0.7 V

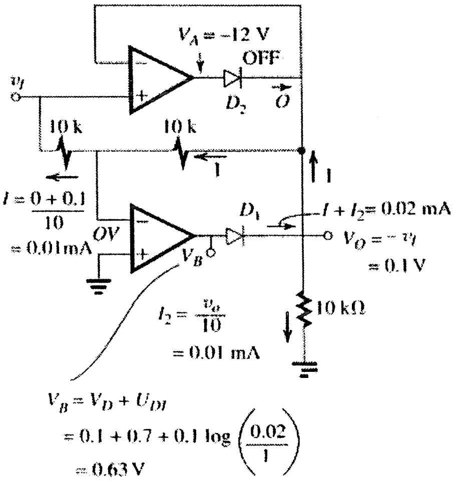
$V_A = v_o + U_{D2} = 10 + 0.7 = 10.7$ V

d) $v_I = 0.1$ V

$V_B = V_D + V_{D1}$

$= 0.1 + 0.7 + 0.1 \log\left(\frac{0.02}{1}\right)$

$= 0.63$ V


$V_A = -12$ V
OFF
v_I
D_2
O
10 k
10 k
I
I
$I = \frac{0 + 0.1}{10}$
= 0.01mA
OV
D_1
$I + I_2 = 0.02$ mA
$V_O = -v_I$
= 0.1 V
V_B
$I_2 = \frac{v_o}{10}$
= 0.01 mA
10 kΩ
$V_B = V_D + U_{DI}$
$= 0.1 + 0.7 + 0.1 \log\left(\frac{0.02}{1}\right)$
= 0.63 V

(e) $v_I = -1$ V - use circuit in (d)

$I = 0.1$ mA

$V_0 = -v_I = 1$ V

$I_2 = 0.1$ mA

$I_{D1} = I + I_2 = 0.2$ mA

$V_B = V_o + V_{D1}$

$= 1 + 0.7 + 0.1 \log\left(\frac{0.2}{1}\right)$

$= 1.63$ V

(f) $v_I = -10$ V use circuit (d)

$I = .0$ mA

$v_0 = -v_1 = 10$ V

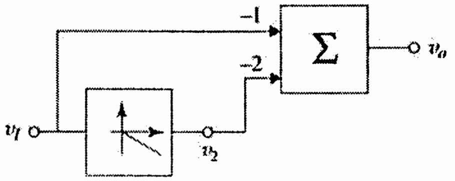
$I_2 = 1.0$ mA

$I_m = 2$ mA

$$V_B = V_0 + V_{D1} = 10 + 0.7 + 0.1\log\left(\frac{2}{1}\right)$$

$= 10.73$ V

Ex: 12.30



For $v_I \geq 0$, i.e. $v_I = |v_I|$,

$v_2 = -|v_I|$ and

$v_0 = -|v_I| - 2 \times -|v_I| = +|v_I|$

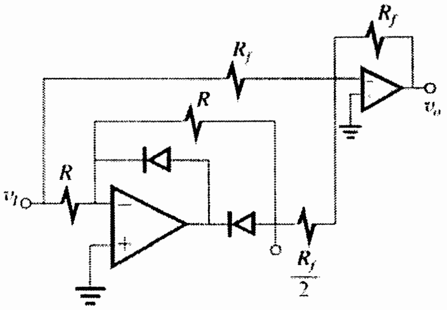
For $v_I \leq 0$, i.e. $v_I = -|v_I|$

$v_2 = 0$, $v_0 = -|x - |v_I||$

$= +|v_I|$

Thus, the block diagram implements the absolute value operation.

Using the circuits of Fig(12.34a), with the diodes reversed, to implement the half-wave rectifier, and a weighted summer results in the circuit shown below.



Use $R = R_f = 10$ kΩ

Ex: 12.31

v_A is a sinusoid of 5-V rms (peak voltage of $5\sqrt{2}$) The average current through the meter will be $\frac{2}{\pi} \times \frac{5\sqrt{2}}{R}$. To obtain full-scale leading. This current must be equal to 1 mA. Thus $\frac{2}{\pi} \times \frac{5\sqrt{2}}{R} =$ 1 mA, which leads to $R = 4.5$ kΩ

V_c will be maximum when V_A is at its positive peak, i.e. $v_A = 5\sqrt{2}$ V. At this value of VA, we obtain

$v_c = V_{D1} + V_M + V_{D3} + V_R$ where

$V_{D1} = V_{D3} \simeq 0.7$ V and

$$V_M = \frac{5\sqrt{2}}{4.5} \times 0.05 = 0.08 \text{ V}$$

Thus

$V_c|_{max} = 0.7 + 0.8 + 0.7 + 5\sqrt{2} = 8.55$ V

Similarly we can calculate :

$v_c|_{min} = -8.55$ V

Ex 13.1

For Q_1

$$I = \frac{V_{CC} - V_{CE\text{sat}}}{R_L} = \frac{15 - 0.2}{1\text{ k}\Omega}$$

$I = 14.8\text{ mA}$

$$R = \frac{-V_D - (-V_{CC})}{14.8} = \frac{-0.7 - (-15)}{14.8}$$

$= 0.97\text{ k}\Omega$

$v_{o\max} = V_{CC} - V_{CE\text{sat}}$

$= 15 - 0.2$

$= 14.8\text{ V}$

$v_{o\min} = -V_{CC} + V_{CE\text{sat}}$

$= -15 + 0.2$

$= -14.8$

Output signal swing is from 14.8 V to -14.8 V

Maximum emitter current $= 2I = 2 \times 14.8$

$= 29.6\text{ mA}$

Ex 13.2

At $v_o = -10\text{ V}$, the load current is -10 mA and the emitter current of Q_1 is $14.8 - 10 = 4.8$ mA.

Thus, $v_{BE1} = 0.6 + 0.025\ \ln\left(\frac{14.8}{1}\right)$

$= 0.64\text{ V}$

Thus, $v_I = -10 + 0.64 = -9.36\text{ V}$

At $v_o = 0\text{ V}$, $i_L = 0$ and $i_{E1} = 14.8\text{ mA}$

Thus, $v_{BE1} = 0.6 + 0.025\ \ln\frac{4.8}{1}$

$= 0.67\text{ V}$

$v_I = +0.67\text{ V}$

At $v_o = +10\text{ V}$, $i_L = 10\text{ mA}$ and $i_{E1} = 24.8\text{ mA}$

Thus, $v_{BE1} = 0.6 + 0.025\ \ln(24.8)$

$= 0.68\text{ V}$

$v_I = 10.68\text{ V}$

To calculate the incremental voltage gain we use

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + r_{e1}}$$

At $v_o = -10\text{ V}$, $i_{E1} = 4.8\text{ mA}$ and

$$r_{e1} = \frac{25}{4.8} = 5.2\ \Omega$$

Thus, $\frac{v_o}{v_i} = \frac{1}{1 + 0.0052} = 0.995\text{ V/V}$

Similarly, at $v_o = 0\text{ V}$, $r_{e1} = \frac{25}{14.8} = 1.7\ \Omega$

and, $\frac{v_o}{v_i} = \frac{1}{1 + 0.0017} = 0.998\text{ V/V}$

At $v_o = +10\text{ V}$, $i_{E1} = 24.8\text{ mA}$ and $r_{e1} = 1\ \Omega$

Thus, $\frac{v_o}{v_i} = \frac{1}{1 + 0.001} = 0.999\text{ V/V}$

Ex 13.3

a. $P_L = \frac{(\hat{V}_o/\sqrt{2})^2}{R_L} = \frac{(8/\sqrt{2})^2}{100} = 0.32\text{ W}$

$P_S = 2V'_{cc} \times I = 2 \times 10 \times 100 \times 10^{-3}$

$= 2\text{ W}$

Efficiency $\eta = \frac{P_L}{P_S} \times 100$

$= \frac{0.32}{2} \times 100$

$= 16\%$

Ex 13.4

(a) $P_L = \frac{1}{2}\frac{\hat{V}_o^2}{R_L}$

$= \frac{1}{2}\frac{(4.5)^2}{4} = 2.53\text{ W}$

(b) $P_+ = P_- = V_{CC} \times \frac{1}{\pi}\frac{\hat{V}_o}{R_L}$

$= 6 \times \frac{1}{\pi} \times \frac{4.5}{4} = 2.15\text{ W}$

(c) $\eta = \frac{P_L}{P_S} \times 100 = \frac{2.53}{2 \times 2.15} \times 100$

$= 59\ \%$

(d) Peak input currents $= \frac{1}{\beta + 1}\frac{\hat{V}_o}{R_L}$

$= \frac{1}{51} \times \frac{4.5}{4}$

$= 22.1\text{ mA}$

(e) Using Eq. 10.22

$$P_{DN\max} = P_{DP\max} = \frac{V_{CC}^2}{\pi^2 R_L}$$

$$= \frac{6^2}{\pi^2 \times 4} = 0.91\text{ W}$$

Ex 13.5

(a) The quiescent power dissipated in each transistor $= I_Q \times V_{CC}$

Total power dissipated in the two transistors

$= 2I_Q \times V_{CC}$

$= 2 \times 2 \times 10^{-3} \times 15$

$= 60\text{ mW}$

(b) I_Q is increased to 10 mA

At $V_o = 0$, $i_N = i_P = 10\text{ mA}$

From equation 13.31

$$R_{\text{out}} = \frac{V_T}{i_P + i_N} = \frac{25}{10 + 10} = 1.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 1.25}$$

$\frac{v_o}{v_i} = 0.988$ at $v_o = 0$ V

At $v_o = 10$ V,

$$i_L = \frac{10\text{ V}}{100\ \Omega} = 0.1\text{ A} = 100\text{ mA}$$

use equation 13.27 to calculate i_N

$i_N^2 - i_N i_L - I_Q^2 = 0$

$i_N^2 - 100\, i_N - 10^2 = 0$

$\Rightarrow i_N = 99.99$ mA

using equation 13.26

$$i_P = \frac{I_Q^2}{I_N} \simeq 1\text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{99.99 + 1} \simeq 0.2475\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 0.2475} \simeq 1$$

$$\%\text{ change} = \frac{1 - 0.988}{1} \times 100 = 1.2\%$$

In example 13.5 $I_Q = 2$ mA, and for $v_o = 0$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{2 + 2} = 6.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} = \frac{100}{100 + 6.25} = 0.94$$

$v_o = 10$ V

$$I_L = \frac{10\text{ V}}{100\ \Omega} = 100\text{ mA}$$

Again calculate i_N (for $I_Q = 2$ mA) using equation 13.27 $i_N = 99.96$ mA

$$i_P = \frac{I_Q^2}{I_N} = \frac{2^2}{99.96} = 0.04\text{ mA}$$

$$R_{\text{out}} = \frac{V_T}{i_N + i_P} = \frac{25}{99.6 + 0.04} = 0.25\ \Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{\text{out}}} \simeq 1$$

$$\%\text{ Change} = \frac{1 - 0.94}{1} \times 100 = 6\%$$

For $I_Q = 10$ mA, change is 1.2%

For $I_Q = 2$ mA, change is 6%

(c) The quiesent power dissipated in each transistor $= I_Q \times V_{CC}$

Total power dissipated $= 2 \times 10 \times 10^{-3} \times 15$
$= 300$ mW

Ex 13.6

From example 13.4 $V_{CC} = 15$ V, $R_L = 100\ \Omega$

Q_N and Q_P matched and $I_S = 10^{-13}$ A and $\beta = 50$,

$I_{\text{Bias}} = 3$ mA

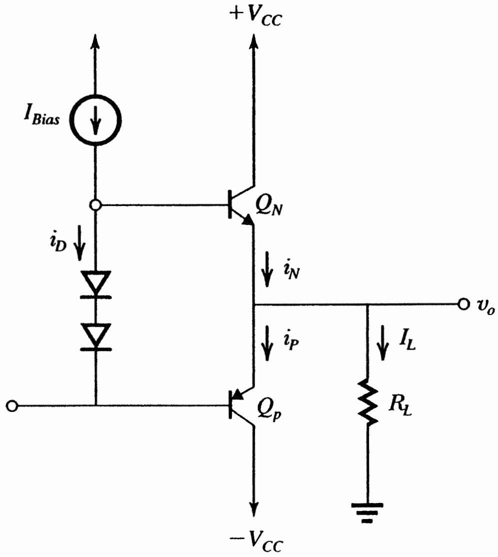
For $v_o = 10$ V, $I_L = \frac{10}{100} = 0.1$ A

As a first approximation $i_N \simeq 0.1$ A, $i_P = 0$, $i_{BN} \simeq \frac{0.1\text{ A}}{50 + 1} \simeq 2$ mA

$i_D = I_{\text{Bias}} - i_{BN} = 3 - 2 = 1$ mA

$$V_{BB} = 2\, V_T \ln\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)$$

This $\frac{1}{3}$ is because biasing diodes have $\frac{1}{3}$ area of the output devices.



But $V_{BB} = V_{BEN} + V_{BEP} =$ (1)

$$V_T \ln\left(\frac{i_N}{I_S}\right) + V_T \ln\left(\frac{i_N - i_L}{I_S}\right)$$

$$= V_T \ln\left[\frac{i_N(i_N - i_L)}{I_S^2}\right] \qquad (2)$$

Equating equations 1 and 2

$$2V_T \ln\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) = V_T \ln\left(\frac{i_N - i_L}{I_S^2}\right)$$

$$\left(\frac{10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)^2 = \frac{i_N(i_N - 0.1)}{(10^{-13})^2}$$

$i_N(i_N - 0.1) = 9 \times 10^{-6}$

If i_N is in mA, then

$i_N(i_N - 100) = 9$

$i_N^2 - 100\, i_N - 9 = 0$

$\Rightarrow i_N = 100.1$ mA

$i_P = i_N - i_L = 0.1$ mA

For $v_O = -10$ V and $i_L = \frac{-10}{100} = -0.1$ A
$= -100$ mA

As a first approximation assume $i_P \cong 100$ mA, $i_N \simeq 0$ since $i_N = 0$, current through diodes = 3 mA

$$\therefore V_{BB} - 2V_T \ln\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) \qquad (3)$$

But $V_{BB} = V_T \ln\left(\frac{i_N}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right)$

$$= V_T \ln\left(\frac{i_P - i_L}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right) \quad (4)$$

Here $i_L = 0.1$ A

Equating equations 3 and 4

$$2V_T \ln\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right) =$$

$$V_T \ln\left(\frac{i_P - 0.1}{10^{-13}}\right) + V_T \ln\left(\frac{i_P}{10^{-13}}\right)$$

$$\left(\frac{3 \times 10^{-3}}{\frac{1}{3} \times 10^{-13}}\right)^2 = \frac{i_P(i_P - 0.1)}{(10^{-13})^2}$$

$i_P(i_P - 0.1) = 81 \times 10^{-6}$

Expressing currents in mA

$i_P(i_P - 100) = 81$

$i_P^2 - 100\, i_P - 81 = 0$

$\Rightarrow i_P = 100.8$ mA

$i_N = i_P - i_L = 0.8$ mA

Ex 13.7

$\Delta I_C = g_m \times 2\ \text{mV}/°\text{C} \times 5\ °\text{C}$, mA

where g_m is in mA / mV

$$g_m = \frac{10\ \text{mA}}{25\ \text{mV}} = 0.4\ \text{mA}/\text{mV}$$

Thus, $\Delta I_C = 0.4 \times 2 \times 5 = 4$ mA

Ex 13.8

Refer to Fig. 10.14

(a) To obtain a terminal voltage of 1.2 V, and since β_1 is very large, it follows, that $V_{R1} = V_{R2} = 0.6$ V.

Thus $I_{C1} = 1$ mA

$$I_R = \frac{1.2\ \text{V}}{R_1 + R_2} = \frac{1.2}{2.4} = 0.5\ \text{mA}$$

Thus, $I = I_{C1} + I_R = 1.5$ mA

(b) For $\Delta V_{BB} = +50$ mV:

$V_{BB} = 1.25$ V $\quad I_R = \frac{1.25}{2.4} = 0.52$ mA

$$V_{BE} = \frac{1.25}{2} = 0.625\ \text{V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = e^{0.025/0.025}$$

$= 2.72$ mA

$I = 2.72 + 0.52 = 3.24$ mA

For $\Delta V_{BB} = +100$ mV

$V_{BB} = 1.3$ V $I_R = \frac{1.3}{2.4} = 0.54$ mA

$$V_{BE} = \frac{1.3}{2} = 0.65\ \text{V}$$

$$I_{C1} = 1 \times e^{\Delta V_{BE}/V_T} = 1 \times e^{0.05/0.025}$$

$= 7.39$

$I = 7.39 + 0.54 = 7.93$ mA

For $\Delta V_{BB} = +200$ mV:

$V_{BB} = 1.4$ V $\quad I_R = \frac{1.4}{2.4} = 0.58$ mA

$V_{BE} = 0.7$ V

$I_{C1} = 1 \times e^{0.1/0.025} = 54.60$ mA

$I = 54.60 + 0.58 = 55.18$ mA

For $\Delta V_{BB} = -50$ mV

$V_{BB} = 1.15$ V $\quad I_R = \frac{1.15}{2.4} = 0.48$ mA

$$V_{BE} = \frac{1.15}{2}$$

$= 0.575$

$I_{C1} = 1 \times e^{-0.025/0.025} = 0.37$ mA

$I = 0.48 + 0.37 = 0.85$ mA

For $\Delta V_{BB} = -100$ mV:

$V_{BB} = 1.1$ V $I_R = \frac{1.1}{2.4} = 0.46$ mA

$V_{BE} = 0.55$ V

$I_{C1} = 1 \times e^{-0.05/0.025} = 0.13$ mA

$I = 0.46 + 0.13 = 0.59$ mA

For $\Delta V_{BB} = -200$ mV:

$V_{BB} = 1.0$ V $\quad I_R = \frac{1}{2.4} = 0.417$ mA

$V_{BE} = 0.5$ V

$I_{C1} = 1 \times e^{-0.1/0.025} = 0.018$ mA

$I = 0.43$ mA

Ex 13.9

Using equation 13.43

$$I_Q = I_{\text{Bias}} \frac{(W/L)_n}{(W/L)_1}$$

$$1 = 0.2\, \frac{(W/L)_n}{(W/L)_P}$$

$$\frac{(W/L)_n}{(W/L)_1} = 5$$

$$Q_1\text{: } I_{\text{Bias}} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 (V_{GS} - V_{tn})^2$$

$$0.2 = \frac{1}{2} \times 0.250 \left(\frac{W}{L}\right)_1 (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 40$$

$$Q_2\text{: } I_{\text{Bias}} = \frac{1}{2}k_P'\left(\frac{W}{L}\right)_2(V_{GS} - |V_t|)^2$$

$$0.2 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_2 \times (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 100$$

$$Q_N\text{: } I_Q = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_N(V_{GS} - V_t)^2$$

$$1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_N 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_N = 200$$

$$Q_P\text{: } I_Q = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_P(V_{GS} - |V_t|)^2$$

$$1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_P \times 0.2^2$$

$$\left(\frac{W}{L}\right)_P = 500$$

Now $V_{GG} = V_{GS1} + V_{GS2}$

$= (V_{\text{ov1}} + V_t) + (V_{\text{ov2}} + |V_t|)$

$= (0.2 + 0.5) + (0.2 + 0.5)$

$= 1.4\text{ V}$

Ex 13.10

$I_N = i_{L\max} = 10\text{ mA}$

$$\therefore 10 = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n V_{\text{ov}}^2$$

$$10 = \frac{1}{2} \times 0.250 \times 200 \times V_{\text{ov}}^2$$

$$\Rightarrow V_{\text{ov}} = 0.63\text{ V}$$

Using equation 13.46

$V_{\text{omax}} = V_{DD} - V_{\text{ov}}|_{\text{Bias}} - V_{tn} - V_{\text{ovN}}$

$= 2.5 - 0.2 - 0.5 - 0.63$

$= 1.17\text{ V}$

Ex 13.11

New values of W/L are

$$\left(\frac{W}{L}\right)_P = \frac{2000}{2} = 1000$$

$$\left(\frac{W}{L}\right)_N = \frac{800}{2} = 400$$

$$I_Q = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_P V_{\text{ov}}^2$$

$$1 \times 10^{-3} = \frac{1}{2} \times 0.1 \times 10^{-3} \times 1000 \times V_{\text{ov}}^2$$

$$\Rightarrow V_{ov} = 0.14\text{ V}$$

Gain Error =

$$-\frac{V_{ov}}{4\mu I_Q R_L} = -\frac{0.14}{4 \times 10 \times 1 \times 10^{-3} \times 100}$$

$= -0.035$

Gain Error $= -0.035 \times 100 = 3.5\%$

$$g_{mn} = g_{mp} = \frac{2I_Q}{V_{ov}} = \frac{2 \times 1 \times 10^{-3}}{0.14}$$

$= 14.14\text{ mA/V}$

$$R_{\text{out}} = \frac{1}{\mu(g_{mp} + g_{mn})} =$$

$$\frac{1}{10 \times (14.14 + 14.14) \times 10^{-3}}$$

$\simeq 3.5\ \Omega$

Ex 13.12

See solution on next page

Ex 13.12

Need to prove when $V_{o2} = 4\,I_Q R_L$ then $V_{GSN2} = V_{tn}$

Assume Q_N off ($V_{GSN} = V_{tn}$) so $i_{N2} = 0$ and $i_{p2} = i_{L2}$

$$i_{p2} = i_{L2} = \frac{V_{O2}}{R_L} = 4I_Q$$

$$4I_Q = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(V_{SGP2} - |V_{tp}|)^2$$

$$\sqrt{4\left(\cancel{\tfrac{1}{2}}\cancel{k_p'}\left(\cancel{\tfrac{W}{L}}\right)_p(V_{SGPQ} - |V_{tp}|)^2\right)}$$

$$= \sqrt{\cancel{\tfrac{1}{2}}\cancel{k_p'}\left(\cancel{\tfrac{W}{L}}\right)_p(V_{SGP2} - |V_{tp}|)^2}$$

$$2(V_{SGPQ} - |V_{tp}|) = (V_{SGP2} - |V_{tp}|)$$

$$V_{SGP2} = 2V_{SGPQ} - 2|V_{tp}| + |V_{tp}|$$

$$= 2V_{SGPQ} - |V_{tp}| \qquad (1)$$

Find V_{i2} for the gate voltage, V_{GP2}:

$$V_{GP2} = (V_{DD} - V_{SGPQ}) + \mu(V_{O2} - V_{i2})$$

$$(V_{GP2} - V_{DD}) = -V_{SGPQ} + \mu(V_{O2} - V_{i2})$$

$$[V_{GSP2}\ \text{OR}] - V_{SGP2} = -V_{SGPQ} + \mu(V_{O2} - V_{i2})$$

using (1):

$$-2V_{SGPQ} + |V_{tp}| = -V_{SGPQ} + \mu(V_{O2} - V_{i2})$$

$$\mu(V_{i2} - V_{O2}) = -V_{SGPQ} + 2V_{SGPQ} - |V_{tp}|$$

$$V_{i2} = +V_{O2} + \frac{(V_{SGPQ} - |V_{tp}|)}{\mu} = V_{O2} + \frac{V_{OVQ}}{\mu}$$

Plug this value for V_{i2} into the value for V_{GN2} and show $V_{GSN2} = V_{tN}$

$$(-\cancel{V_{SS}} + V_{GSNQ}) + \mu(V_{O2} - V_{i2}) = V_{GN2} - (-\cancel{V_{SS}})$$

$$V_{GSNQ} + \cancel{\mu}\left(\cancel{V_{O2}} - \cancel{V_{O2}} - \frac{V_{OVQ}}{\cancel{\mu}}\right) = V_{GSN2}$$

where

$$V_{OVQ} = (V_{GSNQ} - V_{tn}) = (V_{SGPQ} - |V_{tp}|)$$

$$\cancel{V_{GSNQ}} - \cancel{V_{GSNQ}} + V_{tn} = V_{GSN2}\ \text{Q.E.D.}$$

Same proof for p transistor.

Ex 13.13

$$T_J - T_A = \theta_{JA} P_D$$

$$200 - 25 = \theta_{JA} \times 50$$

$$\theta_{JA} = \frac{175}{50} = 3.5°\text{C}/\text{W}$$

But, $\theta_{JA} = \theta_{JC} + \theta_{CS} + \theta_{SA}$

$$3.5 = 1.4 + 0.6 + \theta_{SA}$$

$$\Rightarrow \quad \theta_{SA} = 1.5°\text{C}/\text{W}$$

$$T_J - T_C = \theta_{JC} \times P_D$$

$$T_C = T_J - \theta_{JC} \times P_D$$

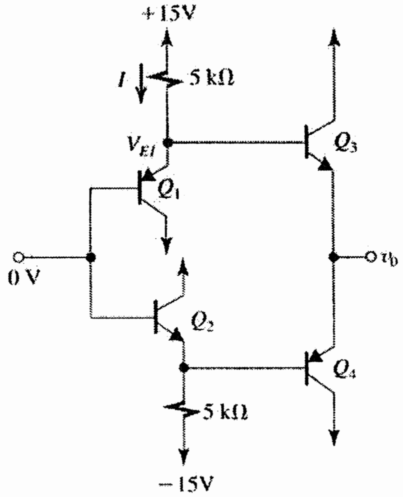
$$= 200 - 1.4 \times 50$$

$$= 130° \text{C}$$

Ex 13.14

(a) From symmetry we see that all transistors will conduct equal currents and have equal V_{BE}'s Thus.

$v_o = 0$ V



If $V_{BE} \simeq 0.7$ V Then

$$V_{E1} = 0.7 \text{ V and } I_1 = \frac{15 - 0.7}{5} = 2.86 \text{ mA}$$

If we neglect I_{B3} then

$I_{C1} \simeq 2.86$ mA

At this current, V_{BE} is given by

$$V_{BE} = 0.025 \ln\left(\frac{2.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right) \simeq 0.63 \text{ V}$$

Thus $V_{E1} = 0.63$ V and $I_1 = 2.87$ mA

No more iterations are required and

$i_{C1} = i_{C2} = i_{C3} = i_{C4} \simeq 2.87$ mA

(b) For $v_I = +10$ V:

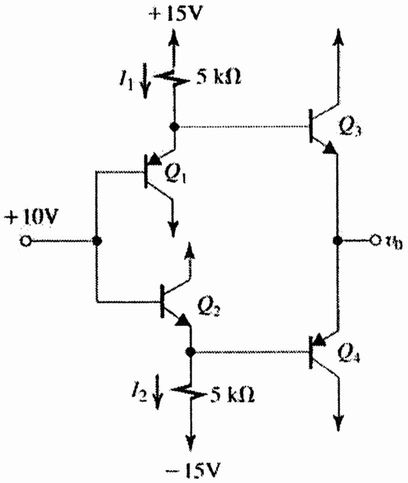
To start the iterations let $V_{BE1} \simeq 0.7$ V

Thus,

$V_{E1} = 10.7$ V

and,

$$I_1 = \frac{15 - 10.7}{5} = 0.86 \text{ mA}$$



Neglecting I_{B3},

$I_{C1} \simeq I_{E1} \simeq I_1 = 0.86$ mA

But at this current

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right)$$

$$= 0.025 \ln\left(\frac{0.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$= 0.6$ V

Thus, $V_{E1} = +10.6$ V and $I_1 = 0.88$ mA No further iterations are required and $I_{C1} \simeq 0.88$ mA.

To find I_{C2} we use an identical procedure :

$V_{BE2} \simeq 0.7$ V

$V_{E2} = 10 - 0.7 = +9.3$ V

$$I_2 = \frac{9.3 - (-15)}{5} = 4.86 \text{ mA}$$

$$V_{BE2} = 0.025 \ln\left(\frac{4.86 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$= 0.643$ V

$V_{E2} = 10 - .643 = +9.357$

$I_2 = 4.87$ mA

$I_{C2} \simeq 4.87$ mA

Finally,

$$I_{C3} = I_{C4} = 3.3 \times 10^{-14} e^{V_{BE}/V_T}$$

Where

$$V_{BE} = \frac{V_{E1} - V_{E2}}{2} = 0.62 \text{ V}$$

Thus, $I_{C3} = I_{C4} = 1.95$ mA

The symmetry of the circuit enables us to find the values for $v_I = -10$ V as follows:

$I_{C1} = 4.87$ mA $I_{C2} = 0.88$ mA

$I_{C3} = I_{C4} = 1.95$ mA

For $v_I = +10$ V, $v_o = V_{E1} - V_{BE3}$

$= 10.6 - 0.62 = +9.98$ V

For $v_I = -10$ V, $v_o = V_{E1} - V_{BE3}$

$= -9.357 - 0.62 = -9.98$ V

(c)For $v_I = +10$ V,

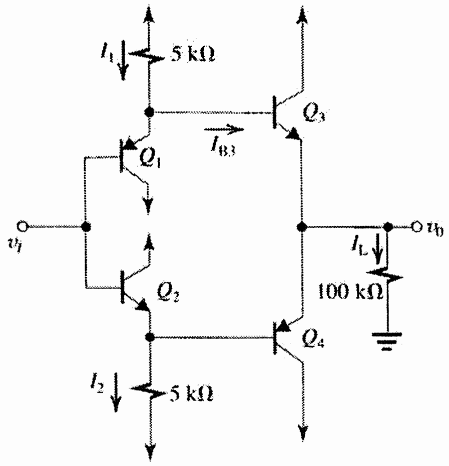
$v_O \simeq 10$ V

$I_L \simeq 100$ mA

$I_{C3} \simeq 100$ mA

$$I_{B3} = \frac{100}{201}$$

$\simeq 0.5$ mA



I_1
5 kΩ
Q_3
Q_1
I_{B3}
v_I
v_O
I_L
Q_2
100 kΩ
Q_4
I_2
5 kΩ

Assuming that V_{BE1} has not changed much from 0.6 V, then

$V_{E1} \simeq 10.6$ V

$$I_1 = \frac{15 - 10.6}{5} = 0.88 \text{ mA}$$

$I_{E1} = I_1 - I_{B3} = 0.88 - 0.5 = 0.38$ mA

$I_{C1} \simeq 0.38$ mA

$$V_{BE1} = 0.025 \ln\left(\frac{0.38 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$= 0.58$ V

$V_{E1} = 10.88$ V

$$I_1 = \frac{15 - 10.58}{5} = 0.88 \text{ mA}$$

Thus, $I_{C1} \simeq 0.30$ mA

Now for Q_2 we have:

$V_{BE2} = 0.643$ V

$V_{E2} = 10 - 0.643 = 9.357$

$I_2 = 4.87$ mA

$I_{B4} \simeq 0$

$I_{C2} \simeq 4.87$ mA (as in (b))

Assuming that $I_{C3} \simeq 100$ mA,

$$V_{BE3} = 0.025 \ln\left(\frac{100 \times 10^{-3}}{3.3 \times 10^{-14}}\right)$$

$= 0.72$ V

Thus, $v_o = V_{E1} - V_{BE3}$

$= 10.58 - 0.72 = +9.86$ V

$V_{BE4} = v_0 - V_{E2}$

$= 9.86 - 9.36 = 0.5$ V

Thus, $I_{C4} = 3.3 \times 10^{-14} e^{0.5/0.025}$

$\simeq 0.02$ mA

For symmetry we find the value for the case $v_I = -10$ V as.

$I_{C1} = 4.87$ mA $I_{C2} = 0.38$ mA

$I_{C3} = 0.02$ mA $I_{C4} = 100$ mA

$v_o = -9.86$ V.

Ex 13.15

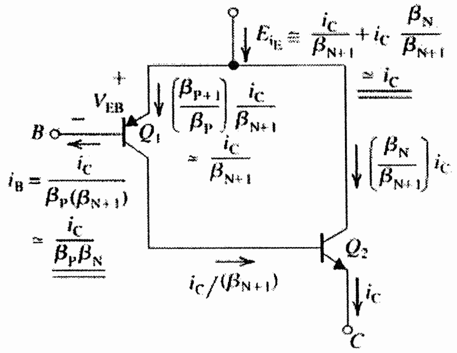
For Q_1:

$$i_{C1} = I_{SP}\, e^{v_{EB}/V_T}$$

$$\frac{i_C}{\beta_N + 1} = I_{SP}\, e^{v_{EB}/V_T}$$

$$i_C \simeq \beta_N I_{SP}\, e^{v_{EB}/V_T}$$

Thus, Effective scale current $= \beta_N I_{SP}$



$E_{i_E} = \frac{i_C}{\beta_{N+1}} + i_C \frac{\beta_N}{\beta_{N+1}}$
$\simeq i_C$
+
V_{EB}
−
B
Q_1
$\left(\frac{\beta_{P+1}}{\beta_P}\right) \frac{i_C}{\beta_{N+1}}$
$\simeq \frac{i_C}{\beta_{N+1}}$
$\left(\frac{\beta_N}{\beta_{N+1}}\right) i_C$
$i_B = \frac{i_C}{\beta_P(\beta_{N+1})}$
$\simeq \frac{i_C}{\beta_P \beta_N}$
Q_2
$i_C/(\beta_{N+1})$
i_C
C

(b) Effective current gain $= \dfrac{i_C}{i_B} = \beta_P \beta_N$

$= 20 \times 50 = 1000$

$100 \times 10^{-3} = 50 \times 10^{-14} e^{v_{EB}/0.025}$

$v_{EB} = 0.025 \ln (2 \times 10^{11})$

$= 0.651$ V

Ex 13.16

See Figure 13.34

When $V_{BE5} = 150 \times 10^{-3} \times R_{E5}$, then $I_{C5} = I_{Bias}$ $= 2$ mA

$$V_{BE5} = V_T \ln\left(\frac{I_{C5}}{I_S}\right)$$

$$= 25 \times 10^{-3} \ln\left(\frac{2 \times 10^{-3}}{10^{-14}}\right)$$

$= 0.651$ V

$150 \times 10^{-3} R_{E1} = 0.651$

$R_{E1} = 4.34\ \Omega$

If peak output current = 100 mA

$V_{BE5} = R_{E1} \times 100\text{ mA} = 4.34 \times 100 \times 10^{-3}$

$= 0.434\text{ V}$

$$i_{C5} = I_S e^{V_{BE5}/V_T}$$

$$= 10^{-14} e^{0.434/25 \times 10^{-3}}$$

$$\simeq 0.35\ \mu\text{A}$$

Ex 13.17

Total current out of mode B $= \dfrac{2v_i}{R_3} + \dfrac{v_o}{R_2}$

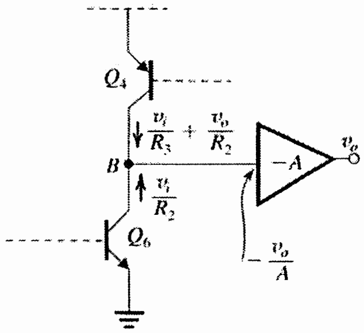
Thus

$$\left(\frac{2v_i}{R_3} + \frac{v_o}{R_2}\right)R = -\frac{v_o}{A}$$

$$\Rightarrow v_o\left(\frac{1}{A} + \frac{R}{R_1}\right) = -\frac{2R}{R_3} v_i$$

$$\frac{v_o}{v_i} = \frac{-\dfrac{2R}{R_3}}{\dfrac{1}{A} + \dfrac{R}{R_1}}$$

$$= \frac{-2R_2/R_3}{1 + (R_2/AR)} \qquad \text{Q.E.D}$$



For $AR >> R_2$

$$\frac{v_o}{v_i} \simeq -\frac{2R_2}{R_3}$$

Ex 13.18

$$P_{D\max} = \frac{T_{J\max} - T_A}{\theta_{JA}}$$

$$= \frac{150 - 50}{35} = 2.9\text{ W}$$

Ex 13.19

For Fig. 10.32 we see that for $P_{\text{dissipation}}$ to be less than 2.9 W, a maximum supply voltage of 20V is called for. The 20-V-supply curve intersects the 3% distoration line at a point for which the output power is 4.2 W. Since

$$P_L = \frac{(\hat{V}_o/\sqrt{2})^2}{R_L}$$

Thus $\hat{V}_o = \sqrt{4.2 \times 2 \times 8} = 8.2\text{ V}$

or 16.4 V peak-to-peak

Ex 13.20

Voltage gain = 2 K

where $K = \dfrac{R_4}{R_3} = 1 + \dfrac{R_2}{R_1} = 1.5$

Thus, $A_v = 3$ V/V

Input resistance $= R_3 = 10\text{ k}\Omega$

Peak-to-Peak $v_o = 3 \times 20 = 60$ V

Peak load current $= \dfrac{30\text{ V}}{8\ \Omega} = 3.75\text{ A}$

$$P_L = \frac{(30/\sqrt{2})^2}{8} = 56.25\text{ W}$$

Ex 13.21

We wish to value

$$\frac{\partial V_{GG}}{\partial T} = -3 - 3 = -6\text{ mV}/°\text{C}$$

but From Eq. 10.58

$$\frac{\partial V_{GG}}{\partial T} = \left(1 + \frac{R_3}{R_4}\right)\frac{\partial V_{BE6}}{\partial T}$$

$$\text{Thus } -6 = \left(1 + \frac{R_3}{R_4}\right) \times -2$$

$$\Rightarrow \frac{R_3}{R_4} = 2$$

Ex 13.22

Refer to Figure 13.44

$$I_{DN} = I_{DP} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(|V_{GS}| - V_t)^2$$

$$100 \times 10^{-3} = \frac{1}{2} \times 2(|V_{GS}| - 3)^2$$

$$0.1 = (|V_{GS}| - V_t)^2$$

$$\Rightarrow V_{GS} = 3.32\text{ V}$$

$V_{GG} = 2\ V_{GS} = 6.64$ V

$$R = \frac{V_{GG}}{20\text{ mA}} = \frac{6.64}{20 \times 10^{-3}} = 3.32\ \Omega$$

Using equation

$$V_{GG} = \left(1 + \frac{R_3}{R_4}\right)V_{BE6} + \left(1 + \frac{R_1}{R_2}\right)V_{BE5} - 4V_{BE}$$

$$6.64 = (1 + 2) \times 0.7 + \left(1 + \frac{R_1}{R_2}\right) \times 0.7 - 4 \times 0.7$$

$$\Rightarrow \frac{R_1}{R_2} \simeq 9.5$$

Ex: 14 .1

In the low-ouput state, the transistor is on and

$$R_{on} = r_{DS} \simeq \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_t)}$$

Therefore, the current drawn from the supply in this state can be calculated as:

$$I_{DD} = \frac{V_{DD}}{R_D + r_{DS}} = 50\ \mu\text{A} \Rightarrow R_D + r_{DS}$$

$$= \frac{2.5\ \text{V}}{50\ \mu\text{A}} = 50\ \text{k}\Omega$$

Also: $V_{OL} = V_{DD}\frac{r_{DS}}{R_D + r_{DS}}$

Substituting for

$R_D + r_{DS}$ $:0.1 = 2.5\frac{r_{DS}}{50\ \text{k}\Omega} \Rightarrow r_{DS} = 2\ \text{k}\Omega$

and hence: $R_D = 48\ \text{k}\Omega$

To obtain $\frac{W}{L}$, we use:

$$r_{DS} = \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_t)} \Rightarrow \frac{W}{L}$$

$$= \frac{1}{2 \times 10^3 \times 125 \times 10^{-6} \times (2.5 - 0.5)}$$

$$\therefore \frac{W}{L} = 2$$

When the switch is closed or in low-output state, the current drawn from the supply is 50 µA.

$$P_{DD} = V_{DD}I_{DD} = 2.5 \times 50 \times 10^{-6} = 125\mu w$$

When the switch is open, no current is drawn from the supply: $P_{DD} = 0$

Ex: 14 .2

When input is low, the output is high and equal to V_{OH}. In this case, the switch is connected to R_{C2}, therefore the current through R_{C1} is zero.

Hence, $V_{OH} = V_{CC} = 5\text{V}$.

When the input is high, the output is low and equal to V_{OL}. The switch is connected to R_{C1}. Hence

$$V_{OL} = V_{CC} - R_{C1}I_{EE} = 5 - 2 \times 1 = 3\text{V}$$

Ex: 14 .3

To determine $\frac{W}{L}$, we use: $K_n R_D = \frac{1}{V_x}$ and substitute $V_x = 0.089\text{V}$, $R_D = 10^{\text{k}\Omega}$,

$k_n' = 300\ \mu\text{A}/\text{V}^2$:

$$300 \times 10^{-6} \times \frac{W}{L} \times 10 \times 10^3 = \frac{1}{0.089} \Rightarrow \frac{W}{L} = 3.75$$

Noise margins stay unchanged, because $V_{OL}, V_{OH}, V_{IL}, V_{TH}$ only depend on V_{DD}, V_t, and V_x. Since V_x has not changed, noise margins stay the same.

Inorder to calculate the power dissipation, we need to first recalculate

$$I_{DD}: I_{DD} = \frac{V_{DD} - V_{OL}}{R_D} = \frac{1.8 - 0.12}{10\ \text{k}\Omega} = 168\ \mu\text{A}$$

$$P_{DD} = V_{DD}I_{DD} = 1.8 \times 168\ \mu\text{A} = 302.4\ \mu\text{W}$$

$$P_{Daverage} = \frac{1}{2}\text{P}_{\text{DD}} = 151\ \mu\text{W}$$

Note that keeping V_x unchanged resulted in higher power consumption, but noise margins stayed the same.

Ex: 14 .4

To determine V_x, we use:

$K_n R_n = \frac{1}{V_x}$ and with $R_D = 10\ \text{k}\Omega$ and K_n unchanged:

$$\frac{V_{x2}}{V_{x1}} = \frac{R_{D1}}{R_{D2}} \Rightarrow V_x = 0.089 \times \frac{25}{10} = 0.22\text{V}$$

To calculate the new noise margins, we have to find V_{OH}, V_{IH}, V_{IL}, V_{OL}.

$V_{OH} = V_{DD} = 1.8\ \text{V}$ unchanged

$$V_{IL} = V_t + V_x = 0.5 + 0.22 = 0.72\ \text{V}$$

$$V_{IH} = V_t + 1.63\sqrt{V_{DD}V_x} - V_x$$

$$= 0.5 + 1.63\sqrt{1.8 \times 0.22} - 0.22 = 1.31\text{V}$$

$$V_{OL} = \frac{V_{DD}}{1 + \frac{V_{DD} - V_t}{V_x}} = \frac{1.8}{1 + \frac{1.8 - 0.5}{0.22}} = 0.26\ \text{V}$$

$$NM_H = V_{OH} - V_{IH} = 1.8 - 1.31 = 0.49\ \text{V}$$

$$NM_L = V_{IL} - V_{OL} = 0.72 - 0.26 = 0.46\ \text{V}$$

The power dissiption becomes:

$$P_{d\ \text{average}} = \frac{1}{2}P_D = \frac{1}{2}V_{DD}I_{DD} = \frac{1}{2}V_{DD}$$

$$= \frac{V_{DD} - V_{OL}}{R_D}$$

$$P_{d\ \text{average}} = \frac{1}{2} \times 1.8 \times \frac{1.8 - 0.26}{10\ \text{k}\Omega} = 139\ \mu\text{W}$$

Note that keeping $\frac{W}{L}$ unchanged resulted in lower noise margins and higher power dissipation.

Ex: 14.5

$$k_r = \sqrt{\frac{(W/L)_1}{(W/L)_2}} = \sqrt{\frac{3}{1/3}} = 3$$

From Eq. 14.20: $V_{OH} = V_{DD} - V_t = 1.3\ \text{V}$

unchanged

From Eq. 14.28

$$V_{OL} \cong \frac{(V_{DD} - V_t)^2}{2k^2_r(V_{DD} - 2V_t)} = \frac{(1.8 - 0.5)^2}{2 \times 3^2(1.8 - 2 \times 0.5)}$$

$$= 0.12\ \text{V}$$

From Eq. 14.22 $V_{IL} = V_t = 0.5\ \text{V}$

unchanged.

From Eq. 14.23

$$V_M = \frac{V_{DD} + (k_r - 1)V_t}{k_r + 1} = \frac{1.8 + (3-1)0.5}{3+1}$$

$$= 0.7\ \text{V}$$

From Eq. 14.26 together with setting

$$\frac{dv_o}{dv_I} = -1:$$

$$2k_r^2\left[(v_I - v_t)v_O - \frac{1}{2}v^2O\right] = (V_{DD} - v_I - v_O)^2$$

$$2k_r^2\left[v_O + (v_I - v_t)\frac{dv_O}{dv_I} - v_O\frac{dv_O}{dv_I}\right]$$

$$= -2(V_{DD} - V_t - v_O)\frac{dv_O}{dv_I}$$

Now if we substitute for $\frac{dv_O}{dv_I} = -1$ then:

$$k_r^2[v_{OL} - v_{IH} + v_t - v_{OL}] = +V_{DD} - V_t - V_{OL}$$

$$\therefore 3^2[0.12 - V_{IH} + 0.5 + 0.12] = 1.8 - 0.5 - 0.12$$

$$9[0.74 - V_{IH}] = 1.18 \Rightarrow V_{IH} = 0.61\text{V}$$

$$NM_H = V_{OH} - V_{IH} = 1.3 - .61 = .69\ \text{V}$$

$$NM_L = V_{IL} - V_{OL} = .5 - .12 = .38\ \text{V}$$

Ex: 14.6

The inverter area is approximately

$A = W_1L_1 + W_2L_2$ Since $\frac{W_1}{L_1} = K_r$ and

$\frac{W_2}{L_2} = \frac{1}{K_r}$, we have $W_1 = k_rL_1$, and

$L_2 = k_rW_2$. Assuming $k_r > 1$, we have

$L_1 = d$ and $W_2 = L$.

Thus:

$$A = k_rL_1L_1 + W_2krW_2 = k_rL^2{}_1 + k_rW^2{}_2$$

$$= k_rd^2 + k_rd^2 = 2k_rd^2$$

Ex: 14.7

From Eq. 14.36 we have: $P_{dyn} = fCV_{DD}{}^2$

$$P_{dyn} = 100 \times 10^6 \times 100 \times 10^{-15} \times 1.8^2$$

$$= 32.4 \times 10^{-6} = 32.4\ \mu\text{w}$$

Ex: 14.8

$$P_{dyn} \alpha C_x V^2{}_{DD} \Rightarrow \frac{P_{dyn1}}{P_{dyn2}} = \frac{C_1V^2{}_{DD1}}{C_2V^2{}_{DD2}}$$

$$= \frac{0.5}{0.13} \times \frac{5^2}{1.2^2} = 66.73 \simeq 66.8$$

Ex: 14.9

$$V_o(t) = V_o(\infty) - (V_o(\infty) - V_o(0^+))e^{-t/\tau}$$

$$\frac{V_{DD}}{2} = V_{DD} - (V_{DD} - O)e^{-t_{PLH}/\left(\frac{V_{DD}}{I}\right)\cdot C}$$

$$\ln\left(-\frac{1}{2}\right) = -\frac{(-t_{PLH})}{\frac{V_{DD}\cdot C}{I}}$$

$$\therefore t_{PLH} = \frac{V_{DD}\cdot C}{I}\cdot 0.69$$

for $t_{PLH} = 10$ psec. with $C = 10$ fF and $V_{DD} = 1.8$ V

$$I = \frac{1.8\cdot 10f}{10\ \text{P}}\cdot 0.69 = 1.2\ \text{mA}$$

Ex: 14.10

For t_{PLH} the output starts at V_{OL} and goes to V_{OH} through the Pu which is 20 kΩ :

$$V_O(t) = V_O(\infty) - (V_O(\infty) - V_O(O^+))e^{-t/\tau}$$

$$\frac{1}{2}(V_{OH} + V_{OL}) = V_{OH} - (V_{OH} - V_{OL})e^{-t_{PLH}/\tau}$$

$$\frac{\left(-\frac{1}{2}V_{OH} + \frac{1}{2}V_{OL}\right)}{-(V_{OH}) + V_{OL}} = e^{-t_{PLH}/\tau}$$

$$\tau\left(-\ln\left(\frac{1}{2}\right)\right) = t_{PLH}$$

t_{PLH} = 0.69 R.C = 0.69 (20 K) (10f) = 138 pSec

For t_{PHL}, the output starts at V_{OH} and goes to V_{OL} through P_O which is 10 kΩ .

$$\frac{1}{2}(V_{OH} + V_{OL}) = V_{OL} - (V_{OL} - V_{OH})e^{-t_{PHL}/\tau}$$

$$\frac{\left(\frac{1}{2}V_{OH} + \frac{1}{2}V_{OL} - \frac{2}{2}V_{OL}\right)}{-V_{OL} + V_{OH}} = e^{-t_{PHL}/\tau}$$

$$\tau\left(-\ln\left(\frac{1}{2}\right)\right) = t_{PHL}$$

$$t_{PHL} = 0.69.\mathrm{R.C} = 0.69 \times 10\ \mathrm{K} \times 10f = 69\ \mathrm{psec}$$

$$t_P = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(138\ \mathrm{p} + 69\ \mathrm{p}) = 10^4\ \mathrm{psec}$$

Ex: 14.11

$V_O(t) = V_O(\infty) - [V_O(\infty) - V_O(O^+)]e^{-t/\tau}$

$V_O(t) = O - [V_{DD} - O]e^{-t/(2\text{ K} \cdot 100\text{ f}) = \tau}$

t_f is when voltage is $= .1V_{DD}$

$.1V_{DD} = -V_{DD}e^{-(0 - t_f)/\tau}$

$-\ln(.1) = t\frac{t_f}{\tau}$

$2.3 \cdot \tau \cong t_f$

$t_f = 2.3 \cdot 2\text{ K} \cdot 100\ f = 0.46$ nsec

Ex: 14.12

a) From Eq. 14.58

$$V_M = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{r+1} \text{ or}$$

$$0.6 = \frac{r(1.2 - 0.4) + 0.4}{1 + r} \Rightarrow 0.6 + 0.6r$$

$$= 0.8r + 0.4 \Rightarrow r = 1$$

$$r = \sqrt{\frac{\mu_p w_p}{\mu_n w_n}} = \sqrt{\frac{1}{4} \times \frac{w_p}{w_n}} = 1 \Rightarrow \frac{w_p}{w_n} = 4$$

$$\Rightarrow (w_p = 4 \times 0.13\ \mu\text{m} = 0.52\ \mu\text{m})$$

b) $V_{OH} = V_{DD} = 1.2\text{V},\ V_{OL} = 0\ \text{V}$

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t) = \frac{1}{8}(5 \times 1.2 - 2 \times 0.4)$$

$$= 0.65\text{V}$$

$$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t) = \frac{1}{8}(3 \times 1.2 + 2 \times 0.4)$$

$$= 0.55\ \text{V}$$

$$NM_H = V_{OH} - V_{IH} = 1.2 - 0.65 = 0.55\ \text{V}$$

$$NM_L = V_{IL} - V_{OL} = 0.55\ \text{V}$$

c) The output resistance of the inverter in the low-output state is:

$$r_{DSN} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{tn})}$$

$$= \frac{1}{430 \times 10^{-6} \times 1(1.2 - 0.4)}$$

$$= 2.9\ \text{k}\Omega$$

Since Q_N and Q_P are matched, the output resistance in the high-output state is the same:

$$r_{DSP} = r_{DSN} = 2.9\ \text{k}\Omega$$

d) For $\left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_n = 1.0$, we have

$$r = \sqrt{\frac{1}{4} \times 1} = 0.5,$$

hence: $V_M = \dfrac{0.5(1.2 - 0.4) + 0.4}{1 + 0.5} = 0.53\ \text{V}$

Ex: 14.13

Using Eq. 14.58 and 14.59

$$V_M = \frac{r(V_{DD} - |V_{tp}| + V_{tn})}{r+1} \Rightarrow 2.5$$

$$= \frac{r(5-1)+1}{r+1} \Rightarrow r = 1$$

$$r = \sqrt{\frac{\mu_p w_p}{\mu_n w_n}} \Rightarrow \frac{w_p}{w_n} = \frac{1}{1/2} = 2$$

When $V_I = V_{DD}$ and $V_O = 0.2\text{V}$, Q_N operates in triode region and hence the circuit is given as:

$$i_D = k_n'\left(\frac{W}{L}\right)_n\left[(V_i - V_{tn})V_O - \frac{1}{2}V_o^2\right]$$

$$\Rightarrow 0.2 \times 10^{-3} = 50 \times 10^{-6} \times \left(\frac{W}{L}\right)_n$$

$$\left[(5-1)0.2 - \frac{1}{2} \times 0.2^2\right] \Rightarrow \left(\frac{W}{L}\right)_n = 5$$

From above: $w_p = 2w_n \Rightarrow \left(\frac{W}{L}\right)_p = 10$

Ex: 14.14

Using Eqs.14.63 to 14.67we have:

$$t_{PHL} = \frac{\alpha_n C}{k_n'\left(\frac{W}{L}\right)_n V_{DD}},$$

$$\alpha_n = 2\frac{7}{4} - \frac{3V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2$$

$$= 2\frac{7}{4} - \frac{3 \times 0.5}{1.8} + \left(\frac{0.5}{1.8}\right)^2 = 1.99$$

Noting that $V_{tn} = |V_{tp}|$, then $\alpha_n = \alpha_p = 1.99$

$$t_{PHL} = \frac{1.99 \times 10 \times 10^{-15}}{300 \times 10^{-6} \times 1.5 \times 1.8} = 24.7\text{ps}$$

$$t_{PLH} = \frac{\alpha_p C}{k_p'\left(\frac{W}{L}\right)_p V_{DD}} \text{ or}$$

$$\frac{t_{PLH}}{t_{PHL}} = \frac{k_n'\left(\frac{W}{L}\right)_n}{k_p'\left(\frac{W}{L}\right)_p} \Rightarrow t_{PLH} = 24.6 \times 4 \times \frac{1.5}{3}$$

$$= 49.4\ \text{ps}$$

$$t_p = \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2}(24.7 + 49.4) = 37\text{ps}$$

Ex: 14.15

From Eq.14.68 we have:

$t_{PHL} = 0.69 R_N C$ and if we substitute for R_N

from Eq. 14.70 i.e. $R_N = \dfrac{12.5}{\left(\frac{W}{L}\right)_n}\ \text{k}\Omega$ then:

$$t_{PHL} = 0.69 \times \frac{12.5}{\left(\frac{W}{L}\right)_n} \times 10^3 \times \text{C} = \frac{8625\text{C}}{\left(\frac{W}{L}\right)_n} \text{ or}$$

$$50 \times 10^{-12} = \frac{8625 \times 20 \times 10^{-15}}{\left(\frac{W}{L}\right)_n}$$

$$\therefore \left(\frac{W}{L}\right)_n = 3.5$$

Similarly, using Eqs.14.69 and 14.71 we obtain:

$$t_{PLH} = 0.69\text{R}_p\, C = 0.69 \times 30 \times 10^3 \frac{C}{\left(\frac{W}{L}\right)_p}$$

or

$$t_{PLH} = 20.7 \times 10^3 \frac{\text{C}}{\left(\frac{W}{L}\right)_p} \Rightarrow 50 \times 10^{-12}$$

$$= 20.7 \times 10^3 \times \frac{20 \times 10^{-15}}{\left(\frac{W}{L}\right)_p} \Rightarrow \left(\frac{W}{L}\right)_p = 8.3$$

Ex: 14.16

t_{PHL} and t_{PLH} are proportional to C

$$\therefore t_{PHL} = t_{PHL} - \text{orginal} * \frac{C_{\text{new}}}{C_{\text{old}}}$$

and

$$t_{PHL} = t_{PLH} - \text{original} * \frac{C_{\text{new}}}{C_{\text{old}}}$$

$$C_{\text{new}} = C_{\text{old}} + .1\text{p} = 6.25\text{f} + .1\text{p}$$

$$C_{\text{new}} = .10625 \text{ pF}$$

$$t_p = \frac{1}{2}(t_{PHL} + t_{PLH}) = t_{p\text{OLD}} * \frac{C_{\text{new}}}{C_{\text{old}}}$$

$$t_p = 28\text{p} * \frac{.10625\text{p}}{6.25\text{f}} = 476 \text{ psec}$$

Ex: 14.17

W_p is reduced from 1.125 μm to 0.375 μm

$$\therefore \frac{0.375}{1.125} \times 100 = 33\% \text{ reduction}$$

$$\text{C}_{gd2} = \text{C}_{gd1} = 0.3375 \text{ fF}$$

$$C_{g3} = C_{g4} = 0.7875\text{fF}$$

$$\text{C} = (4 \times 0.3375\text{f}) + 1\text{f} + 1\text{f} + (2 \times .7875\text{f}) + .2\text{f}$$

$$= 4.225 \text{ fF}$$

$$t_{PHL} = 24.6 \times 10^{-12} \times \left(\frac{4.225 f}{6.25 f}\right) = 16.6 \text{ psec}$$

$$t_{PLH} = 31.5 \times 10^{-12} \times \left(\frac{4.225 f}{6.25 f}\right) = 21.3 \text{ psec}$$

$$t_p = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(16.6\text{p} + 21.3\text{p}) = 19 \text{ psec}$$

Ex: 14.18

$$f_{\max} = \frac{1}{2t_p} = \frac{1}{2(28 \times 10^{-12})} = 17.9 \text{ GHz}$$

The minimum period at which the inverter can reliability operate is

$T_{min} = t_{PHL} + t_{PLH}$. Thus,

$$F_{max} = \frac{1}{T_{min}} = \frac{1}{t_{PHL} + t_{PLH}} = \frac{1}{2\text{t}_\text{p}} = \frac{1}{2 \times 28}$$

$$= 17.96 \text{ Hz}$$

Ex: 14.19

a) As mentioned on page of the Text, C_{int} is the contribution of intrinsic capacitances of Q_N and Q_P. Therefore,

$$C_{\text{int}} = 2e_{gd1} + 2C_{gd2} + C_{db} + C_{db2}$$

$$\therefore C_{\text{int}} = 2 \times 0.1125 + 2 \times 0.3375 + 1 + 1 = 2.9 \text{ fF}$$

$$C_{\text{ext}} = Cg_3 + Cg_4 + C_w = 0.7875 + 2.3625 + 0.2$$

$$= 3.35\text{fF}$$

b) From Eq. 14.79 we have:

$t_p = 0.69\left(R_{\text{eq}}C_{\text{at}} + \frac{1}{S}R_{\text{eq}}C_{\text{ext}}\right)$ The extrinsic part of t_p is: $0.69\frac{1}{\text{S}}R_{\text{eq}}C_{\text{ext}}$ and in order to reduce the extrinsic part by a factor of 2, S has to be increased by a factor of 2. Note that $S = \frac{R_{\text{eg}}}{R_{\text{eq}}}$.

Therefore, R_{eq} has to be reduced by a factor of 2 or equivalently $\left(\frac{W}{L}\right)_n$ and $\left(\frac{W}{L}\right)_p$ have to be increased by factor of 2.

c) From Eq. 14.79

$= 0.69\left(R_{\text{eq0}}C_{\text{int}\phi} + \frac{1}{S}R_{\text{eq}\phi}C_{\text{ext}}\right)$. Hence,

$$\frac{t_{p\text{new}}}{t_{p\text{old}}} = \frac{C_{\text{int}} + C_{\text{at}}/\text{s}}{C_{\text{int}} + C_{\text{ext}}}$$

$$\text{For } S = 2 : t_{p\text{new}} = \frac{2.9 + 3.35/2}{2.9 + 3.35} \times 28 \text{ ps}$$

$$= 20.5 \text{ ps}$$

d) $A = W \times L$ and since $\left(\frac{W}{L}\right)$ is doubled and L is constant, then A or area is also doubled.

Ex: 14.20

Using Eq. 14.35

$$P_{dyn} = fCV_{DD}^2 = 1 \times 10^9 \times 6.25 \times 10^{-15} \times 2.5^2$$

$$= 39.1 \ \mu\text{w}$$

The maximum possible operating frequency is:

$f_{max} = \frac{1}{2\text{t}_\text{p}}$. Hence,

$$PDP = P_{dyn} \times t_p = f_{max}CV^2_{DD} \times t_p$$

$$= \frac{1}{2t_p} \times C \times V^2_{DD} \times t_p = \frac{CV^2_{DD}}{2}$$

$$= \frac{6.25 \times 10^{-15} \times 2.5^2}{2} = 19.5 \text{ fF/J}$$

Ex: 14.21

a) For NMOS devices:

$$\frac{W}{L} = n = \frac{0.18}{0.18} \times 1.5 = \frac{0.27}{0.18}$$

For PMOS devices:

$$\frac{W}{L} = 4p = \frac{0.18}{0.18} \times 4 \times 3 = \frac{2.16}{0.18}$$

b) For NMOS devices:

$$\frac{W}{L} = 4n = \frac{0.18}{0.18} \times 4 \times 1.5 = \frac{1.08}{0.18}$$

For PMOS devices:

$$\frac{W}{L} = p = \frac{0.18}{0.18} \times 3 = \frac{0.54}{0.18}$$

Ex: 14.22

(a) The minimum current available to charage a load capacitance is that provided by a single PMOS device. The maximum current available to charge a load capacitance is that provided by four PMOS transistors. Thus, the ratio is 4.

(b) There is only one possible configuration (or path) for capacitor discharge. Thus the minimum and maximum currents are the same

⇒ ratio is 1.

Ex: 14.23

Since dynamic power dissipation is scaled by $\frac{1}{S^2}$ and propagation delay is scaled by $\frac{1}{S}$, hence, PDP is scaled by $\frac{1}{S^2} \times \frac{1}{S} = \frac{1}{S^3} = \frac{1}{8}$ So PDP decreases by a factor of 8.

Ex: 14.24

If V_{DD} and Vt are kept constant, the entries in Table 14.2 that change are as follows:

Obviously, V_{DD} and V_t do not scale by $\frac{1}{S}$ any-more. They are kept constant!

$t_p \propto \frac{\alpha C}{k' V_{DD}}$ since $\alpha \propto \frac{V_t}{V_{DD}}$, thus α remains unchanged, while C is scaled by $\frac{1}{S}$, and K' is scaled by S, therefore t_p is scaled by $\frac{1/S}{S} = \frac{1}{S^2}$

Energy/Switching cycle, i.e., $CV^2{}_{DD}$, is scaled by $\frac{1}{S}$

$P_{dyn} \propto \frac{CV^3{}_{DD}}{2t_p}$ and thus is scaled by $\frac{1/S}{1/S^2} = S$

thus P_{dyn} increases.

The power density, i.e., $\frac{P_{dyn}}{\text{deive area}}$ is scaled by $\frac{S}{1/S^2} = S^3$

Ex: 14.25

Using Eq. 14.94 we have:

$$V_{DSat} = \frac{L}{\mu_n} V_{sat} = \frac{0.25 \times 10^{-6}}{400 \times 10^{-4}} \times 10^7 \times 10^{-2}$$

$= 0.63\text{V}$

Ex: 14.26

For the NMOS transistor, $V_{GS} = 1.2\text{ V}$ results in $V_{GS} - V_{tn} = 1.2 - 0.4 = 0.8\text{ V}$ which is greater than $V_{Dsat} = 0.34\text{ V}$. Also, $V_{DS} = 1.2\text{ V}$ is greater than V_{Dsat}, thus both conditions in Eq. 14.101 are satisfied and the NMOS transistor will be operating in the velocity-saturation region and thus i_D is given by Eq. 14.100

$$i_D = 430 \times 10^{-6} \times 1.5 \times 0.34\left(1.2 - 0.4 - \frac{1}{2} \times 0.34\right)$$

$(1 + 0.1 \times 1.2) = 154.7\ \mu\text{A}$

if velocity-saturation-were absent, the current would be:

$$i_D = \frac{1}{2} \times 430 \times 10^{-6} \times 1.5(1.2 - 0.4)^2$$

$\times (1 + 0.1 \times 1.2) = 231.2\ \mu\text{A}$

Saturation is obtained over the range

$V_{DS} = 0.34\text{V}$ to 1.2V compared to

$V_{DS} = V_{OV} = (1.2 - 0.4) = 0.8\text{ V}$ to 1.2 V

in the absence of velocity saturation.

For the PMOS transistor, we see that since $|V_{GS}| - |V_{tp}| = 0.8\text{ V}$ and $|V_{DS}| = 1.2\text{ V}$ are both larger than $|V_{Dsat}| = 0.6\text{ V}$ the device will be operating in velocity saturation and

$$i_D = 110 \times 10^{-6} \times 1.5 \times 0.6\left(1.2 - 0.4 - \frac{1}{2} \times 0.6\right)$$

$(1 + 0.1 \times 1.2) = 55.4\ \mu\text{A}$

$0.6 \le V_{DS} \le 1.2\text{ V}$

without velocity saturation

$$i_D = \frac{1}{2} \times 110 \times 1.5 \times 0.6(1.2 - 0.4)^2(1 + 0.1 \times 1.2)$$

$= 59.1\ \mu\text{A}$

$V_{ov} \le V_{DS} \le 1.2\text{ V}$ or $0.8\text{V} \le V_{DS} \le 1.2\text{V}$

Note that the velocity saturation reduces the NMOS current by 33% and the PMOS current by ~ 7%.

Ex: `14.27`

a) Using Eq. `14.102` we have

$i_D = I_{se}V_{GS}/nV_T$

Thus, $\log i_D = \log I_S + \frac{V_{GS}}{nV_T}\log(e)$

Therefore, the slope of the straight line representing subthreshold conduction is given by:

$\frac{nV_T}{\log(e)} = 2.3\text{n}\text{V}_\text{T}$

b) $V_T = 25$ mV for $i_D = 100$ nA

at $V_{GS} = .21$V

$100\text{n} = Ise^{.21/1.22(25\text{m})}$

$I_s = .1$ nA

$i_D = .1\text{n}e^{0/nV_T} = .1\text{nA}$

c) For $V_{GS} = 0$, $i_D = .1\text{nA}$

$I_{total} = 500 \times 10^6 \times .1 \times 10^{-9} = 50$ mA

$P_{diss} = I_{total} \times V_{DD} = 50\text{ m} \times 1.2 = 60$ mW

15.1

$(W/L)_n = 1.5 \ (W/L)_p = 0.32$

$t_{PLH} = 0.5$ ns

$t_{PLH} = 0.03$ ns

THE NOISE MARGINS WILL NOT CHANGE

15.2

Using eq. (15.11

$$V_{OL} = (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{\gamma}}\right] = (2.5 - 0.5) \times$$

$$\left[1 - \sqrt{1 - \frac{1}{4}}\right]$$

$V_{OL} = 0.27$ V

using eq. (15.13 and (15.14

$$NM_L = V_t - (V_{DD} - V_t)\left[1 - \sqrt{1 - \frac{1}{\gamma} - \frac{1}{\sqrt{\gamma(\gamma + 1)}}}\right]$$

$$NM_L = 0.5 - (2)\left[1 - \sqrt{1 - \frac{1}{4} - \frac{1}{\sqrt{4(5)}}}\right] = 0.7 \text{ V}$$

$$NM_H = (V_{DD} - V_t)\left(1 - \frac{2}{\sqrt{3\gamma}}\right) = (2) \times$$

$$\left(1 - \frac{2}{\sqrt{3.4}}\right) = 0.85 \text{ V}$$

$$\gamma = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}{\mu_p C_{ox}\left(\frac{W}{L}\right)_p} = \frac{115\ \mu\left(\frac{0.375}{0.25}\right)}{30\ \mu\left(\frac{W}{L}\right)_p} = 4$$

$$\therefore \left(\frac{W}{L}\right)_p = 1.44$$

using eq. (15.12

$$I_{stat} = \frac{1}{2}(30\ \mu)(1.44)(2.5 - 0.5)^2 = 86.4\ \mu\text{A}$$

$$P_D = I_{stat}V_{DD} = 86.4\ \mu\ 2.5 = 0.22\ \text{mW}$$

using eq15.15 and (15.16

$$\alpha_p = 2\bigg/\left[\frac{7}{4} - 3\left(\frac{0.5}{2.5}\right) + \left(\frac{0.5}{2.5}\right)^2\right] = 1.68$$

$$t_{PLH} = \frac{1.68 \times 7 \times 10^{-15}}{30 \times 10^{-6} \times 1.44 \times 2.5} = 0.11 \text{ nsec}$$

using eq. 15.17 and (15.18

$$\alpha_n = 2\bigg/\left[1 + \frac{3}{4}\left(1 - \frac{1}{\gamma}\right) - \left(3 - \frac{1}{\gamma}\right)\left(\frac{0.5}{2.5}\right) + \left(\frac{0.5}{2.5}\right)^2\right]$$

$$= 1.9$$

$$t_{PHL} = \frac{1.9 \times 7 \times 10^{-15}}{115 \times 10^{-6} \times \left(\frac{.375}{.25}\right) \times 2.5} = 0.03 \text{ nsec}$$

$$t_p = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(.11\text{n} + 0.03\text{n}) = 0.07 \text{ nsec}$$

15.3

$$V_t = V_{t0} + \gamma(\sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f})$$

since $V_{OH} = V_{DD} - V_t$,

$$V_t = V_{t0} + \gamma(\sqrt{V_{DD} - V_t + 2\phi_f} - \sqrt{2\phi_f})$$

Substituting values, we get

$$V_t = 0.5 + 0.3\text{V}^{1/2}$$

$$(\sqrt{1.8 \text{ V} - V_t + 0.85 \text{ V}} - \sqrt{0.85 \text{ V}})$$

$$V_t = 0.5 + 0.3\text{V}^{1/2}$$

$$\sqrt{2.65\text{V} - V_t} - 0.3\text{V}^{1/2}\sqrt{0.85\text{V}}$$

$$V_t - 0.223 = 0.3\sqrt{2.65 - V_t}$$

Squaring both sides yields

$$V_t^2 - 0.446V_t + 0.05 = 0.09(2.65 - V_t)$$

so that, $V_t^2 - 0.356V_t - 0.189 = 0$

Solving this quadratic equation, yields one practical value for V_t :

$V_t = 0.648$ V

$V_{OH} = V_{DD} - V_t = 1.8 \text{ V} - 0.648 \text{ V}$

$= 1.15$ V

15.4

(a) Referring to Fig 15.12 without loading,

$V_{OH} \rightarrow 5$ V

$V_{OL} \rightarrow 0$ V

(b) Referring to Fig.15.12 (a),

$$i_{DN}(0) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{t0})^2$$

$$= \frac{1}{2}(50\ \mu\text{A}/\text{V}^2)\left(\frac{4\ \mu\text{m}}{2\ \mu\text{m}}\right)(5\text{ V} - 1\text{ V})^2 = 800\ \mu\text{A}$$

$$i_{DP}(0) = \frac{1}{2}k_P'\left(\frac{W}{L}\right)_p(V_{DD} - V_{t0})^2$$

$$= \frac{1}{2}(20\mu\text{A}/\text{V}^2)\left(\frac{4}{2}\right)(5\text{V} - 1\text{V})^2 = 320\ \mu\text{A}$$

Capacitor current is

$$i_C(o) = i_{DN}(0) + i_{DP}(0) = 800\mu\text{A} + 320\ \mu\text{A}$$

$= 1120\ \mu$A

To obtain $i_{DN}(t_{PLH})$, we note that this situation is identical to that in Example15.2 and we can use the result of part (c) :

$i_{DN}(t_{PLH}) = 50\ \mu$A

$$i_{DP}(t_{PLH}) = k_P'\left(\frac{W}{L}\right)_P \times \gamma$$

$$\left[(V_{DD} - V_{t0})\frac{V_{DD}}{2} - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= (20\ \mu\text{A}/\text{V}^2)\left(\frac{4}{2}\right)\left[(5\text{ V} - 1\text{ V})\left(\frac{5\text{ V}}{2}\right) - \frac{1}{2}\left(\frac{5\text{ V}}{2}\right)^2\right]$$

$= 275\ \mu\text{A}$

Thus, $i_c(t_{PLH}) = 50\mu\text{A} + 275\mu\text{A} = 325\ \mu\text{A}$

$$i_C|_{av} = \frac{1}{2}(1120\mu\text{A} + 325\mu\text{A}) = 722.5\ \mu\text{A}$$

$$t_{PLH} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_C|_{av}} = \frac{70(10^{-15})F\left(\frac{5\text{ V}}{2}\right)}{722.5(10^{-6})\text{A}}$$

$= 0.24$ ns

(c) Referring to Fig. 15.12(b),

$$i_{DN}(o) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_{DD} - V_{to})^2$$

$$= \frac{1}{2}(50\mu\text{A}/V^2)\left(\frac{4}{2}\right)(5\text{ V} - 1\text{ V})^2 = 800\ \mu\text{A}$$

$$i_{DP}(o) = \frac{1}{2}k_P'\left(\frac{W}{L}\right)_p (V_{DD} - V_{to})^2$$

$$= \frac{1}{2}(20\mu\text{A}/\text{V}^2)\left(\frac{4}{2}\right)(5V - 1V)^2 = 320\mu\text{A}$$

$i_C(o) = i_{DN}(o) + i_{DP}(o) = 800\mu\text{A} + 320\mu\text{A}$

$= 1120\mu\text{A}$

$$i_{DD}(t_{PHL}) = k_n'\left(\frac{W}{L}\right)_n \times$$

$$\left[(V_{DD} - V_{to})\frac{V_{DD}}{2} - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= 50\ \mu\text{A}/V^2\left(\frac{4}{2}\right)\left[(5\text{ V} - 1\text{ V})\left(\frac{5\text{ V}}{2}\right) - \frac{1}{2}\left(\frac{5\text{ V}}{2}\right)^2\right]$$

$= 688\mu\text{A}$

To find $i_{DP}(t_{PHL})$, we first determine V_{tp} when $v_o = \frac{V_{DD}}{2}$ which corresponds to $V_{SG} = \frac{V_{DD}}{2}$

$$|V_{tp}| = V_{to} + \gamma\left[\sqrt{\frac{V_{DD}}{2} + 2\phi_f} - \sqrt{2\phi_f}\right]$$

$$= 1\text{ V} + 0.5\text{ V}^{1/2}\left[\sqrt{\frac{5\text{ V}}{2} + 0.6\text{ V}} - \sqrt{0.6\text{ V}}\right] = 1.49\text{ V}$$

$$\text{Thus, } i_{DP}(t_{PHL}) = \frac{1}{2}k_P'\left(\frac{W}{L}\right)_p\left[\frac{V_{DD}}{2} - |V_{tp}|\right]^2$$

$$= \frac{1}{2}(20\mu\text{A}/V^2)\left(\frac{4}{2}\right)\left[\frac{5V}{2} - 1.49V\right]^2 = 20\ \mu\text{A}$$

$i_C(t_{PHL}) = i_{DN}(t_{PHL}) + i_{DP}(t_{PHL})$

$= 688\ \mu\text{A} + 20\ \mu\text{A} = 708\ \mu\text{A}$

$$i_C|_{av} = \frac{1120\ \mu\text{A} + 708\ \mu\text{A}}{2} = 914\ \mu\text{A}$$

So,

$$t_{PHL} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_C|_{av}} = \frac{70(10^{-15})F\left(\frac{5\text{V}}{2}\right)}{914(10^{-6})\text{A}} = 0.19\text{ ns}$$

Q_p will turn off when $V_o = |V_{tp}|$
where

$$|V_{tp}| = V_{to} + \gamma[\sqrt{V_{DD} - |V_{tp}| + 2\phi_f} - \sqrt{2\phi_f}]$$

Solving for $|V_{tp}|$,

$$|V_{tp}| = 1\text{V} + 0.5\text{V}^{1/2}$$

$$[\sqrt{5\text{ V} - |V_{tp}| + 0.6\text{ V}} - \sqrt{0.6\text{ V}}]$$

$$|V_{tp}| - 0.613\text{V} = 0.5\text{V}^{1/2}\sqrt{5.6\text{V} - |V_{tp}|}$$

squaring both sides and setting one side equal to zero, we have the quadratic equation,

$$|V_{tp}|^2 - 0.976|V_{tp}| - 1.024\text{V}^2 = 0$$

solving, we get $|V_{tp}| = 1.6$ V

(d)

$$t_p = \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2}(0.24\text{ ns} + 0.19\text{ ns})$$

$\simeq 0.22$ ns

15.5

$$R_{TG_{AV}} = \frac{R_{TG1} + R_{TG2}}{2} = \frac{4.5\text{ k}\Omega + 6.5\text{ k}\Omega}{2}$$

$= 5.5\text{k}\Omega$

$$t_{PLH} = 0.69RC = 0.69(5.5\text{k}\Omega)(70)(10^{-15})F$$

$t_{PLH} = 0.27$ns which is close to the value of 0.24ns obtained in Exercise 15.14

15.6

$$\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p = 1.5$$

Using Eq. 15.36

$$R_{TG} \approx \frac{12.5}{(W/L)_n} = \frac{12.5}{1.5} = 8.3\text{ k}\Omega$$

15.7

Using Eq. 14.71

$$R_{P1} = \frac{30}{\left(\frac{W}{L}\right)_p}\text{ k}\Omega = \frac{30}{(2)}\text{ k}\Omega = 15\text{ k}\Omega$$

with Eq. 15.36 we see that

$$R_{TG} = \frac{12.5}{\left(\frac{W}{L}\right)_n}\text{ k}\Omega = \frac{12.5}{(1)}\text{ k}\Omega = 12.5\text{ k}\Omega$$

Using Eq. 15.38

$$t_p = 0.69[(C_{out1} + C_{TG1})R_1 + (C_{in2} + C_{TG2})$$

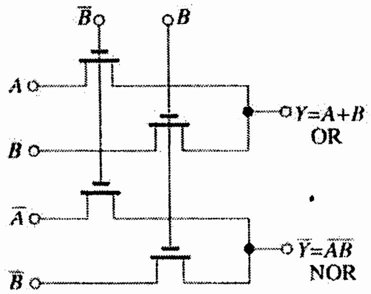
$$\times (R_1 + R_2)$$

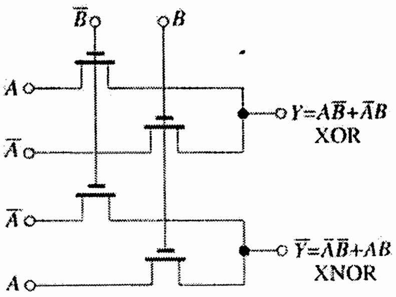
$$= 0.69[(10\text{fF} + 5\text{fF})(15\text{k}\Omega) + (10\text{fF} + 5\text{fF})$$

$$\times (15\text{ k}\Omega + 12.5\text{ k}\Omega)]$$

$t_p = 0.64$ ns

15.8





15.9

Since $i_D(V_{DD}) = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)_{eq}(V_{DD} - V_t)^2$,

doubling $\left(\frac{W}{L}\right)$ will double $\left(\frac{W}{L}\right)_{eq}$ and $i_D(V_{DD})$

so $i_D(V_{DD}) = 2(76.1\ \mu\text{A}) = 152.2\ \mu\text{A}$

This new $\left(\frac{W}{L}\right)_{eq}$ will also double $i_D\left(\frac{V_{DD}}{2}\right)$:

$$i_D\left(\frac{V_{DD}}{2}\right) = 2(68.9\mu\text{A}) = 137.8\ \mu\text{A}$$

This doubles I_{av} to $2(72.5\ \mu\text{A}) = 145\ \mu\text{A}$

the new t_{PHL} is

$$t_{PHL} = \frac{C\left(V_{DD} - \frac{V_{DD}}{2}\right)}{I_{av}}$$

$$= \frac{30(10^{-15})F(1.8\text{V} - 0.9\text{V})}{145(10^{-6})\text{A}} = 0.19\ \text{ns}$$

15.10

Refer to Fig. E15.10

(a) $\left(\frac{W}{L}\right)_{eq1} = \frac{1}{2}\left(\frac{W}{L}\right) = \frac{1}{2} \times \frac{4}{2} = 1$

$\left(\frac{W}{L}\right)_{eq2} = \frac{1}{2}\left(\frac{W}{L}\right) = \frac{1}{2} \times \frac{4}{2} = 1$

(b) $i_{D1}(v_{I1} = V_{DD}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq1}(V_{DD} - V_t)^2$

$$= \frac{1}{2} \times 50 \times 1(5 - 1)^2$$

$$= 400\mu\text{A}$$

$$i_{D1}(v_{Y1} = V_t) = k_n'\left(\frac{W}{L}\right)_{eq1}\left[(V_{DD} - V_t)V_t - \frac{1}{2}v_t^2\right]$$

$$= 50 \times 1\left[(5 - 1)1 - \frac{1}{2} \times 1\right]$$

$$= 175\mu\text{A}$$

$$i_{D1}|_{av} = \frac{400 + 175}{2} = 288\mu\text{A}$$

(c) $i_{D1}|_{av}\Delta t = C_{L1}\,\Delta v_{Y1}$

$$\Delta t = \frac{C_{L1}(V_{DD} - V_t)}{i_{D1}|_{av}} = \frac{40 \times 10^{-15} \times 4}{288 \times 10^{-6}}$$

$$= 0.56\ \text{ns}$$

(d) Following the hint we assume that Q_{eq} remains saturated during Δt.

$$i_{D2}|_{av} = i_{D2}(v_{Y1} = 3\text{V}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq2}(3 - 1)^2$$

$$i_{D2}|_{av} = \frac{1}{2} \times 50 \times 1(3 - 1)^2$$

$$= 100\mu\text{A}$$

(e) $\Delta v_{Y2} = -\frac{i_{D2}|_{av} \cdot \Delta t}{C_{L2}}$

$$= -\frac{100 \times 10^{-6} \times 0.56 \times 10^{-9}}{40 \times 10^{-15}}$$

$$= -1.4\ \text{V}$$

Thus, v_{Y2} decrease to 3.6 V.

15.11

$V_{OH} = 0$

$V_{OL} = -0.88\ \text{V}$

SHOULD BE SHIFTED BY − 0.88 V

$V_{OH} = -0.88\text{V}$ AFTER SHIFTING

$V_{OL} = -1.76\text{V}$ AFTER SHIFTING

15.12

Refer to Fig. 15.12 Neglecting the base current of Q_1, the current through R_1, D_1, D_2 and R_2 is

$$I = \frac{5.2 - V_{D1} - V_{D2}}{R_1 + R_2}$$

$$= \frac{5.2 - 0.75 - 0.75}{0.907 + 4.98} = 0.6285\ \text{mA}$$

Thus, $V_B = -I_{R1} = -0.57\ \text{V}$

$V_R = V_B - V_{BE1} = -0.57 - 0.75 = -1.32\ \text{V}$

15.13

Refer to Fig. 15.26

$$I_E = \frac{V_R - V_{BE}|_{QR} - (-V_{EE})}{R_E}$$

$$= \frac{-1.32 - 0.75 + 5.2}{0.779} \simeq 4 \text{ mA}$$

$$V_C|_{QR} = -\gamma \times 4 \times R_{C2} \cong -4 \times 0.245 = -1 \text{ V}$$

$V_C|_{Q_A Q_B} = 0\text{V}$ (because the current through R_{C1} is zero)

15.14

Refer to Fig. 15.28

For $V_I = V_{IL}$, $I_{QR} = 99\, I_{QA}$,

$$I_E = \frac{-1.32 - V_{BE|QR} + 5.2}{0.779}$$

Assume $V_{BE}|_{QR} = 0.75\text{V}$, $I_E = 4.018$ mA

$I_{QR} \simeq 0.99 \times 4.018 = 3.98$ mA

Thus a better estimate of $V_{BE}|_{QR}$ is

$$V_{BE}|_{QR} = 0.75 + 0.025 \ln\left(\frac{3.98}{1}\right)$$

$= 0.785$ V

and correspondingly,

$$I_E = \frac{-1.32 - 0.785 + 5.2}{0.779} = 3.97 \text{ mA}$$

For $V_I = -1.32\text{V}$, $I_{QR} = I_{QA} = I_E/2$,

$$I_E = \frac{-1.32 - 0.75 + 5.2}{0.779} = 4.018 \text{ mA}$$

Thus a better estimate for $V_{BE}|_{QR}$ is

$$V_{BE}|_{QR} = 0.75 + 0.025 \ln\left(\frac{2.009}{1}\right)$$

$= 0.767$ V

and correspodingly,

$I_E = 4.00$ mA

For $V_I = V_{IH} = -1.205$ V,

$I_{QA} = 99 I_{QR}$,

$$I_E = \frac{-1.205 - 0.75 + 5.2}{0.779} = 4.166 \text{ mA}$$

Thus a better estimate for $V_{BE}|_{QA}$ is

$$V_{BE}|_{QA} = 0.75 + 0.025 \ln\left(\frac{0.99 \times 4.166}{1}\right)$$

$= 0.788$ V.

and correspondingly

$$I_E = \frac{-1.205 - 0.788 + 5.2}{0.799} = 4.12 \text{ mA}$$

At $V_I = V_R$, $I_{QR} = \frac{1}{2} I_E = 2$ mA

Thus, $V_C|_{QR} = -2 \times 0.245 = -0.49$ V

$v_{OR} = -0.49 - 0.75 = -1.24$ V

$$I_E|_{Q2} = \frac{-1.24 + 2}{0.05} = 15.2 \text{ mA}$$

A better estimate for $V_{BE}|_{Q2}$ is

$$V_{BE}|_{Q2} = 0.75 + 0.025 \ln\left(\frac{15.2}{1}\right)$$

$= 0.818$ V

Thus a better estimate for v_{OR} is

$v_{OR} = -0.49 - 0.818 = -1.31$ V

15.15

REFER TO FIG. 15.32for

$V_I = V_{IH} = -1.205$

The value of I_E we found in Exercise 15.14to be 4.12 mA. The $V_C|_{QR} \simeq -0.22 \times 4.12$

$= -0.906$ V

$v_{NOR} \simeq -0.906 - 0.75 = -1.656$ V

$$I|_{Q3} = \frac{-1.656 + 2}{0.05} = 6.88 \text{ mA}$$

A better estimate for $V_{BE}|_{Q3}$ is

$$V_{BE}|_{Q3} = 0.75 + 0.025 \ln\left(\frac{6.88}{1}\right)$$

$= 0.798$ V

and correspondingly

$V_{NOR} = -0.906 - 0.798 = -1.704$ V

(b)For $v_I = V_{OH} = -0.88$ V,

$$I_E \simeq \frac{-0.88 - 0.75 + 5.2}{0.779} = 4.58 \text{ mA}$$

A better estimate for $V_{BE}|_{QA}$ is

$$V_{BE}|_{QA} = 0.75 + 0.025 \ln\left(\frac{4.58}{1}\right) = 0.788 \text{ V}$$

$$\text{Thus,} I_E = \frac{-0.88 - 0.788 + 5.2}{0.779} = 4.53 \text{ mA}$$

$V_C|_{QA} = -0.22 \times 4.53 = -1$ V

$V_{NOR} = -1 - 0.75 = -1.75$ V

$$I|_{RT} = \frac{-1.75 + 2}{0.05} = 5 \text{ mA}$$

$$V_{BE}|_{Q3} = 0.75 + 0.025 \ln\left(\frac{5}{1}\right)$$

$= 0.79$ V

$V_{NOR} = -1 - 0.79 = -1.79$ V

(c) The input resistance into the base of Q_3 is

$(\beta + 1)[r_{e3} + R_T]$

$$= 101\left[\frac{25}{5} + 50\right] = 5.55 \text{ k}\Omega$$

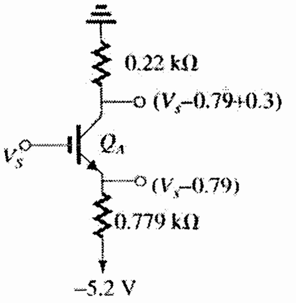
$$\frac{V_C|_{QA}}{V_i} = -\frac{(5.55\ \text{k}\Omega \parallel 0.22\ \text{k}\Omega)}{r_e|_{QA} + R_E}$$

$$= \frac{-5.55 \parallel 0.22}{\left(\frac{25}{4.53} + 771\right) \times 10^{-3}} = -0.269$$

$$\frac{v_{NOR}}{v_C|_{QA}} = \frac{50\ \Omega}{50\ \Omega + 5\ \Omega} = 0.909$$

Thus, $\frac{v_{NOR}}{v_C|_{QA}} = -0.269 \times 0.909 = -0.24\text{V} / \text{V}$

d) See figure below. Assume $V_{BE} \approx 0.79\text{V}$



(because the current will be 4 to 5 mA). At the range of saturation,

$I_C = \alpha I_E = 0.99 I_E$

Thus, $\frac{0 - V_s + 0.79 - 0.3}{0.22} = 0.99 \times$

$$\frac{V_s - 0.79 + 5.2}{0.779}$$

$\Rightarrow V_S = -0.58\ \text{V}$

15.16

Refer to Fig.15.26 For the reference circuit, the current through R_1, D_1, D_2, and R_2

is $\frac{5.2 - 2 \times 0.75}{4.98 + 0.907} = 0.629\ \text{mA}$

$V_B|_{Q1} = -0.57\ \text{V}$

$V_R = -0.57 - 0.75 = -1.32\ \text{V}$

$I_E|_{Q1} = \frac{-1.32 + 5.2}{6.1} = 0.636\ \text{mA}$

Thus the reference circuit draws a current of $(0.629 + 0.636) = 1.265$ mA from the 5.2 V supply. It follows that the power dissipated in the reference circuit is $1.265 \times 5.2 = 6.6$ mW. Since the reference circuit supplies four gates, the dissipation attributed to a gate is $\frac{6.6}{4} = 1.65\ \text{mW}$

In addition, the gate draws a current $I_E \approx 4$ mA from the 5.2V supply. Thus the total power dissipation / gate is

$P_D = 4 \times 5.2 + 1.65 = 22.4\ \text{mW}$

Ex 16.1

Refer to Fig.16.5(a)

when $v_{\phi} = v_s = \frac{V_{DD}}{2}$ then Q_{eq} will be saturatted $v_{\bar{Q}} = \frac{V_{DD}}{2}$ and Q_2 will be in triode

$$Q_{eq}: \left(\frac{W}{L}\right) = \frac{1}{2}\left(\frac{W}{L}\right)_{5,6}$$

$$\frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq}\left(\frac{V_{DD}}{2} - V_t\right)^2 = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_2$$

$$\left[(V_{DD} - V_t)\frac{V_{DD}}{2} - \frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

From Example 16.1

$k_n' = 4k_p' = 300\ \mu A / V^2$;

$|V_t| = 0.5;\ V_{DD} = 1.8$

$$\frac{1}{2}(300 \times 10^{-6})\left(\frac{1}{2}\right)\left(\frac{W}{L}\right)_5\left(\frac{1.8}{2} - 0.5\right)^2$$

$$= \frac{1}{2}\left(\frac{300\ \mu}{4}\right)\left(\frac{1.08}{0.18}\right)\left[(1.8 - 0.5)\left(\frac{1.8}{2}\right) - \frac{1}{2}(.9)^2\right]$$

$$(12 \times 10^{-6})\left(\frac{W}{L}\right)_5 = 172 \times 10^{-6}$$

$$\left(\frac{W}{L}\right)_5 = 14.3 \cong \frac{2.6\ \mu m}{0.18\ \mu m}$$

Ex 16.2

Bits for row address:

$2^M = 1,024$

$\log_2(2^M) = \log_2(1,024)$

$\mu * \log_2(2) = \log_2(1,024)$

$$\mu = \frac{\log_2(1,024)}{\log_2(2)} = 10$$

Bits for column address:

$2^N = 128$

$$N = \frac{\log_2(128)}{\log_2(2)} = 7$$

Bits for block address:

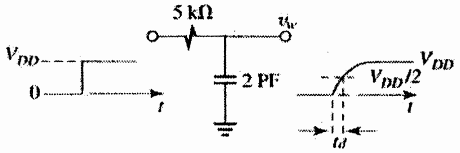
$2^{Bits} = 32$

$$\text{Bits} = \frac{\log_2(32)}{\log_2(2)} = 5$$

Ex 16.3

$v_w = V_{DD}(1 - e^{-t/CR})$

$$\frac{V_{DD}}{2} = V_{DD}(1 - e^{-t_d/CR})$$



$t_d = CR\ln 2$

$= 2 \times 10^{-12} \times 5 \times 10^3 \times 0.69$

$= 6.9$ ns

Ex 16.4

$$\left.\frac{\left(\frac{W}{L}\right)_a}{\left(\frac{W}{L}\right)_n}\right|_{max} = \frac{1}{\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2} - 1$$

$$\left.\left(\frac{W}{L}\right)_a\right|_{max} = 1.5 \times \left[\frac{1}{\left(1 - \frac{0.5}{1.8 - 0.5}\right)^2} - 1\right] = 2.5$$

$$\Rightarrow \left(\frac{W}{L}\right)_a \leq 2.5$$

Ex 16.5

$\Delta t = \frac{C_B \times \Delta V}{I_5}$: To find I_5, we use

$$I_5 = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{DD} - V_{tn} - V_{\bar{Q}})^2$$

(a) $\left(\frac{W}{L}\right)_a = 2.5$

$$I_5 = \frac{1}{2} \times 300 \times 10^{-6} \times 2.5 \times (1.8 - 0.5 - 0.5)^2$$

$= 240\ \mu A$

$$\Delta t = \frac{2 \times 10^{-12} \times 0.2}{240 \times 10^{-6}} = 1.7 \text{ ns}$$

(b) $\left(\frac{W}{L}\right)_a = 1.5$

$$I_5 = \frac{1}{2} \times 300 \times 10^{-6} \times 1.5 \times (1.8 - 0.5 - 0.5)^2$$

$= 144\ \mu A$

$$\Delta t = \frac{2 \times 10^{-12} \times 0.2}{144\ \mu A} = 2.8 \text{ ns}$$

or: $\Delta t\ \alpha\ \frac{1}{I_5}\ \alpha\ \frac{1}{\left(\frac{W}{L}\right)_a}$ therefore.

$$\Delta t = 1.7 \text{ ns} \times \frac{2.5}{1.5} = 2.8 \text{ ns}$$

Ex 16.6

$$\left(\frac{W}{L}\right)_p \leq \left(\frac{W}{L}\right)_a \times \frac{\mu_n}{\mu_p}\left[1-\left(1-\frac{V_{tn}}{V_{DD}-V_{tn}}\right)^2\right]$$

$$\left(\frac{W}{L}\right)_p \leq \left(\frac{W}{L}\right)_a \times 4 \times \left[1-\left(1-\frac{0.5}{1.8-0.5}\right)^2\right]$$

$$\left(\frac{W}{L}\right)_p \leq 2.5\left(\frac{W}{L}\right)_a \text{ or}$$

$$\left(\frac{W}{L}\right)_p \leq 2.5 \times 2.5 \Rightarrow \left(\frac{W}{L}\right)_p \leq 6.25$$

For minimum area: select

$W_n = W_p = W_a = 0.18\mu\text{m}$

Ex 16.7

From Eqs. 16.14 and 16.15 we have:

$\Delta V_{(1)} =$

$$\frac{C_S}{C_B} \times \frac{V_{DD}}{2} = \frac{30 \times 10^{-15}}{0.3 \times 10^{-12}} \times \frac{1.2}{2} = 6.0 \text{ mV}$$

$\Delta V_{(0)} =$

$$-\frac{C_S}{C_B} \times \frac{V_{DD}}{2} = \frac{-30 \times 10^{-15}}{0.3 \times 10^{-12}} \times \frac{1.2}{2} = -0.06 \text{ V}$$

$$= -60 \text{ mV}$$

Ex 16.8

Area of the storage array

$= 64 \times 1024 \times 1024 \times 2 = 134217728 \ \mu\text{m}^2$

$= 134.2 \text{ mm}^2$ or equivalently

$= 11.6 \text{ mm} \times 11.6 \text{ mm}$

Total chip area

$= 1.3 \times 134.2 = 174.46 \text{ mm}^2 = 13.2 \times 13.2 \text{ mm}^2$

Ex 16.9

Refer to Example 16.2

Since Δt is proportional to $\tau = \frac{C}{G_m}$, we can reduce Δt by a factor of 2 by decreasing τ by the same factor. $\Delta t \ \alpha \ \tau \ \alpha \ \frac{1}{G_m}$

Hence, G_m has to be doubled. $G_m = g_{mn} + g_{mp}$ and both g_{mn} and g_{mp} have to be increased by a factor of 2. The increase in g_m can be achieved by increasing the correspondng $\frac{W}{L}$, thus:

$$\left(\frac{W}{L}\right)_n = 2 \times \frac{0.54}{0.18} = 6$$

$$\left(\frac{W}{L}\right)_p = 2 \times \frac{2.16}{0.18} = 24$$

Ex 16.11

From Eq. 16.18 $\Delta t = \frac{CV_{DD}}{I}$ or

$$I = \frac{CV_{DD}}{\Delta t} = \frac{50^{\text{pF}} \times 1.8}{0.5 \text{ ns}} = 180 \ \mu\text{A}$$

$$P = V_{DD}I = 1.8 \times 180 \ \mu\text{A} = 324 \ \mu\text{W}$$

Ex 16.12

Refer to Fig. 13.26

Our decoder is an extension of that show:

We have M bits is the address (as opposed to 3) and correspondingly there will be 2^M word lines. Now, each of the 2^M word lines is connected to M NMOS devices and to one PMOS transistor. Thus the total number of devices required is

$$M2^M(\text{NMOS}) + 2^M(\text{PMOS})$$

$$= 2^M(M + 1)$$

Ex 16.13

Refer to Fig. 13.28 Our tree decoder will have 2^N bit lines. Thus it will have N levels: At the first levels there will be 2 transistors, at the second $2^2, \ldots,$ at the Nth level there will be 2^N transistors. Thus the total number of transistors, can be find as

$$\text{Number} = 2 + 2^2 + 2^3 + \cdots + 2^N$$

$$= 2\underbrace{(1 + 2 + 2^2 + \cdots + 2^{N-1})}$$

Geometric series r = 2

$$\text{Sum} = \frac{r^N - 1}{r - 1} = \frac{2^N - 1}{2 - 1}$$

$$= 2^N - 1$$

Thus,

Number $= 2(2^N - 1)$

Ex 16.14

$$f = \frac{1}{2 \times 5t_p}$$

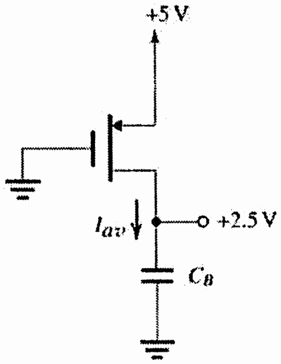
$$= \frac{1}{2 \times 5 \times 10^{-9}} = 100 \text{ MHz}$$

Ex 16.15

(a) $I_{av} = k_p'\left(\frac{W}{L}\right)_p\left[(5-1)2.5 - \frac{1}{2}2.5^2\right]$

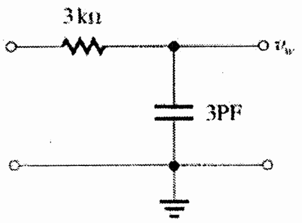
$= 20 \times \frac{24}{2}[10 - 3.125]$

$= 1.65\text{ mA}$


+5 V
I_{av}
+2.5 V
C_B

Thus, $t_{\text{Charging}} = \dfrac{2 \times 10^{-12} \times 2}{1.65 \times 10^{-3}} = 6.1\text{ ns}$

(b)

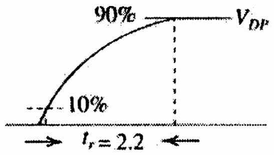


$t_r \simeq 2.25$

$= 2.2 \times 3 \times 10^{-12} \times 3 \times 10^3$

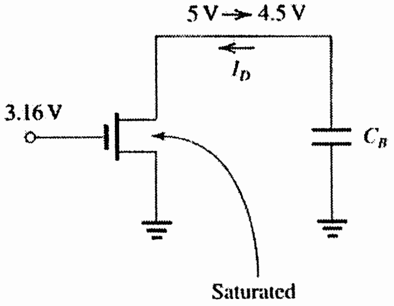
$= 19.8\text{ ns}$

(c) In one time-constant the voltage reached is


90%
V_{DP}
10%
$t_r = 2.2$

$V_{DD}(1 - e^{-1}) = 0.632 V_{DD}$

$= 3.16\text{ V}$


5 V → 4.5 V
I_D
3.16 V
C_B
Saturated

$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(3.16-1)^2$

$= \frac{1}{2} \times 50 \times \frac{6}{2} \times 2.16^2$

$= 0.35\text{ mA}$

$\Delta t = \dfrac{C_B\,\Delta V}{I_D}$

$= \dfrac{2 \times 10^{-12} \times 0.5}{0.35 \times 10^{-3}} = 2.9\text{ ns}$

1.1

(a) $I = \frac{V}{R} = \frac{10\text{ V}}{1\text{ k}\Omega} = 10\text{ mA}$

(b) $R = \frac{V}{I} = \frac{10\text{ V}}{1\text{ mA}} = 10\text{ k}\Omega$

(c) $V = IR = 10\text{ mA} \times 10\text{ k}\Omega = 100\text{ V}$

(d) $I = \frac{V}{R} = \frac{10\text{ V}}{100\ \Omega} = 0.1\text{ A}$

Note: Volts, milliamps, and kilo-ohms constitute a consistent set of units.

1.2

(a) $V = IR = 10\text{ mA} \times 1\text{ k}\Omega = 10\text{ V}$

$P = I^2R = (10\text{ mA})^2 \times 1\text{ k}\Omega = 100\text{ mW}$

(b) $R = V/I = 10\text{ V}/1\text{ mA} = 10\text{ k}\Omega$

$P = VI = 10\text{ V} \times 1\text{ mA} = 10\text{ mW}$

(c) $I = P/V = 1\text{ W}/10\text{ V} = 0.1\text{ A}$

$R = V/I = 10\text{ V}/0.1\text{ A} = 100\ \Omega$

(d) $V = P/I = 0.1\text{ W}/10\text{ mA}$

$= 100\text{ mW}/10\text{ mA} = 10\text{ V}$

$R = V/I = 10\text{ V}/10\text{ mA} = 1\text{ k}\Omega$

(e) $P = I^2R \Rightarrow I = \sqrt{P/R}$

$I = \sqrt{1000\text{ mW}/1\text{ k}\Omega} = 31.6\text{ mA}$

$V = IR = 31.6\text{ mA} \times 1\text{ k}\Omega = 31.6\text{ V}$

Note: V, mA, kΩ, and mW constitute a consistent set of units.

1.3

Thus, there are 17 possible resistance values.

1.4

Shunting the 10 kΩ by a resistor of value of R result in the combination having a resistance R_{eq},

$$R_{eq} = \frac{10R}{R + 10}$$

Thus, for a 1% reduction,

$$\frac{R}{R+10} = 0.99 \Rightarrow R = 990\text{ k}\Omega$$

For a 5% reduction,

$$\frac{R}{R+10} = 0.95 \Rightarrow R = 190\text{ k}\Omega$$

For a 10% reduction,

$$\frac{R}{R+10} = 0.90 \Rightarrow R = 90\text{ k}\Omega$$

For a 50% reduction,

$$\frac{R}{R+10} = 0.50 \Rightarrow R = 10\text{ k}\Omega$$

Shunting the 10 kΩ by

(a) 1 MΩ result in

$$R_{eq} = \frac{10 \times 1000}{1000 + 10} = \frac{10}{1.01} = 9.9\text{ k}\Omega,$$

a 1% reduction;

(b) 100 kΩ results in

$$R_{eq} = \frac{10 \times 100}{100 + 10} = \frac{10}{1.1} = 9.09\text{ k}\Omega,$$

a 9.1% reduction;

(c) 10 kΩ results in

$$R_{eq} = \frac{10}{10 + 10} = 5\text{ k}\Omega,$$

a 50% reduction.

1.5

$$V_O = V_{DD}\frac{R_2}{R_1 + R_2}$$

To find R_O, we short circuit V_{DD} and look back into node X,

$$R_O = R_2 \parallel R_1 = \frac{R_1R_2}{R_1 + R_2}$$

1.6

Use voltage divider to find V_o

$$V_o = 9\frac{3.3}{3.3 + 6.8} = 2.94\text{ V}$$

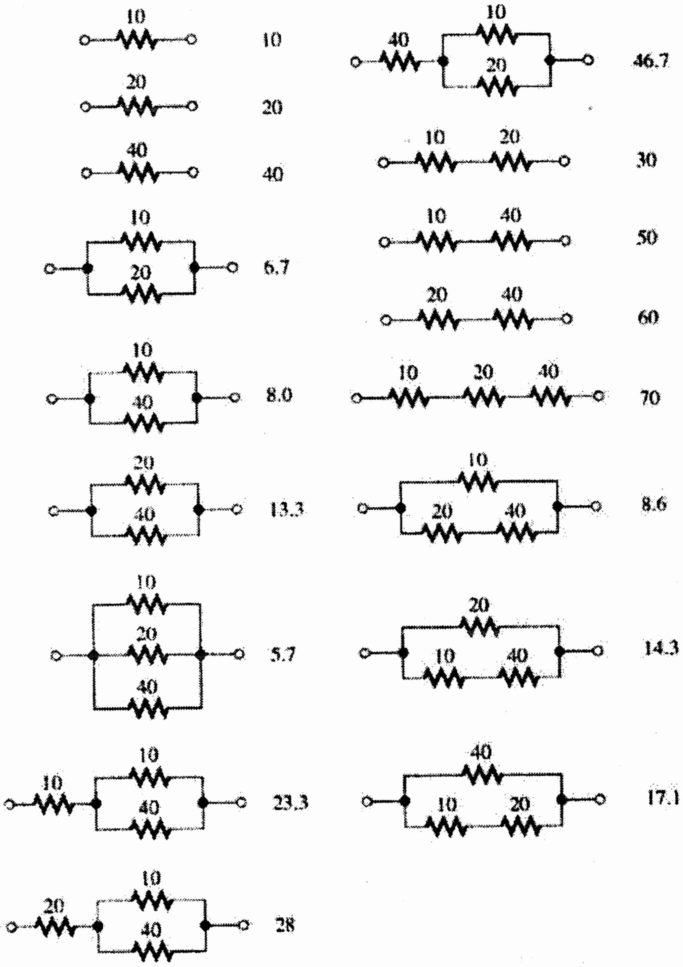
Equivalent output resistance R_O is

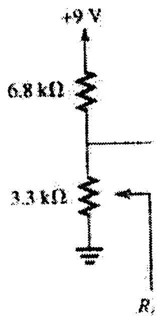
$$R_O = (3.3\text{ k}\Omega \parallel 6.8\text{ k}\Omega) = 2.22\text{ k}\Omega$$

The extreme values of V_o for ±5 % tolerance resistor are

$$V_{O-\min} = 9\frac{3.3(1 - 0.05)}{3.3(1 - 0.05) + 6.8(1 + 0.05)}$$
$$= 2.75\text{ V}$$

This figure is for 1.3



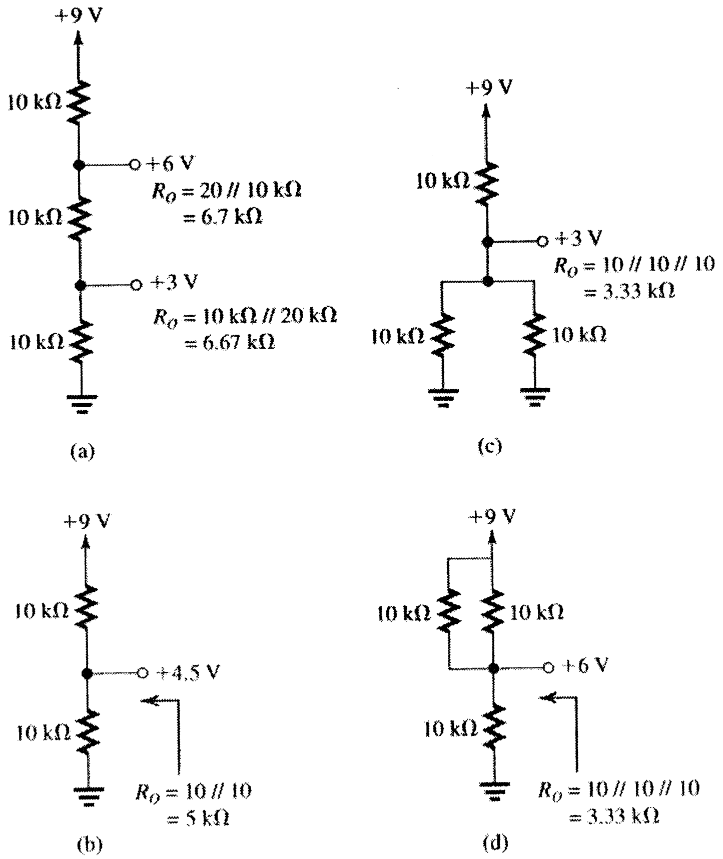

+9 V
6.8 kΩ
3.3 kΩ
R_o

$$V_{O\ \max} = 9\frac{3.3(1+0.05)}{3.3(1+0.05)+6.8(1-0.05)}$$
$$= 3.14\ \text{V}$$

The extreme values of R_O for ±5 % tolerance resistors are

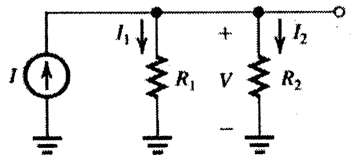
$$R_{O-\min} = \frac{3.3(1-0.05)\times 6.8(1-0.05)}{3.3(1-0.05)+6.8(1-0.05)}$$
$$= 2.11\ \text{k}\Omega$$
$$R_{O-\max} = \frac{3.3(1+0.05)\times 6.8(1+0.05)}{3.3(1+0.05)+6.8(1+0.05)}$$
$$= 2.33\ \text{k}\Omega$$

1.7


+9 V
10 kΩ
+6 V
R_O = 20 // 10 kΩ = 6.7 kΩ
10 kΩ
+3 V
R_O = 10 kΩ // 20 kΩ = 6.67 kΩ
10 kΩ
(a)
+9 V
10 kΩ
+3 V
R_O = 10 // 10 // 10 = 3.33 kΩ
10 kΩ
10 kΩ
(c)
+9 V
10 kΩ
+4.5 V
10 kΩ
R_O = 10 // 10 = 5 kΩ
(b)
+9 V
10 kΩ
10 kΩ
+6 V
10 kΩ
R_O = 10 // 10 // 10 = 3.33 kΩ
(d)

Voltage generated
+3V (two ways: (a) and (c) with (c) having lower output resistance)
+4.5 V (b)
+6V (two ways: (a) and (d) with (d) having a lower output resistance)

1.8


I_1
I_2
+
I
R_1
V
R_2
−

$$V = I(R_1 \parallel R_2)$$
$$= I\frac{R_1 R_2}{R_1 + R_2}$$
$$I_1 = \frac{V}{R_1} = I\frac{R_2}{R_1 + R_2}$$
$$I_2 = \frac{V}{R_2} = I\frac{R_1}{R_1 + R_2}$$

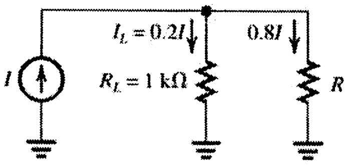
1.9

Connect a resistor R in parallel with R_L.

To make $I_L = 0.2I$ (and thus the current through R, $0.8I$), R should be such

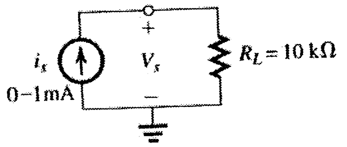
$0.2I \times 1\ \text{k}\Omega = 0.8IR$

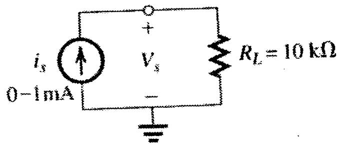
$\Rightarrow R = 250\ \text{k}\Omega$


$I_L = 0.2I$
$0.8I$
I
$R_L = 1\ \text{k}\Omega$
R

1.10

For $R_L = 10\ \text{k}\Omega$, when signal source generates 0 – 1 mA, a voltage of 0 – 10 V may appear across the source


i_s
0–1mA
+
V_s
–
$R_L = 10\ \text{k}\Omega$


i_s
0–1mA
+
V_s
–
$R_L = 10\ \text{k}\Omega$

To limit $V_S \le 1$ V, the net resistance has to be $\le 1\ \text{k}\Omega$. To achieve this we have to shunt R_L with a resistor R so that $(R \parallel R_L) \le 1\ \text{k}\Omega$.

$R \parallel R_L \le 1\ \text{k}\Omega$.

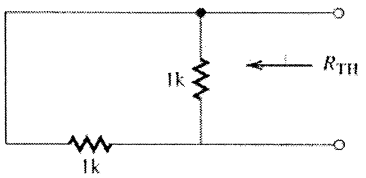
$$\frac{RR_L}{R + R_L} \le 1\ \text{k}\Omega$$

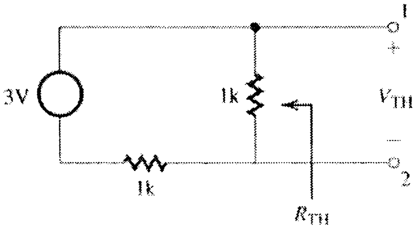
For $R_L = 10$ K

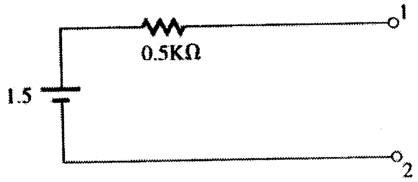
$R \simeq 1.11\ \text{k}\Omega$

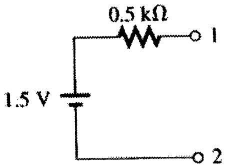
The resulting circuit needs only one additional resistance of $1.11\ \text{k}\Omega$ in parallel with R_l so that $V_S \le 1$ V

1.11




3V
1k
1k
1
+
V_{TH}
–
2
R_{TH}



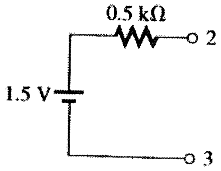


(a)

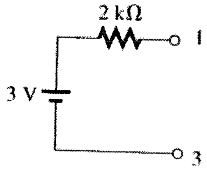
$$V_{TH} = 3\left(\frac{1\ \text{k}}{1\ \text{k} + 1\ \text{k}}\right) = 1.5\ \text{V}$$

$R_{TH} = 1\ \text{k} \parallel 1\ \text{k} = 0.5\ \text{k}$

Same procedure is used for b) & c)

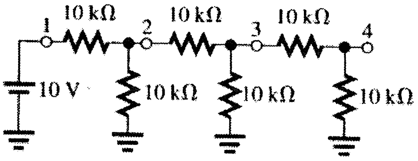


(b)

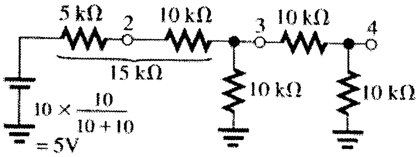


(c)

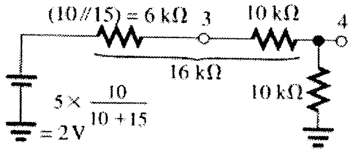
1.11


1
10 kΩ
2
10 kΩ
3
10 kΩ
4
10 V
10 kΩ
10 kΩ
10 kΩ

⇓

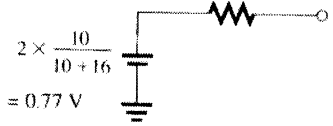

5 kΩ
2
10 kΩ
3
10 kΩ
4
15 kΩ
10 kΩ
10 kΩ
$10 \times \frac{10}{10+10}$
= 5V

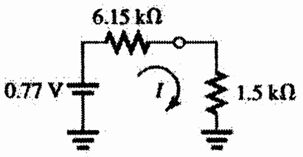
⇓


(10//15) = 6 kΩ
3
10 kΩ
4
16 kΩ
10 kΩ
$5 \times \frac{10}{10+15}$
= 2V

⇓

Thévenin equivalent: (10//16) = 6.15 kΩ 4


$2 \times \frac{10}{10+16}$
= 0.77 V



Now, when a resistance of 1.5 kΩ is connected between 4 and ground,

$$I = \frac{0.77}{6.15 + 1.5}$$

$$= 0.1 \text{ mA}$$

1.12

(a) Node equation at the common mode yields

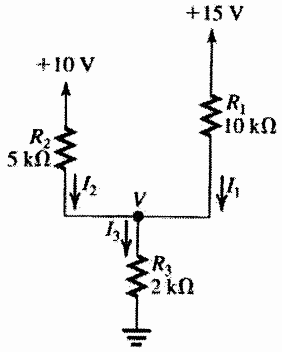
$$I_3 = I_1 + I_2$$

Using the fact that the sum of the voltage drops across R_1 and R_3 equals 15 V, we write

$$15 = I_1R_1 + I_3R_3$$

$$= 10I_1 + (I_1 + I_2) \times 2$$

$$= 12I_1 + 2I_2$$



That is,

$$12I_1 + 2I_2 = 15 \qquad (1)$$

Similarly, the voltage drops across R_2 and R_3 add up to 10 V, thus

$$10 = I_2R_2 + I_3R_3$$

$$= 5I_2 + (I_1 + I_2) \times 2$$

which yields

$$2I_1 + 7I_2 = 10 \qquad (2)$$

Equations (1) and (2) can be solved together by multiplying (2) by 6,

$$12I_1 + 42I_2 = 60 \qquad (3)$$

Now, subtracting (1) from (3) yields

$$40I_2 = 45$$

$$\Rightarrow I_2 = 1.125 \text{ mA}$$

Substituting in (2) gives

$$2I_1 = 10 - 7 \times 1.125 \text{ mA}$$

$$\Rightarrow I_1 = 1.0625 \text{ mA}$$

$$I_3 = I_1 + I_2$$

$$= 1.0625 + 1.1250$$

$$= 1.1875 \text{ mA}$$

$$V = I_3R_3$$

$$= 1.1875 \times 2 = 2.3750 \text{ V}$$

To summarize:

$I_1 \simeq 1.06$ mA $I_2 \simeq 1.13$ mA

$I_3 \simeq 1.19$ mA $V \simeq 2.38$ V

(b) A node equation at the common node can be written in terms of V as

$$\frac{15 - V}{R_1} + \frac{10 - V}{R_2} = \frac{V}{R_3}$$

Thus,

$$\frac{15 - V}{10} + \frac{10 - V}{5} = \frac{V}{2}$$

$$\Rightarrow 0.8\,V = 3.5$$

$$\Rightarrow V = 2.375 \text{ V}$$

Now, I_1, I_2, and I_3 can be easily found as

$$I_1 = \frac{15 - V}{10} = \frac{15 - 2.375}{10}$$

$$= 1.0625 \text{ mA} \simeq 1.06 \text{ mA}$$

$$I_2 = \frac{10 - V}{5} = \frac{10 - 2.375}{5}$$

$$= 1.125 \text{ mA} \simeq 1.13 \text{ mA}$$

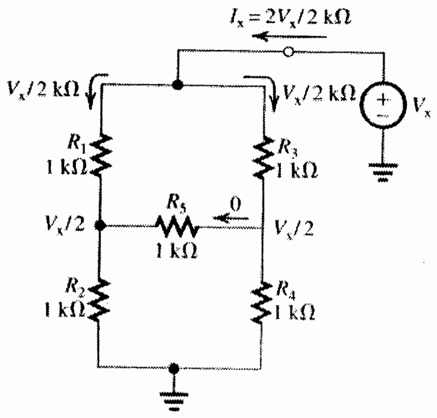
$$I_3 = \frac{V}{R_3} = \frac{2.375}{2} = 1.1875 \text{ mA} \simeq 1.19 \text{ mA}$$

Method (b) is much preferred; faster, more insightful and less prone to errors. In general, one attempts to identify the least possible number of variables and write the corresponding minimum number of equations.

1.13

From the symmetry of the circuit, there will be no current in R_5. (Otherwise the symmetry would be violated.) Thus each branch will carry a current $V_x/2$ kΩ and I_x will be the sum of the two current,

$$I_x = \frac{2V_x}{2 \text{ kΩ}} = \frac{V_x}{1 \text{ kΩ}}$$



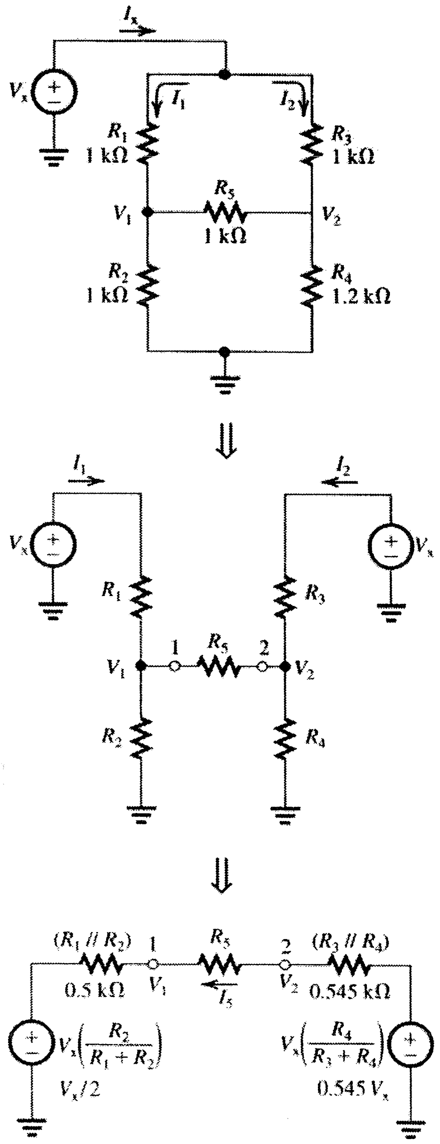
Thus,

$$R_{eq} = \frac{V_x}{I_x} = 1\ \text{k}\Omega$$

Now, if R_4 is raised to 1.2 kΩ the symmetry will be broken. To find I_x we use Thévenin's theorem as follows:

$$I_5 = \frac{0.545V_x - 0.5V_x}{0.5 + 1 + 0.545} = 0.022V_x$$

$$V_1 = \frac{V_x}{2} + 0.022\ V_x \times 0.5$$


I_x
V_x
I_1
I_2
R_1 1 kΩ
R_3 1 kΩ
R_5
1 kΩ
V_1
V_2
R_2 1 kΩ
R_4 1.2 kΩ
⇓
I_1
I_2
V_x
V_x
R_1
R_3
1
R_5
2
V_1
V_2
R_2
R_4
⇓
$(R_1 // R_2)$
1
R_5
2
$(R_3 // R_4)$
0.5 kΩ
V_1
I_5
V_2
0.545 kΩ
$V_x\left(\frac{R_2}{R_1+R_2}\right)$
$V_x/2$
$V_x\left(\frac{R_4}{R_3+R_4}\right)$
$0.545\,V_x$

$$= 0.5V_x \times 1.022 = 0.511V_x$$

$$V_2 = V_1 + I_5R_5 = 0.533V_x$$

$$I_1 = \frac{V_x - V_2}{1\ \text{k}\Omega} = 0.489\ V_x$$

$$I_2 = \frac{V_x - V_2}{1\ \text{k}\Omega} = 0.467V_x$$

$$I_x = I_1 + I_2 = 0.956V_x$$

$$\Rightarrow R_{eq} = \frac{V_x}{I_x} = 1.05\ \text{k}\Omega$$

1.14

(a) $Z = 1\ \text{k}\Omega$ at all frequencies

(b) $Z = 1/j\omega C = -j\dfrac{1}{2\pi f \times 10 \times 10^{-9}}$

At $f = 60$ Hz, $Z = -j265\ \text{k}\Omega$

At $f = 100$ kHz, $Z = -j159\ \Omega$

At $f = 1$ GHz, $Z = -j0.016\ \Omega$

(c) $Z = 1/j\omega C = -j\dfrac{1}{2\pi f \times 2 \times 10^{-12}}$

At $f = 60$ Hz, $Z = -j1.33\ \text{G}\Omega$

At $f = 100$ kHz, $Z = -j0.8\ \text{M}\Omega$

At $f = 1$ GHz, $Z = -j79.6\ \Omega$

(d) $Z = j\omega L = j2\pi fL = j2\pi f \times 10 \times 10^{-3}$

At $f = 60$ Hz, $Z = j3.77\ \Omega$

At $f = 100$ kHz, $Z = j6.28\ \text{k}\Omega$

At $f = 1$ GHz, $Z = j62.8\ \Omega$

(e) $Z = j\omega L = j2\pi fL = j2\pi f(1 \times 10^{-9})$

$f = 60$ Hz, $Z = j3.77 \times 10^{-7} = j0.377\ \mu\Omega$

$f = 100$ kHz,

$Z = j6.28 \times 10^{-4} = j0.628\ \text{m}\Omega$

$f = 16$ Hz, $Z = j62.8\ \Omega$

1.15

(a) $Z = R + \dfrac{1}{j\omega C}$

$$= 10^3 + \frac{1}{j2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}}$$

$$= (1 - j1.59)\ \text{k}\Omega$$

(b) $Y = \dfrac{1}{R} + j\omega C$

$$= \frac{1}{10^3} + j2\pi \times 10 \times 10^3 \times 0.01 \times 10^{-6}$$

$$= 10^{-3}(1 + j0.628)\ \Omega$$

$$Z = \frac{1}{Y} = \frac{1000}{1 + j0.628}$$

$$= \frac{1000(1 - j0.628)}{1 + 0.628^2}$$

$$= (717.2 - j45.04)\ \Omega$$

(c) $Y = \dfrac{1}{R} + j\omega C$

$$= \frac{1}{100 \times 10^3} + j2\pi \times 10 \times 10^3 \times 100 \times 10^{-12}$$

$$= 10^{-5}(1 + j0.628)$$

$$Z = \frac{10^5}{1 + j0.628}$$

$$= (71.72 - j450.4)\ \text{k}\Omega$$

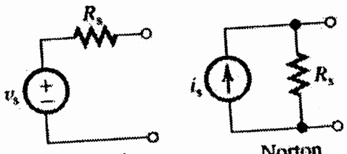
(d) $Z = R + j\omega L$

$$= 100 + j2\pi \times 10 \times 10^3 \times 10 \times 10^{-3}$$

$$= 100 + j6.28 \times 100$$

$$= (100 + j628)\ \Omega$$

1.16



Thévenin Equivalent

Norton Equivalent

$$v_{OC} = v_S$$
$$i_{SC} = i_S$$
$$v_S = i_S R_S$$

Thus,

$$R_S = \frac{v_{OC}}{i_{SC}}$$

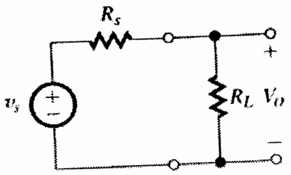
(a) $v_S = v_{OC} = 10\text{ V}$

$$i_S = i_{SC} = 100\ \mu A$$
$$R_S = \frac{v_{OC}}{i_{SC}} = \frac{10\text{ V}}{100\ \mu A} = 0.1\text{ M}\Omega = 100\text{ k}\Omega$$

(b) $v_S = v_{OC} = 0.1\text{ V}$

$$i_S = i_{SC} = 10\ \mu A$$
$$R_S = \frac{v_{OC}}{i_{SC}} = \frac{0.1\text{ V}}{10\ \mu A} = 0.01\text{ M}\Omega = 10\text{ k}\Omega$$

1.17



$$\frac{v_O}{v_s} = \frac{R_L}{R_L + R_S}$$
$$v_O = v_S / \left(1 + \frac{R_S}{R_L}\right)$$

Thus,

$$\frac{v_S}{1 + \frac{R_S}{100}} = 30$$

and

$$\frac{v_S}{1 + \frac{R_S}{10}} = 10$$

Dividing (1) by (2) gives

$$\frac{1 + (R_S/10)}{1 + (R_S/100)} = 3$$
$$\Rightarrow R_S = 28.6\text{ k}\Omega$$

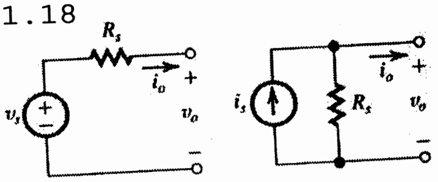
Substituting in (2) gives

$$v_s = 38.6\text{ mV}$$

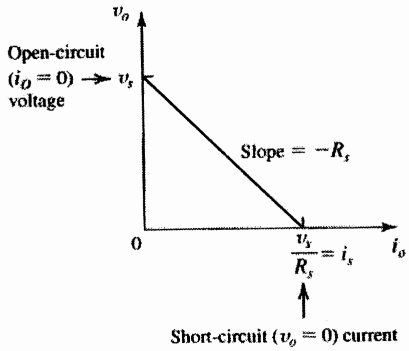
The Norton current i_s can be found as

$$i_S = \frac{v_S}{R_S} = \frac{38.6\text{ mV}}{28.6\text{ k}\Omega} = 1.35\ \mu A$$

1.18



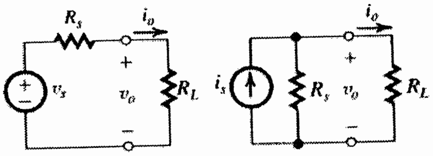
$$v_O = v_s - i_O R_s$$
$$v_O = (i_s - i_O)R_s$$
$$= i_s R_s - i_O R_s$$
$$v_O = v_s - i_O R_s$$



1.19

(1) 1.26

(2)



R_L represents the input resistance of the processor

For $v_O = 0.9\ v_s$

$$0.9 = \frac{R_L}{R_L + R_S} \Rightarrow R_L = 9R_S$$

For $i_o = 0.9\ i_s$

$$0.9 = \frac{R_S}{R_S + R_L} \Rightarrow R_L = R_S / 9$$

1.20

Case	ω (rad/s)	f (Hz)	T (s)
a	6.28×10^9	1×10^9	1×10^{-9}
b	1×10^9	1.59×10^8	6.28×10^{-9}
c	6.28×10^{10}	1×10^{10}	1×10^{-10}
d	3.77×10^2	60	1.67×10^{-2}
e	6.28×10^3	1×10^3	1×10^{-3}
f	6.28×10^6	1×10^6	1×10^{-6}

1.21

(a) $V_{peak} = 117 \times \sqrt{2} = 165$ V

(b) $V_{rms} = 33.9 \times \sqrt{2} = 24$ V

(c) $V_{peak} = 220 \times \sqrt{2} = 311$ V

(d) $V_{peak} = 220 \times \sqrt{2} = 311$ kV

1.22

(a) $v = 10 \sin(2\pi \times 10^4 t)$, V

(b) $v = 120\sqrt{2} \sin(2\pi \times 60)$, V

(c) $v = 0.1 \sin(1000t)$, V

(d) $v = 0.1 \sin(2\pi \times 10^{+3} t)$, V

1.23

The two harmonics have the ratio 126/98 = 9/7. Thus, these are the 7th and 9th harmonics. From Eq. 1.2 we note that the amplitudes of these two harmonics will have the ratio 7 to 9, which is confirmed by the measurement reported. Thus the fundamental will have a frequency of 98/7 or 14 kHz and peak amplitude of 63 × 7 = 441 mV. The rms value of the fundamental will be $441/\sqrt{2} = 312$ mV. To find the peak-to-peak amplitude of the square wave we note that $4V/\pi = 441$ mV. Thus,

Peak-to-peak amplitude

$$= 2V = 441 \times \frac{\pi}{2} = 693 \text{ mV}$$

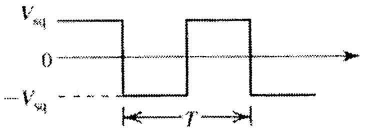
$$\text{Period } T = \frac{1}{f} = \frac{1}{14 \times 10^3} = 71.4\ \mu s$$

1.24

To be barely audible by a relatively young listener, the 5th harmonic must be limited to 20 kHz; thus the fundamental will be 4 kHz. At the low end, hearing extends down to about 20 Hz. For the fifth and higher to be audible the fifth must be no lower than 20 Hz. Correspondingly, the fundamental will be at 4 Hz.

1.25

If the amplitude of the square wave is V_{sq} then the power delivered by the square wave to a resistance R will be V_{sq}^2/R. If this power is to equal that delivered by a sine wave of peak amlitude $\hat{V}$ then



$$\frac{V_{sq}^2}{R} = \frac{(\hat{V}/\sqrt{2})^2}{R}$$

Thus, $V_{sq} = \hat{V}/\sqrt{2}$. This result is independant of frequency.

1.26

Decimal	Binary
0	0
5	101
8	1000
25	11001
57	111001

1.27

b_3	b_2	b_1	b_0	Value Represented
0	0	0	0	+0
0	0	0	1	+1
0	0	1	0	+2
0	0	1	1	+3
0	1	0	0	+4
0	1	0	1	+5
0	1	1	0	+6
0	1	1	1	+7
1	0	0	0	−0
1	0	0	1	−1
1	0	1	0	−2
1	0	1	1	−3
1	1	0	0	−4
1	1	0	1	−5
1	1	1	0	−6
1	1	1	1	−7

Note that there are two possible representation of zero: 0000 and 1000. For a 0.5-V step size, analog signals in the range ± 3.5 V can be represented

Input	Steps	Code
+2.5 V	+5	0101
−3.0 V	−6	1110
+2.7	+5	0101
−2.8	−6	1110

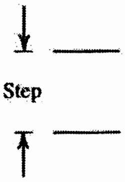
1.28

(a) For N bits there will be 2^N possible levels, from 0 to V_{FS}. Thus there will be $(2^N - 1)$ discrete steps from 0 to V_{FS} with the step size given by

$$\text{Step size} = \frac{V_{FS}}{2^N - 1}$$

This is the analog change corresponding to a change in the LSB. It is the value of the resolution of the ADC.

(b) The maximum error in conversion occurs when the analog signal value is at the middle of a step. Thus the maximum error is



$$\frac{1}{2} \times \text{step size} = \frac{1}{2}\frac{V_{FS}}{2^N - 1}$$

This is known as the quantization error.

(c) $\dfrac{10\ V}{2^N - 1} \le 5\ \text{mV}$

$2^N - 1 \geq 2000$

$2^N \geq 2001 \Rightarrow N = 11,$

For $N = 11$

$$\text{Resolution} = \frac{10}{2^{11} - 1} = 4.9\ \text{mV}$$

$$\text{Quantization error} = \frac{4.9}{2} = 2.4\ \text{mV}$$

1.29

There will be 44,100 samples per second with each sample represented by 16 bits. Thus the through-put or speed will be 44,100 × 16 = 7.056×10^5 bits per second.

1.30

(a) $A_v = \dfrac{v_O}{v_i} = \dfrac{10\ \text{V}}{100\ \text{mV}} = 100\ \text{V/V}$

or, 20 log 100 = 40 dB

$$A_i = \frac{i_O}{i_i} = \frac{v_O/R_L}{i_i} = \frac{10\ \text{V}/100\ \Omega}{100\ \mu A} = \frac{0.1\ A}{100\ \mu A}$$

$= 1000$ A/A

or, 20 log 1000 = 60 dB

$$A_P = \frac{v_O i_O}{v_i i_i} = \frac{v_O}{v_i} \times \frac{i_O}{i_i} = 100 \times 1000$$

$= 10^5$ W/W

or 10 log 10^5 = 50 dB

(b) $A_v = \dfrac{v_O}{v_i} = \dfrac{2\ V}{10\ \mu V} = 2 \times 10^5\ \text{V/V}$

or, 20 log 2×10^5 = 106 dB

$$A_i = \frac{i_O}{i_i} = \frac{v_O/R_L}{i_i} = \frac{2\ V/10\ \text{k}\Omega}{100\ \text{nA}}$$

$$= \frac{0.2\ \text{mA}}{100\ \text{nA}} = \frac{0.2 \times 10^{-3}}{100 \times 10^{-9}} = 2000\ A/A$$

or 20 log A_i = 66 dB

1.31

$$A_P = \frac{v_O i_O}{v_i i_i} = \frac{v_O}{v_i} \times \frac{i_O}{i_i}$$

$= 2 \times 10^5 \times 2000$

$= 4 \times 10^8$ W/W

or 10 log A_P = 86 dB

(c) $A_v = \dfrac{v_O}{v_i} = \dfrac{10\ \text{V}}{1\ \text{V}} = 10\ \text{V/V}$

or, 20 log 10 = 20 dB

$$A_i = \frac{i_O}{i_i} = \frac{v_O/R_L}{i_i} = \frac{10\ \text{V}/10\ \Omega}{1\ \text{mA}}$$

$$= \frac{1\ \text{A}}{1\ \text{mA}} = 1000\ A/A$$

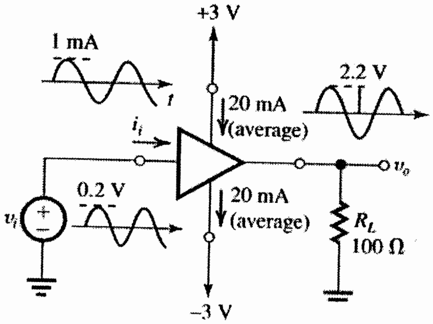
or, 20 log 1000 = 60 dB

$$A_P = \frac{v_O i_O}{v_i i_i} = \frac{v_O}{v_i} \times \frac{i_O}{i_i}$$

$= 10 \times 1000 = 10^4$ W/W

or $10 \log_{10} A_P$ = 40 dB

1.32



$$A_v = \frac{v_O}{v_i} = \frac{2.2}{0.2}$$

$= 11$ V/V

or 20 log 11 = 20.8 dB

$$A_i = \frac{i_O}{i_i} = \frac{2.2\ \text{V}/100\ \Omega}{1\ \text{mA}}$$

$$= \frac{22\ \text{mA}}{1\ \text{mA}} = 22\ A/A$$

or, 20 log A_i = 26.8 dB

$$A_p = \frac{P_O}{P_I} = \frac{(2.2/\sqrt{2})^2/100}{\dfrac{0.2}{\sqrt{2}} \times \dfrac{10^{-3}}{\sqrt{2}}}$$

$= 242$ W/W

or, 10 log A_p = 23.8 dB

Supply power = 2 × 3 V × 20 mA = 120 mW

Output power =

$$\frac{v_{o\,rms}^2}{R_L} = \frac{(2.2/\sqrt{2})^2}{100\ \Omega} = 24.2\ \text{mW}$$

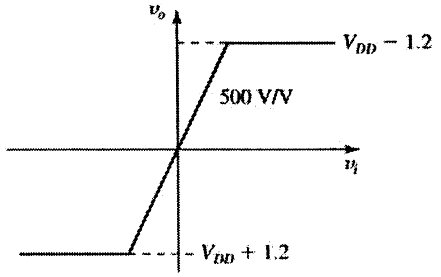
Input power $= \dfrac{24.2}{242} = 0.1$ mW (negligible)

Amplifier dissipation ≃ Supply power − Output power

$= 120 - 24.2 = 95.8$ mW

$$\text{Amplifier efficiency} = \frac{\text{Output power}}{\text{Supply power}} \times 100$$

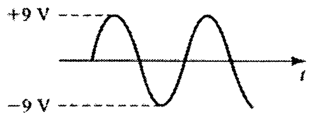
$$= \frac{24.2}{120} \times 100 = 20.2\%$$



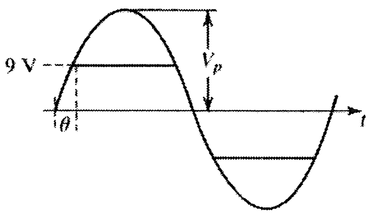
For $V_{DD} = 15$ V, the largest undistorted sine-wave output is of 13.8-V peak amplitude or 9.8 V_{rms}. The input needed is 9.8 V/500 = 19.6 mV_{rms}.

1.33

(a) For an output whose extremes are just at the edge of clipping, i.e., an output of 9-V_{peak}, the input must have 9 V/1000 = 9 mV_{peak}.

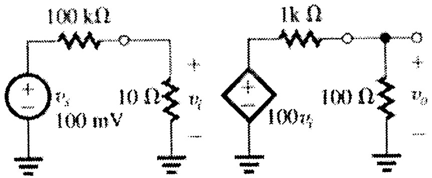


(b) For an output that is clipping 90% of the time, $\theta = 0.1 \times 90^\circ = 9^\circ$ and V_P sin $9^\circ = 9\ V \Rightarrow V_P = 57.5\ V$ which of course does not occur as the output saturates at ± 9 V. To produce this result, the input peak must be 57.5/1000 = 57.5 mV.



(c) For an output that is clipping 99% of the time,
$\theta = 0.01 \times 90^\circ = 0.9^\circ$
$V_P \sin 0.9^\circ = 9$ V
$\Rightarrow V_P = 573$ V and the input must be 573 V/1000 or 0.573 V_{peak}.

1.34



$$\frac{v_O}{v_S} = \frac{R_i}{R_i + R_S} \times A_{v_o} \times \frac{R_L}{R_L + R_O}$$

(a) $$\frac{v_O}{v_S} = \frac{10R_S}{10R_S + R_S} \times A_{v_o} \times \frac{10R_O}{10R_O + R_O}$$

$$= \frac{10}{11} \times 10 \times \frac{10}{11} = 8.26\ V/V$$

or, 20 log 8.26 = 18.3 dB

(b) $$\frac{v_O}{v_S} = \frac{R_S}{R_S + R_S} \times A_{v_o} \times \frac{R_O}{R_O + R_O}$$

= 0.5 × 10 × 0.5 = 2.5 V/V

or, 20 log 2.5 = 8 dB

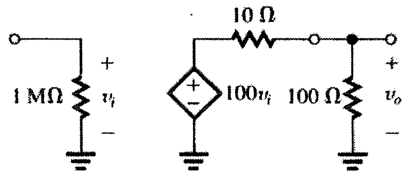
(c)

$$\frac{v_O}{v_S} = \frac{R_S/10}{(R_S/10) + R_S} \times A_{v_o} \times \frac{R_O/10}{(R_O/10) + R_O}$$

$$= \frac{1}{11} \times 10 \times \frac{1}{11} = 0.083\ V/V$$

or 20 log 0.083 = −21.6 dB

1.35



$20 \log A_{v_o} = 40\ \text{dB} \Rightarrow A_{vO} = 100\ V/V$

$$A_v = \frac{v_O}{v_i}$$

$$= 100 \times \frac{100}{100 + 10}$$

= 90.9 V/V

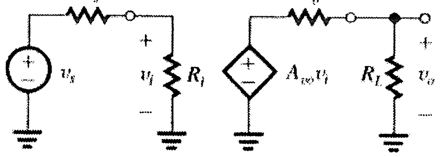
or, 20 log 90.9 = 39.1 dB

$$A_P = \frac{v_O^2 / 100\ \Omega}{v_i^2 / 1\ \text{M}\Omega} = A_v^2 \times 10^4 = 8.3 \times 10^7 W/W$$

or 10 log (8.3 × 10^7) = 79.1 dB.

For a peak output sine-wave current of 100 Ω, the peak output voltage will be 100 mA × 100 Ω = 10 V. Correspondingly v_i will be a sine wave with a peak value of 10 V/A_v = 10/90.9 or an rms value of $10/(90.9 \times \sqrt{2}) = 0.08$ V.

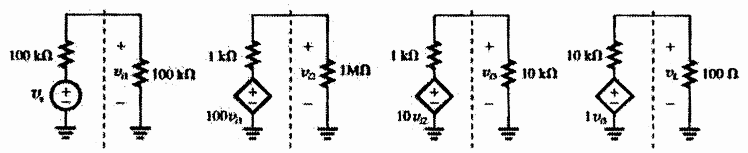
Corresponding output power =
$(10/\sqrt{2})^2 / 100\ \Omega$
= 0.5 W



$$\frac{v_O}{v_S} = \frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 100\ \text{k}\Omega} \times 1000 \times \frac{100\ \Omega}{100\ \Omega + 1\ \text{k}\Omega}$$

$$= \frac{10}{110} \times 1000 \times \frac{100}{1100} = 8.26\ V/V$$

This figure is for 1.37


100 kΩ
v_s
+
v_{i1}
−
100 kΩ
1 kΩ
$100v_{i1}$
+
v_{i2}
−
1MΩ
1 kΩ
$10v_{i2}$
+
v_{i3}
−
10 kΩ
10 kΩ
$1v_{i3}$
+
v_L
−
100 Ω

1.36

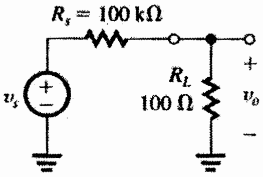
The signal loses about 90% of its strength when connected to the amplifier input (because $R_i = R_s/10$). Also, the output signal of the amplifier loses approximately 90% of its strength when the load is connected (because $R_L = R_o/10$). Not a good design! Nevertheless, if the source were connected directly to the load,

$$\frac{v_O}{v_S} = \frac{R_L}{R_L + R_S}$$

$$= \frac{100\ \Omega}{100\ \Omega + 100\ \text{k}\Omega}$$

$$\simeq 0.001\ \text{V/V}$$

$$R_S = 100\ \text{k}\Omega$$


$R_s = 100\ \text{k}\Omega$
v_s
R_L
100 Ω
+
v_o
−

which is clearly a much worse situation. Indeed inserting the amplifier increases the gain by a factor 8.3/0.001 = 8300.

1.37

In example 1.3 when the first and the second stages are interchanged, the circuit looks like the figure above

$$\frac{v_{i1}}{v_S} = \frac{100\ \text{k}\Omega}{100\ \text{k}\Omega + 100\ \text{k}\Omega} = 0.5\ \text{V/V}$$

$$A_{v1} = \frac{v_{i2}}{v_{i1}} = 100 \times \frac{1\ \text{M}\Omega}{1\ \text{M}\Omega + 1\ \text{k}\Omega}$$

$$= 99.9\ \text{V/V}$$

$$A_{v2} = \frac{v_{i3}}{v_{i2}} = 10 \times \frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 1\ \text{k}\Omega}$$

$$= 9.09\ \text{V/V}$$

$$A_{v3} = \frac{v_L}{v_{i3}} = 1 \times \frac{100\ \Omega}{100\ \Omega + 10\ \Omega} = 0.909\ \text{V/V}$$

$$\text{Total gain} = A_v = \frac{v_L}{v_{i1}} = A_{v1} \times A_{v2} \times A_{v3}$$

$$= 99.9 \times 9.09 \times 0.909 = 825.5\ \text{V/V}$$

The voltage gain from source to load is

$$\frac{v_L}{v_S} = \frac{v_L}{v_{i1}} \times \frac{v_{i1}}{v_S} = A_v \cdot \frac{v_{i1}}{v_S}$$

$$= 825.5 \times 0.5$$

$$= 412.7\ \text{V/V}$$

The overall voltage has reduced appreciably. It is due to the reason because the input impedance of the first stage, R_{in}, is comparable to the source resistance R_s. In example 1.3 the input impedance of the first stage is much larger than the source resistance

1.38

a. Case S-A-B-L

$$\frac{V_O}{V_S} = \frac{V_O}{V_{ib}} \times \frac{V_{ib}}{V_{ia}} \times \frac{V_{ia}}{V_S} =$$

$$\left(1 \times \frac{100}{100 + 100}\right) \times \left(100 \times \frac{100}{100 + 10}\right) \times \left(\frac{10}{100 + 10}\right)$$

$\frac{V_O}{V_S} = 4.13\ \text{V/V}$ and gain in dB 20 log 4.1 = 12.32 dB (See figure below)

b. Case S-B-A-L

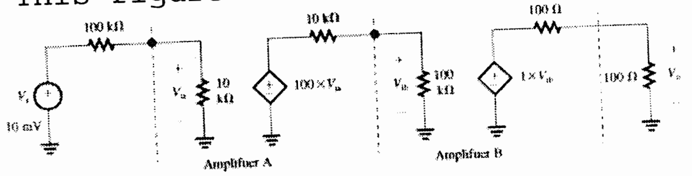
$$\frac{V_O}{V_S} = \frac{V_O}{V_{ia}} \cdot \frac{V_{ia}}{V_{ib}} \cdot \frac{V_{ib}}{V_S}$$

$$= \left(100 \times \frac{100}{100 + 10\ \text{K}}\right) \times \left(1 \times \frac{10\ \text{K}}{10\ \text{K} + 100}\right) \times$$

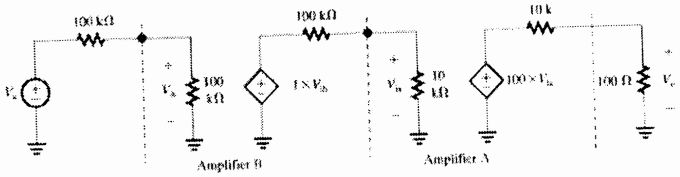
$$\left(\frac{100\ \text{K}}{100\ \text{K} + 100}\right)$$

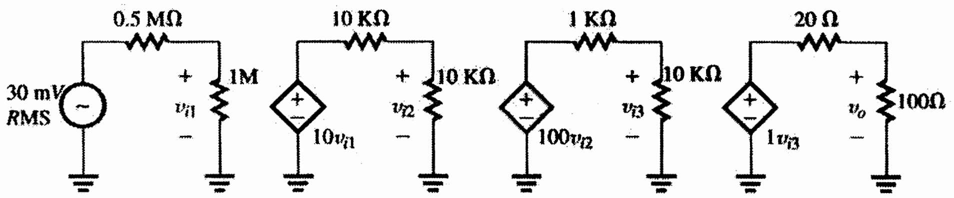
$\frac{V_O}{V_S} = 0.49\ V/S$ and gain in dB is 20 log 0.49 = −6.19 dB case a is preferred as it provides higher voltage gain.

This figure is for 1.38(a)


100 kΩ
V_s
10 mV
+
V_{ia}
10 kΩ
10 kΩ
$100 \times V_{ia}$
+
V_{ib}
100 kΩ
100 Ω
$1 \times V_{ib}$
100 Ω
+
V_o
Amplifuer A
Amplifuer B

This figure is for 1.38(b)

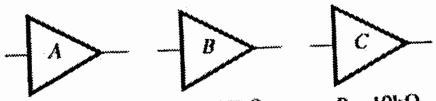

100 kΩ
V_s
+
V_{ib}
100 kΩ
−
100 kΩ
$1 \times V_{ib}$
+
V_{ia}
10 kΩ
−
10 k
$100 \times V_{ia}$
100 Ω
+
V_o
−
Amplifier B
Amplifier A



This figure is for 1.39

1.39

Deliver 0.5W to a 100Ω load
Source is 30mV RMS with 0.5MΩ source resistance. Choose from 3 amplifiers types



R_i= 1MΩ	R_i= 10kΩ	R_i= 10kΩ
A_v= 10 v/v	A_v= 100 v/v	A_v= 1 v/v
R_o= 10kΩ	R_o= 1kΩ	R_o= 20Ω

Choose order to eliminate loading on input and output
A -1st-to minimize loading on 0.5 MΩ source
B - 2nd-to boost gain
C - 3rd - to minimize loading at 100Ω output.
(See figure below)

$$\frac{v_O}{v_S} = \frac{2\ V}{30mV} = 235.7 < \left(\frac{1\mu}{0.5\mu + 1\mu}\right)(10)$$

$$\left(\frac{10}{10+10}\right)(100)\left(\frac{10}{10+1}\right)(1)\left(\frac{100}{20+100}\right)$$

$$235.7 < 253.6$$

$$v_O = (253.6)(30\text{mV}) = 7.61\text{v RMS}$$

$$P = \frac{v_O^2}{R_L} = \frac{(7.61)^2}{100} = 0.58\ \text{W}$$

1.40

(a) Required voltage gain $= \dfrac{v_O}{v_S}$

$$= \frac{3\ \text{V}}{0.01\ \text{V}} = 300\ \text{V/V}$$

(b) The smallest R_i allowed is obtained from

$$0.1\ \mu A = \frac{10\ \text{mV}}{R_S + R_i} \Rightarrow R_S + R_i = 100\ \text{k}\Omega$$

Thus R_i = 90 kΩ.
For R_i = 90 kΩ, i_i = 0.1 μA peak, and

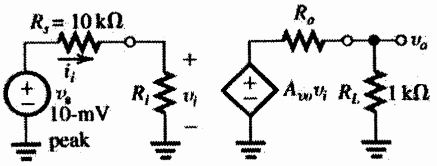
Overall current gain $= \dfrac{v_O / R_L}{i_i}$

$$= \frac{3\ \text{mA}}{0.1\ \mu A} = 3 \times 10^4\ \text{A/A}$$

Overall power gain $= \dfrac{v_{O rms}^2 / R_L}{v_{s(rms)} \times i_{i(rms)}}$

$$= \frac{\left(\frac{3}{\sqrt{2}}\right)^2 / 1000}{\left(\frac{10 \times 10^{-3}}{\sqrt{2}}\right) \times \left(\frac{0.1 \times 10^{-6}}{\sqrt{2}}\right)}$$

$= 9 \times 10^6$ W/W
(This takes into acct. the power dissipated in the internal resistance of the source.)
(c) If (A_{v_o} v_i) has its peak value limited to 5 V, the largest value of R_o is found from



$$= \times \frac{R_L}{R_L + R_O} = 3 \Rightarrow R_O = \frac{2}{3}R_L = 667\ \Omega$$

(If R_o were greater than this value, the output voltage across R_L would be less than 3 V.)

(d) For R_i = 90 kΩ and R_o = 667 Ω, the required value A_{vo} can be found from

$$300\ \text{V/V} = \frac{90}{90+10} \times A_{v_o} \times \frac{1}{1 + 0.667}$$

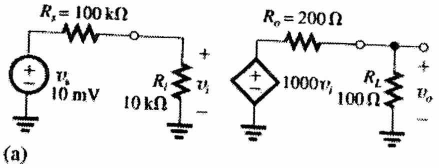
$$\Rightarrow A_{v_o} = 555.7\ \text{V/V}$$

(e) R_i = 100 kΩ (1×10^5 Ω)
R_o = 100 Ω (1×10^2 Ω)

$$300 = \frac{100}{100+10} \times A_{v_o} \times \frac{1000}{1000+100}$$

$$\Rightarrow A_{v_o} = 363\ \text{V/V}$$

1.41



(a)

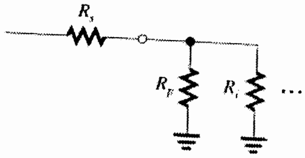
$$v_O = 10\ \text{mV} \times \frac{10}{10+100} \times 1000 \times \frac{100}{100+200}$$

= 303 mV

(b) $\dfrac{v_O}{v_S} = \dfrac{303\ \text{mV}}{10\ \text{mV}} = 30.3\ \text{V/V}$

(c) $\dfrac{v_O}{v_i} = 1000 \times \dfrac{100}{100+200} = 333.3\ \text{V/V}$

(d)



Connect a resistance R_P in parallel with the input and select its value from

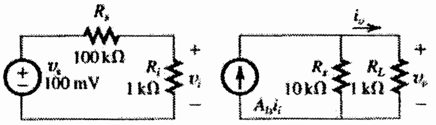
$$\frac{(R_P \parallel R_i)}{(R_P \parallel R_i) + R_S} = \frac{1}{2}\frac{R_i}{R_i + R_S}$$

$$\Rightarrow 1 + \frac{R_S}{R_P \parallel R_i} = 22 \Rightarrow R_P \parallel R_i = \frac{R_S}{21} = \frac{100}{21}$$

$$\Rightarrow \frac{1}{R_P} + \frac{1}{R_i} = \frac{21}{100}$$

$$R_P = \frac{1}{0.21 - 0.1} = 9.1\ \text{k}\Omega$$

1.42


R_s
100 kΩ
v_s
100 mV
R_i
1 kΩ
+
v_i
−
i_o
$A_{is}i_i$
R_o
10 kΩ
R_L
1 kΩ
+
v_o
−

(a) Current gain $= \dfrac{i_O}{i_i}$

$$= A_{is}\frac{R_O}{R_O + R_L}$$

$$= 100\frac{10}{11}$$

$$= 90.9\frac{\text{A}}{\text{A}} = 39.2\ \text{dB}$$

(b) Voltage gain $= \dfrac{v_O}{v_S}$

$$= \frac{i_O}{i_i}\frac{R_S}{R_S + R_i}$$

$$= 90.9 \times \frac{1}{101}$$

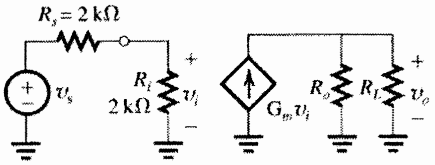
$$= 0.9\ \text{V/V} = -0.9\ \text{dB}$$

(c) Power gain $= A_P = \dfrac{v_O i_O}{v_S i_i}$

$$= 0.9 \times 90.9$$

$$= 81.8\ \text{W/W} = 19.1\ \text{dB}$$

1.43


$R_s = 2\ \text{k}\Omega$
v_s
R_i
2 kΩ
+
v_i
−
$G_m v_i$
R_o
R_L
+
v_o
−

$G_m = 40\ \text{mA/V}$
$R_o = 20\ \text{k}\Omega$
$R_L = 1\ \text{k}\Omega$

$$v_i = v_S\frac{R_i}{R_S + R_i}$$

$$= v_S\frac{2}{2 + 2} = \frac{v_S}{2}$$

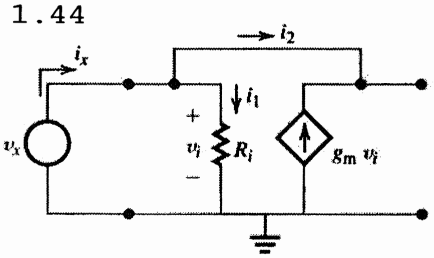
$$v_O = G_m v_i(R_L \parallel R_O)$$

$$= 40\frac{20 \times 1}{20 + 1}v_i$$

$$= 40\frac{20}{21}\frac{v_S}{2}$$

Overall voltage gain $= \dfrac{v_O}{v_S} = 19.05\ \text{V/V}$

1.44


i_2
i_x
i_1
+
v_i
−
v_x
R_i
$g_m v_i$

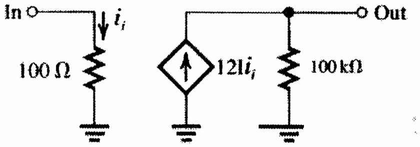
$$\left.\begin{aligned} i_x &= i_1 + i_2 \\ i_1 &= v_i / R_i \\ i_2 &= g_m + v_i \\ v_i &= V_x \end{aligned}\right\}$$

$$i_x = v_x / R_i + g_m v_x$$

$$i_x = v_x + \left(\frac{1}{R_i} + g_m\right)$$

$$\frac{v_x}{i_x} = \frac{1}{1/R_i + g_m}$$

$$= \frac{R_x}{1 + g_m R_i} = R_{\text{in}}$$


In
i_i
100 Ω
$121 i_i$
100 kΩ
Out

1.45

Transresistance amplifier

To limit Δv_o to 10% corresponding to R_s varying in the range 1 to 10 kΩ, we select R_i sufficiently low;

$$R_i \le \frac{R_{s\,\text{min}}}{10}$$

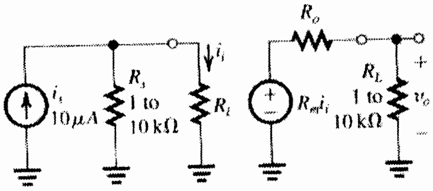
Thus, $R_i = 100\ \Omega$

To limit Δv_o to 10% while R_L varies over the range 1 to 10 kΩ, we select R_o sufficiently low;

$$R_O \le \frac{R_{L\,\text{min}}}{10}$$

Thus, $R_O = 100\ \Omega$

Now, for $i_S = 10\ \mu A$,

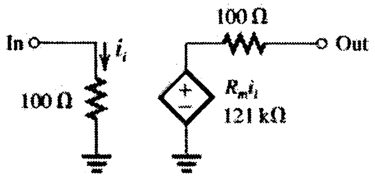

i_s
10 μA
R_s
1 to
10 kΩ
i_i
R_i
$R_m i_i$
R_o
R_L
1 to
10 kΩ
+
v_o
−

$$v_{O\min} = 10^{-5}\frac{R_{s\min}}{R_{s\min}+R_i}R_m\frac{R_{L\min}}{R_{L\min}+R_O}$$

$$1 = 10^{-5}\frac{1000}{1000+100}R_m\frac{1000}{1000+100}$$

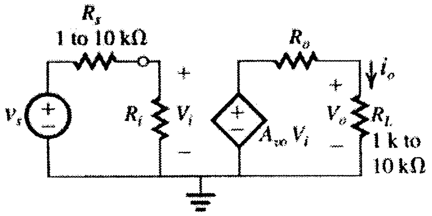
$$\Rightarrow R_m = 1.21\times 10^5$$

$$= 121\ \text{k}\Omega$$



1.46

Voltage Amplifier



For R_s varying in the range 1 K to 10 kΩ and Δi_o variation limited to 10%, select R_i to be sufficiently large:

$R_i \geq 10\ R_{S\max}$

$R_i = 10\times 10\ \text{k}\Omega = 100\ \text{k}\Omega = 1\times 10^5\ \Omega$

For R_L varying in the range 1 to 10 kΩ, the load current variation limited to 10%, select R_o sufficiently low:

$$R_O \leq \frac{R_{L\max}}{10}$$

$$R_O = \frac{1\ \text{k}\Omega}{10} = 100\ \Omega = 1\times 10^2\ \Omega$$

Now find A_{vo}

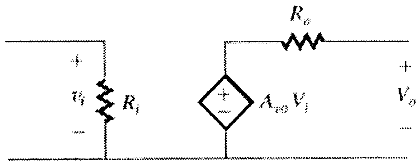
$$i_{o\min} = 10\ \text{mV}\times\frac{R_i}{R_i+R_{S\max}}\times A_{vo}\frac{1}{R_O+R_{L\max}}$$

$$1\times 10^{-3} = 10\times 10^{-3}\times\frac{100\ \text{k}\Omega}{100\ \text{k}\Omega+10\ \text{k}\Omega}$$

$$\times A_{vo}\times\frac{1}{100\ \Omega+10\ \text{k}\Omega}$$

$$1\times 10^{-3} = 10\times 10^{-3}\times\frac{100}{110}\times A_{vo}\times\frac{1}{1100}$$

$$A_{vo} = 121\ \text{V/V}$$



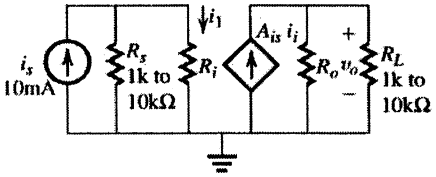
Voltage amplifier equivalent circuit is

$R_i = 1\times 10^5\ \Omega$, $A_{vo} = 121$ V/V and

$R_O = 1\times 10^2\ \Omega$

1.47

Current Amplifier



For R_s varying in the range 1 kΩ to 10 kΩ range and load voltage variation limited to 10%, select R_i to be sufficiently low:

$$R_i \leq \frac{R_{S\min}}{10}$$

$$R_i = \frac{1\ \text{k}\Omega}{10} = 100\ \Omega = 1\times 10^2\ \Omega$$

For R_L varying in the range 1 kΩ to 10 kΩ and load voltage variation limited to 10%, R_o is selected sufficiently large:

$R_o \geq 10\ R_{L\max}$

$R_o = 10\times 10\ \text{k}\Omega$

$= 100\ \text{k}\Omega = 1\times 10^5\ \Omega$

Now we find A_{is}

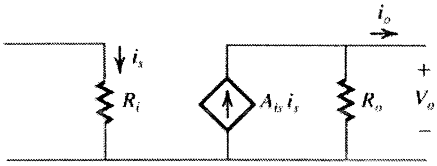
$$V_{O\min} = 10\ \mu A\times\frac{R_{S\min}}{R_{S\min}+R_i}\times A_{is}\times R_O \| R_{L\min}$$

$$= 10\times 10^{-6}\frac{R_{S\min}}{R_{S\min}+R_i}\times A_{is}\frac{R_O R_{L\min}}{R_O+R_{L\min}}$$

$$= 10\times 10^{-6}\frac{1\ \text{k}\Omega}{1\ \text{k}\Omega+100\ \Omega}\times A_{is}\frac{100\ \text{K}\times 1\ \text{K}}{100\ \text{K}+1\ \text{K}}$$

$$\Rightarrow A_{is} = 111.1\ \text{A/A}$$

Current amplifier equivalent circuit is



$R_i = 1\times 10^2\ \Omega$, $A_{is} = 111.1$ A/s,

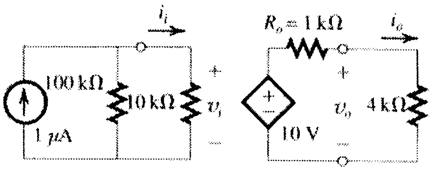
$R_O = 1\times 10^5\ \Omega$

1.48

$$R_O = \frac{\text{Open-circuit output voltage}}{\text{Short-circuit output current}} = \frac{10\ \text{V}}{10\ \text{mA}}$$

$= 1\ \text{k}\Omega$

$$v_O = 10\times\frac{4}{1+4} = 8\ \text{V}$$



$$A_v = \frac{v_O}{v_I} = \frac{8}{1 \times 10^{-3} \times (100 \| 10) \times 10^3}$$

$= 888$ V/V or 58.9 dB

$$A_i = \frac{i_o}{i_i} = \frac{v_O / R_L}{10^{-3} \times \frac{100}{100 + 10}} = \frac{8/(4 \times 10^3)}{10^{-3} \times \frac{100}{110}}$$

$= 2200$ A/A or 66.8 dB

$$A_i = \frac{v_O^2 / R_L}{i_i^2 R_i} = \frac{8^2/(4 \times 10^3)}{\left(10^{-3} \times \frac{100}{100+10}\right)^2 10 \times 10^3}$$

$= 19.36 \times 10^5$ W/W or 62.9 dB

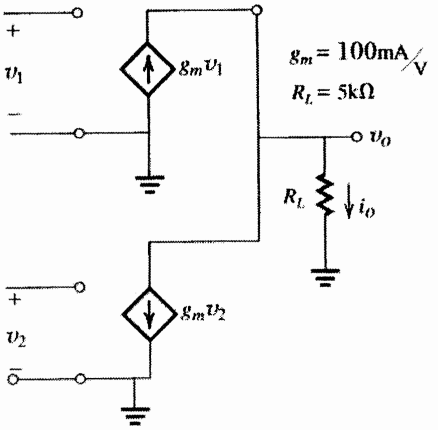
Overall current gain $= \frac{i_o}{1\ \mu A}$

$$= \frac{v_O / R_L}{1\ \mu A} = \frac{8/(4 \times 10^3)}{10^{-3}}$$

$= 2000$ A/A or 66 dB

1.49

Using the voltage divider rule



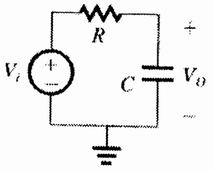
a. $i_o = g_m v_1 - g_m v_2$

$v_O = i_o R_L = g_m R_L (v_1 - v_2) = v_0$

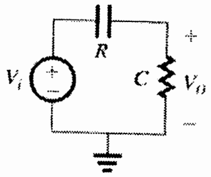
b. $v_1 = v_2 \therefore v_0 = 0V$

$$\left.\begin{array}{l} v_1 = 1.01 \\ v_2 = 0.99 \end{array}\right\} \therefore v_0 = 10V$$

1.50



(a)



(b)

for (a) $V_0 = V_I\left(\frac{1/SC}{1/SC + R}\right)$

$$\frac{V_0}{V_I} = \frac{1}{1 + SCR}$$

where k = 1

$\omega_0 = \frac{1}{RC}$ from table 1.2 it is low pass.

for (b) $V_0 = V_I\left(\frac{R}{R + \frac{1}{SC}}\right)$

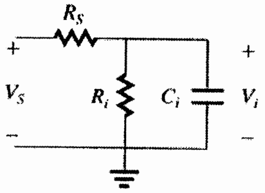
$$\frac{V_0}{V_I} = \frac{SRC}{1 + SCR}$$

$$\frac{V_0}{V_I} = \frac{S}{S + \frac{1}{RC}}$$

where k = 1

$\omega_0 = \frac{1}{RC}$ from table 1.2 it is high pass.

1.51



$$\frac{V_i}{V_S} = \frac{\dfrac{R_i \frac{1}{sC_i}}{R_i + \frac{1}{sC_i}}}{R_S + \left(\dfrac{R_i \frac{1}{sC_i}}{R_i + \frac{1}{sC_i}}\right)} = \frac{\dfrac{R_i}{1 + sC_i R_i}}{R_S + \left(\dfrac{R_i}{1 + sC_i R_i}\right)}$$

$$= \frac{R_i}{R_S + sC_i R_i R_s + R_i}$$

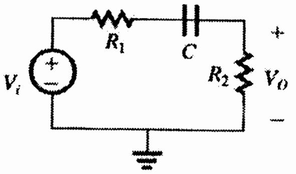
$$\frac{V_i}{V_S} = \frac{R_i}{(R_S + R_i) + sC_i R_i R_S} = \frac{\dfrac{R_i}{(R_S + R_i)}}{1 + S\left(\dfrac{C_i R_i R_S}{R_S + R_i}\right)}$$

Where K $= \frac{R_i}{(R_S + R_i)}$

$\omega = \frac{R_S + R_i}{C_i R_i R_S}$ from table 1.2 low pass for given values $\omega_0 = 12.5$ MHz

1.52

Using the voltage-divider rule.



$$T(s) = \frac{V_O}{V_i} = \frac{R_2}{R_2 + R_1 + \frac{1}{sC}}$$

$$T(s) = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{s}{s + \frac{1}{C(R_1 + R_2)}}\right)$$

which is from Table 1.2 is of the high-pass type with

$$K = \frac{R_2}{R_1 + R_2} \quad \omega_0 = \frac{1}{C(R_1 + R_2)}$$

As a further verification that this is a high-pass network and $T(s)$ is a high-pass transfer function, we assume as $s \Rightarrow 0$, $T(s) \Rightarrow 0$; and as $s \to \infty$, $T(s) = R_2/(R_1 + R_2)$. Also, from the circuit observe as $s \to \infty$, $(1/sC) \to 0$ and $V_O/V_i = R_2/(R_1 + R_2)$. Now, for $R_1 = 10\ \text{k}\Omega$, $R_2 = 40\ \text{k}\Omega$, and $C = 0.1\ \mu F$.

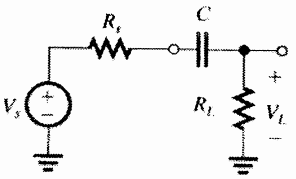
$$f_o = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \times 0.1 \times 10^{-6}(10 + 40) \times 10^3}$$

$$= 31.8\ \text{Hz}$$

$$|T(j\omega_0)| = \frac{K}{\sqrt{2}} = \frac{40}{10 + 40}\frac{1}{\sqrt{2}} = 0.57\ \text{V/V}$$

1.53

Using the voltage divider rule,



$$\frac{V_O}{V_S} = \frac{R_L}{R_L + R_S + \frac{1}{sC}}$$

$$= \frac{R_L}{R_L + R_S}\,\frac{s}{s + \frac{1}{C(R_L + R_S)}}$$

which is of the high-pass STC type (see Table 1.2) with

$$K = \frac{R_L}{R_L + R_S} \quad \omega_0 = \frac{1}{C(R_L + R_S)}$$

For $f_o \le 10$ Hz

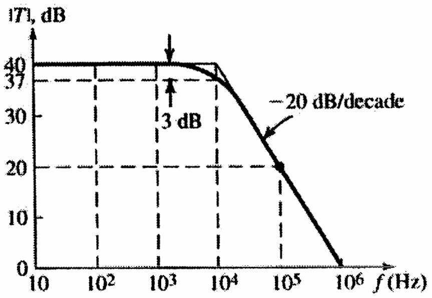
$$\frac{1}{2\pi C(R_L + R_S)} \le 10$$

$$\Rightarrow C \ge \frac{1}{2\pi \times 10(20 + 5) \times 10^3}$$

Thus, the smallest value of C that will do the job is $C = 0.64\ \mu F$.

1.54

The given measured data indicate that this amplifier has a low-pass STC frequency response with a low-frequency gain of 40 dB, and a 3-dB frequency of 10^4 Hz. From our knowledge of the Bode plots for low-pass STC networks (Figure 1.23a) we can complete the Table entries and sketch the amplifier frequency response



f(Hz)	|T|(dB)	∠T(°)
0	40	0
100	40	0
1000	40	0
10^4	37	−45°
10^5	20	−90°
10^6	0	−90°

1.55

Since the overall transfer function is that of three identical STC LP circuits in cascade (but with no loading effects since the buffer amplifiers have input and zero output resistances) the overall gain will drop by 3 dB below the value at dc at the frequency for which the gain of each STC circuit is 1 dB down. This frequency is found as follows: The transfer function of each STC circuit is

$$T(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

where

$$\omega_0 = 1/CR$$

Thus,

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_{1dB}}{\omega_0}\right)^2}} = -1$$

$$\Rightarrow 1 + \left(\frac{\omega_{1dB}}{\omega_0}\right)^2 = 10^{0.1}$$

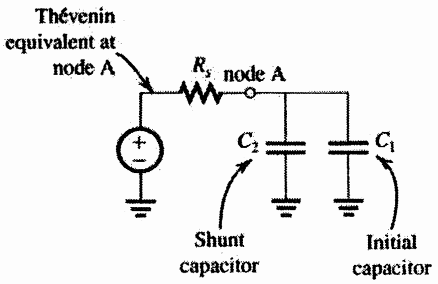
$$\omega_{1dB} = 0.51\,\omega_0$$

$$\omega_{1dB} = 0.51/CR$$

1.56

$R_S = 100\text{ k}\Omega$, since the 3-dB frequency is reduced by a very high factor (from 6 MHz to 120 kHz) C_2 must be much larger than C_1. Thus, neglecting C_1 we find C_2 from

$$120\text{ kHz} \simeq \frac{1}{2\pi C_2 R_S}$$



$$= \frac{1}{2\pi C_2 \times 10^5}$$

$$\Rightarrow C_2 = 13.3\text{ pF}$$

If the original 3-dB frequency (6 MHz) is attributable to C_1 then

$$6\text{ MHz} = \frac{1}{2\pi C_1 R_S}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 6 \times 10^6 \times 10^5}$$

$$= 0.26\text{ pF}$$

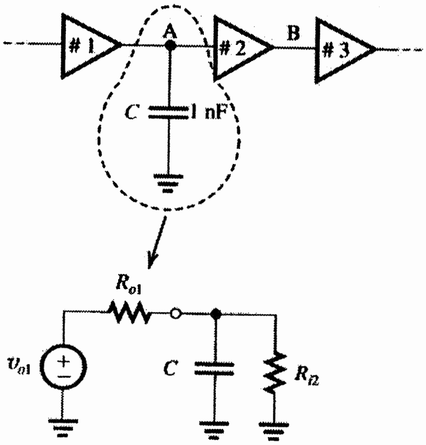
1.57

Since when C is connected the 3-dB frequency is reduced by a large factor, the value of C must be much larger than whatever parasitic capacitance originally existed at node A (i.e., between A and ground). Furthermore, it must be that C is now the dominant determinant of the amplifier 3-dB frequency (i.e., it is dominating over whatever may be happening at node B or anywhere else in the amplifier). Thus, we can write

$$150\text{ kHz} = \frac{1}{2\pi C(R_{o1} \parallel R_{i2})}$$

$$\Rightarrow (R_{o1} \parallel R_{i2}) = \frac{1}{2\pi \times 150 \times 10^3 \times 1 \times 10^{-9}}$$

$$= 1.06\text{ k}\Omega$$



Now $R_{i2} = 100\text{ k}\Omega$.

Thus $R_{o1} = 1.07\text{ k}\Omega$

Similarly, for node B,

$$15\text{ kHz} = \frac{1}{2\pi C(R_{o2} \parallel R_{i3})}$$

$$\Rightarrow R_{o2} \parallel R_{i3} = \frac{1}{2\pi \times 15 \times 10^3 \times 1 \times 10^{-9}}$$

$$= 10.6\text{ k}\Omega$$

$R_{o2} = 11.9\text{ k}\Omega$

She should connect a capacitor of value C_p to node B where C_p can be found from,

$$10\text{ kHz} = \frac{1}{2\pi C_p(R_{o2} \parallel R_{i3})}$$

$$\Rightarrow C_p = \frac{1}{2\pi \times 10 \times 10^3 \times 10.6 \times 10^3}$$

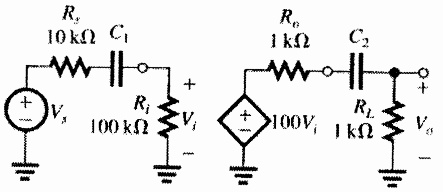
$= 1.5$ nF

Note that if she chooses to use node A she would need to connect a capacitor 10 times larger!

1.58

For the input circuit, the corner frequency f_{o1} is found from

$$f_{o1} = \frac{1}{2\pi C_1(R_S + R_i)}$$



For $f_{o1} \le 100$ Hz,

$$\frac{1}{2\pi C_1(10 + 100) \times 10^3} \le 100$$

$$\Rightarrow C_1 \geq \frac{1}{2\pi \times 110 \times 10^3 \times 10^2} = 1.4 \times 10^{-8}$$

Thus we select $C_1 = 1 \times 10^{-7}\ F = 0.1\ \mu F$.

The actual corner frequency resulting from C_1 will be

$$f_{o1} = \frac{1}{2\pi \times 10^{-7} \times 110 \times 10^3} = 14.5 \text{ Hz}$$

For the output circuit,

$$f_{o2} = \frac{1}{2\pi C_2(R_O + R_L)}$$

For $f_{o2} \leq 100$ Hz,

$$\frac{1}{2\pi C_2(1+1) \times 10^3} \leq 100$$

$$\Rightarrow C_2 \geq \frac{1}{2\pi \times 2 \times 10^3 \times 10^2} = 0.8 \times 10^{-6}$$

Select $C_2 = 1 \times 10^{-6} = 1\ \mu$F

This will place the corner frequency at

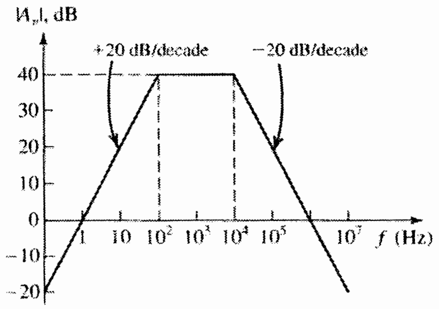
$$f_{02} = \frac{1}{2\pi \times 10^{-6} \times 2 \times 10^3} = 80 \text{ Hz}$$

$$T(s) = 100\frac{s}{\left(1 + \frac{s}{2\pi f_{01}}\right)\left(1 + \frac{s}{2\pi f_{02}}\right)}$$

1.59

The LP factor $1/(1 + jf/10^4)$ results in a Bode plot like that in Fig. 1.23(a) with the 3dB frequency $f_O = 10^4$ Hz. The high-pass factor $1/(1 + 10^4/jf)$ results in a Bode plot like that in Fig.1.24(a) with the 3dB frequency $f_O = 10^4$ Hz.

The Bode plot for the overall transfer function can be obtained by summing the dB values of the two individual plots and then raising the resulting plot vertically by 40 dB (corresponding to the factor 100 in the numerator). The result is as follows:



f =	10	10^2	10^3	10^4	10^5	10^6	10^7	(Hz)
$\lvert A_v\rvert \simeq$	20	40	40	40	20	0	−20	(dB)
↑ 37	↑ 37							

Better approximation
(3-dB frequencies)
Bandwidth = $10^4 - 10^2$ = 9900 Hz

1.60

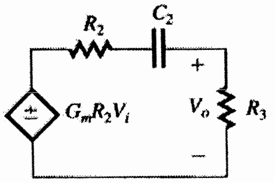
$$T_i(s) = \frac{V_i(s)}{V_S(s)} = \frac{1/sC_1}{1/sC_1 + R_1} = \frac{1}{sC_1R_1 + 1}$$

LP

3 dB frequency

$$= \frac{1}{2\pi C_1 R_1} = \frac{1}{2\pi 10^{-11} 10^6} = 15.9 \text{ Hz}$$

For $T_O(S)$, the following quivalent circuit can be used:



$$T_O(S) = -G_mR_2\frac{R_3}{R_2 + R_3 + 1/sC_2}$$

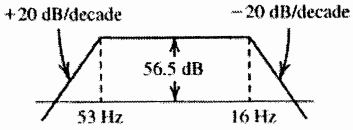
$$= -G_m(R_2 \| R_3)\frac{S}{S + \frac{1}{C_2(R_2 + R_3)}}$$

$$\text{3 dB frequency} = \frac{1}{2\pi C_2(R_2 + R_3)}$$

$$= \frac{1}{2\pi 100 \times 10^{-9} \times 30 \times 10^3} = 53 \text{ Hz}$$

$$\therefore T(S) = T_i(S)T_O(S)$$

$$= \frac{1}{1 + \frac{S}{2\pi \times 15.9 \times 10^3}} \times -666.7 \times \frac{S}{S + (2\pi \times 53)}$$



Bandwidth = 16 kHz − 53 Hz ≃ 16 Hz

1.61

$$V_i = V_S\frac{R_i}{R_S + R_i}$$

a) To satisfy constraint (1), namely

$$V_i \geq \left(1 + \frac{x}{100}\right)V_S$$

We substitute in Eq.(1) to obtain

$$\frac{R_i}{R_S + R_i} \geq 1 - \frac{x}{100}$$

Thus

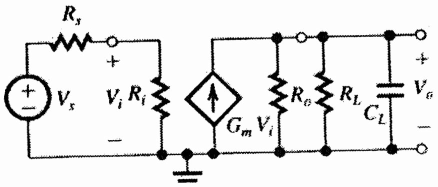
$$\frac{R_S + R_i}{R_i} \le \frac{1}{1 - \frac{x}{100}}$$

$$\frac{R_S}{R_i} \le \frac{1}{1 - \frac{x}{100}} - 1 = \frac{\frac{x}{100}}{1 - \frac{x}{100}}$$

which can be expressed as

$$\frac{R_i}{R_S} \ge \frac{1 - \frac{x}{100}}{\frac{x}{100}}$$

resulting in



$$R_i \ge R_S\left(\frac{100}{x} - 1\right)$$

b) The 3-dB frequency is determined by the parallel RC circuit at the output

$$f_O = \frac{1}{2\pi}\omega_O = \frac{1}{2\pi}\frac{1}{C_L(R_L \parallel R_O)}$$

Thus,

$$f_O = \frac{1}{2\pi C_L}\left(\frac{1}{R_L} + \frac{1}{R_O}\right)$$

To obtain a value for f_0 greater than a specified value f_{3dB} we select R_0 so that

$$\frac{1}{2\pi C_L}\left(\frac{1}{R_L} + \frac{1}{R_O}\right) \ge f_{3dB}$$

$$\frac{1}{R_L} + \frac{1}{R_O} \ge 2\pi C_L f_{3dB}$$

$$\frac{1}{R_O} \ge 2\pi C_L f_{3dB} - \frac{1}{R_L}$$

$$R_O \le \frac{1}{2\pi f_{3dB} + C_L - \frac{1}{R_L}} \qquad (2)$$

c) To satisfy constraint (3), we first determine the dc gain as

$$\text{dc gain} = \frac{R_i}{R_S + R_i} G_m(R_O \parallel R_L)$$

For the dc gain to be greater than a specified value A_O,

$$\frac{R_i}{R_S + R_i} G_m(R_O \parallel R_L) \ge A_O$$

The first factor on the LHS is (from constaint (1)) greater or equal to $(1 - x/100)$. Thus

$$G_m \ge \frac{A_O}{\left(1 - \frac{x}{100}\right)(R_O \parallel R_L)} \qquad (3)$$

Substituting $R_S = 10\text{ k}\Omega$ and $x = 20\%$ in (1) results in

$$R_S \ge 10\left(\frac{100}{20} - 1\right) = 40\text{ k}\Omega$$

Substituting $f_{3dB} = 3$ MHz, $C_L = 10$ pF and RL = 10 kΩ in Eq. (2) result in

$$R_O \le \frac{1}{2\pi \times 3 \times 10^6 \times 10 \times 10^{-12} - \frac{1}{10^4}}$$

$$= 11.3\text{ k}\Omega$$

Substituting $A_o = 80$, $x = 20\%$, $R_L = 10\text{ k}\Omega$, and $R_o = 11.3\text{ k}\Omega$, eq. (3) results in

$$G_m \ge \frac{80}{\left(1 - \frac{20}{100}\right)(10 \parallel 11.3) \times 10^3} = 18.85\text{ mA/V}$$

If the more practical value of $R_o = 10\text{ k}\Omega$ is used then

$$G_m \ge \frac{80}{\left(1 - \frac{20}{100}\right)(10 \parallel 10) \times 10^3} = 20\text{ mA/V}$$

1.62

Using the voltage-divider rule we obtain

$$\frac{V_O}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

where

$$Z_1 = R_1 \parallel \frac{1}{sC_1} \text{ and } Z_2 = R_2 \parallel \frac{1}{sC_2}.$$

It is obviously more convenient to work in terms of admittances. Therefore we express V_o/V_i in the alternate form

$$\frac{V_O}{V_i} = \frac{Y_1}{Y_1 + Y_2}$$

and substitute $Y_1 = (1/R_1) + sC_1$ and $Y_2 = (1/R_2) + sC_2$ to obtain

$$\frac{V_O}{V_i} = \frac{\frac{1}{R_1} + sC_1}{\frac{1}{R_1} + \frac{1}{R_2} + s(C_1 + C_2)}$$

$$= \frac{C_1}{C_1 + C_2}\frac{s + \frac{1}{C_1 R_1}}{s + \frac{1}{(C_1 + C_2)}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

This transfer function will be independent of frequency (s) if the second factor reduces to unity.

This in turn will happen if

$$\frac{1}{C_1R_1} = \frac{1}{C_1 + C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

which can be simplified as follows

$$\frac{C_1 + C_2}{C_2} = R_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad (1)$$

$$1 + \frac{C_2}{C_1} = 1 + \frac{R_1}{R_2}$$

or

$$C_1R_1 = C_2R_2$$

When this condition applies, the attenuator is said to be compensated, and its transfer function is given by

$$\frac{V_O}{V_i} = \frac{C_1}{C_1 + C_2}$$

which, using Eq. (1) can be expressed in the alternate form

$$\frac{V_O}{V_i} = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{R_2}{R_1 + R_2}$$

Thus when the attenuator is compensated ($C_1R_1 = C_2R_2$) its transmission can be determined either by its two resistors R_1, R_2 or by its two capacitors, C_1, C_2, and the transmission is *not* a function of frequency.

1.63

The HP STC circuit whose response determines the frequency response of the amplifier in the low-frequency range has a phase angle of 11.4° at $f = 100$ Hz. Using the equation for $\angle T(j\omega)$ from Table 1.2 we obtain

$$\tan^{-1}\frac{f_o}{100} = 11.4° \Rightarrow f_o = 20.16 \text{ Hz}$$

The LP STC circuit whose response determines the amplifier response at the high-frequency end has a phase angle of $-11.4°$ at $f = 1$ kHz. Using the relationship for $\angle T(j\omega)$ given in Table 1.2 we obtain for the LP STC circuit.

$$-\tan^{-1}\frac{10^3}{f_o} = -11.4° \Rightarrow f_o = 4959.4 \text{ Hz}$$

At $f = 100$ Hz the drop in gain is due to the HP STC network, and thus its value is

$$20\log\frac{1}{\sqrt{1 + \left(\frac{20.16}{100}\right)^2}} = -0.17 \text{ dB}$$

Similarly, at $f = 1$ kHz the drop in gain is caused by the LP STC network. The drop in gain is

$$20\log\frac{1}{\sqrt{1 + \left(\frac{1000}{4959.4}\right)^2}} = -0.17 \text{ dB}$$

The gain drops by 3 dB at the corner frequencies of the two STC networks, that is, at $f = 20.16$ Hz and $f = 4959.4$ Hz.

1.64

Using the expression in (3.2) using $B = 7.3 \times 10^{15}\ \text{Cm}^{-3}\text{K}^{-3/2}$;
$k = 8.62 \times 10^{-5}$ eV/K; $E_g = 1.12$ V, we have:

$T = -70°\text{C} = 203$ K:

$$n_i = 2.67 \times 10^4\ \text{cm}^{-3};\ \frac{n_i}{N} = 5.33 \times 10^{-19}$$

That is, one out of every 5.33×10^{18} silicon atoms is ionized at this temperature.

$T = 0°\text{C} = 273$ K:

$$n_i = 1.52 \times 10^9\ \text{cm}^{-3};\ \frac{n_i}{N} = 3.05 \times 10^{-14}$$

$T = 20°\text{C} = 293$ K:

$$n_i = 8.60 \times 10^9\ \text{cm}^{-3};\ \frac{n_i}{N} = 1.72 \times 10^{-13}$$

$T = 100°\text{C} = 373$ K:

$$n_i = 1.43 \times 10^{12}\ \text{cm}^{-3};\ \frac{n_i}{N} = 2.87 \times 10^{-11}$$

$T = 125°\text{C} = 398$ K:

$$n_i = 4.72 \times 10^{12}\ \text{cm}^{-3};\ \frac{n_i}{N} = 9.45 \times 10^{-11}$$

1.65

Hole concentration in intrinsic $S_i = n_i$

$$n_i = BT^{3/2}e^{-Eg/KT}$$

$$= 7.3 \times 10^{15}(300)^{3/2}e^{-(1.12/(2 \times 8.62 \times 10^{-5} \times 300))}$$

$$= 1.5 \times 10^{10} \text{ holes/cm}^3$$

In phosphorus doped *Si*, hole concentration drops below intrinsic level by a factor of 10^7

∴ Hole concentration in *P* doped *Si* is

$$p_n = \frac{1.5 \times 10^{10}}{10^7} = 1.5 \times 10^3\ \text{cm}^{-3}$$

Phosphorus doped *Si*, so

$$n_n \simeq N_D \quad p_n n_n = n_i^2$$

$$n_n = n_i^2/p_n = \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^3}$$

1.66

$$N_D = n_n = 1.5 \times 10^{17}\ P \text{ atoms/cm}^3$$

$$T = 27°\text{C} = 273 + 27 = 300 \text{ K}$$

At 300 K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$

Phosphorous doped *Si*

$$n_n = N_D = 10^{16}/\text{cm}^3$$

$$p_n \cong \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \times \text{cm}^3$$

Hole concentration $= p_n = 2.25 \times 10^4/\text{cm}^3$

$T = 125°\text{C} = 273 + 125 = 398$ K

At 398 K, $n_i = BT^{3/2}e^{-Eg/(2KT)}$

$= 7.3 \times 10^{15} \times (398)^{3/2}e^{-1.12/(2 \times 8.62 \times 10^{-5} \times 398)}$

$= 4.72 \times 10^{12}/\text{cm}^3$

$$p_n \cong \frac{n_i^2}{N_D} = 2.23 \times 10^9/\text{cm}^3$$

At 398 K, hole concentration

$p_n = 2.23 \times 10^9/\text{cm}^3$

1.67

(a) The resistivity of silicon is given For intrinsic silicon,

$p = n = n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$

Using $\mu_n = 1350\ \text{cm}^2/\text{Vs}$ and

$\mu_p = 480\ \text{cm}^2/\text{Vs}$, we have:

$\rho = 2.28 \times 10^5\ \Omega\text{-cm}$.

Using $R = \rho \cdot \frac{L}{A}$ with $L = 0.01$ cm and

$A = 3 \times 10^{-8}\ \text{cm}^2$, we have

$R = 7.59 \times 10^9\ \Omega$.

(b) $n_n \approx N_D = 10^{16}\ \text{cm}^{-3}$:

$$p_n = \frac{n_i^2}{n_n} = 2.25 \times 10^4\ \text{cm}^{-3}$$

Using $\mu_n = 1110\ \text{cm}^2/\text{Vs}$ and

$\mu_p = 400\ \text{cm}^2/\text{Vs}$, we have:

$\rho = 0.56\ \Omega\text{-cm}$; $R = 18.8\ \text{k}\Omega$.

(c) $n_n \approx N_D = 10^{18}\ \text{cm}^{-3}$;

$$p_n = \frac{n_i^2}{n_n} = 2.25 \times 10^2\ \text{cm}^{-3}$$

Using $\mu_n = 1110\ \text{cm}^2/\text{Vs}$ and

$\mu_p = 400\ \text{cm}^2/\text{Vs}$, we have:

$\rho = 5.63 \times 10^{-3}\ \Omega\text{-cm}$; $R = 188\ \Omega$.

As expected, since ND is increased by 100, the resistivity decreases by the same factor.

(d) $p_p \approx N_A = 10^{16}\ \text{cm}^{-3}$; $n_p = \frac{n_i^2}{n_n}$

$= 2.25 \times 10^4\ \text{cm}^{-3}$

$\rho = 1.56\ \Omega\text{-cm}$; $R = 52.1\ \text{k}\Omega$

(e) Since ρ is given to be $2.8 \times 10^{-6}\ \Omega\text{-cm}$, we directly calculate $R = 9.33 \times 10^{-2}\ \Omega$.

1.68

$J_{\text{drift}} = q\,(n\mu_n + p\mu_p)\,E$

Here $n = N_D$ and since it is η - Si, one can assume $p << n$ and ignore the term $p\mu_p$ Also

$$E = \frac{1\ \text{V}}{10\ \mu\text{m}} = \frac{1\ \text{V}}{10 \times 10^{-4}\text{cm}} = 10^3\ \text{V/cm}$$

Need $J_{\text{drift}} = 1\ \text{mA}/\mu\text{m}^2 = q\,N_D\,\mu n\,E$

$$\frac{10^{-3}\text{A}}{10^{-8}\text{cm}^2} = 1.6 \times 10^{-19}\ N_D \times 1350 \times 10^3$$

$\Rightarrow N_D = 4.63 \times 10^{17}/\text{cm}^3$

1.69

$$p_{no} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4/\text{cm}^3$$

From Figure p3.10

$$\frac{dp}{dx} = -\frac{10^8 p_{no} - p_{no}}{W} \simeq -\frac{10^8 p_{no}}{0.1 \times 10^{-4}}$$

since $0.1\ \mu\text{m} = 0.1 \times 10^{-4}$ cm

$$\frac{dp}{dx} = \frac{10^8 \times 2.25 \times 10^4}{0.1 \times 10^{-4}}$$

$= 2.25 \times 10^{17}$

Hence

$$J_p = -qD_p\frac{dp}{dx}$$

$= -1.6 \times 10^{-19} \times 12 \times (-2.25 \times 10^{17})$

$= 0.432\ \text{A/cm}^2$

1.70

$N_A = N_D = 10^{16}\ \text{cm}^{-3}$ and $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$ we have $V_o = 695$ mV. Using (3.26) and $\epsilon_s = 11.7 \times 8.85 \times 10^{-14}$ F/cm, we have $W = 4.24 \times 10^{5}$ cm $= 0.424\ \mu$m. The extension of the depletion width into the n and p regions is given in (3.27) and (3.28) respectively:

$$x_n = W \cdot \frac{N_A}{N_A + N_D} = 0.212\ \mu\text{m}$$

$$x_p = W \cdot \frac{N_D}{N_A + N_D} = 0.212\ \mu\text{m}$$

Since both regions are doped equally, the depletion region is symmetric.

Using (3.29) and $A = 10^{-6}\ \text{cm}^2$ the charge magnitude on each side of the junction is:

$Q_J = 3.39 \times 10^{-14}$ C.

1.71

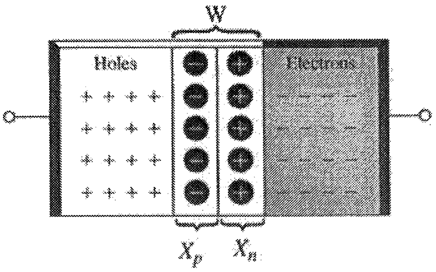
V_T at 300° K = 25.8 mV

built in voltage V_o

$$V_o = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 25.8 \times 10^{-13}$$

$$\ln\left(\frac{10^{16} \times 10^{15}}{(1.5 \times 10^{10})^2}\right)$$

$= 0.633$ V



Depletion with

$$W = \sqrt{\frac{2\epsilon s}{q}\left(\frac{1}{N_A} + \frac{1}{N_O}\right) V_O} \leftarrow \text{equation 3.26}$$

$$W = \sqrt{\frac{2 \times 1.04 \times 10^{-17}}{1.6 \times 10^{-19}}\left(\frac{1}{10^6} + \frac{1}{10^{15}}\right) \times 0.633}$$

$= 0.951 \times 10^{-4}$cm $= 0.951$ µm

to find X_n and X_p

$$X_n = W\frac{N_A}{N_A + N_D} = 0.951 \times \frac{10^{16}}{10^{16} + 10^{15}}$$

$= 0.8642$ µm

$$X_p = W\frac{N_D}{N_A + N_D} = 0.951 \times \frac{10^{15}}{10^{16} + 10^{15}}$$

$= 0.8642$ µm

to calculate charge stored on either side

$$Q_J = Aq\left(\frac{N_A N_D}{N_A + N_D}\right) \text{W where junction area}$$

$= 400$ µm^2 $= 400 \times 10^{-8}$cm^2

$$= 400 \times 10^{-8} \cdot 1.6 \times 10^{-19}\left(\frac{10^{16} \cdot 10^{15}}{10^{16} + 10^{15}}\right).$$

$$= 0.951 \times 10^{-4}$$

Hence,

$Q_J = 5.53 \times 10^{-14}$ C

1.72

Charge stored $Q_J = q\text{AXN}$

Here X= 0.1 µm = 0.1 × 10^{-4} cm

A= 10 µm × 10 µm = 10 × 10^{-4} cm × 10 × 10^{-4} cm

= 100 × 10^{-4} cm^2

So, $Q_J = 1.6 \times 10^{-19} \times 100 \times 10^{-8} \times 0.1 \times 10^{-4} \times 10^{16}$

=16 fC

1.73

$$V_O = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

If N_A or N_D is increased by a factor of 10, then new value of V_O will be

$$V_O = V_T \ln\left(\frac{10\, N_A N_D}{n_i^2}\right)$$

The change in the value of V_O is

$$= \frac{\ln(10)}{\ln\left(\frac{N_A N_D}{n_i^2}\right)}$$

1.74

with $N_A = 10^{16}$ cm^{-3}, $N_D = 10^{16}$ cm^{-3}, and $n_i = 1.5 \times 10^{10}$, we have $V_O = 635$ mV.

and $V_R = 5$ V, we have $W = 2.83 \times 10^{-4}$ cm = 2.83 µm.

with A = 4 × 10^{-6} cm², we have $Q_J = 4.12 \times 10^{-14}$ C.

1.75

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_O + V_R)}$$

$$= \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right) V_O\left(1 + \frac{V_R}{V_O}\right)}$$

$$= W_O\sqrt{1 + \frac{V_R}{V_O}}$$

$$\therefore \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right) V_O} = W_O$$

$$Q_j = A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right) \cdot (V_O + V_R)}$$

$$= A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right) V_O \cdot \left(1 + \frac{V_R}{V_O}\right)}$$

$$= A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right) V_O \cdot \left(1 + \frac{V_R}{V_O}\right)}$$

$$= Q_{JO}\sqrt{1 + \frac{V_R}{V_O}}$$

$$\because A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right) V_O} = Q_{JO}$$

1.76

$$I_s = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$$

$A = 200$ µm^2 $= 200 \times 10^{-8}$ cm^2

$$I_s = 200 \times 10^{-8} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2$$

$$\frac{10}{5 \times 10^{-4} \times 10^{17}} + \frac{18}{10 \times 10^{-14} \times 10^{16}}$$

$= 1.44 \times 10^{-6}$ A

$$I \cong I_s e^{V/V_T}$$

$$= 1.44 \times 10^{-16} \times e^{700/25.9}$$

$$\cong 79\ \mu\text{A}$$

1.77

$$n_i = BT^{3/2} e^{-E_O/(2KT)}$$

At 300 K,

$$n_i = 7.3 \times 10^{15} \times (300)^{3/2} \times e^{-1.12/\left(2 \times 8.62 \times 10^{-5} \times 300\right)}$$

$$= 1.4939 \times 10^{10}/\text{cm}^2$$

$$n_i^2(\text{at } 300\text{ K}) = 2.232 \times 10^{20}$$

At 305 K,

$$n_i = 7.3 \times 10^{15} \times (305)^{3/2} \times e^{-1.12/\left(2 \times 8.62 \times 10^{-5} \times 305\right)}$$

$$= 2.152 \times 10^{10}$$

$$n_i^2(\text{at } 305\text{ K}) = 4.631 \times 10^{20}$$

$$\text{so } \frac{n_i^2(\text{at } 305\text{ K})}{n_i^2(\text{at } 300\text{ K})} = 2.152$$

1.78

$$I = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)(e^{V/V_T} - 1)$$

$$\text{So } I_p = Aqn_i^2 \frac{D_p}{L_p N_D}(e^{V/V_T} - 1)$$

$$I_n = Aqn_i^2 \frac{D_n}{L_n N_A}(e^{V/V_T} - 1)$$

For $p^+ - n$ junction $N_A >> N_D$

$$\therefore I \simeq I_p = Aqn_i^2 \frac{D_p}{L_p N_D}(e^{V/V_T} - 1)$$

For this case

$$I_s \simeq Aqn_i^2 \frac{D_p}{L_p N_D} = 10^4 \times 10^{-8}\text{ cm}^2 \times 1.6 \times 10^{-19}$$

$$\times (1.5 \times 10^{10})^2 \frac{10}{10 \times 10^{-4} \times 10^{16}}$$

$$= 3.6 \times 10^{-15}\text{ A}$$

$$I = I_s(e^{V/V_T} - 1) = 0.5 \times 10^{-3}$$

$$3.6 \times 10^{-15}\left(e^{V/\left(25.9 \times 10^{-3}\right)} - 1\right) = 0.5 \times 10^{-3}$$

$$\Rightarrow V = 0.6645\text{ V}$$

$$C_j = \frac{C_{jo}}{\left(1 + \frac{V_R}{V_O}\right)^m}$$

$$\text{For } V_R = 1\text{ V, } C_j = \frac{0.6\text{ pF}}{\left(1 + \frac{1}{0.75}\right)^{\frac{1}{3}}}$$

$$= 0.45\text{ pF}$$

$$\text{For } V_R = 10\text{ V, } C_j = \frac{0.6\text{ pF}}{\left(1 + \frac{10}{0.75}\right)^{\frac{1}{3}}}$$

$$= 0.25\text{ pF}$$

1.80

$$C_d = \left(\frac{\tau_T}{V_T}\right)I$$

$$10\text{ pF} = \left(\frac{\tau_T}{25.9 \times 10^{-3}}\right) \times 1 \times 10^{-3}$$

$$\tau_T = 10 \times 10^{-12} \times 25.9$$

$$= 259\text{ pS}$$

For I = 0.1 mA

$$C_d = \left(\frac{\tau_T}{V_T}\right) \times I$$

$$= \left(\frac{259 \times 10^{-12}}{25.9 \times 10^{-3}}\right) \times 0.1 \times 10^{-3}$$

$$= 1\text{ pF}$$

1.81

$$\tau_p = \frac{L_p^2}{D_p} = \frac{(10 \times 10^{-4})^2}{10}$$

note $1\ \mu\text{m} = 10^{-4}\text{cm}$

$$= 100\text{ ns}$$

$$Q_P = \tau_P I_P$$

$$= 100 \times 10^{-9} \times 0.2 \times 10^{-3}$$

$$= 20 \times 10^{-12}\text{ C}$$

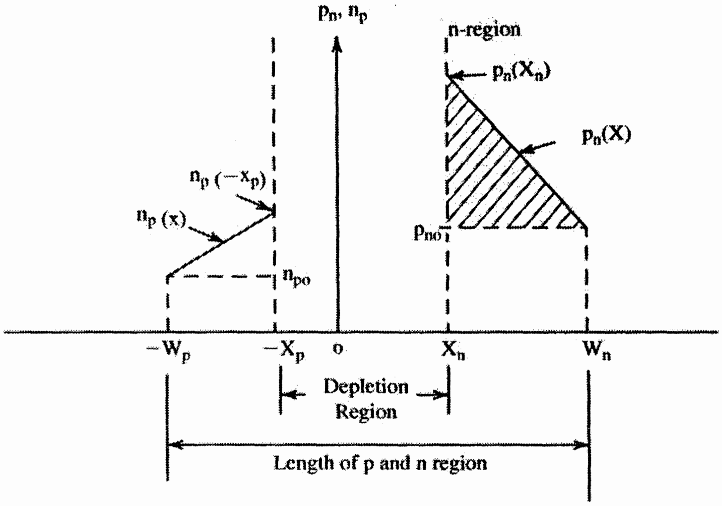
$$C_d = \left(\frac{\tau_P}{V_T}\right)I$$

$$= \left(\frac{100 \times 10^{-9}}{25.9 \times 10^{-3}}\right) \times 0.2 \times 10^{-3}$$

$$= 772\text{ pF}$$

1.82

a.



b. The current $I = I_p + I_n$

Find current component I_p

$$p_n(x_n) = p_{no}e^{V/V_T} \text{ and } p_{no} = \frac{n_i^2}{N_D}$$

$$I_p = AJ_p = AqD_p\frac{dp}{dx}$$

$$\frac{dp}{dx} = \frac{p_n(x_n) - p_{no}}{W_n - X_n} = \frac{p_{no}e^{V/V_T} - p_{no}}{W_n - X_n}$$

$$= p_{no}\frac{(e^{V/V} - 1)}{W_n - X_n}$$

$$= \frac{n_i^2}{N_D}\frac{(e^{V/V_T} - 1)}{(W_n - X_n)}$$

$$\therefore I_p = AqD_p\frac{dp}{dx}$$

$$= Aqn_i^2\frac{D_p}{(W_n - X_n)N_D} \times (e^{V/V_T} - 1)$$

Similarly

$$I_n = Aqn_i^2\frac{D_n}{(W_p - X_p)N_A} \times (e^{V/V_T} - 1)$$

$$I = I_p + I_n$$

$$= Aqn_i^2\left[\frac{D_p}{(W_n - X_n)N_D} + \frac{D_n}{(W_p - X_p)N_A}\right]$$

$$(e^{V/V_T} - 1)$$

The excess change, Q_p, can be obtained by multiplying the area of the sheded triangle of the $p_n(x)$ distibution graph by Aq.

$$Q_p = Aq \times \frac{1}{2}[p_n(X_n) - p_{no}](W_n - X_n)$$

$$= \frac{1}{2}Aq[p_{no}e^{V/V_T} - p_{no}](W_n - X_n)$$

$$= \frac{1}{2}Aqp_{no} + (e^{V/V_T} - 1)(W_n - X_n)$$

$$= \frac{1}{2}Aq\frac{ni^2}{N_D}(W_n - X_n)(e^{V/V_T} - 1)$$

$$= \frac{1}{2}\frac{(W_n - X_n)^2}{D_p} \cdot I_p$$

$$\simeq \frac{1}{2}\frac{W_n^2}{D_p} \cdot I_p \text{ for } W_x >> X_p$$

c. $C_d = \dfrac{dQ}{dV} = \tau_T\dfrac{dI}{dV}$

But $I = I_S(e^{V/V_T} - 1)$

$$\frac{dI}{dV} = \frac{I_S e^{V/V_T}}{V_T}$$

$$\simeq \frac{I}{V_T}$$

so $C_d \cong \tau_T \cdot \dfrac{I}{V_T}$

d. $C_d = \dfrac{1}{2}\dfrac{W_n^2}{10}\dfrac{1 \times 10^{-3}}{25.9 \times 10^{-3}} = 8 \times 10^{-12}$ F

Solve for W_n

$W_n = 63.25\ \mu\text{m}$

2.1

$$v_+ = v_I \frac{1K\Omega}{1M\Omega + 1K\Omega} = \frac{4}{1001} V$$

$$v_O = A v_+ \Rightarrow A = \frac{4}{4/1001} = 1001$$

2.2

The voltage at the positive input has to be -3.000V.

$$v_+ = -3.020 V \text{ , } A = \frac{v_o}{(v_+ - v_-)} = \frac{-2}{-3.020-(-3)} = 100$$

2.3

#	v_1	v_2	$v_d = v_2 - v_1$	v_0	v_0/v_d
1	0.00	0.00	0.00	0.00	–
2	1.00	1.00	0.00	0.00	–
3	(a)	1.00	(b)	1.00	
4	1.00	1.10	0.10	10.1	101
5	2.01	2.00	−0.01	−0.99	99
6	1.99	2.00	0.01	1.00	100
7	5.10	(c)	(d)	−5.10	

experiments 4,5,6 show that the gain is approximately 100 V/V. The missing entry for experiment #3 can be predicted as follows:

(b) $v_d = \frac{v_o}{A} = \frac{1.00}{100} = 0.01$ V.

(a) $v_1 = v_2 - v_d = 1.00 - 0.01 = 0.99$ V

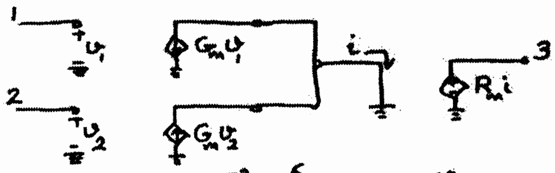
The missing entries for experiment #7:

(d) $v_d = \frac{-5.10}{100} = -0.051$ V

(c) $v_2 = v_1 + v_d = 5.10 - 0.051 = 5.049$ V

All the resluts seen to be reasonable.

2.4



$$A = G_m R_m = 100 \times 10^{-3} \times 10^{6} = 100{,}000 \text{ V/V}$$

2.5

$v_{CM} = 1\, v \,\text{Sin}\,(2\pi 60)t = \frac{1}{2}(v_1 + v_2)$

$v_d = 0.01\,\text{Sin}\,(2\pi 1000)t = v_1 - v_2$

$v_1 = v_{CM} - v_d/2 = \text{Sin}(120\pi)t - 0.005\,\text{Sin}\,2000\pi t$

$v_2 = v_{CM} + v_d/2 = \text{Sin}\,120\pi t + 0.005\,\text{Sin}\,2000\pi t$

2.6

Circuit	v_o/v_i(V/V)	R_{in} (kΩ)
a	$\frac{-100}{10} = -10$	10
b	−10	10
c	−10	10
d	−10	10

virtual ground no current in 10 kΩ

2.7

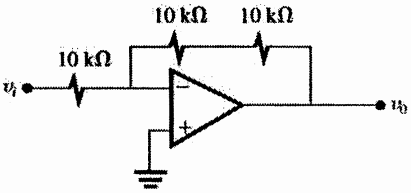
closed loop gain = -1 V/V. for $v_i = 5V \Rightarrow v_o = -5$ V

Gain would be in the range of $\frac{-0.95}{1.05}$ to $\frac{-1.05}{0.95}$: $-0.9 < G < -1.1$

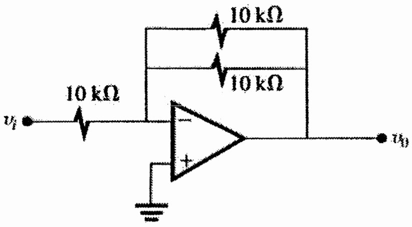
for $v_i = 5 \Rightarrow -4.5 < v_o < -5.5V$

2.8

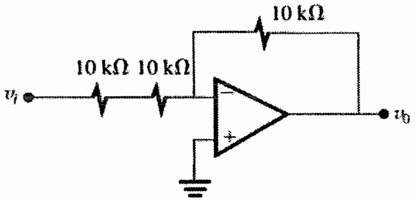
There are four possibilities:


10 kΩ
10 kΩ
10 kΩ
v_i
v_0
−
+

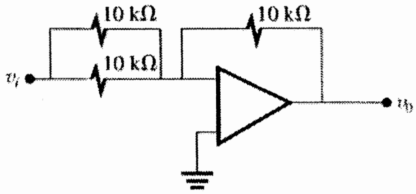
$\frac{v_o}{v_i} = -2$ V/V $R_{in} = 10$ kΩ


10 kΩ
10 kΩ
10 kΩ
v_i
v_0
−
+

$\frac{v_o}{v_i} = -0.5$ V/V $R_{in} = 10$ kΩ


10 kΩ
10 kΩ 10 kΩ
v_i
v_0
−
+

$\frac{v_o}{v_i} = -0.5$ V/V $R_{in} = 20$ kΩ


10 kΩ
10 kΩ
10 kΩ
v_i
v_0

$\frac{v_o}{v_i} = -2$ V/V $R_{in} = 5$ kΩ

2.9

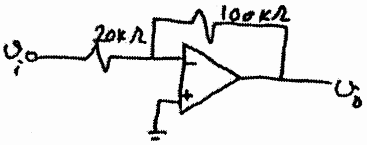
a.	G = −1 V/V	b.	G = −10 V/V
c.	G = −0.1 V/V	d.	G = −100 V/V
e.	G = −10 V/V		

2.10

$\frac{v_o}{v_i} = -5 = \frac{-R_2}{R_1} \Rightarrow R_2 = 5R_1$

$R_1 + R_2 = 120\,k\Omega \Rightarrow 5R_1 + R_1 = 120\,k\Omega \Rightarrow$

$R_1 = 20\,k\Omega \Rightarrow R_2 = 100\,k\Omega$


v_i
20kΩ
100kΩ
−
+
v_o

2.11

$20\log|G| = 26\,dB \Rightarrow G = -19.95\,V/V = \frac{v_o}{v_i} = -\frac{R_2}{R}$

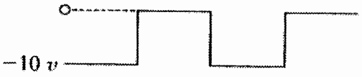
$\Rightarrow R_2 = 19.95 R_1 \leq 10\,M\Omega$

For largest possible input resistance, select

$R_2 = 10\,M\Omega \Rightarrow R_1 \simeq 500\,k\Omega$

$R_{in} = 500\,k\Omega$

2.12



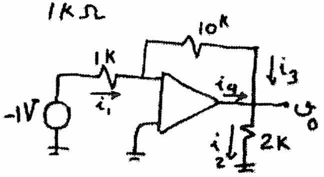
$G = \frac{v_o}{v_i} = \frac{-R_2}{R_1} = \frac{-100}{10} = -10$

$v_{low} = 10$ V, $v_{high} = 0$, $v_{avg} = -5$ V

2.13

$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \Rightarrow v_o = -1 \times -\frac{10\,k\Omega}{1\,k\Omega} = 10\,V$

$i_2 = \frac{v_o}{2\,k\Omega} = 5\,mA$


10K
1K
i_3
i_4
i_1
−1V
v_o
i_2
2K

$i_1 = i_3 = \frac{v_o}{10\,k\Omega} = 1\,mA$

$i_4 = i_2 - i_3 = 4\,mA$ This additional current comes from the output of the opamp.

2.14

$$|\text{Gain}| = \frac{R_2}{R_1} = \frac{R_2(1+x/100)}{R_1(1+x/100)} \simeq \frac{R_2}{R_1}\left(1 \pm \frac{2x}{100}\right)$$

for small x

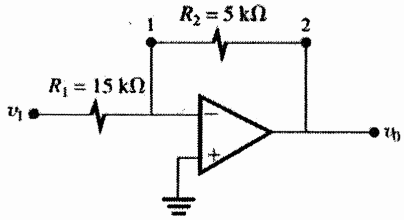
$\Rightarrow 2x\%$ is the tolerance on the closed loop gain (G).

$G = -100\,V/V$, $x = 5 \Rightarrow -110 < G < -90$

or more precisely: $-100 \times \frac{105}{95} < G < -100\,\frac{95}{105}$

$-110.5 < G < -90.5$

2.15



$$G = \frac{v_o}{v_i} = \frac{-R_2}{R_1} \Rightarrow \frac{R_2}{R_1} = \frac{5}{15}$$

$v_1 = 0V$, $v_2 = v_o = 5V$

For ±1% on R_1, R_2: $R_1 = 15 \pm 0.15\,k\Omega$

$R_2 = 5 \pm 0.05\,k\Omega$

$$v_o = v_i\frac{-R_2}{R_1} = 15\frac{R_2}{R_1} \Rightarrow 15 \times \frac{4.95}{15.15}$$

$$\le v_o \le 15 \times \frac{5.05}{14.85}$$

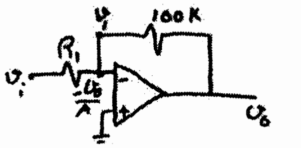
$\Rightarrow 4.9\ V \le v_o \le 5.1\ V$

For $v_i = -15 \pm 0.15\ V$ $14.85 \times \frac{4.95}{15.15}$

$$\le v_o \le 15.15 \times \frac{5.05}{14.85}$$

$\Rightarrow 4.85\ v \le v_o \le 5.15\ V$

2.16



$$v_1 = \frac{-v_o}{A} = -\frac{v_o}{200}$$

$$\frac{v_o}{v_i} = -50\,V/V$$

$$\frac{v_i - \left(\frac{-v_o}{A}\right)}{R_1} = \frac{\left(\frac{-v_o}{A} - v_o\right)}{100k\Omega} \Rightarrow R_1 = 100K \times \frac{\frac{v_o}{200} - \frac{v_o}{50}}{\frac{-v_o}{200} - v_o}$$

$$\Rightarrow R_1 = 100K \times \frac{3}{201} = 1.49k\Omega$$

Shunt Resisror R_a : $R_a \parallel 2k\Omega = 1.49K$

$$\frac{R_a \times 2}{R_a + 2} = 1.49 \Rightarrow R_a = 5.84k\Omega$$

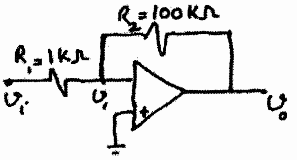
2.17

ⓐ

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \Rightarrow -100\,V/V = \frac{-R_2}{1k\Omega} \Rightarrow R_2 = 100k\Omega$$

ⓑ $A = 1000\,V/V$

$v_1 = \frac{-v_o}{A}$



$$\frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_2}$$

$$\frac{v_o}{v_i} = \frac{-R_2/R_1}{1 + (1 + \frac{R_2}{R_1})/A} = \frac{-100}{1 + \frac{101}{1000}} = -90.8\,V/V$$

$$\Rightarrow \frac{v_o}{v_i} = 90.8\ V/V$$

ⓒ Assume $R_1' = R_x \parallel R_1$ when $R_1 = 1k\Omega$

$$\frac{v_o}{v_i} = -100\ V/V$$

$$\frac{v_i - v_1}{R_1'} = \frac{v_1 - v_o}{R_2} \Rightarrow R_1' = R_2 \times \left(\frac{-v_o}{100} - \frac{-v_o}{1000}\right) \Big/ \left(\frac{-v_o}{1000} - v_o\right)$$

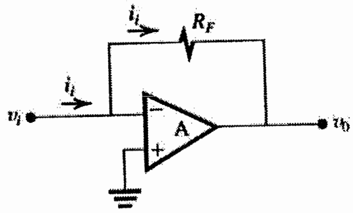
$$R_1' = \frac{1 - 0.1}{1.001} = 0.899\,K\Omega = \frac{R_1 R_x}{R_1 + R_x} = \frac{R_x}{1 + R_x}$$

$$\Rightarrow R_x = 8.9\,K\Omega \simeq 8.87\,K\Omega \pm 1\%$$

2.18

Voltage of the inverting input terminal will vary from $\frac{-10V}{1000}$ to $\frac{+10V}{1000}$. Thus the virtual ground will depart from the ideal voltage of zero by a maximum of ±10mV.

2.19



a) For A = ∞: $v_i = 0$

$$v_o = -i_i R_F$$

$$R_m = \frac{v_o}{i_i} = -R_F$$

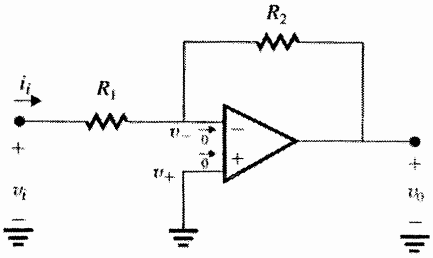
$$R_{in} = \frac{v_i}{i_i} = 0$$

b) For A-finite: $v_i = -\frac{v_o}{A}$, $v_o = v_i - i_i R_F$

$$\Rightarrow v_o = \frac{-v_o}{A} - i_i R_F \Rightarrow R_m = \frac{v_o}{i_i} = -\frac{R_F}{1 + \frac{1}{A}}$$

$$R_i = \frac{v_i}{i_i} = \frac{R_F}{1 + A}$$

2.20



$$v_o = -Av_- = v_- - i_i R_2$$

$$i_i R_2 = v_-(1 + A)$$

$$v_- = \frac{i_i R_2}{1 + A}$$

Again $v_i = i_i R_1 + v_-$

$$= i_i R_1 + i_i \frac{R_2}{1 + A}$$

So $R_{in} = \frac{v_i}{i_i} = R_1 + \frac{R_2}{A + 1}$

2.21

$$R_1' = R_1 \parallel R_c \qquad G' = \frac{-R_2/R_1'}{1 + \frac{1 + R_2/R_1'}{A}}$$

In order for $G' = G$:

$$G = \frac{-R_2/R_1'}{1 + \frac{1 + R_2/R_1'}{A}} = \frac{-R_2}{R_1}$$

$$R_1' = \frac{R_1 R_c}{R_1 + R_c}$$

$$\Rightarrow \frac{R_1 + R_c}{R_1 R_c} = \frac{1}{R_1}\left(1 + \frac{R_2 \frac{(R_1 + R_c)}{R_1 R_c}}{A}\right)$$

$$(R_1 + R_c)A = AR_c + R_c + \frac{R_2}{R_1}(R_1 + R_c)$$

$$R_1 A = R_c + GR_1 + GR_c$$

$$\frac{R_c}{R_1} = \frac{A - G}{1 + G}$$

2.22

$$G = \frac{-R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}} \qquad G_{nominal} = \frac{-R_2}{R_1}$$

$$\epsilon = \left| \frac{G - G_{nominal}}{G_{nominal}} \right| = \left| \frac{G}{G_{nominal}} - 1 \right|$$

$$\epsilon = \left| \frac{1}{1 + \frac{1 + R_2/R_1}{A}} - 1 \right| = \left| \frac{-\frac{1 + R_2/R_1}{A}}{1 + \frac{1 + R_2/R_1}{A}} \right| = \frac{1}{\frac{A}{1 + \frac{R_2}{R_1}} + 1}$$

which can be rearranged to yield:

$$\frac{A}{1 + \frac{R_2}{R_1}} + 1 = \frac{1}{\epsilon} \Rightarrow A = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{\epsilon} - 1\right)$$

$$\text{or} \quad A = (1 - G_{nominal})\left(\frac{1}{\epsilon} - 1\right)$$

For $G_{nominal} = -100$ V/V and $\epsilon = 10\% = 0.1$

$$A = (1 + 100)\left(\frac{1}{0.1} - 1\right) = 909 \text{ V/V}$$

This is the minimum required value for A.

2.23

(a) $\dfrac{\Delta|G|/|G|}{\Delta A/A} = \dfrac{1+R_2/R_1}{A}$

(b) $\dfrac{\Delta|G|}{G} = 0.5\%,\ \dfrac{\Delta A}{A} = 50\%,\ \dfrac{R_2}{R_1} = 100$

$$\frac{0.005}{.5} = \frac{1+100}{A}$$

$$A = \frac{101(.5)}{(.005)} = 10.1\text{ k}$$

2.24

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}\left(1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right)$$

For $R_1 = R_2 = R_4 = 1\,M\Omega \Longrightarrow \dfrac{v_o}{v_i} = -\left(1+1+\dfrac{1}{R_3}\right)$

a) $\dfrac{v_o}{v_i} = -10\,V/V \Longrightarrow 10 = 2+\dfrac{1}{R_3} \Longrightarrow R_3 = \dfrac{1}{8}M\Omega = 125\,k\Omega$

b) $\dfrac{v_o}{v_i} = -100\,V/V \Longrightarrow 100 = 2+\dfrac{1}{R_3} \Longrightarrow R_3 = \dfrac{1}{98}M\Omega = 10.2\,k\Omega$

c) $\dfrac{v_o}{v_i} = -2\,V/V \Longrightarrow 2 = 2+\dfrac{1}{R_3} \Longrightarrow R_3 = \infty$; eliminate R_3.

2.25

$R_2/R_1 = 1000,\ R_2 = 100\text{ k}\Omega \Rightarrow R_1 = 1000\ \Omega$

a) $R_{in} = R_1 = 100\ \Omega$

b) $\dfrac{v_O}{v_I} = \dfrac{-R_2}{R_1}\left(1+\dfrac{R_4}{R_2}+\dfrac{R_4}{R_3}\right) = -1000$

If $R_2 = R_1 = R_4 = 100\text{ K} \Rightarrow R_3 = \dfrac{100\text{ K}}{1000-2} \simeq$

$100\ \Omega$

$R_{in} = R_1 = 100\text{ k}\Omega$

2.26

$v_x = 0 - i_1 R_2\ ,\ i_1 = \dfrac{v_I}{R_1} \Longrightarrow v_x = -v_I\dfrac{R_2}{R_1}$

$$\frac{v_x}{v_I} = -\frac{R_2}{R_1}$$

$$v_x = v_o\,\frac{R_2\|R_3}{R_2\|R_3+R_4} = v_o\,\frac{R_2R_3}{R_2R_3+R_4R_2+R_4R_3}$$

$$\frac{v_o}{v_x} = \frac{R_2R_3+R_2R_4+R_3R_4}{R_2R_3} = 1+\frac{R_4}{R_3}+\frac{R_4}{R_2}$$

$$\frac{v_o/v_x}{v_I/v_x} = \frac{v_o}{v_i} = \frac{(1+R_4/R_3+R_4/R_2)}{-R_1/R_2} \Longrightarrow$$

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}\left(1+\frac{R_4}{R_3}+\frac{R_4}{R_2}\right)$$

2.27

a) $R_1 = R$

$$R_2 = (R \parallel R)+\frac{R}{2} = \frac{R}{2}+\frac{R}{2} = R$$

$$R_3 = (R_2 \parallel R)+\frac{R}{2} = (R \parallel R)+\frac{R}{2} = R$$

$$R_4 = (R_3 \parallel R)+\frac{R}{2} = (R \parallel R)+\frac{R}{2} = R$$

b) $v = RI = RI_1 \Rightarrow I_1 = I$

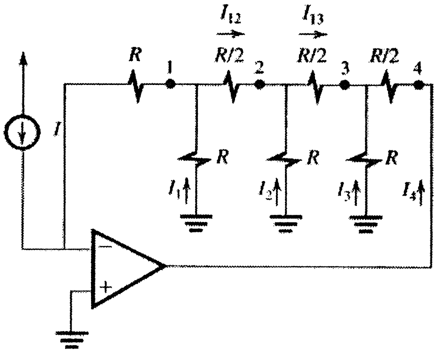
$I_{12} = I+I = 2I \Rightarrow v_1 + 2I\times\dfrac{R}{2} = RI_2$

$RI+RI = RI_2 \Rightarrow I_2 = 2I$

$I_{13} = I_2+I_{12} = 4I \Rightarrow v_2 + 4I\times\dfrac{R}{2} = RI_3$

$R\times 2I + 4I\times\dfrac{R}{2} = RI_3 \Rightarrow I_3 = 4I,$

$I_4 = -(4I+4I)\ I_4 = 8I$



c) $v_1 = I_1R = -IR$

$v_2 = -I_2R = -2IR$

$v_3 = -I_3R = -4IR$

$$v_4 = -I_3R + I_4\frac{R}{2} = -4IR - 8I\frac{R}{2} = -8IR$$

2.28

a) $I_1 = \frac{1V}{10k\Omega} = 0.1mA$

$I_2 = I_1 = 0.1\ mA$, $I_2 \times 10k\Omega = I_3 \times 100\Omega \Rightarrow I_3 = 10^{mA}$

$v_x = 10mA \times 100\Omega = 1V$

b) $v_x = R_L I_L + v_o$, $I_L = I_2 + I_3 = 10.1mA$

$1V = R_L \times 10.1mA + v_o$

$R_L = \frac{1 - v_o}{10.1} \Rightarrow R_{Lmax} = \frac{1 - v_{omin}}{10.1} = \frac{14}{10.1}$

$R_{Lmax} =$

c) $100\Omega \leq R_L \leq 1K\Omega$

I_L stays fixed at 10.1mA

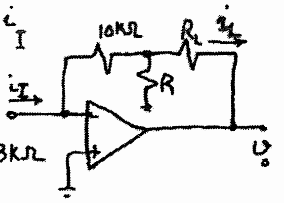
$v_o = v_x - R_L I_L = 1 - R_L \times 10.1 \Rightarrow -9.1^V \leq v_o \leq -0.01^V$

2.29

a) $\frac{i_L}{i_I} = 20 \Rightarrow i_L = 20\, i_I$

$-10k\Omega \times i_I = R(i_I - i_L)$

$R = \frac{10k\Omega \times i_I}{20 i_I - i_I} = 0.53k\Omega$



b) $R_L = 1K\Omega \quad -12^V \leq v_o \leq 12^V$

$v_o = R_L i_L + 10K\Omega \times i_I = i_I (1K\Omega \times \frac{i_L}{i_I} + 10^{K\Omega})$

$v_o = i_I (1 \times 20 + 10) = 30\, i_I$

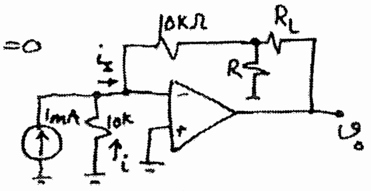
$i_I = \frac{v_o}{30} \Rightarrow \frac{-12}{30} \leq i_I \leq \frac{12}{30} \Rightarrow -0.4_{mA} \leq i_I \leq 0.4_{mA}$

c) $R_I = \frac{v_I}{i_I} = \frac{0}{i_I} = 0$

$v_- = 0 \Rightarrow i = 0$

$\Rightarrow i_I = 1mA$

From part a: $i_L = 20 \times i_I = 20\ mA$



2.30

$R_I = 100\ k\Omega - 10 \leq \frac{v_o}{v_i} \leq -1\ \frac{V}{V}$

$R_i = R_1 = 100\ k\Omega$

$\frac{v_o}{v_i} = \frac{-R_2}{R_1}\left(\frac{R_4}{R_3} + \frac{R_4}{R_2} + 1\right)$

$R_4 = 0 \Rightarrow \frac{v_o}{v_i} = \frac{-R_2}{R_1} = -1 \Rightarrow R_2 = 100\ k\Omega$

$R_4 = 10\ k\Omega \Rightarrow \frac{v_o}{v_i} = -10$

$= -1 \times \left(\frac{10\ k\Omega}{R_3} + \frac{10\ k\Omega}{100\ k\Omega} + 1\right)$

$+10 = \left(\frac{10}{R_3} + 1.1\right) \Rightarrow R_3 = 1.12\ k\Omega$

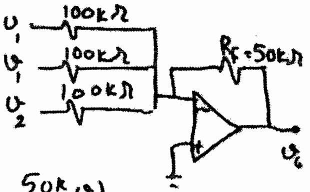
Potentiometer in the middle:

$\frac{v_o}{v_i} = -1\left(\frac{5}{5 + R_3} + \frac{5}{100} + 1\right)$

$\frac{v_o}{v_i} = -1.87\ V/V$

2.31

$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_n} v_n\right)$



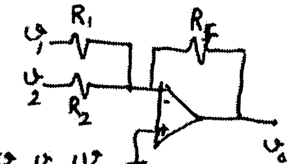
$v_o = -\left(\frac{50k}{100k} v_1 + \frac{50k}{100k} v_1 + \frac{50k}{100k} v_2\right)$

$v_o = -\left(v_1 + \frac{v_2}{2}\right) \qquad v_1 = 3, v_2 = -3 \Rightarrow v_o = -1.5V$

2.32

we choose the weighted sommer configuration

$v_o = -\left[4v_1 + \frac{v_2}{3}\right]$


v_1
R_1
R_F
v_2
R_2
v_o

$i_1 = \frac{v_1}{R_1}$ $\quad i_2 = \frac{v_2}{R_2}$

$i_1, i_2 \leqslant 0.1\ mA$ for $v_1, v_2 = 1V$

$R_1, R_2 \geqslant 10 k\Omega$

$\frac{R_F}{R_1} = 4$, if $\underline{R_1 = 10k\Omega} \Rightarrow \underline{R_F = 40k\Omega}$

$\frac{R_F}{R_2} = \frac{1}{3} \Rightarrow \underline{R_2 = 120k\Omega}$

2.33

$v_o = -(2v_1 + 4v_2 + 8v_3)$

$R_1, R_2, R_3 \geqslant 10\ k\Omega$

$\frac{R_F}{R_1} = 2,\ \frac{R_F}{R_2} = 4\ \frac{R_F}{R_3} = 8$

$R_3 = 10\ k\Omega \Rightarrow R_F = 80\ k\Omega$

$R_2 = 20\ k\Omega$

$R_1 = 40\ k\Omega$

2.34

The output signal should be:

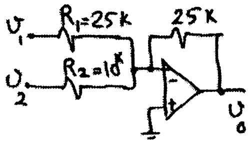
$v_o = -5\sin\omega t - 5$

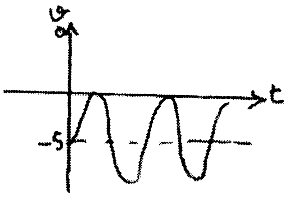
if we assume : $v_1 = 5\sin\omega t$, $v_2 = 2V$ $\Big\}\ v_o = -(v_1 + 2.5v_2)$

In a weighted summer configuration:

$\frac{R_F}{R_1} = +1$ $\qquad \frac{R_F}{R_2} = 2.5$

$R_2 = 10K\Omega \Rightarrow R_F = 25K = R_1$


v_1
$R_1 = 25K$
25K
v_2
$R_2 = 10^K$
v_o



2.35

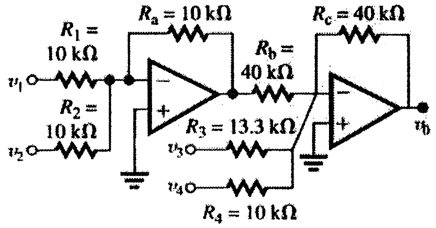
$v_o = v_1 + 2v_2 - 3v_3 - 4v_4$:

For a weighted summer circuit:

$$v_o = v_1\frac{R_aR_c}{R_1R_b} + v_2\frac{R_aR_c}{R_2R_b} - v_3\frac{R_c}{R_3} - v_4\frac{R_c}{R_4}$$

$$\frac{R_a}{R_1}\frac{R_c}{R_b} = 1,\ \frac{R_a}{R_2}\frac{R_c}{R_b} = 1,\ \frac{R_c}{R_3} = 3,\ \frac{R_c}{R_4} = 4$$

assume:


$R_a = 10\ k\Omega$
$R_c = 40\ k\Omega$
$R_1 =$ 10 kΩ
$R_b =$ 40 kΩ
v_1
$R_2 =$ 10 kΩ
$R_3 = 13.3\ k\Omega$
v_2
v_3
v_4
v_O
$R_4 = 10\ k\Omega$

$R_4 = 10\ k\Omega \Rightarrow R_c = 40\ k\Omega \Rightarrow R_3 = \frac{40}{3}$

$= 13.3\ k\Omega$

$\frac{R_a}{R_1} \times \frac{40}{R_b} = 1\ \ \frac{R_a}{R_2} \times \frac{40}{R_b} = 1$

$R_b = 40\ k\Omega,\qquad R_1 = R_2 = R_a = 10\ k\Omega$

2.36

$v_1 = 3\sin(2\pi \times 60t) + 0.01\sin(2\pi \times 1000t)$

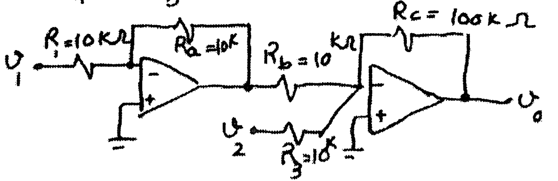
$v_2 = 3\sin(2\pi \times 60t) - 0.01\sin(2\pi \times 1000t)$

we want to have : $v_o = 10v_1 - 10v_2$

$v_o = v_1\frac{R_a}{R_1}\frac{R_c}{R_b} - v_3\frac{R_c}{R_3}$

$\frac{R_a}{R_1}\frac{R_c}{R_b} = 10$, $\frac{R_c}{R_3} = 10$, if $R_3 = 10^{K\Omega} \Rightarrow R_c = 100^{K\Omega}$

$\Rightarrow \frac{R_a}{R_1} \times \frac{100^{K\Omega}}{R_b} = 10 \Rightarrow R_a = R_1 = R_b = 10k\Omega$


$R_c = 100k\Omega$
$R_1 = 10K\Omega$
$R_a = 10^K$
$R_b = 10^{K\Omega}$
v_1
v_o
v_2
$R_3 = 10^K$

$v_o = 10v_1 - 10v_2 = 10 \times 0.02\ \sin 2\pi \times 1000t$

$v_o = 0.2\sin(2\pi \times 1000t)$ $\quad -0.2^v \leqslant v_o \leqslant 0.2^v$

2.37

This is a weighted summer circuit:

$$v_o = -\left(\frac{R_F}{R_0} v_o + \frac{R_F}{R_1} v_1 + \frac{R_F}{R_2} v_2 + \frac{R_F}{R_3} v_3\right)$$

we may write: $v_0 = 5v \times a_0$, $v_2 = 5v \times a_2$
$v_1 = 5v \times a_1$ $v_3 = 5v \times a_3$

$$v_o = -R_F\left(\frac{50}{80\text{ k}\Omega} + \frac{5}{40\text{ k}\Omega} a_1 + \frac{5}{20\text{ k}\Omega} a_2 + \frac{5}{10\text{ k}\Omega} a_3\right)$$

$$v_0 = -R_F\left(\frac{a_0}{16} + \frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{2}\right)$$

$$v_o = -\frac{R_F}{16}(2^0 a_0 + 2^1 a_1 + 2^2 a_2 + 2^3 a_3)$$

2.38

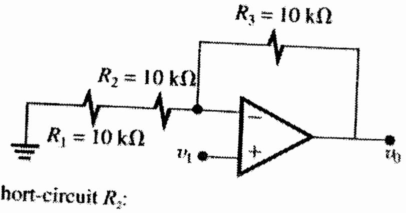
a) $\frac{v_o}{v_i} = 1 = 1 + \frac{R2}{R_1} \Longrightarrow R_2 = 0\ ,\ R_1 = 10\text{k}\Omega$

b) $\frac{v_o'}{v_i} = 2 = 1 + \frac{R_2}{R} \Longrightarrow R_1 = R_2 = 10\text{K}\Omega$

c) $\frac{v_o}{v_i} = 101\ V/V = 1 + \frac{R2}{R_1} \Longrightarrow \text{if } R_1 = 10^{\text{k}\Omega} \Longrightarrow R_2 = 1^{\text{M}\Omega}$

d) $\frac{v_o'}{v_i} = 100\ V/V = 1 + \frac{R_2}{R_1} \longrightarrow \text{if } R_1 = 10^{\text{k}\Omega} \Longrightarrow R_2 = 990\ \text{k}\Omega$

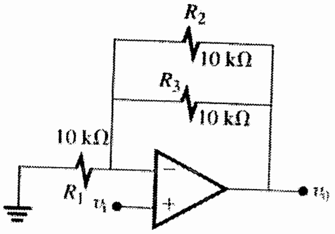
2.39


$R_3 = 10\text{ k}\Omega$
$R_2 = 10\text{ k}\Omega$
$R_1 = 10\text{ k}\Omega$
$-$
$+$
v_1
v_0

Short-circuit R_2:

$$\frac{v_o}{v_i} = 2$$

Short-circuit R_3:



$$\frac{v_o}{v_i} = 1$$

2.40

$v_+ = v_- = v_o = R \times i,\ i = 100\ \mu\text{A}$ when

$v = 10\text{V}$

$$\Rightarrow R = \frac{10}{0.1\text{ mA}} = 100\text{ k}\Omega$$

As indicated, i only depends on R and v and the meter resistance does not affect i.

2.41

$v_o = v_{I1} + 3v_{I2} - 2(v_{I3} + 3v_{I4})$

$$\frac{R_F}{R_{N3}} = 2 \text{ if } R_{N3} = 10\text{ k}\Omega \Rightarrow R_F = 20\text{ k}\Omega$$

$$\frac{R_F}{R_{N4}} = 6 \Rightarrow R_{N4} = \frac{20}{6} = 3.3\text{ k}\Omega$$

$$R_N = R_{N3} \parallel R_{N4} = 10\text{ K} \parallel 3.3\text{ k} = 2.48\text{ k}\Omega$$

$$\left(1 + \frac{R_F}{R_N}\right)\frac{R_P}{R_o} = 1 \Rightarrow \left(1 + \frac{20}{2.48}\right)\frac{R_P}{R_{P1}}$$

$$= 1 \Rightarrow 9.06\ R_P = R_{P1}$$

$$R_P = R_{P1} \parallel R_{P2} \parallel R_{P3} \Rightarrow R_P$$

$$= \frac{1}{\frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P0}}}$$

$$\left(1 + \frac{R_F}{R_N}\right)\frac{R_P}{R_{P2}} = 3 \Rightarrow 9.06\frac{R_P}{R_{P2}} = 3 \Rightarrow R_{P2} \simeq 3R_P$$

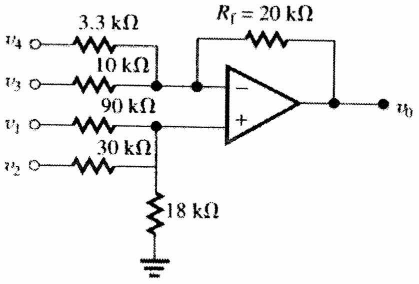
$$R_{P1} \parallel R_{P2} = \frac{9 \times 3R_P}{9 + 3} = 2.25R_P,$$

$$R_P = 2.25R_P \parallel R_{P0}$$

$$\Rightarrow R_P + R_{PO} = 2.25\ R_{PO} \Rightarrow R_{PO} = 1.8\ R_P$$

$$R_P = 10\text{ k}\Omega \Rightarrow R_{PO} = 18\text{ k}\Omega$$

$R_{P1} = 9 \times 10\text{ k} = 90\text{ k}\Omega$
$R_{P2} = 3 \times 10\text{ k} = 30\text{ k}\Omega$



2.42

$$v_+ = v_I \frac{R_4}{R_3 + R_4} = v$$

$$\frac{v}{R_1} = \frac{v_o - v_-}{R_2} \Rightarrow v_o = v_-\left(1 + \frac{R_2}{R_1}\right)$$

From the two above equations:

$$\frac{v_o}{v_I} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) = \frac{1 + R_2/R_1}{1 + R_3/R_4}$$

2.43

Setting $v_2 = 0$, we obtain the output component due to v_1 as:

$v_{o1} = -20\ v$

Setting $v_1 = 0$, we obtain the output component due to v_2 as:

$$v_{O2} = v_2\left(1 + \frac{20R}{R}\right)\left(\frac{20R}{20R + R}\right) = 20\ v_2$$

The total output voltage is:

$v_o = v_{o1} + v_{o2} = 20\,(v_2 - v_1)$

For $v_1 = 10\text{Sin}\,2\pi \times 60t - 0.1\ \text{Sin}\,(2\pi \times 1000\ t)$

$v_2 = 10\text{Sin}\,2\pi \times 60t + 0.1\ \text{Sin}\,(2\pi \times 1000\ t)$

$v_o = 4\text{Sin}\,(2\pi \times 1000\ t)$

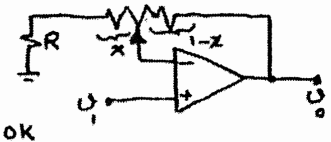
2.44

$$\frac{v_o}{v_I} = 1 + \frac{R_2}{R_1} = 1 + \frac{(1-x)}{x} = 1 + \frac{1}{x} - 1 = \frac{1}{x}$$

$$0 < x \leq 1 \Longrightarrow 1 \leq \frac{v_o}{v_I} < \infty$$

if we add a resistor on the ground path:

$$\frac{v_o}{v_i} = 1 + \frac{(1-x) \times 10K}{x \times 10K + R}$$

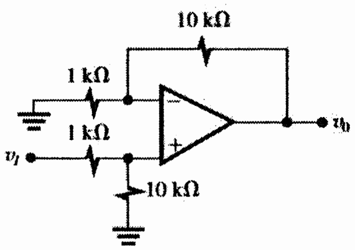


$\text{Gain}_{\text{max}} = 21$ when

$$x = 0 \Longrightarrow 21 = 1 + \frac{10K}{R}$$

$$\Longrightarrow R = \frac{10K}{20} = 0.5\text{k}\Omega$$

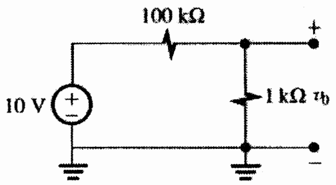
2.45



$$v_O = v_I \frac{10}{1 + 10}\left(1 + \frac{10}{1}\right)$$

$$v_O = 10\ v_I$$

2.46



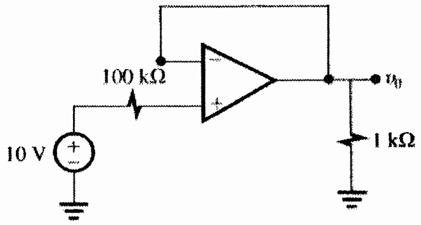
a) Source is connected directly.

$$v_O = 10 \times \frac{1}{101} = 0.099\ \text{V}$$

$$i_L = \frac{v_O}{1\ \text{k}\Omega} = \frac{0.099}{1} = 0.099\ \text{mA}$$

Current supplied by the source is 0.099 mA.

b) inserting a buffer



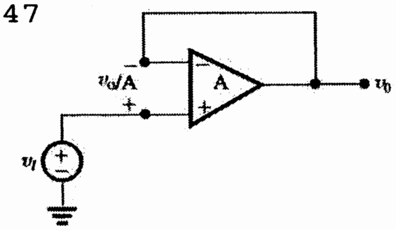
$$v_O = 10\ \text{V}$$

$$i_L = \frac{10\ \text{V}}{1\ \text{K}} = 10\ \text{mA}$$

current supplied by the source is 0.

The load current i_L comes from the power supply of the op-amp.

2.47



$$v_O = v_I - \frac{v_O}{A}$$

$$\frac{v_O}{v_I} = \frac{1}{1+\frac{1}{A}}$$

error of Gain magnitude

$$\frac{\left|\frac{v_O}{v_I} - 1\right|}{1} = -\frac{1}{A+1}$$

$A\left(\frac{V}{V}\right)$	1000	100	10
$\frac{V_O}{V_i}\left(\frac{V}{V}\right)$	0.999	0.990	0.909
Gain error	−0.1%	−1%	−9.1%

2.48

$$A = 50 \text{ V/V} \quad 1+\frac{R_2}{R_1} = 10 \text{ V/V}$$

if $R_1 = 10\ \text{k}\Omega \Rightarrow R_2 = 90\ \text{k}\Omega$

$$G = \frac{v_O}{v_i} = \frac{1+\frac{R_2}{R_1}}{1+\frac{1+R_2/R_1}{A}}$$

$$G = \frac{1+98/10}{1+\frac{1+90/10}{50}} = \frac{10}{1.2} = 8.33 \text{ V/V}$$

In order to compensate the gain drop, we can shunt a resistor with R_1.

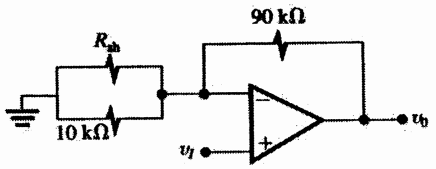
Compensated:

$$R_{sh}\text{: } 10 = \frac{1+\left(\frac{90}{10}+\frac{90}{R_{sh}}\right)}{1+\frac{1+\frac{90}{10}+\frac{90}{R_{sh}}}{50}} \Rightarrow$$

$$10 \times (510R_{sh} + 90R_{sh} + 900)$$

$$= 50 \times (10R_{sh} + 90R_{sh} + 900)$$

$$100R_{sh} = 3600 \Rightarrow R_{sh} = 36\ \text{k}\Omega$$



If A = 100 then:

$$G_{\text{uncompensated}} = \frac{1+\frac{90}{10}}{1+\frac{1+90/10}{100}} = \frac{10}{1.1} = 9.09 \text{ V/V}$$

$$G_{\text{compensated}} = \frac{1+\frac{90}{10}+\frac{90}{36}}{1+\frac{1+\frac{90}{10}+\frac{90}{36}}{100}}$$

$$= \frac{125}{1.125} = 11.1 \text{ V/V}$$

2.49

$$G = \frac{G_O}{1+\frac{G_O}{A}},\ \frac{G_O-G}{G_O} \times 100 = \frac{G_O/A \times 100}{1+\frac{G_O}{A}} \le x$$

$$\text{or } \frac{1+\frac{G_O}{A}}{\frac{G_O}{A}} \ge \frac{100}{x} \Rightarrow \frac{A}{G_O} \ge \underbrace{\left(\frac{100}{x}-1\right)}_{F}$$

$$\Rightarrow A \ge G_O F \text{ where } F = \frac{100}{x} - 1 \simeq \frac{100}{x}$$

x	0.01	0.1	1	10
F	10^4	10^3	10^2	10

Thus for:

$x = 0.01$:					
G_O (V/V)	1	10	10^2	10^3	10^4
A(V/V)	10^4	10^5	10^6	10^7	10^8

$\underbrace{10^7 \quad 10^8}_{}$ too high to be practical

$x = 0.1$:					
G_O (V/V)	1	10	10^2	10^3	10^4
A(V/V)	10^3	10^4	10^5	10^6	10^7

$x = 1$:					
G_O (V/V)	1	10	10^2	10^3	10^4
A(V/V)	10^2	10^3	10^4	10^5	10^6

$x = 10$:					
G_O (V/V)	1	10	10^2	10^3	10^4
A(V/V)	10	10^2	10^3	10^4	10^5

2.50

for non-inverting amplifier

$$G = \frac{G_O}{1 + \frac{G_O}{A}} \quad , \epsilon = \frac{G_O - G}{G_O} \times 100$$

for inverting amplifier

$$G = \frac{G_O}{1 + \frac{1 - G_O}{A}} \quad , \epsilon = \frac{G_O - G}{G_O} \times 100$$

case	G_O (V/V)	A(V/V)	G (V/V)	ϵ%
a	−1	10	−0.83	16
b	1	10	0.91	9
c	−1	100	−0.98	2
d	10	10	5	50
e	−10	100	−9	10
f	−10	1000	−9.89	1.1
g	+1	2	0.67	33

2.51

when potentiometer is set to the bottom:

$$v_O = v_+ = -15 + \frac{30 \times 20}{20 + 100 + 20} = -10.714 \text{ V}$$

when set to the top:

$$v_O = -15 + \frac{30 \times 20}{20 + 100 + 20} = 10.714 \text{ V}$$

$$\Rightarrow -10.714 \leq v_O \leq +10.714$$

pot has 20 turn, each turn:

$$\Delta v_O = \frac{2 \times 10.714}{20} = 1.07 \text{ V}$$

2.52

Notica that

we have : $\frac{R_4}{R_3} = \frac{R_2}{R_1} = \frac{100}{10}$.

therefore

$$v_0 = \frac{R_2}{R_1} v_{Id} \Rightarrow A = \frac{R_2}{R_1} = 10 \text{ V/V}$$

$$R_{id} = 2R_1 = 20\text{k}\Omega$$

If $\frac{R_2}{R_1}$, $\frac{R_4}{R_3}$ were different by 1% :

$$\frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3}$$

2.53

If we assume $R_3 = R_1$, $R_4 = R_2$, then

$$R_{id} = 2R_1 \Rightarrow R_1 = \frac{20}{2} = 10 \text{ k}\Omega$$

a) $A_d = \frac{R_2}{R_1} = 1 \text{ V/V} \Rightarrow R_2 = 10 \text{ k}\Omega$

$R_1 = R_2 = R_3 = R_4 = 10 \text{ k}\Omega$

b) $A_d = \frac{R_2}{R_1} = 2 \text{ V/V} \Rightarrow R_2 = 20 \text{ k}\Omega = R_4$

$R_1 = R_3 = 10 \text{ k}\Omega$

c) $A_d = \frac{R_2}{R_1} = 100 \text{ V/V} \Rightarrow R_2 = 1 \text{ M}\Omega = R_4$

$R_1 = R_3 = 10 \text{ k}\Omega$

d) $A_d = \frac{R_2}{R_1} = 0.5 \text{ V/V} \Rightarrow R_2 = 5 \text{ k}\Omega = R_4$

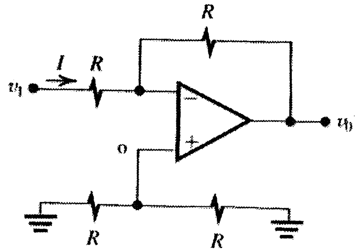
$R_1 = R_3 = 10 \text{ k}\Omega$

2.54

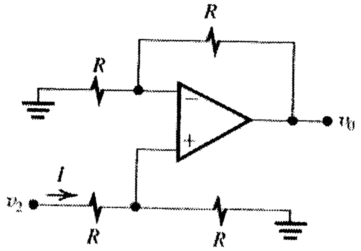
Considering that $v_- = v_+$:

$$v_1 + \frac{v_O - v_1}{2} = \frac{v_2}{2} \Rightarrow v_O = v_2 - v_1$$

v_1 only: $R_I = \frac{v_1}{I} = R$



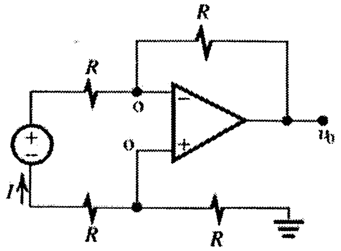
v_2 only: $R_I = \frac{v_2}{I} = 2R$



v_S between 2 terminals:

$$R_I = \frac{v}{I} = 2R$$

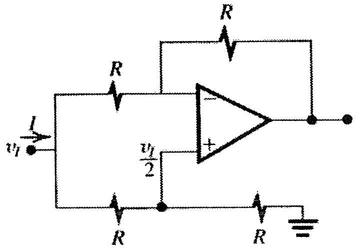
$$v_+ = v_- = 0$$



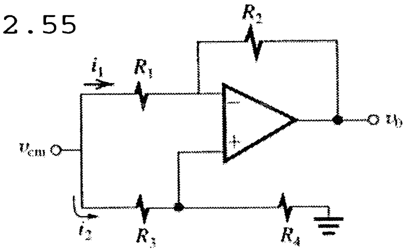
v_3 connected to both v_1 & v_2:

$$R_I = \frac{v}{I} = R$$

$$v_+ = v_- = \frac{v_I}{2}$$



2.55



$$v_+ = v_{cm}\frac{R_4}{R_3 + R_4}$$

$$v_+ = v_-$$

$$i_2 = \frac{v_{cm}}{R_3 + R_4}$$

$$i_1 = \frac{v_{cm}}{R_1} - \frac{v_{cm}R_4}{R_3 + R_4} = \frac{1}{R_1} = \frac{v_{cm}}{R_1}\frac{R_3}{R_3 + R_4}$$

$$i = i_1 + i_2 = \frac{v_{cm}}{R_1}\frac{R_3}{R_3 + R_4} + \frac{v_{cm}}{R_3 + R_4}$$

if we replace $\frac{R_4}{R_3}$: with $\frac{R_2}{R_1}$: $\left(\frac{R_4}{R_3} = \frac{R_2}{R_1}\right)$

$$\frac{1}{R_I} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$$

$$\Rightarrow R_5 = (R_1 + R_2) \parallel (R_3 + R_4)$$

2.56

$$A_{cm} = \frac{v_o}{v_{ICM}} = \frac{R_4}{R_3 + R_4}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

The worst case is when A_{cm} has its maximum value.

$$A_{cm} = \frac{1}{\frac{R_3}{R_4} + 1}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

Max A_{cm} ⇒ $\frac{R_3}{R_4}$ has to be at its minimum value and also $\frac{R_2}{R_1}$ has to be minimum.

$$\frac{100-x}{100+x} \leqslant \frac{R_3}{R_4} \leqslant \frac{100+x}{100-x} \qquad \frac{100-x}{100+x} \leqslant \frac{R_2}{R_1} \leqslant \frac{100+x}{100-x}$$

so if $\frac{R_3}{R_4} = \frac{100-x}{100+x}$ & $\frac{R_2}{R_1} = \frac{100-x}{100+x}$

$$A_{cm\,Max} = \frac{1}{\frac{100-x}{100+x} + 1}\left(1 - \frac{100-x}{100+x}\,\frac{100-x}{100+x}\right)$$

$$A_{cm}Max = \frac{1}{200}\,\frac{(100+x)^2 - (100-x)^2}{100+x} = \frac{2x}{100+x} \simeq \frac{x}{50}$$

x	0.1	1	5
$A_{cm\,Max}$	0.002	0.02	0.1

$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right|$. Now we have to calculate A_d based on values we chose for $R_1 - R_4$ that gave us $A_{cm\,Max}$.

$R_2 = R_3 = 100 - x$ $\qquad R_1 = R_4 = 100 + x$

$v_o = v_{o1} + v_{o2}$ by applying superposition

$$v_o = -\frac{R_2}{R_1}v_1 + v_2\frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right)$$

$$v_o = -\frac{100-x}{100+x}v_1 + v_2\frac{100+x}{200}\left(1 + \frac{100-x}{100+x}\right)$$

$$v_o = -\frac{100-x}{100+x}v_1 + v_2$$

if we consider $\frac{100-x}{100+x} \simeq 1 \Rightarrow \frac{v_o}{v_{id}} \simeq 1$

Cont.

$$CMRR = 20\log\frac{Ad}{Acm} = 20\log\frac{1}{x/50} = 20\log\frac{50}{x}$$

x	0.1	1	5
CMRR	54db	34db	20db

2.57

$$A_{cm} = \frac{R_4}{R_3+R_4}\left(1-\frac{R_2R_3}{R_1R_4}\right)$$

In order to calculate A_d, we use Superposition principle:

$$v_O = v_{01} + v_{02} = \frac{-R_2}{R_1}v_1 + v_2\frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)$$

then replace $v_1 = v_{cm} - \frac{v_d}{2}$

$$v_2 = v_{cm} + \frac{v_d}{2}$$

$$v_O = -\frac{R_2}{R_1}v_{cm} + \frac{R_2}{R}v_{d/2} + v_{cm}$$

$$\frac{1+\frac{R_2}{R_1}}{1+\frac{R_3}{R_4}} + \frac{v_O}{2}\,\frac{1+R_2/R_1}{1+\frac{R_3}{R_4}}$$

$$v_O = \underbrace{\frac{R_2}{2R_1}\left[1+\frac{R_1/R_2+1}{R_3/R_4+1}\right]}_{Ad}v_d + \frac{R_2}{R_1}\left[-1+\frac{R_1/R_2+1}{R_3/R_4+1}\right]v_{cm}$$

$$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = 20\log\frac{\frac{R_2}{2R_1}\left[1+\frac{R_1/R_2+1}{R_3/R_4+1}\right]}{\frac{1}{\frac{R_3}{R_4}+1}\left(1-\frac{R_2R_3}{R_1R_4}\right)}$$

$$CMRR = 20\log\left|\frac{\frac{1}{2}\frac{R_2}{R_1}\left[2+\frac{R_1}{R_2}+R_3/R_4\right]}{1-\frac{R_2}{R_1}\cdot\frac{R_3}{R_4}}\right|$$

$$CMRR = 20\log\left|\frac{1+\frac{1}{2}\frac{R_1}{R_2}+\frac{1}{2}\frac{R_3}{R_4}}{\frac{R_1}{R_2}\cdot\frac{R_3}{R_4}}\right|$$

for worst case, minimum CMRR we have co maximize the denominator, which means:

$R_1 = R_{in}(1+\epsilon)$ $\quad R_3 = R_{gn}(1-\epsilon)$

$R_2 = R_{2n}(1-\epsilon)$ $\quad R_4 = R_{gn}(1+\epsilon)$

also $\frac{R_{2n}}{R_{1n}} = \frac{R_{gn}}{R_{3n}} = K$

$$CMRR = 20\log\left|k\frac{1+\frac{1}{2K}\frac{1+\epsilon}{1-\epsilon}+\frac{1}{2K}\frac{1-\epsilon}{1+\epsilon}}{\frac{1+\epsilon}{1-\epsilon}-\frac{1-\epsilon}{1+\epsilon}}\right|$$

$$CMRR = 20\log\left|\frac{k(1-\epsilon^2)+(1+\epsilon^2)}{4\epsilon}\right| \simeq$$

$$20\log\left|\frac{k+1}{4\epsilon}\right|$$

for $\epsilon^2 << 1$.

if $k = A_{d\,\text{ideal}} = 100$, $\epsilon = 0.01$

$$CMRR = 20\log\frac{101}{0.04} = 68\text{ db}$$

2.58

$$A_{cm} = \frac{R_4}{R_3+R_4}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

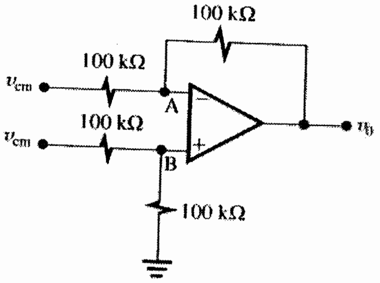
$$= \frac{100}{100+100}\left(1-\frac{100.100}{100.100}\right)$$

$A_{cm} = 0$

Refer to 2.17: $\frac{R_2}{R_1} = \frac{R_4}{R_3}$

$\Rightarrow A_d = \frac{R_2}{R_1} = 1$

b) Since $A_{cm} = 0$,



then if we apply V_{cm} to V_{I1} and V_{I2}, $v_o = 0$.

Therefore, $V_A = v_{cm}\frac{100}{100+100}$

$$V_A = \frac{v_{cm}}{2}$$

Similarly, $v_B = \frac{v_{cm}}{2}$

we know $V_A = V_B$ and $-2.5 \le v_A \le 2.5$

$\Rightarrow -5 \le v_{cm} \le 5$

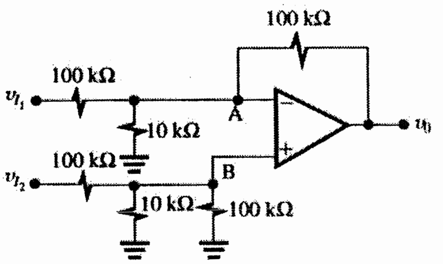
c) we aply the Superposition principle to calculate A_d.

v_{o1} is the output voltage when $v_{I2} = 0$

v_{o2} is the output voltage when $v_{I1} = 0$

$v_o = v_{o1} + v_{o2}$

$$v_{o1} = \frac{-R_2}{R_1} v_{I1} = -v_{I1}$$



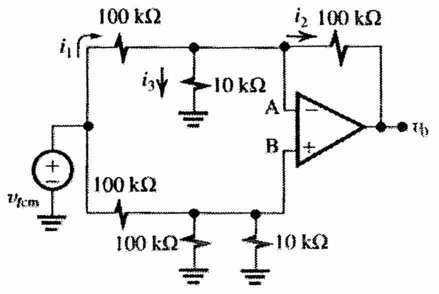
$$v_{o2} = v_{I2} \frac{100\text{ k}\Omega \parallel 10\text{ k}\Omega}{100\text{ k}\Omega \parallel 10\text{ k}\Omega + 100}$$

$$\left(1 + \frac{100\text{ k}\Omega}{100\text{ k}\Omega \parallel 10\text{ k}\Omega}\right)$$

$v_{o2} = v_{I2} \times 1$

$\Rightarrow v_o = v_{o1} + v_{o2} = -v_{I1} + v_{I2} \Rightarrow A_d = 1$

Now we calculate A_{cm}:



$$v_B = v_{ICM} \frac{100\text{ k}\Omega \parallel 10\text{ k}\Omega}{100\text{ k}\Omega \parallel 10\text{ k}\Omega + 100\text{ k}\Omega}, \; v_A = v_B$$

$$i_1 = \frac{v_{ICM} - v_A}{100\text{ k}\Omega}$$

$v_o = v_A - 100\text{ k}\Omega \times i_2$ and

$$i_2 = i_1 - i_3 = i_1 - \frac{v_A}{10\text{ k}\Omega}$$

$v_o = v_A - 100\text{ k}\Omega \times i_1 + 10 \times v_A$

$v_o = v_A - v_{ICM} + v_A + 10 \times v_A$

$v_A = v_B \Rightarrow v_o$

$$= v_{ICM}\left(-1 + 12\frac{100\text{ k}\Omega \parallel 10\text{ k}\Omega}{(100\text{ k}\Omega \parallel 10\text{ k}\Omega) + 100\text{ k}\Omega}\right)$$

$$\frac{v_o}{v_{ICM}} = A_{cm} = 0$$

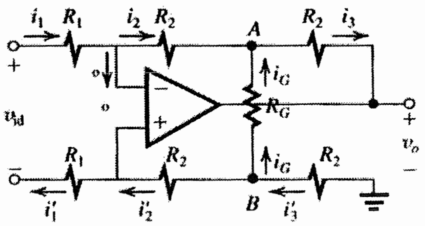
Now we calculate v_{ICM} range:

$-25 \le v_B \le 2.5 \Rightarrow$

$$-2.5 < v_{ICM} \times \frac{100\text{ k}\Omega \parallel 10\text{ k}\Omega}{(100\text{ k}\Omega \parallel 10\text{ k}\Omega) + 100\text{ k}\Omega} < 2.5$$

$-30\text{ V} \le v_{ICM} \le 30\text{ V}$

2.59



$v_+ = v_-$ so we can consider v_+, v_- a virtual short: $i_1 = v_{Id}/2R_1 \Rightarrow i_2 = \dfrac{v_{id}}{2R_1}$

$$i_1' = i_2' = \frac{v_{id}}{2R_1}$$

then:

$$i_2R_2 + v_{AB} + i_2'R_2 = 0 \Rightarrow v_{AB} = \frac{-v_{Id}}{R_1}R_2$$

$$i_G = \frac{v_{id}}{R_G} \times \frac{R_2}{R_1}$$

$$i_3 = i_2 + i_G = \frac{v_{id}}{2R_1} + \frac{v_{id}}{R_G}\frac{R_2}{R_1}$$

$$i_3' = i_G + i_2' = i_3$$

$$\Rightarrow v_o = -[i_3'R_2 + v_{BA} + i_3R_2]$$

$$v_o = -[2i_3R_2 + v_{BA}]$$

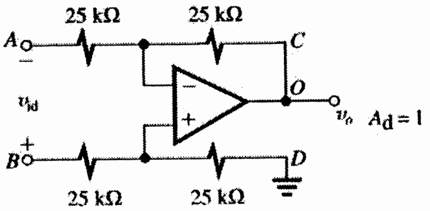
$$v_o = -\left[\frac{2v_{id}}{2R_1}R_2 + 2v_{id}\frac{R_2}{R_1}\frac{R_2}{R_G} + \frac{v_{id}}{R_1}R_2\right]$$

$$\frac{v_o}{v_{id}} = A_d = -2\frac{R_2}{R_1}\left[1 + \frac{R_2}{R_G}\right]$$

2.60

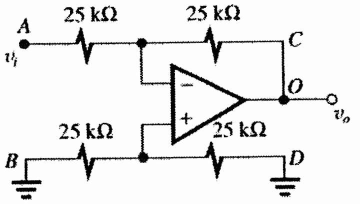
a) $A_d = \frac{R_2}{R_1} = 1$. Connect c and o together

a)

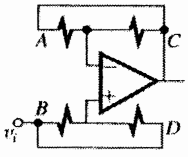

25 kΩ
25 kΩ
A
C
O
v_{id}
−
+
B
D
25 kΩ
25 kΩ
v_o
Ad = 1

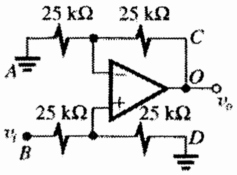
b) $\frac{v_o}{v_i} = -1$ V/V

i)


A
v_i
25 kΩ
25 kΩ
C
O
v_o
25 kΩ
25 kΩ
B
D

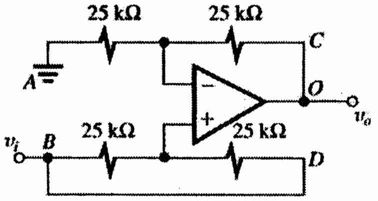
ii) $\frac{v_o}{v_i} = +1$ V/V

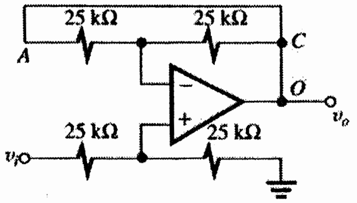



25 kΩ
25 kΩ
A
C
O
v_o
25 kΩ
25 kΩ
v_i
B
D

The circuit on the left ideally has infinite input resistance

iii) $\frac{v_o}{v_i} = +2$ V/V


25 kΩ
25 kΩ
C
A
O
v_o
v_i
B
25 kΩ
25 kΩ
D

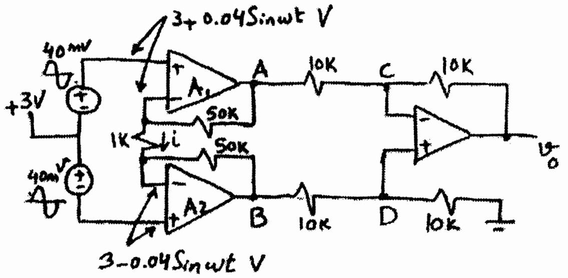

25 kΩ
25 kΩ
C
A
O
v_o
25 kΩ
25 kΩ
v_i

iv) $\frac{v_o}{v_i} = +\frac{1}{2}$ V/V

$$v_+ = \frac{v_i}{2} = v_o$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{1}{2}$$

2.61


3 + 0.04 Sinωt V
40mV
+3V
A₁
A
10K
C
10K
50K
1K
i
50K
v_o
40mV
A2
B
10K
D
10K
3 − 0.04 Sinωt V

$$i = \frac{3 + 0.04\,\text{Sin}\,\omega t - (3 - 0.04\,\text{Sin}\,\omega t)}{1\text{K}} = 0.08\,\text{Sin}\,\omega t,\ \text{mA}$$

$$v_A = 3 + 0.04\,\text{Sin}\,\omega t + 50\text{K} \times i = 3 + 4.04\,\text{Sin}\,\omega t,\ V$$

$$v_B = 3 - 0.04\,\text{Sin}\,\omega t - 50\text{K} \times i = 3 - 4.04\,\text{Sin}\,\omega t,\ V$$

$$v_C = v_D = \frac{1}{2}v_B = 1.5 - 2.02\,\text{Sin}\,\omega t,\ V$$

$$v_o = v_B - v_A = -8.08\,\text{Sin}\,\omega t,\ V$$

2.62 a.

The gain of the first stage is: $\left(1 + \frac{R_2}{R_1}\right) = 101$ If the opamps of the first stage saturate at ±14 V :

$-14\text{ V} \leq v_I \leq +14\text{ V} \Rightarrow -14\text{ V} \leq 101\ v_{icm} \leq +14\text{ V}$

$\Rightarrow -0.14\text{ V} \leq v_{icm} \leq 0.14\text{ V}$

As explained in the text, the disadvantage of circuit in Fig. 2.20a is that v_{icm} is amplified by a gain equal to $v_{icm}\left(1 + \frac{R_2}{R_1}\right)$ in the first stage and therefore a very small v_{icm} range is acceptable to avoid saturation.

b) In Fig. 2.20b, when v_{icm} is applied, v_- for both A_1 & A_2 is the same and therefore no current flows through $2R_1$. This means voltage at the output of A_1 and A_2 is the same as v_{icm}.

$-14 \leq v_o \leq 14 \Rightarrow -14 \leq v_{icm} \leq 14$

This circuit allow for bigger range of v_{icm}.

2.63

$A_d = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) = \frac{100K}{100K}\left(1 + \frac{100k}{5K}\right) = 21\ V/V$

$A_{cm} = 0$

$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = \infty$

If all resistors are ±1% :

$A_d \simeq 21$

In order to calculate A_{cm}, apply V_{cm} to both inputs and note that V_{cm} will appear at both output terminals of the first stage. Now we can evaluate v_o by analyzing the second stage as was done in problem 2.65. In P2.65 we showed that if each 100K resistor has ±x% tolerance, A_{cm} of the differential amplifier is : $A_{cm} = \frac{v_o}{v_{cm}} = \frac{x}{50}$. Therefore the overall A_{cm} is also $\frac{x}{50}$.

$x = 1 \Rightarrow A_{cm} = \frac{1}{50} = 0.02$

$CMRR = 20\log\frac{21}{0.02} = \underline{60\,db}$

If $2R_1 = 1k\Omega$: $A_d = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) = 201\ V/V$

$A_{cm} = 0.02$ unchanged

$CMRR = 20\log\frac{201}{0.02} = \underline{80\,db}$

Conclusion: Large CMRR can be achieved by having relatively large A_d in the first stage.

2.64

$A_{d(2)}$ of the second stage is $\frac{R_4}{R_3} = 0.5$

$R_4 = 100\text{ k}\Omega$, $R_3 = 200\text{ k}\Omega$

we use a series configuration of R_{1F} and R_1 (Pot): $R_1 = 100\text{ k}\Omega$Pot (Fixed)

Minimum gain =

$$\left(1 + \frac{R_2}{R_1}\right) = 0.5\left(1 + \frac{R_2}{\frac{100\text{ K} + R_1}{2}}\right)$$

$$1 \leq A_d \leq 100 \Rightarrow 1 = 0.5\left(1 + \frac{2R_2}{R_{1F} + 100\text{ k}\Omega}\right)$$

$\Rightarrow R_{1F} + 100 = 2R_2$ (1)

Maximum gain $= 100 = 0.5\left(1 + \frac{R_2}{R_{1F}/2}\right) \Rightarrow$

$2R_2 = 199\ R_{1F}$ (2)

(1), (2) $\Rightarrow R_{1F} = 0.505\text{ k}\Omega \simeq 0.5\text{ k}\Omega$

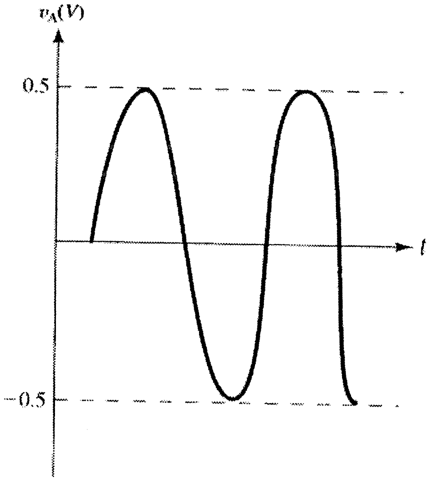
$R_2 = 50.25\text{ k}\Omega \simeq 50\text{ k}\Omega$

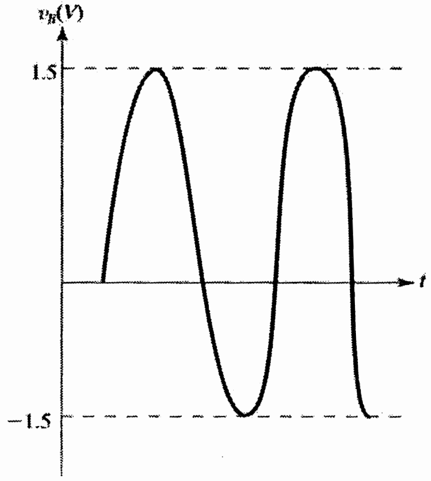
2.65

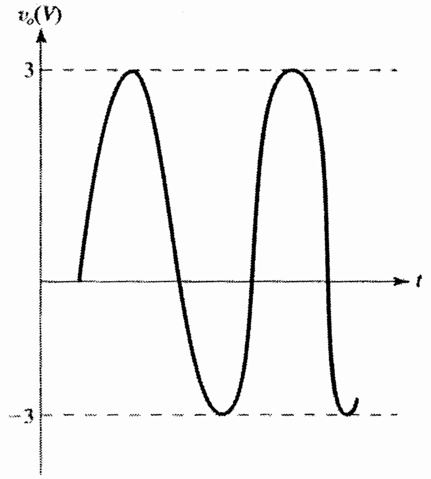
a) $\frac{v_B}{v_A} = 1 + \frac{20}{10} = 3\text{ V/V}$,

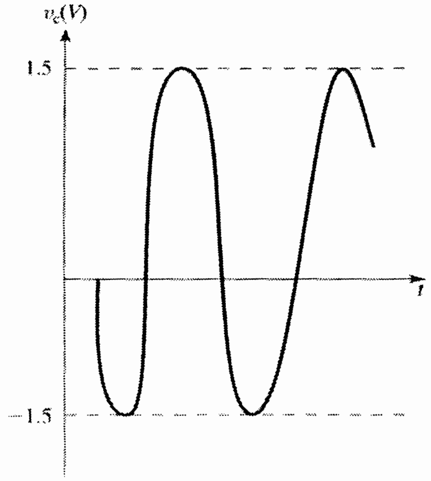
$\frac{v_C}{v_A} = -\frac{30}{10} = -3\text{ V/V}$

b) $v_o = v_B - v_C = 6V_A \Rightarrow \frac{v_o}{v_A} = 6\text{ V/V}$





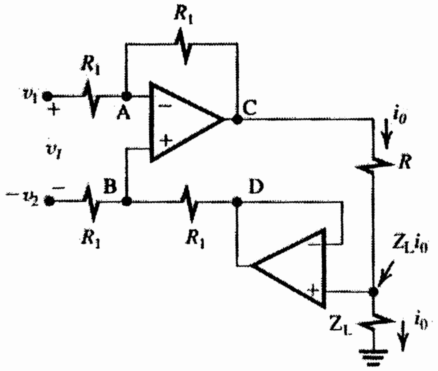




c) v_B and v_C can be ±14 V or 28 V P-P.
$-28 \leq v_o \leq 28$ or 56 VP-P.

$$v_{orms} = 19.8\text{ V} = \frac{28}{\sqrt{2}}$$

2.66



Since the inputs of the op-amp do not draw any current, v_I appears across R_1

$$i_o = \frac{v_1}{R}$$

$$v_D = Z_L i_o$$

we use superposition:

$$v_1 = v_1 - v_2$$

$$v_1 \text{ only: } V_B = \frac{V_O}{2} = \frac{z_L i_{o1}}{2}$$

$$\frac{v_1 - \frac{z_L i_{o1}}{2}}{R_1} = \frac{\frac{z_L i_{o1}}{2} - i_{o1}(z_L + R)}{R_1}$$

$$\Rightarrow v_1 = i_{o1}R \Rightarrow i_{o1} = \frac{v_1}{R}$$

Now if only $(-v_2)$ is applied:

$$v_B = \frac{-v_2 + z_L i_{o2}}{2}, \quad v_A = \frac{i_{o2} \times (R + z_L)}{2}$$

$$v_A = v_B \Rightarrow -v_2 + z_L i_{o2} = i_{o2}R + i_{o2}z_L$$

$$-v_2 = i_{o2}R \Rightarrow i_{o2} = \frac{-V_2}{R}$$

The total current due to both sources is:

$$i_o = i_{o1} + i_{o2} = \frac{v_1}{R} - \frac{v_2}{R} = \frac{v_1}{R}$$

The circuit has ideally infinite input resistance, and if requires that both terminals of Z_L be available, while the other circuit has finite input resistance with one side of Z_L grounded.

2.67

$$\frac{v_o}{v_i} = \frac{-1}{sCR} = \frac{-1}{j\omega CR} = \frac{1}{-j\omega \times 10\times 10^{-9}\times 100\times 10^{3}}$$

$$\frac{v_o}{v_i} = -\frac{10^3}{j\omega}$$

a) $\frac{v_o}{v_i} = 1 \Rightarrow \omega = 1^{K\,Rad/s} \Rightarrow \underline{f = 159\,HZ}$

b) $\frac{1}{j}$ indicates 90° lag, but since its $\frac{-1}{j}$, it results in output leading the input by 90°

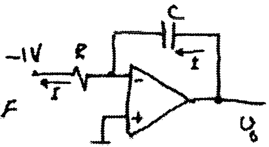
c) $\frac{v_o}{v_i} = \frac{-10^3}{j\omega}$ if frequency if lowered by a factor of 10, then the output would in crease by a factor of 10.

d) The phase doesnot change and the output still leads the input by 90°

2.68

$R_{in} = R = 100K\Omega$

$CR = 1S \Rightarrow C = \frac{1}{100\times 10^3} = 10\mu F$



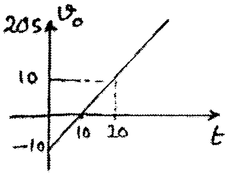
with a −1V dc input applied, the capacitor charges with a constant current:

$I = \frac{1V}{R} = 0.01\,mA$ and its voltage rises linearly:

$$v_o(t) = -10 + \frac{1}{C}\int_0^t I\,dt = -10 + \frac{I}{C}t = -10 + \frac{t}{RC}$$

the voltage reaches 0V at $t = 10RC = 10s$

and it reaches 10V at $t = 20s$



2.69

$|T| = \frac{1}{\omega RC}$ if |T| = 100 V/V for f = 1 kHz,

then for |T| = 1 V/V, f has to be 1 kHz× 100 = 100 kHz.

Also

$$RC = \frac{1}{\omega T} = \frac{1}{2\pi \times 1\text{ kHz} \times 100} = 1.59\ \mu s$$

2.70

$R_{in} = R$, Thus $R = 100\text{ k}\Omega$.

$$|T| = \frac{1}{\omega RC} = 1 \text{ at } \omega = \frac{1}{RC}.$$

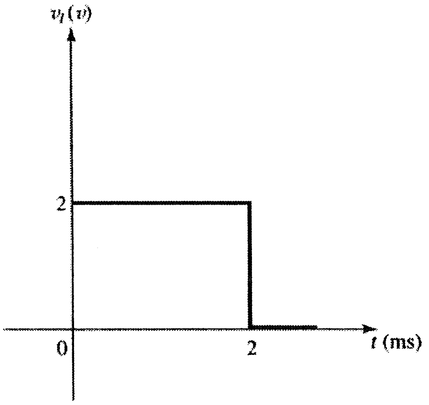
$$\omega = 1000\text{ rad}/s = \frac{1}{RC} \Rightarrow C = \frac{1}{1000 \times 100^2} = 10\text{mF}$$

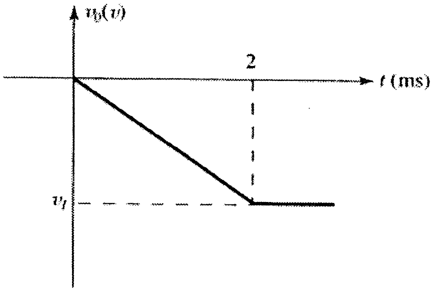
with a 2V-2ms pulse at the input, the output falls linearly until t = 2ms at which

$$v_o = v_1,\ v_0 = \frac{-I}{C}t = \frac{-2}{RC}t = -2t \text{ Volts}$$

where t in ms

Thus $v_1 = -4$ V





with V_I = 2sin 1000t applied at the input,

$$v_o(t) = 2 \times \frac{1}{1000 \times 10^{-3}} \sin(1000t + 90°)$$

$$v_o(t) = 2\sin(1000t + 90°)$$

2.71

$R_{in} = R = 20\ \text{k}\Omega$

$|T| = \frac{1}{\omega RC} = 1$ at

$\omega = 2\pi \times 10\ \text{kHz} \Rightarrow C = \frac{1}{2\pi \times 10\ \text{kHz} \times 20\ \text{K}}$

$C = 0.796\ \text{nF}$

$\frac{v_o}{v_i} = \frac{R_F / R}{1 + S\,CR_F}$ and the finite dc gain is $\frac{-R_F}{R}$.

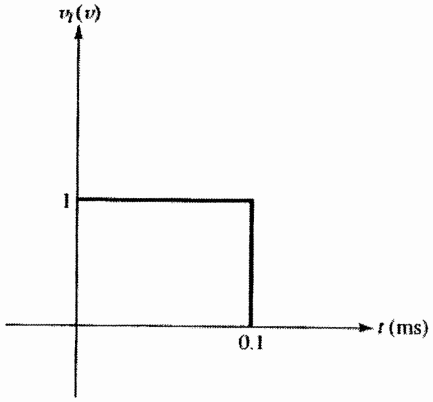
There fore for 40db gain or equivalently 100 V/V

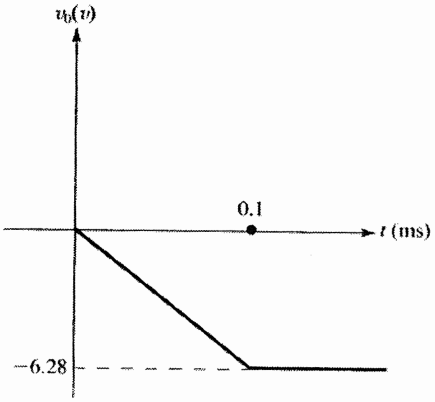
we have: $\frac{-R_F}{R} = -100$ V/V

$\Rightarrow R_F = 100 \times 20\ \text{k} = 2\ \text{M}\Omega$

The corner frequency $\frac{1}{C\ \ /R_F}$ is:

$\frac{1}{0.796\text{m} \times 2\text{M}} = 628\ \text{Hz}$





a) when no R_F

$v_o(t) = \frac{-1}{RC}\int_0^t 1.\text{dt} = -62.8t \quad 0 \leqslant t \leqslant 0.1\ \text{ms}$

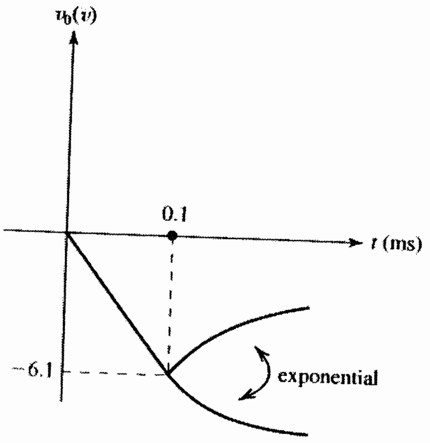
$v_o(0.1) = -6.28\ \text{V}$

b) with R_F: $v_o(t) = v_o(\infty)(i - e^{-t/CR_F})$

(Refer to pg. 112)

$v_o(\infty) = -I \times R_F = -\frac{1\ \text{V}}{20\ \text{K}} \times 2\ \text{M} = -100\ \text{V}$

$v_o(t) = -100(1 - e^{-t/1.5})$

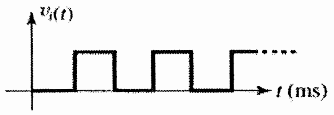


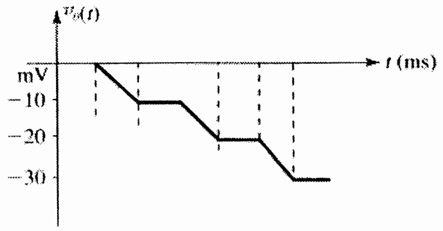
2.72

Each pulse lowers the output voltage by:

$\Delta v_o = \frac{1}{RC}\int_0^{10\text{ms}} 1.\text{dt} = \frac{10\ \mu\text{s}}{RC} = \frac{10\ \mu\text{s}}{1\ \text{ms}} = 10\ \text{mV}$

Therefore a total of 100 pulses are required to cause a change of 1 V in $v_o(t)$.





2.73

$$\frac{v_o}{v_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{1/R_1}{1/R_2 + sC} = -\frac{R_2/R_1}{1+sCR_2}$$

which is an STC LP circuit with a dc gain of $-\frac{R_2}{R_1}$ and a 3-db frequency $\omega_0 = \frac{1}{CR_2}$.

The input resistance equal to R_1. So for:

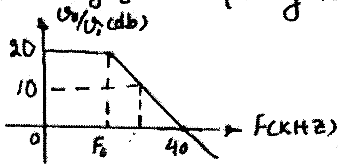
$R_i = 1K \Rightarrow \underline{R_1 = 1K\Omega}$ and for dc-gain of 20db or

10 : $\frac{R_2}{R_1} = 10 \Rightarrow \underline{R_2 = 10\ K\Omega}$

for 3db frequency of 4KHZ: $\omega_0 = 2\pi \times 4 \times 10^3 = \frac{1}{CR_2}$

$\Rightarrow C \approx \underline{4nF}$

the unity gain frequency is (0db) is 40 KHZ



2.74

$$\frac{v_O}{v_i}(s) = -sRC = -s \times 0.01 \times 10^{-6} \times 10 \times 10^3$$

$$= -10^{-4}s$$

$$\frac{v_O}{v_i}(jw) = -jw \times 10^{-4} \Rightarrow \left|\frac{v_O}{v_i}\right| = -w \times 10^{-4}$$

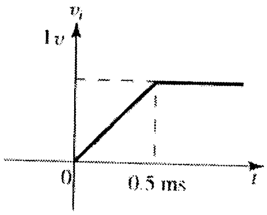
$$\Rightarrow \left|\frac{v_O}{v_i}\right| = 1 \text{ when } w = 10^4 \text{rad/s}$$

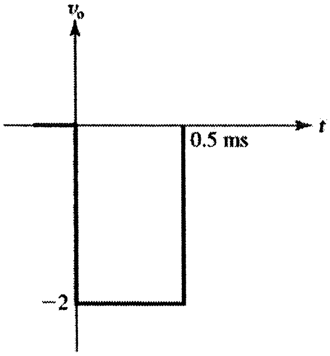
or $f = 1.59$ kHz

for an input 10 times this frequency, the output will be 10 times as large as the input: 10V peak-to-peak. The (-j) indicates that the output lags the input by 90°. Thus

$$v_O(t) = -5\sin(10^5 t + 90°) \text{ Volts}$$

2.75





$$v_O = -CR\frac{dv_i}{dt}$$

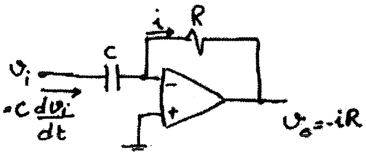
therefore:

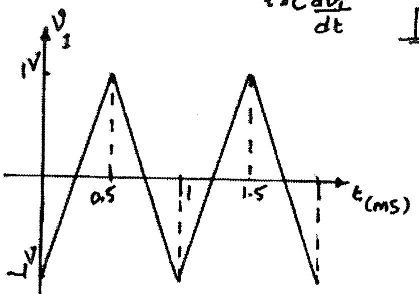
For $0 \le t \le 0.5$:

$$v_O = -1\text{ms} \times \frac{1\text{ V}}{0.5\text{ ms}} = -2\text{ V}$$

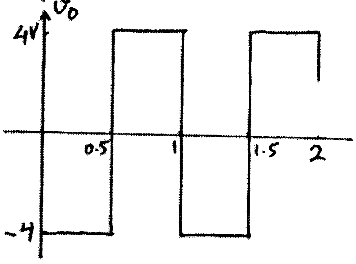
and $v_O = 0$ otherwise

2.76





$$\text{Slope} = \frac{2V}{0.5ms} = 4V/ms$$



$$C\frac{dv_i}{dt} = 0.1 \times 10^{-6} \times \frac{4}{10^{-3}} = 0.4mA$$

Thus the peak value of the output square wave is $0.4\text{mA}\times 10^{k\Omega} = 4V$. The frequency of the output is the same as the input (1KHz).

The average value of the output is 0.

To increase the value of the output to 10^V, R has to be increased to $\frac{10}{4} = 2.5$, i.e $25^{k\Omega}$.

When a 1-KHz, 1V peak input sine wave is applied

$v_i = \text{Sin}(2\pi\times 1000\, t)$

a sinusoidal signal appears at the output.

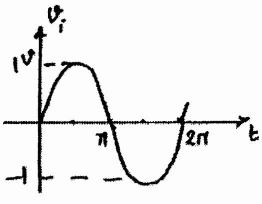
It can be determined by one of the following methods:

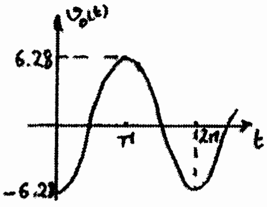
a) $v_o(t) = -RC\,\frac{dv_i}{dt} = -0.1\times 10^{-6}\times 10\times 10^{3}\frac{dv_i}{dt} = -10^{-3}\frac{dv_i}{dt}$

$v_o(t) = -10^{-3}\times 2\pi\times 1000\times \text{Cos}(2\pi\times 1000t)$

$v_o(t) = -2\pi\,\text{Cos}(2\pi\times 1000t)$

Thus the peak amplitude is 6.28V and the negative peaks occur at $t = 0, \frac{2\pi}{2\pi\times 1000}, \ldots$





b) $\frac{v_o}{v_i} = -SRC \Rightarrow \frac{v_o}{v_i}(j\omega) = -j\omega RC \Rightarrow v_o(t) = -j\omega RC\, v_i(t)$

the output is inverted and has 90° phase shift, due to (-j) factor.

$v_o(t) = -(\omega RC)\times 1\ \text{Sin}(2\pi\times 1000t + 90^\circ)$

$v_o(t) = -6.28\ \text{Sin}(2\pi\times 1000t + 90^\circ)$

$v_o(t) = -6.28\ \text{Cos}(2\pi\times 1000t)$

Same as before.

c) The peaks of the output waveform are equal to RC×(maximum slope of input wave). Since the maximum slope occurs at the zero crossings, its value is $2\pi\times 1000$. Thus the peak output $= 2\pi\times 1000\times RC = 6.28V$

The negative peak occurs at $\omega t = 0, 2\pi, \ldots$

2.77

$RC = 10^{-3}$s when

$C = 10\text{mF} \Rightarrow R = 100\ \text{k}\Omega$

$\frac{v_O}{v_i} = -\text{sRC},\ \frac{v_O}{v_i}(jw) = -jwRC$

$\varphi = -90°$ always

$\left|\frac{v_O}{v_i}\right| = 1 \Rightarrow w = \frac{1}{\text{unity } RC} = 1\text{krad/s}$

Gain is 10 times the unity gain, when = the frequency is 10 times the unity gain frequency. Similarly for $w = \frac{1}{10}$ krad/s, gain is 0.1 V/V . (for $w = 10$ krad/s, gain $= 10$ V/V)

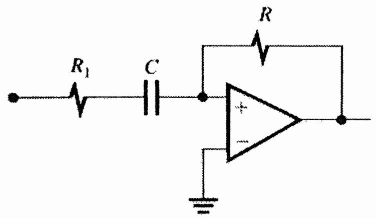
for high frequency C is short circuited,

$\frac{v_O}{v_i} = \frac{-R}{R_1} = -100 \Rightarrow R_1 = 1\ \text{k}\Omega$

$\frac{v_O}{v_i} = \frac{-RCs}{R_1Cs + 1} = \frac{-10^{-3}s}{10^{-5}s + 1}$

$\Rightarrow w_{3db} = 100\ \text{krad/s}$ or

$f_{3db} = 15.9\ \text{kHz}$



for unity gain:

$|10^{-3}s| = |10^{-5}s + 1| \Rightarrow w_H = 1.01\ \text{krad/s}$

if $w = 10.1\text{krad/s}$: $\left|\frac{v_O}{v_i}\right| = \frac{10.1}{1.01} = 10.$

$\varphi = -95.77°$

2.78

$$\frac{v_o}{v_i} = -\frac{Z_2}{Z_1} = \frac{-R_2}{R_1 + \frac{1}{sC}} = \frac{-\left(\frac{R_2}{R_1}\right)s}{s + \frac{1}{R_1C}}$$ which is the

transfer function of an STC HP filter with a high frequency gain $K = -\frac{R_2}{R_1}$ and a 3-db frequency $\omega_0 = \frac{1}{R_1C}$

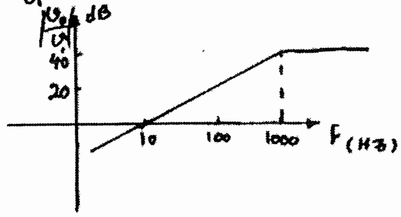
The high-frequency input impedance approaches R_1. (as $\frac{1}{j\omega C}$ becomes neglibibly small) So we can select $\underline{R_1 = 10k\Omega}$

To Obtain a high-frequency gain of 40db (i.e. 100) : $\frac{R_2}{R_1} = 100 \Rightarrow \underline{R_2 = 1M\Omega}$.

For a 3-db frequency of 1000 Hz :

$\frac{1}{R_1C} = 2\pi \times 1000 \Rightarrow C = 15.9nF$

from the Bode-diagram below, we see that $\left|\frac{v_o}{v_i}\right|$ reduces to unity at $f = 0.01 f_0 = 10\,Hz$



2.79

$$\frac{v_O}{v_i} = -\frac{Z_2}{Z_1} = -\frac{1}{Z_1Y_2} = -\frac{1}{\left(R_1 + \frac{1}{sC_1}\right)\left(\frac{1}{R_2} + sC_2\right)}$$

$$\frac{v_O}{v_i} = -\frac{R_2/R_1}{\left(1 + \frac{1}{R_1C_1s}\right)(1 + sR_2C_2)}$$

$$\frac{v_O}{v_i}(jw) = \frac{-R_2/R_1}{\left(1 + \frac{1}{jwR_1C_1}\right)(1 + jwR_2C_2)}$$

$$= \frac{-R_2/R_1}{\left(1 + \frac{w_1}{jw}\right)\left(1 + j\frac{w}{w_2}\right)}$$

where $w_1 = \frac{1}{R_1C_1}$, $w_2 = \frac{1}{R_2C_2}$

a) for $w = w_1 << w_2$

$$\frac{v_O}{v_i}(jw) \simeq \frac{-R_2/R_1}{\left(1 + \frac{w}{jw}\right)} \simeq \frac{-R_2R_1}{w_1/jw} \simeq -j\frac{R_2}{R_1}\frac{w}{w_1}$$

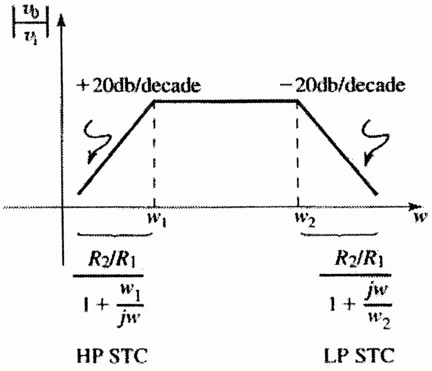
b) for $w_1 << w << w_2$

$$\frac{v_O}{v_i}(jw) \simeq -\frac{R_2}{R_1}$$

c) for $w >> w_2$ and $w_2 >> w_1$:

$$\frac{v_O}{v_i}(jw) \simeq \frac{-R_2R_1}{1 + jw/w_2} \simeq \frac{-R_2R_1}{jw/w_2} = j\left(\frac{R_2}{R_1}\right)\left(\frac{w_2}{w}\right)$$

from the results of a), b) and c) we can draw the Bode-plot:


$\left|\frac{v_o}{v_i}\right|$
+20db/decade
−20db/decade
w_1
w_2
w
$\frac{R_2/R_1}{1 + \frac{w_1}{jw}}$
$\frac{R_2/R_1}{1 + \frac{jw}{w_2}}$
HP STC
LP STC

Design: $\frac{R_2}{R_1} = 1000$ (60 dB gain in the mid-frequency range)

R_{in} for $w >> w_1$

$= R_1 = 1\,k\Omega \Rightarrow R_2 = 1\,M\Omega$

$f_1 = 100\text{ Hz} \Rightarrow w_1 = 2\pi \times 100 = \frac{1}{R_1C_1}$

$\Rightarrow C_1 = 1.59\,\mu F$

$f_2 = 10\text{ Hz} \Rightarrow w_2 = 2\pi \times 10 \times 10^3 = \frac{1}{R_1C_1}$

$\Rightarrow C_2 = 15.9\,pF$

2.80

$v_{OS} = \pm 2\text{ mV}$

$v_O = 0.01\sin\omega t \times 200 + v_{OS} \times 200$

$= 2\sin\omega t \pm 0.4\text{ V}$

2.81

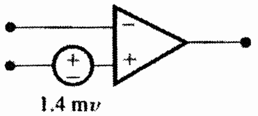
Output DC offset, $v_{OS} = 3\text{mV} \times 1000 = 3\text{V}$

Therefore the maximum amplitude of an input sinusoid is the one that results in an output peak amplitude of $13 - 3 = 10\text{V} \Rightarrow v_i = \frac{10}{1000} = \underline{10\text{ mV}}$

If the amplifier is capacity coupled, then:

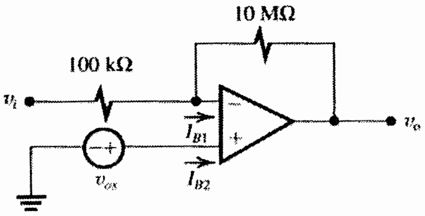
$v_{i\,max} = \frac{13}{1000} = 13\text{mV}$

2.82



$v_{OS} = \frac{1.4}{100} = 1.4\text{ mV}$

2.83



a) $I_B = (I_{B1} + I_{B2})/2$

open input:

$v_O = v_+ + R_2 I_{B1} = v_{OS} + R_2 I_{B1}$

$9.31 = v_{OS} + 10000 I_{B1}$ (1)

input connected to ground:

$v_O = v_+ + R_2\left(I_{B1} + \frac{v_{OS}}{R_1}\right)$

$= v_{OS}\left(1 + \frac{R_2}{R_1} + R_2 I_{B1}\right)$

$9.09 = v_{OS} \times 101 + 10000\, I_{B1}$ (2)

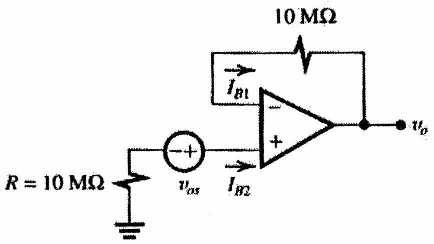
(1), (2)

$\Rightarrow 100 v_{OS} = -0.22 \Rightarrow v_{OS} = -2.2\text{ mV}$

$\Rightarrow I_{B1} = 930\text{ nA}$

$I_B \simeq I_{B1} = 930\text{ nA}$

b) $v_{OS} = -2.2\text{ mV}$



c) In this case, Since R is too large, we may ignore v_{OS} compare to the voltage drop across R.

$v_{OS} << R I_B$, Also Eq 2.46 holds:

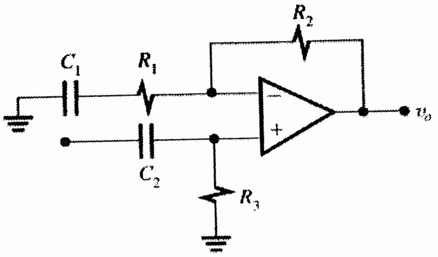
$R_3 = R_1 \| R_2$

therefore from Eq. 2.47:

$v_O = I_{OS} \times R_2 \Rightarrow I_{OS} = \frac{0.8}{10\text{ M}\Omega}$

$I_{OS} = -80\text{ nA}$

2.84



$R_2 = R_3 = 100\text{ k}\Omega$

$1 + \frac{R_2}{R_1} = 200$

$R_1 = \frac{100\text{ k}}{199} = 502\ \Omega$

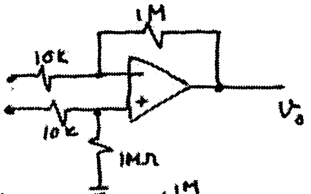
$\frac{1}{R_1 C_1} = 2\pi \times 100 \Rightarrow C_1 = \frac{1}{500 \times 2\pi \times 100}$

$= 3.18\ \mu\text{F}$

$\frac{1}{R_3 C_2} = 2\pi \times 10 \Rightarrow C_2 = \frac{1}{100\text{ K} \times 2\pi \times 100}$

$= 0.16\ \mu\text{F}$

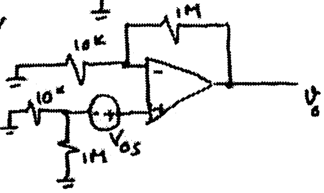
2.85


1M
10K
10K
1MΩ
V_o

The output component due to V_{os} is :

$V_{o1} = V_{os}(1+\frac{1M}{10K})$

$V_{o1} = 4(1+100) = \underline{404mV}$


1M
10K
10K
1M
V_{os}
V_o

The output component due to I_B or input bias current is :

$I_{B_1} = I_B + \frac{I_{os}}{2}$, $I_{B2} = I_B - \frac{I_{os}}{2}$

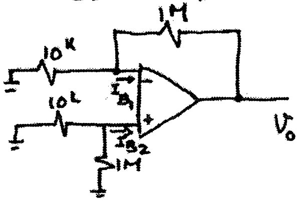
$I_{B_1} = 0.3 + \frac{0.05}{2} = 0.325\mu A$ $\quad I_{B2} = 0.275\mu A$

$V_+ = -I_{B_2} \times (10^K || 1M)$

$V_+ = -2.72mV$

$V_{o2} = V_+ + (1^M \times (I_{B_1} + \frac{V_+}{10k}))$

$\underline{V_{o2} = 50mV}$


1M
10K
10K
I_{B_1}
I_{B_2}
1M
V_o

The worst case (largest) DC offset voltage at the output is $404 + 50 = \underline{454mV}$

2.86

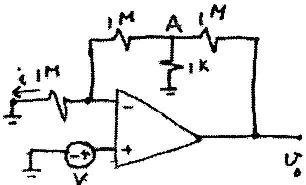
$V_- = V_+ = V_{os} \Rightarrow V_A = 2V_{os} = 8mV$

$i = \frac{V_{os}}{1M} = V_{os}\ (\mu A)$

$V_o = V_A + 1M \times (i + \frac{V_A}{1K})$

$V_o = 2V_{os} + 1M(\frac{V_{os}}{1M} + \frac{2V_{os}}{1K}) = 2003V_{os} = 2003 \times 4\overset{mV}{} = 8^V$

$\underline{V_o = 8V}$


1M
A
1M
1K
i
1M
V_{os}
V_o

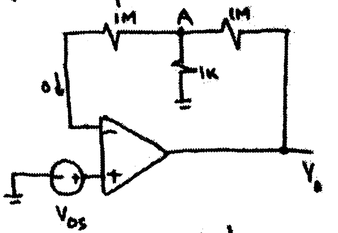
for capacitively coupled input:

$V_+ = V_- = V_{os}$

$V_A = V_{os}$

$V_o = V_A + 1M \times \frac{V_{os}}{1K}$

$V_o = 1001 V_{os} = \underline{\pm 4.004V}$


1M
A
1M
1K
V_{os}
V_o

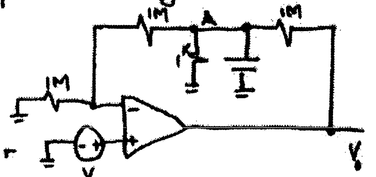
for capacitively coupled 1K to ground:

$V_+ = V_- = V_{os}$

$V_A = 2V_{os}$

$V_o = 3V_{os} = \pm 12mV$

This is much smaller than capacitively coupled input case.


1M
A
1M
1K
1M
V_{os}
V_o

2.87

At 0°C, we expect

$\pm 10 \times 25 \times 1000\ \mu = \pm 250$ mV

At 75°C, we expect

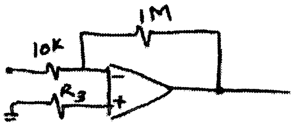
$\pm 10 \times 50 \times 1000\ \mu = \pm 500$ mV

We expect these quantities to have opposite polarities.

2.88

$R_3 = R_1 || R_2 = 9.9^{K\Omega}$

(Refer to 2.46)

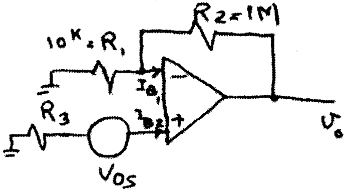


$V_o = I_{os} R_2$ $\quad$ Eq. 2.47

$V_o = 0.21 = I_{os} \times 1^M \Rightarrow I_{os} = 0.21\mu A$

If $V_{os} = 1mV$

$V_+ = -I_{B2} R_3 \mp V_{os}$


$R_2 = 1M$
$10^K = R_1$
I_{B_1}
I_{B2}
R_3
V_{os}
V_o

$$I_{B_1} = \frac{R_3 I_{B_2} \pm V_{os}}{R_1} + \frac{0.21 + R_3 I_{B_2} \pm V_{os}}{R_2}$$

$$I_{B_1} = R_3 I_{B_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \pm V_{os}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

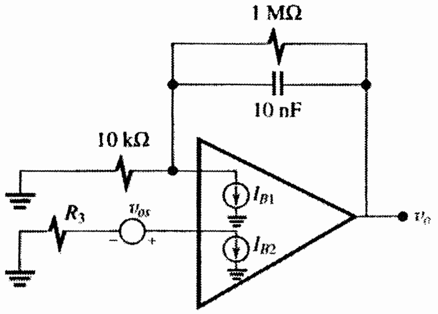
$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow I_{B_1} - I_{B_2} = \pm V_{os}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\Rightarrow I_{os} = \frac{\pm 1\,\text{mV}}{9.9\text{K}} = \pm 0.1\,\mu\text{A}$$

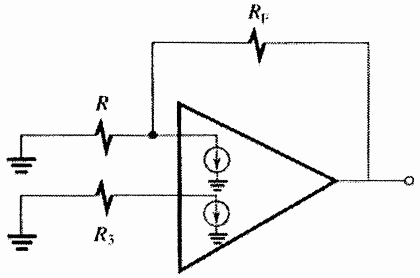
If we apply the same current as I_{os} to the other end of R_3, then it will cancel out the offset current effect on the output. $\underline{\pm 0.1\,\mu A}$

2.89

a) To compensate for the effect of dc bias current I_B, we can consider the following model



$$R_3 = R \,\|\, R_F = 10\,\text{k}\Omega \,\|\, 1\,\text{M}\Omega \Rightarrow R_3 = 9.9\,\text{k}\Omega$$



b) the dc output voltage of the integrator when the input is grounded is: $V_O = V_{OS}\left(1 + \frac{R_F}{R}\right) + I_{OS}R_F$

$$V_O = 3\,\text{mV}\left(1 + \frac{1\,\text{M}\Omega}{10\,\text{k}\Omega}\right) + 10\,\text{nA} \times 1\,\text{M}\Omega$$

$$= 0.303\ \text{V} + 0.01\ \text{V}$$

$$V_O = 0.313\ \text{V}$$

2.90

$$w_t = A_O w_b$$

$$\Rightarrow f_t = A_O f_b$$

A_O	f_b(Hz)	f_t(Hz)
10^5	10^2	10^7
10^6	1	10^6
10^5	10^3	10^8
10^7	10^{-1}	10^6
2×10^5	10	2×10^6

2.91

$$A = \frac{A_o}{1 + j\,\omega/\omega_b} \Rightarrow |A| = \frac{|A_o|}{\sqrt{1 + \left(\frac{f}{f_b}\right)^2}}$$

$A_o = 86\,\text{db}$, $A = 40\,\text{db}$ @ $f = 100\,\text{KHZ}$

$$20\log\sqrt{1+\left(\frac{f}{f_b}\right)^2} = 20\log\frac{|A_o|}{|A|} = 20\log A_o - 20\log A$$

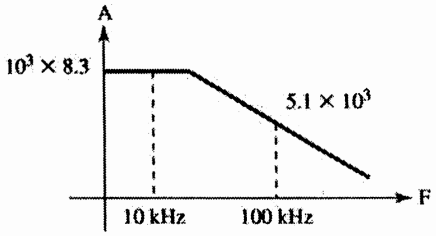
$$= 86 - 40 = 46\,\text{db}$$

$$1 + \left(\frac{100\,\text{KHZ}}{f_b}\right)^2 = (199.5)^2 \Rightarrow f_b = 0.501\,\text{KHZ}$$

$$\underline{f_b = 501\,\text{HZ}}$$

$$f_t = A_o f_b = \underbrace{1.995 \times 10^4}_{86\text{db}} \times 501 = 9.998\,\text{MHZ} \underline{\simeq 10\,\text{MHZ}}$$

2.92



$A_O = 8.3 \times 10^3$ V/V

Eq. 2.25: $A = \dfrac{A_O}{1 + j\dfrac{f}{f_b}}$

$f_t = A_O f_b$

$$5.1 \times 10^3 = \frac{8.3 \times 10^3}{\sqrt{1 + \left(\dfrac{100\ \text{kH}}{f_b}\right)^2}} \Rightarrow 1 + \left(\frac{100\ \text{kHz}}{f_b}\right)^2$$

$= 2.65$

$f_b = 60.7$ kHz

$f_t = A_O f_b = 8.3 \times 10^3 \times 60.7\text{kHz} = 503$ MHz

2.93

we have:

$A_O(\text{db}) = 20\ \text{dB} + \text{A(db)}$

$20\ \text{db} = 20 \log 10 \Rightarrow A_O = 10\text{A}$

a) $A_O = 10 \times 3 \times 10^5 = 3 \times 10^6$ Hz V/V

$$\text{A} = \frac{A_O}{1 + jf/f_b} \Rightarrow \left|1 + j\frac{f}{f_b}\right| = \frac{A_O}{A} = 10 \Rightarrow$$

$$\frac{6 \times 10^2}{f_b} = \sqrt{99}$$

$\Rightarrow f_b = 60.3$ Hz

$f_t = A_O f_b = 3 \times 10^6 \times 60.3 = 180.9$ MHz

b)

$A = 50 \times 10^5 \times 10\ \text{V/V} \Rightarrow A_O = 10 \times 50 \times 10^5$

$= 50 \times 10^6$ V/V

$$\left|1 + \frac{jf}{f_b}\right| = \frac{A_O}{\text{A}} = 10 \Rightarrow \frac{10\ \text{Hz}}{f_b} = \sqrt{99} \Rightarrow f_b = 1\ \text{Hz}$$

$f_t = A_O f_b = 50$ MHz

c) $A = 1500\ \text{V/V} \Rightarrow A_O = 1500$ V/V

$$\left|1 + \frac{jf}{f_b}\right| = 10 \Rightarrow \frac{0.1 \times 10^9}{f_b} = \sqrt{99} \Rightarrow f_b = 10\ \text{kHz}$$

$f_b = 15000 \times 10\ \text{K} = 150$ MHz

d) $A_O = 10 \times 100 = 1000$ V/V

$$\left|1 + \frac{jf}{f_b}\right| = 10 \Rightarrow \frac{0.1 \times 10^9}{f_b} = \sqrt{99} \Rightarrow f_b = 10\ \text{MHz}$$

$f_t = 1000 \times 10\ \text{MHz} = 10$ GHz

e) $A = 25\ \text{V/mV} \times 10 = 25 \times 10^4$ V/V

$$\left|1 + \frac{jf}{f_b}\right| = 10 \Rightarrow \frac{2.5\ \text{kHz}}{f_b} = \sqrt{99} \Rightarrow f_b = 2.51\ \text{kHz}$$

$f_t = A_O f_b = 25 \times 10^4 \times 2.51 \times 10^3 = 627.5$ MHz

2.95

$G_{\text{Nom}} = -\dfrac{R_2}{R_1} = -20 \quad A_O = 10^4$ V/V

$f_b = 10^6$ Hz

$$w_{3\text{db}} = \frac{w_t}{1 + R_2 R_1} = \frac{2\pi \times 10^6}{1 + 20}$$

$= 2\pi \times 47.6$ kHz

$f_{3\text{db}} = 47.6$ kHz

$$\frac{v_O}{v_i} \simeq \frac{-R_2/R_1}{1 + \dfrac{s}{w_t\left(1 + \dfrac{R_2}{R_1}\right)}} = \frac{-20}{1 + \dfrac{215}{2\pi \times 10^6}}$$

$$f = 0.1 f_{3\text{db}} \Rightarrow \left|\frac{v_O}{v_i}\right| = \frac{-20}{\sqrt{1 + (0.1)^2}} = 19.9\ \text{V/V}$$

$$f = 10 f_{3\text{db}} \Rightarrow \left|\frac{v_O}{v_i}\right| = \frac{-20}{\sqrt{1 + 100}} = 19.9\ \text{V/V}$$

2.96

$1 + \dfrac{R_2}{R_1} = 100\ \text{V/V}$, $f_t = 20$ MHZ

$f_{3db} = \dfrac{f_t}{1 + \dfrac{R_2}{R_1}} = 200$ KHZ

$G_{(j\omega)} = \dfrac{100}{1 + j\,f/f_{3db}} \Rightarrow \varphi = -\tan^{-1} \dfrac{f}{f_{3db}} =$

$\varphi = -6° \Rightarrow f = f_{3db} \times \tan 6° = 21$ KHZ

$\varphi = -84° \Rightarrow f = f_{3db} \times \tan 84° = 1.9$ MHZ

2.96

a) $\frac{R_2}{R_1} = -100 \text{ V/V}, f_{3db} = 100 \text{ kHz}$

$$w_t = w_{3db}\left(1 + \frac{R_2}{R_1}\right) \Rightarrow f_t = 100 \text{ k} \times 101 = 10.1 \text{ MHz}$$

b) $1 + \frac{R_2}{R_1} = 100 \text{ V/V}, f_{3db} = 100 \text{ kHz}$

$$f_t = f_{3db}\left(1 + \frac{R_2}{R_1}\right) = 10 \text{ MHz}$$

c) $1 + \frac{R_2}{R_1} = 2 \text{ V/V}, f_{3db} = 10 \text{ kHz}$

$f_t = 10 \text{ MHz} \times 2 = 20 \text{ MHz}$

d) $-\frac{R_2}{R_1} = -2 \text{ V/V}, f_{3db} = 10 \text{ kHz}$

$f_t = 10 \text{ MHz}(1 + 2) = 30 \text{ MHz}$

e) $-\frac{R_2}{R_1} = -1000 \text{ V/V}, f_{3db} = 20 \text{ kHz}$

$f_t = 20 \text{ kHz}(1 + 100) = 20.02 \text{ MHz}$

f) $1 + \frac{R_2}{R_1} = 1 \text{ V/V}, f_{3db} = 1 \text{ MHz}$

$f_t = 1\text{M} \times 1 = 1\text{MHz}$

g) $-\frac{R_2}{R_1} = -1, f_{3db} = 1 \text{ MHz}$

$f_t = 1 \text{ M}(1 + 1) = 2 \text{ MHz}$

2.97

$1 + \frac{R_2}{R_1} = 100 \text{ V/V} \qquad f_{3db} = 8 \text{ kHz}$

$f_t = 8 \times 100 = 800 \text{ kHz}$

for $f_{3db} = 20 \text{ kHz}$: $G_o = \frac{800}{20} = 40 \text{ V/V}$

2.98

$f_{3db} = f_t = 1 \text{ MHz}$

$$|G| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3db}}\right)^2}} = \frac{1}{\sqrt{1 + f^2}} \; f_{\text{in}} \text{ MHz}$$

$|G| = 0.99 \Rightarrow f = 0.142 \text{ MHz}$

The follower behaves like a low-pass STC circuit with a time constant $\tau = \frac{1}{2\pi \times 10^6} = \frac{1}{2\pi} \mu\text{s}$

$t_r = 2.20 = 0.35 \mu\text{s}$ (Refer to Appendix F)

2.99

a) Assume two identical stages, each with a gain function: $G = \frac{G_O}{1 + j\frac{w}{w_1}} = \frac{G_O}{1 + jf/f_1}$

$$G = \frac{G_O}{\sqrt{1 + \left(\frac{f}{f_1}\right)^2}}$$

overall gain of the cascade is $\frac{G_O^2}{\sqrt{1 + \left(\frac{f}{f_1}\right)^2}}$

The gain will drop by 3db when:

$1 + \left(\frac{f_{3db}}{f_1}\right)^2 = \sqrt{2}$, Note 3db $= 20\log\sqrt{2}$

$f_{3db} = F_1\sqrt{\sqrt{2} - 1}$

b) $40 \text{ db} = 20 \log G_O \Rightarrow G_O = 100 = 1 + \frac{R_2}{R_1}$

$$f_{3db} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{1 \text{ MHz}}{100} = 10 \text{ kHz}$$

c) Each stage should have 20db gain or

$1 + \frac{R_2}{R_1} = 10$ and therefore a 3db frequency of:

$f_1 = \frac{10^6}{10} = 10^5 \text{ Hz}$.

The overall $f_{3db} = 10^5\sqrt{\sqrt{2} - 1} = 64.35 \text{ kHz}$ which is 6 time greater than the bandwidth achieved using single op amp.
(case b above)

2.100

$f_t = 100 \times 5 = 500 \text{ MHz}$ if single op-amp is used.

with op-amp that has only $f_t = 40 \text{ MHz}$, the possible closed 100 p gain at 5 MHz is:

$|A| = \frac{40}{5} = 8 \text{ V/V}$

To obtain a overall gain of 100, three such amplifier cascaded, would be required. Now, if each of the 3 stages, has a low-frequency (d) closed loop gain K, then its 3-db frequency will $\frac{40}{k}$ MHz.

Thus for each stage the closed loop gain is:

$$|G| = \frac{k}{\sqrt{1+\left(\frac{f}{\frac{40}{k}}\right)^2}}$$

which at $f = 5$ MHz becomes:

$$|G_{5\text{MHz}}| = \frac{k}{\sqrt{1+\left(\frac{k}{8}\right)^2}}$$

The overall gain of 100: $100 = \left[\frac{k}{\sqrt{1+\left(\frac{k}{8}\right)^2}}\right]^3$

$k = 5.7$

Thus for each cascade stage: $f_{3db} = \frac{40}{5.7}$

$f_{3db} = 7$ MHz

The 3-db frequency of the overall amplifier f_1, can be calculated as:

$$\left[\frac{5.7}{\sqrt{1+\left(\frac{f}{7}\right)^2}}\right]^3 = \frac{(5.7)^3}{\sqrt{2}} \Rightarrow f_1 = 3.6 \text{ MHz}$$

2.101

a) $\frac{R_2}{R_1} = k,\ f_{3db} = \frac{f_t}{1+\frac{R_2}{R_1}} = \frac{f_t}{1+k}$

GBP = Gain × f_{3db}

$$\text{GBP} = k\frac{f_b}{1+k}$$

b) $1+\frac{R_2}{R_1} = k\ \ f_{3db} = \frac{f_t}{k}$

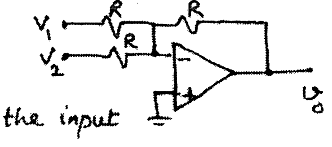
$$\text{GBP} = k\frac{f_t}{k} = f_t$$

The non-inverting amplifier realizes a higher GBP and it's independent of k.

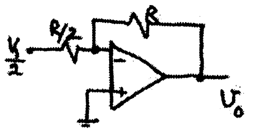
2.102

To find f_{3db} we use superposition:

Set $V2 = 0$



Now using Thevenin's Theorem to simplify the input circuit results in:



Thevenin's equivalent

$$\frac{V_o}{V_1/2} = \frac{-R/R/2}{1+s\,\frac{1+R/R/2}{\omega_t}}$$

which gives,

$$\frac{V_o}{V_1} = \frac{-1}{1+s/(\omega_t/3)}$$

$f_{3db} = \frac{f_t}{3}$. Similar results can be obtained for $\frac{V_o}{V_2}$.

2.103

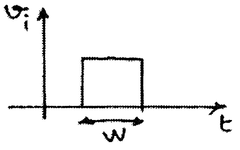
The peak value of the largest possible sine wave that can be applied at the input without output clipping is: $\frac{\pm 12\text{V}}{100} = 0.12\text{V} = 120$ mV rms

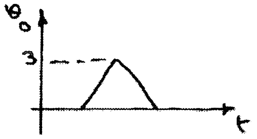
value $= \frac{120}{\sqrt{2}} = 85$ mV

2.104

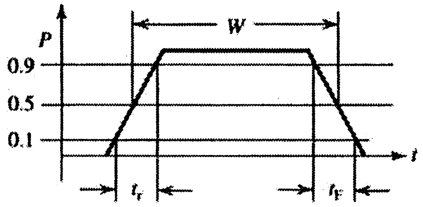
The output is triangular with the slew rate of 20 V/μs. In order to reach 3V, it takes $\frac{3}{20}$ μs = 0.15 μs = 150 ns.

Therefore the minimum pulse width is 150 ns.





2.105



$W = 2\ \mu s$
$t_r + t_F = 0.2\ W = 0.4\ \mu s$
$t_r = t_F = 0.2\ \mu s$

$$SR = \frac{(0.9 - 0.1)P}{t_r} = \frac{0.8 \times 10}{0.2} = 40\ V/\mu s$$

2.106

Slope of the triangle wave $= \frac{20\ V}{T/2} = SR$

Thus $\frac{20}{T} \times 2 = 10\ V/\mu s$

$\Rightarrow T = 4\ \mu s$ or $f = \frac{1}{T} = 250\ kHz$

For a sine wave $v_o = v_o \sin(2\pi \times 250 \times 10^3 t)$

$$\left.\frac{dv_o}{dt}\right|_{max} = 2\pi \times 250 \times 10^3\ \hat{v}_o = SR$$

$$\Rightarrow \hat{v}_o = \frac{10 \times 10^6}{2\pi \times 10^3 \times 250} = 6.37 jV$$

2.107

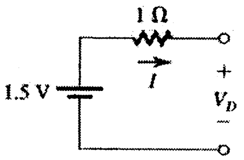
$$v_o = 10 \sin\omega t \Rightarrow \frac{dv_o}{dt} = 10\omega\cos\omega t \Rightarrow \left.\frac{dv_o}{dt}\right|_{max}$$
$$= 10\ \omega$$

The highest frequency at which this output is possible is that for which:

$$\left.\frac{dv_o}{dt}\right|_{max} = SR \Rightarrow 10\omega_{max} = 60 \times 10^{+6} \Rightarrow \omega_{max}$$
$$= 6 \times 10^5$$

$\Rightarrow f_{max} = 45.5\ kHz$

3.1



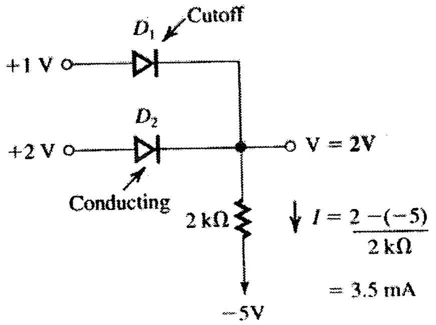
The diode can be reverse-biased and thus no current would flow, or forward-biased where current would flow.

(a) Reverse biased I = OA $V_D = 1.5$ V

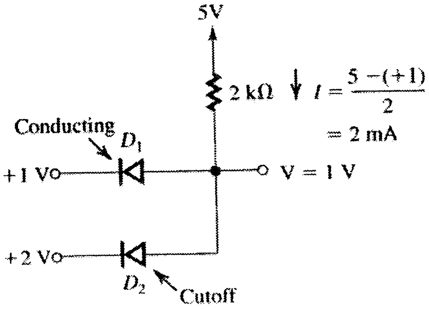
(b) Foward biased $I = 1.5$ A $V_D = 0$ V

3.2

(a)

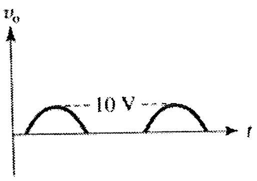

D_1
Cutoff
+1 V
D_2
+2 V
V = 2V
Conducting
2 kΩ
$I = \frac{2-(-5)}{2\text{ k}\Omega}$
= 3.5 mA
−5V

(b)


5V
2 kΩ
$I = \frac{5-(+1)}{2}$
= 2 mA
Conducting
D_1
+1 V
V = 1 V
+2 V
D_2
Cutoff

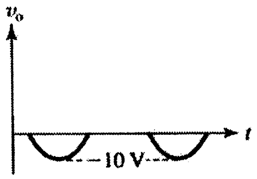
3.3

(a)



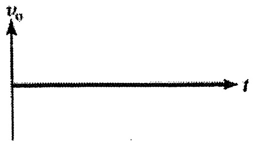
$V_{p+} = 10$ V $V_{p-} = 0$ V
$f = 1$ kH$_3$

(b)



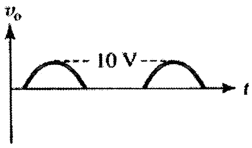
$V_{p+} = 0$ V $V_{p-} = -10$ V
$f = 1$ kH$_3$

(c)



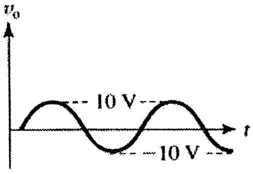
$v_O = 0$ V
Neither D_1 nor D_2 conducts so there is no output.

(d)



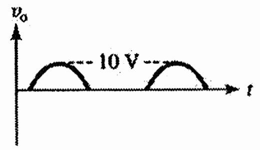
$V_{p+} = 10$ V $V_{p-} = 0$ V $f = 1$ kH$_3$
Both D_1 and D_2 conduct when $V_I > 0$

(e)



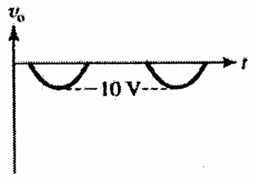
$V_{p+} = 10$ V $V_{p-} = -10$ V $f = 1$ kH$_3$
D_1 conducts when $v_I > 0$ and D_2 conducts when $v_I < 0$. Thus the output follows the input.

(f)



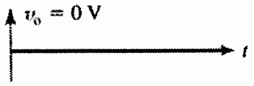
$V_{p+} = 10$ V $V_{p-} = 0$ V $f = 1$ kH$_3$
–D_1 is cutoff when $v_I < 0$

(g)



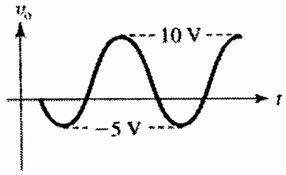
$V_{p+} = 0$ V $V_{p-} = -10$ V $f = 1$ kH$_3$
D_1 shorts to ground when $v_I > 0$ and is cut off when $v_I < 0$ whereby the output follows v_I.

(h)



$v_O = 0$ V – The output is always shorted to ground as D_1 conducts when $v_I > 0$ and D_2 conducts when $v_I < 0$.

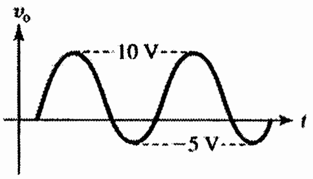
(i)



$V_{p+} = 10$ V $V_{p-} = -5$ V $f = 1$ kH$_3$
When $v_I > 0$, D_1 is cutoff and v_O follows v_I
When $v_I < 0$, D_1 is conducting and the circuit becomes a voltage divider where the negative peak is

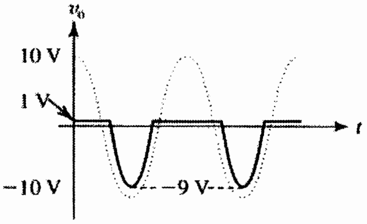
$$\frac{1\ \text{k}\Omega}{1\ \text{k}\Omega + 1\ \text{k}\Omega} \bullet -10\ \text{V} = -5\ \text{V}$$

(j)



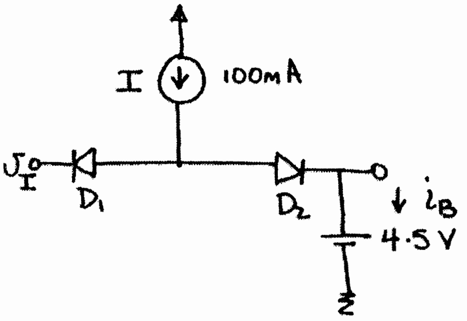
$V_{p+} = 10$ V $V_{p-} = -5$ V $f = 1$ kH$_3$
When $v_I > 0$, the output follows the input as D_1 is conducting .
When $v_I < 0$, D_1 is cut off and the circuit becomes a voltage divider.

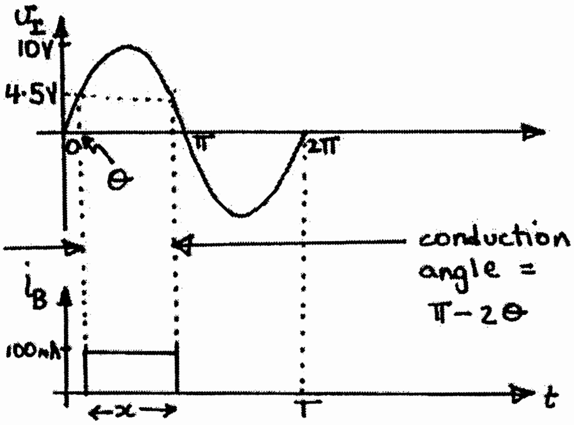
(k)



$V_{p+} = 1$ V $V_{p-} = -9$ V $f = 1$ kH$_3$
When $v_I > 0$, D_1 is cutoff and D_2 is conducting.The output becomes 1 V.
When $v_I < 0$, D_1 is conducting and D_2 is cutoff.
The output becomes:-
$v_O = v_I + 1\text{V}$

3.4




v_I
10V
4.5V
0
θ
π
2π
conduction angle = $\pi - 2\theta$
i_B
100mA
$\leftarrow x \rightarrow$
T
t

- When $v_I < 4.5V$ D_1 conducts and D_2 is cutoff so $i_B = 0A$. For $v_I > 4.5V$ D_2 conducts and D_1 is cut off thus disconnecting the input v_I. All of the current then flows through the battery.

$$10 \sin\theta = 4.5\ V$$
$$\theta = \sin^{-1}(4.5/10)$$
$$\text{conduction angle} = \pi - 2\theta$$

Fraction of cycle that $i_B = \underline{\underline{100mA}}$ is given by:-

$$x = \frac{\pi - 2\theta}{2\pi} = 0.35$$

$$i_{B\,avg} = \frac{1}{T}\int_T i_B\, dt$$

$$= \frac{1}{T}\left[100 \cdot 0.35T\right]$$

$$= \underline{\underline{35\,mA}}$$

If v_I is reduced by 10% the peak value of i_B remains the same

$$i_{B\,peak} = \underline{\underline{100mA}}$$

but the fraction of the cycle for conduction changes

$$x = \frac{\pi - 2\theta}{2\pi} = \frac{\pi - 2\sin^{-1}(4.5/9)}{2\pi}$$

$$= \frac{1}{3}$$

Thus:

$$i_{B\,avg} = \frac{1}{T}\left[100 \cdot \frac{T}{3}\right]$$

$$= \underline{\underline{33.3\,mA}}$$

3.5

$$\frac{5-0}{R} \leqslant 0.1\,mA$$

$$R \geqslant 5/0.1 = \underline{\underline{50k\Omega}}$$

3.6

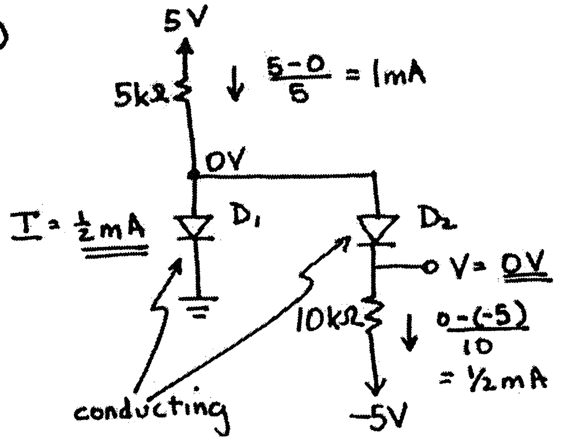
The maximum input current occurs when one input is low and the other two are high.

$$\frac{5-0}{R} \le 0.1\ \text{mA}$$

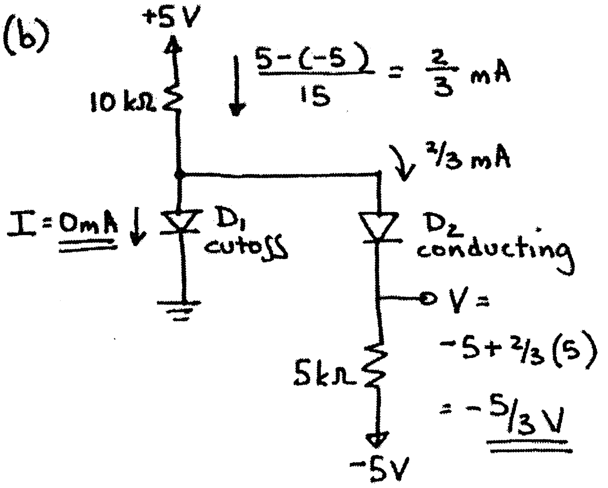
$$R \ge 50\ \text{k}\Omega$$

3.7

(a)

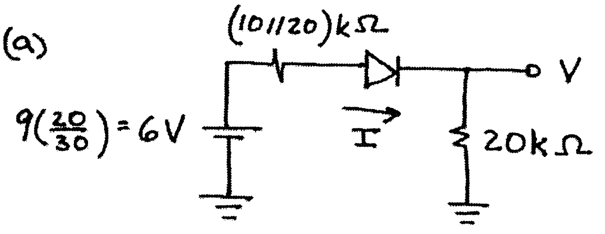

5V
5kΩ
$\frac{5-0}{5} = 1\text{mA}$
0V
I = ½ mA
D₁
D₂
V = 0V
10kΩ
$\frac{0-(-5)}{10} = ½\text{mA}$
conducting
−5V

(b)


+5V
10 kΩ
$\frac{5-(-5)}{15} = \frac{2}{3}$ mA
⅔ mA
I = 0mA
D₁ cutoff
D₂ conducting
V = −5 + ⅔(5) = −5/3 V
5kΩ
−5V

3.8

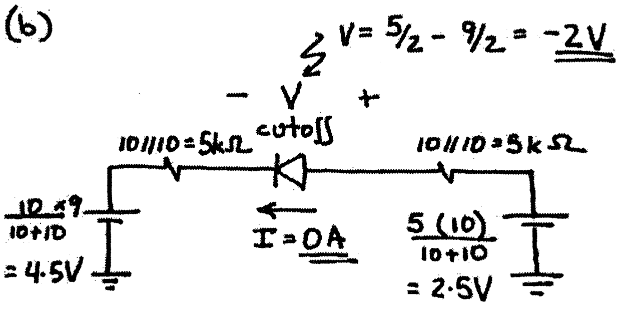
(a)


(10//20)kΩ
V
$9(\frac{20}{30}) = 6V$
I
20kΩ

$$I = \frac{6}{(10//20)+20} = \underline{\underline{0.225\,mA}}$$

$$V = \frac{20}{(10//20)+20} \times 6 = \underline{\underline{4.5V}}$$

(b)


V = 5/2 − 9/2 = −2V
− V +
10//10 = 5kΩ
cutoff
10//10 = 5kΩ
$\frac{10 \times 9}{10+10} = 4.5V$
I = 0A
$\frac{5(10)}{10+10} = 2.5V$

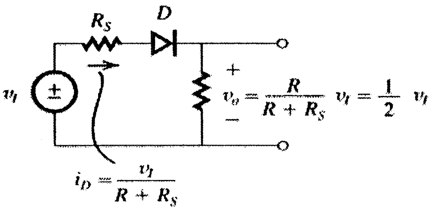
3.9

$R \geq \frac{120\sqrt{2}}{50} \geq 3.4\text{ k}\Omega$

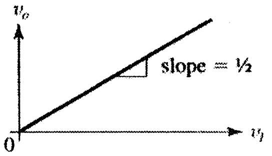
The largest reverse voltage appearing across the diode is equal to the peak input voltage

$120\sqrt{2} = 169.7$ V

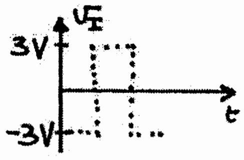
3.10

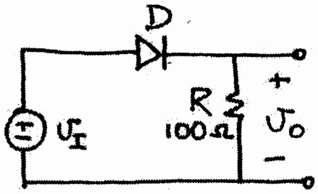

R_S
D
v_I
+
$v_o = \frac{R}{R+R_S}\,v_I = \frac{1}{2}\,v_I$
−
$i_D = \frac{v_I}{R+R_S}$

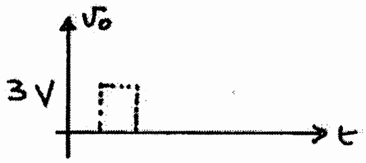
D starts to conduct when $v_I > 0$



3.11







$v_{O,\text{peak}} = 3\text{V}$

$$v_{O,\text{avg}} = \frac{1}{T}\int v_O\,dt$$

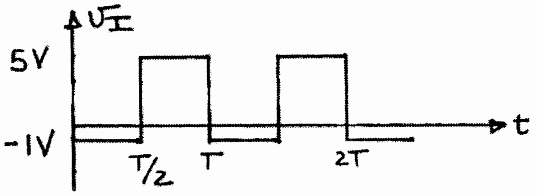
$$= \frac{1}{T}\left[3\,\frac{T}{2}\right] = \underline{\underline{3/2\text{ V}}}$$

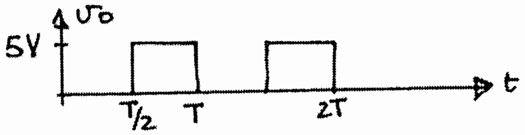
$i_{D,\text{peak}} = 3/100 = \underline{\underline{30\text{ mA}}}$

$i_{D,\text{avg}} = \frac{3/2}{100} = \underline{\underline{15\text{ mA}}}$

The maximum reverse diode voltage is $\underline{\underline{3\text{ V}}}$

3.12





$v_{O,\text{peak}} = \underline{\underline{5\text{V}}}$

$v_{O,\text{avg}} = \underline{\underline{2.5\text{V}}}$

$i_{D,\text{peak}} = \frac{v_{O,\text{peak}}}{100} = \underline{\underline{50\text{mA}}}$

$i_{D,\text{avg}} = i_{D,\text{peak}}/2 = \underline{\underline{25\text{mA}}}$

maximum reverse voltage = $\underline{\underline{1\text{V}}}$

3.13

V	RED	GREEN	
3 V	ON	OFF	- D_1 conducts
0	OFF	OFF	- No current flows
–3 V	OFF	ON	- D_2 conducts

3.14

$$i_1 = I_s\, e^{0.7/V_T} = 10^{-3}$$

$$i_2 = I_s\, e^{0.5/V_T}$$

$$\frac{i_2}{i_1} = \frac{i_2}{10^{-3}} = e^{\frac{0.5-0.7}{0.025}}$$

$$i_2 = 0.335\ \mu\text{A}$$

3.15

$$i = I_S\, e^{v/nV_T} = I_S\, e^{0.7/0.025} = 5(10^{-3})$$

$$I_S = 5(10^{-3})\, e^{-0.7/0.025} = \underline{\underline{3.46\times10^{-15}\text{A}}}$$

v	i
0.71 V	7.46 mA
0.8 V	273.21 mA
0.69 V	3.35 mA
0.6 V	91.65 μA

Let $i_1 = I_S\, e^{v_1/0.025}$

$$i_2 = 10 i_1 = I_S\, e^{v_2/0.025}$$

$$\frac{i_2}{i_1} = 10 = e^{\frac{v_2 - v_1}{0.025}}$$

$$\therefore\ \Delta v = v_2 - v_1 = \underline{\underline{57.56\text{ mV}}}$$

3.16

To calculate I_s use

$$I_s = I e^{-V/nV_T} = I e^{-V/n\times 0.025}$$

To calculate the voltage at 1% of the measured current use

$i_2 = 0.01\, i_1$ so,

$$\frac{i_2}{i_1} = 0.01 = e^{\frac{V_2 - V_1}{nV_T}}$$

$$V_2 = V_1 + nV_T \ln 0.01$$

$$= V + n(0.025)\ln(0.01)$$

V	I	I_s		V	V
[V]		n=1 [A]	n=2 [A]	n=1 [V]	n=2 [V
0.7	1A	6.91×10^{-13}	8.32×10^{-7}	0.585	0.470
0.650	1mA	5.11×10^{-15}	2.26×10^{-9}	0.535	0.420
0.650	10μA	5.11×10^{-17}	2.26×10^{-11}	0.535	0.420
0.7	10mA	6.91×10^{-15}	8.32×10^{-9}	0.584	0.470

3.17

Let $I_1 = I_S e^{V_1/nV_T}$ and

$$I_2 = I_S e^{V_2/nV_T} = I_1/10$$

Calculate n by :-

$$\frac{I_2}{I_1} = e^{\frac{V_2 - V_1}{nV_T}}$$

$$n = \frac{1}{V_T}\left[\frac{V_2 - V_1}{\ln I_2/I_1}\right] = \frac{1}{0.025}\left[\frac{V_2 - V_1}{\ln 0.1}\right]$$

Calculate I_s by :-

$$I_s = I_1 e^{-V_1/nV_T}$$

Calculate the diode voltage at $10I_1$ by :- $V_3 = nV_T \ln \frac{10I_1}{I_s}$

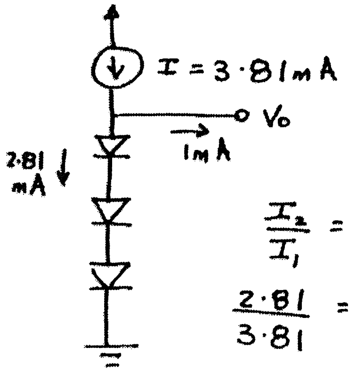
I	V_1 [V]	V_2 [V]	n	I_s [A]	V_3 [V]
10mA	0.7	0.6	1.737	10^{-9}	0.8
1mA	0.7	0.6	1.737	10^{-10}	0.8
10A	0.8	0.7	1.737	10^{-7}	0.9
1 mA	0.7	0.58	2.085	1.47×10^{-9}	0.82
10μA	0.7	0.64	1.042	2.15×10^{-17}	0.7

3.18

$\therefore$ The voltage across each diode is $V_0/3$

$$I = I_s e^{\frac{V_0/3}{nV_T}} = 10^{-14} e^{\frac{2/3}{0.025}}$$

$$= \underline{\underline{3.81\,\text{mA}}}$$

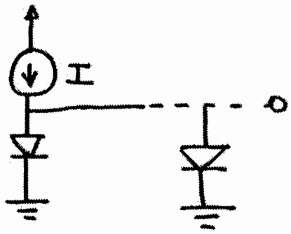


$$\frac{I_2}{I_1} = e^{\frac{(V_2 - V_1)/3}{0.026}}$$

$$\frac{2.81}{3.81} = e^{\frac{(V_2 - 2)/3}{0.025}}$$

$$\Delta V = V_2 - 2 = \underline{\underline{-22.8\,\text{mV}}}$$

3.19



With one diode the current through it is

$$I = I_s e^{V_1/nV_T}$$

With two diodes in parallel, the current splits between each diode so that the diodes each has half the current

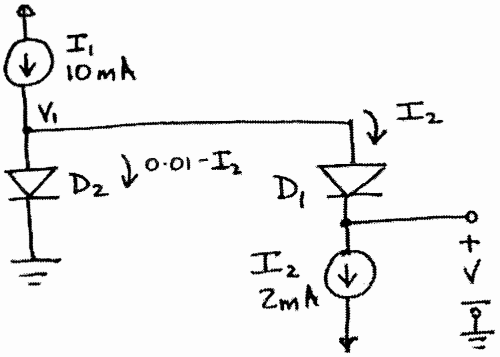
$$\frac{I}{2} = I_s e^{V_2/nV_T}$$

$$\therefore \frac{I/2}{I} = e^{\frac{V_2 - V_1}{nV_T}}$$

The change in voltage is

$$\Delta V = V_2 - V_1 = nV_T \ln\left(\tfrac{1}{2}\right) = \underline{\underline{-17.3\text{mV}}}$$

3.20



The current through D_1 is

$$10 I_s e^{\frac{V_1 - V}{nV_T}} = I_2 \quad Ⓐ$$

The current through D_2 is

$$I_s e^{\frac{V_1}{nV_T}} = 0.01 - I_2$$

$$I_s = (0.01 - I_2)\, e^{\frac{-V_1}{nV_T}} \quad Ⓑ$$

Ⓑ → Ⓐ

$$10(0.01 - I_2)\, e^{\frac{-V}{nV_T}} = I_2$$

$$V = -V_T \ln\left(\frac{I_2}{10(0.01 - I_2)}\right)$$

$$= 0.025 \ln\left(\frac{2}{10(8)}\right) = \underline{\underline{92.2\text{mV}}}$$

For $V = 50\text{mV}$

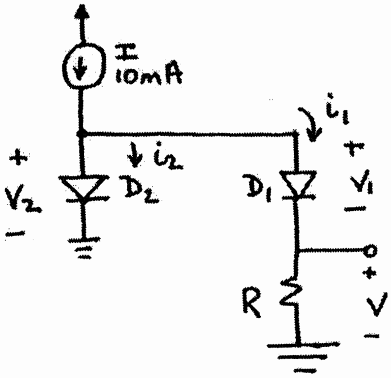
$$-V_T \ln\left(\frac{I_2}{10(10 - I_2)}\right) = 50 \times 10^{-3}$$

$$I_2 = 10(10 - I_2)\, e^{-2}$$

$$I_2(1 + 10e^{-2}) = 100e^{-2}$$

$$I_2 = \underline{\underline{5.75\text{mA}}}$$

3.21



Given for each diode

$i = I_s e^{V/nV_T} \Rightarrow \quad 10\times10^{-3} = I_s e^{0.7/n\times 0.025}$ ①

$100\times10^{-3} = I_s e^{0.8/n\times0.025}$ ②

②/① $\quad 10 = e^{0.1/n(0.025)}$

$n = 1.737$

$V = V_2 - V_1 = nV_T \ln(i_2/i_1) = 80\text{mV}$

$1.737(25\times10^{-3}) \ln\left(\frac{0.01 - i_1}{i_1}\right) = 80$

$i_1 = 1.4\text{ mA}$

$R = 80/i_1 = 80/1.4 = \underline{\underline{57.1\Omega}}$

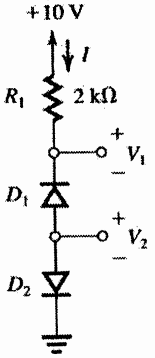
3.22

For a diode conducting a constant current, the diode voltage decreases by approximately 2 mV per increase of 1° C.

$T = -20°\text{C}$ corresponds to a temperature decrease of 40° C ,which results in an increase of the diode voltage by 80 mV. Thus $V_D = 770$ mV.

$T = +70°$ C corresponds to a temperature increase of 50° C , which results in a decrease of the diode voltage by 100 mV. Thus $V_D = 590$ mV.

3.23



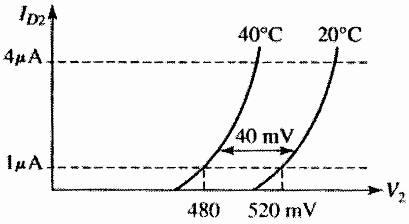
At 20°C:

$V_{R1} = V_2 = 520$ mV

$R_1 = 520$ kΩ

$I = \dfrac{520\text{ mV}}{520\text{ k}\Omega} = 1\ \mu\text{A}$

At 40°C, $I = 4\ \mu$A



$V_2 = 480 + 2.3 \times 1 \times 25 \log 4$

$= 514.6$ mV

$V_{R1} = 4\ \mu\text{A} \times 520\text{ k}\Omega = 2.08$ V

At 0°C, $I = \dfrac{1}{4}\ \mu$A

$V_2 = 560 - 2.3 \times 1 \times 25 \log 4$

$= 525.4$ mV

$V_{R1} = \dfrac{1}{4} \times 520 = 0.13$ V

3.24

The voltage drop = 700 − 580 = 120 mV. Since the diode voltage decreases by approximately 2 mV for every 1°C increase in temperature, the junction temperature must have increased by

$$\frac{120}{2} = \underline{\underline{60\,^\circ\text{C}}}$$

Power being dissipated =

$$580\times10^{-3}\times15 = \underline{\underline{8.7\,\text{W}}}$$

Thermal Resistance = temperature rise/watt

$$= 60/8.7 = \underline{\underline{6.9\,^\circ\text{C/W}}}$$

3.25

$$i = I_S\, e^{v/nV_T}$$

$$10 = I_S\, e^{0.8/2(0.025)}$$

$$I_S = 1.12\times10^{-6}\ \text{A}$$

For current varying between $i_1 = 0.5$ mA to $i_2 = 1.5$ mA, the voltage varies from

$$v_1 = 2(0.025)\ln\left(\frac{0.5\times10^{-3}}{1.12\times10^{-6}}\right) = \underline{\underline{0.305\,\text{V}}}$$

to:

$$v_2 = 2(0.025)\ln\left(\frac{1.5\times10^{-3}}{1.12\times10^{-6}}\right) = \underline{\underline{0.360\,\text{V}}}$$

∴ the voltage decreases by approximately 2 mV for every 1°C increase in temperature, the voltage may vary by ∓50 mV for the ±25°C temperature variation.

3.26

$$i = I_S\, e^{v/nV_T}$$

$$\frac{I_{S2}}{I_{S1}} = \frac{1}{0.1\times10^{-3}} = 10^4$$

For identical currents

$$I_{S1}\, e^{v_1/nV_T} = I_{S2}\, e^{v_2/nV_T}$$

$$e^{\frac{v_1-v_2}{nV_T}} = 10^4$$

$$v_1 - v_2 = nV_T \ln 10^4$$

$$= 25\times10^{-3}\ln 10^4$$

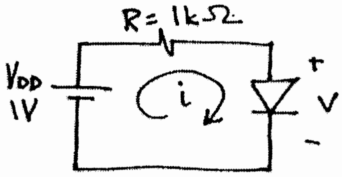
$$= +\underline{\underline{0.23\,\text{V}}}$$

I.E. THE VOLTAGE DIFFERENCE BETWEEN THE TWO DIODES IS +0.23 V INDEPENDENT OF THE CURRENT. HOWEVER, SINCE THE TWO CURRENTS CAN VARY BY A FACTOR OF 3 (0.5 mA TO 1.5 mA) THE DIFFERENCE VOLTAGE WILL BE:

$0.23\ \text{V} \pm nV_T \ln 3 = 0.23\ \text{V} \pm 2.75$ mV

SINCE TEMPERATURE CHANGE AFFECTS BOTH DIODES SIMILARLY THE DIFFERENCE VOLTAGE REMAINS CONSTANT.

3.27



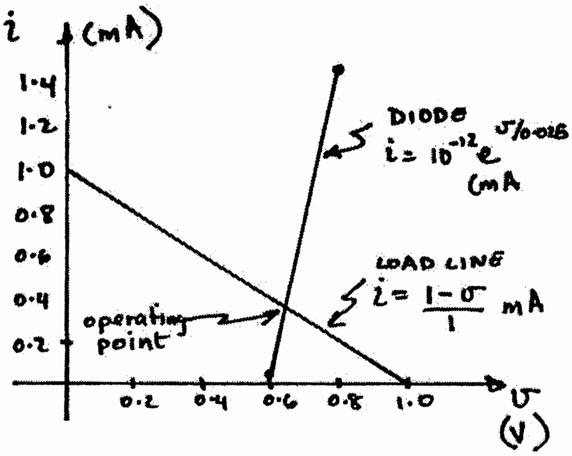
$i = 10^{-15}\, e^{v/V_T}$

where $n = 1$

$v = 0.7$ V $i = 1.45$ mA

$v = 0.6$ V $i = 0.026$ mA

A sketch of the graphical construction to determine the operating point is shown below.



From the above sketch we see that the operating point must lie between $v = 0.6$ and 0.7 V and $i \approx 0.3$ to 0.4 mA. To find the point more accurately an enlarged graph is plotted.

For $i = 0.3\,\text{mA} = 10^{-12} e^{v/0.025}$

$\Rightarrow v = 660.7$ mV

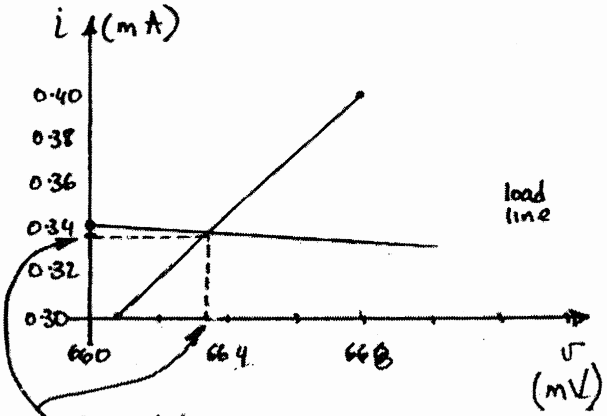
For $i = 0.4\,\text{mA} = 10^{-12} e^{v/0.025}$

$\Rightarrow v = 667.9$ mV

For the load line:

$v = 660\,\text{mV} \Rightarrow i = 0.34$ mA

$v = 670\,\text{mV} \Rightarrow i = 0.33$ mA



Graphical Point $i = 0.337$ mA

$v = 663.4$ mV

Comparing the graphical results to the exponential model gives:

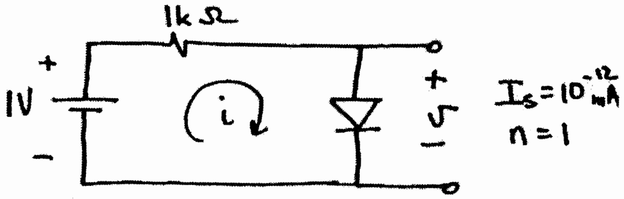
At $i = 0.337\,\text{mA} = 10^{-12} e^{v/0.025}$

$\Rightarrow v = 663.6$ mV

which is only $(663.6 - 663.4) = \underline{\underline{0.2\,\text{mV}}}$ greater than the value found graphically!

3.28

Iterative Analysis:



#1 $v = 0.7$ V $\quad i = \frac{1-0.7}{1} = 0.3$ mA

#2 $v = 0.25 \ln\left(\frac{0.3}{10^{-12}}\right) = 0.6607$ V

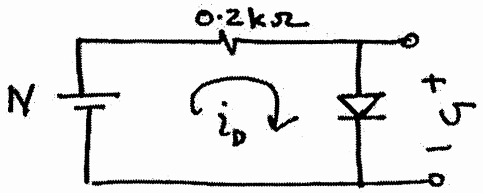
$i = \frac{1-0.6607}{1} = 0.3393$ mA

#3 $v = 0.25 \ln\left(\frac{0.3393}{10^{-12}}\right) = \underline{\underline{0.6638\,\text{V}}}$

$i = \frac{1-0.6638}{1} = \underline{\underline{0.3362}}$

∴ i did not change by much stop here.

3.29



(a) $i_D = \frac{1-0.7}{0.2} = \underline{\underline{1.5mA}}$

(b) Iterative Analysis given $v_D = 0.7V$ at $i_D = 1mA$

#1 $v = 0.7V$ $\quad i_D = \frac{1-0.7}{0.2} = 1.5mA$

#2 $\because i = I_s e^{v/nV_T}$ $\quad n=2$

$\frac{i_2}{i_1} = e^{\frac{v_2 - v_1}{0.05}}$

thus $v_2 = v_1 + 0.05 \ln i_2/i_1$

∴ for $i = 1.5$ mA

$v = 0.7 + 0.05 \ln \frac{1.5}{1}$ & $i_D = \frac{1-0.720}{0.2}$

$= 0.720V$ $\quad = 1.4mA$

#3

$v = 0.720 + 0.05 \ln\left(\frac{1.4}{1.5}\right)$ & $i_D = \frac{1-0.716}{0.2}$

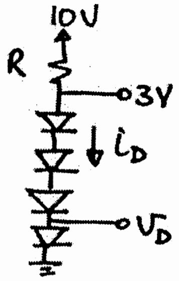
$= 0.716V$ $\quad = 1.42mA$

#4

$v = 0.716 + 0.05 \ln\left(\frac{1.42}{1.4}\right)$ & $i_D = \underline{\underline{1.42mA}}$

$= \underline{\underline{0.716V}}$

3.30



$v_D = \frac{3}{4} = 0.75V$

$i_D = I_s e^{v_D/nV_T}$

$\frac{i_{D2}}{i_{D1}} = e^{\frac{V_{D2}-V_{D1}}{nV_T}}$

$$\therefore i_D = i_{D2} = i_{D1} e^{\frac{V_{D2}-V_{D1}}{nV_T}}$$

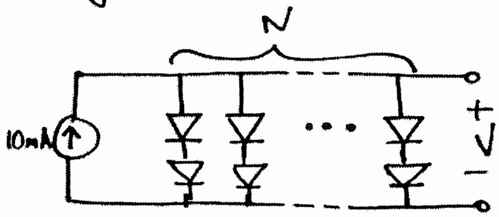
$$= 1 \times e^{\frac{0.75-0.7}{1\times0.025}}$$

$$= 7.389 \text{ mA}$$

$$\therefore R = \frac{10-3}{I_D} = \frac{10-3}{7.389} = \underline{\underline{0.947k\Omega}}$$

3.31

Since $2V_D = 1.4V$ is close to the required 1.25V, use N parallel pairs of diodes to split the current evenly.



$$\therefore V = 2\left[0.7 + 0.1 \log \frac{10/N}{20}\right] = 1.25V$$

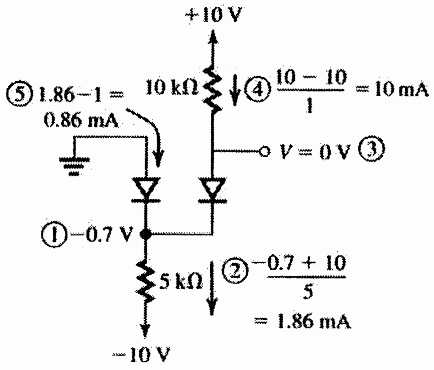
$N = 2.8$ ⇒ Use $\underline{\underline{\text{3 sets of diodes}}}$

$$V = 2\left(0.7 + 0.1 \log \frac{10/3}{20}\right) = \underline{\underline{1.244V}}$$

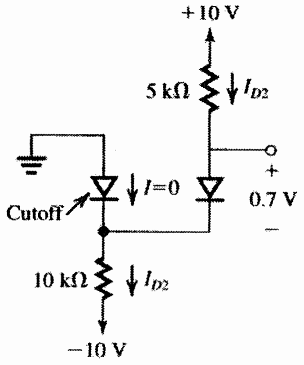
3.32

– CONSTANT VOLTAGE DROP MODEL

(a)


+10 V
⑤ 1.86−1 = 0.86 mA
10 kΩ
④ $\frac{10-10}{1}$ = 10 mA
V = 0 V ③
① −0.7 V
5 kΩ
② $\frac{-0.7+10}{5}$ = 1.86 mA
−10 V

(b)

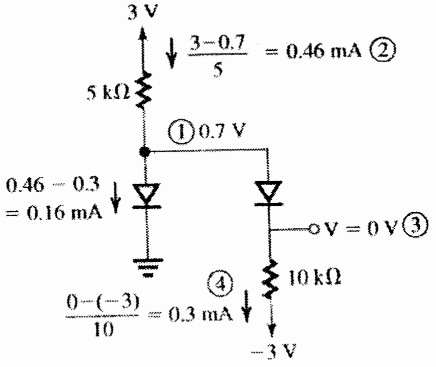

+10 V
5 kΩ
I_{D2}
I=0
Cutoff
+
0.7 V
−
10 kΩ
I_{D2}
−10 V

$$I_{D2} = \frac{10-(-10)-0.7}{15} = 1.29 \text{ mA}$$

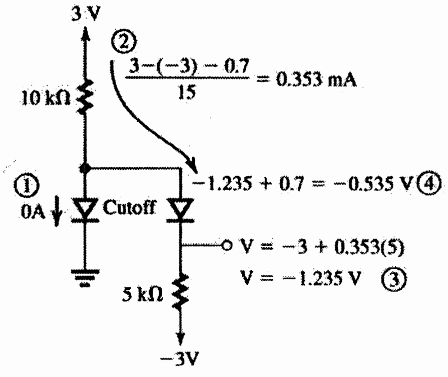
$$v_D = -10 + 1.29(10) + 0.7 = 3.6 \text{ V}$$

3.33

(a)

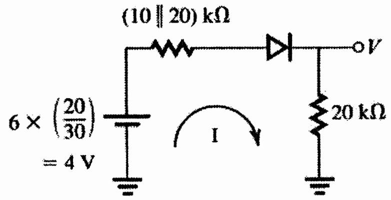

3 V
$\frac{3-0.7}{5}$ = 0.46 mA ②
5 kΩ
① 0.7 V
0.46 − 0.3 = 0.16 mA
V = 0 V ③
④
10 kΩ
$\frac{0-(-3)}{10}$ = 0.3 mA
−3 V

(b)


3 V
②
$\frac{3-(-3)-0.7}{15}$ = 0.353 mA
10 kΩ
①
0A
Cutoff
−1.235 + 0.7 = −0.535 V ④
V = −3 + 0.353(5)
V = −1.235 V ③
5 kΩ
−3V

3.34

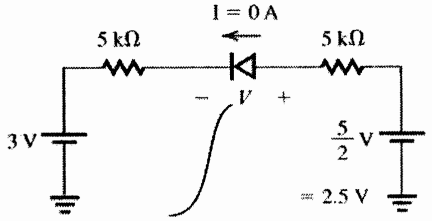
(a)


(10 ‖ 20) kΩ
V
$6 \times \left(\frac{20}{30}\right)$ = 4 V
I
20 kΩ

$$I = \frac{4-0.7}{(10\,\|\,20)+20} = 0.124 \text{ mA}$$

$$V = 20I = 2.48 \text{ V}$$

(b)

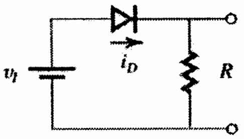

I = 0 A
5 kΩ
5 kΩ
−
V
+
3 V
$\frac{5}{2}$ V = 2.5 V

cutoff $\because \frac{5}{2} < \frac{3}{2}$

$\therefore I = 0\text{A}$

$V = \frac{5}{2} - \frac{3}{2} = -0.5 \text{ V}$

3.35



$$i_{D,\text{peak}} = \frac{v_{I,\text{peak}} - 0.7}{R} \le 50$$

$$R \ge \frac{120\sqrt{2} - 0.7}{50} = 3.38\text{ k}\Omega$$

Reverse voltage $= 120\sqrt{2} = 169.7$ V.

The design is essentially the same since the supply voltage >> 0.7 V

3.36

Using the exponential model

$$i_D = I_S e^{\Delta V/nV_T}$$

FOR A +10mV CHANGE

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta V/nV_T} = e^{0.01/n(0.025)}$$

$$= \begin{cases} 1.492 & \sim n=1 \\ 1.221 & \sim n=2 \end{cases}$$

$$\%\text{ CHANGE} = \frac{i_{D2}-i_{D1}}{i_{D1}} \times 100$$

$$= \begin{cases} (1.492-1)\times 100 = \underline{\underline{+49.2\%}} & n=1 \\ (1.221-1)\times 100 = \underline{\underline{22.1\%}} & n=2 \end{cases}$$

FOR A −10mV CHANGE

$$\frac{i_{D2}}{i_{D1}} = 10^{-0.01/n(0.025)} = \begin{cases} 0.670 & n=1 \\ 0.819 & n=2 \end{cases}$$

$$\%\text{CHANGE} = \begin{cases} (0.670-1)100 = \underline{\underline{-33\%}} & n=1 \\ (0.819-1)100 = \underline{\underline{-18\%}} & n=2 \end{cases}$$

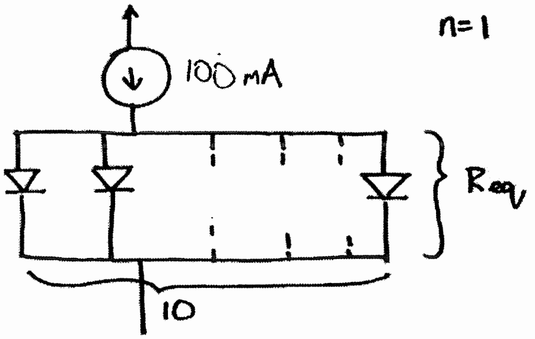
For a current change limited to ±10%

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta V/n\times 0.025} = 0.9 \text{ to } 1.1$$

$$\Delta V = \begin{cases} \underline{\underline{-2.634\text{mV to } 2.383\text{mV} \quad n=1}} \\ \underline{\underline{-5.268\text{mV to } 4.766\text{mV} \quad n=2}} \end{cases}$$

3.37

$n=1$



Each diode has the current

$$i_D = \frac{0.1}{10} = 0.01\text{ A}$$

Each diode has a small-signal resistance

$$r_d = \frac{nV_T}{I_D} = \frac{0.025}{0.01} = \underline{\underline{2.5\Omega}}$$

$$R_{eq} = r_d/10 = \underline{\underline{0.25\,\Omega}}$$

(c) $I = 10\mu A$ $\quad I_2 = 990\mu A$

$r_{d1} = \frac{0.025}{10\times10^{-6}}$ $\quad r_{d2} = \frac{0.025}{990\times10^{-6}}$

$= 2.5k\Omega$ $\quad = 25.25\Omega$

$$v_o/v_I = \underline{\underline{0.01V/V}}$$

(d) $I = 100\mu A$ $\quad I_2 = 900\mu A$

$r_{d1} = \frac{0.025}{100\times10^{-6}}$ $\quad r_{d2} = \frac{0.025}{990\times10^{-6}}$

$= 250\Omega$ $\quad = 27.78\Omega$

$$\frac{v_o}{v_I} = \underline{\underline{0.1\,V/V}}$$

(e) $I = 500\mu A$ $\quad I_2 = 500\mu A$

$r_{d1} = r_{d2} = \frac{0.025}{500\times10^{-6}} = 50\Omega$

$$\frac{v_o}{v_I} = \underline{\underline{\frac{1}{2}\,V/V}}$$

(f) $I = 600\mu A$ $\quad I_2 = 400\mu A$

$r_{d1} = \frac{0.025}{600\times10^{-6}}$ $\quad r_{d2} = \frac{0.025}{400\times10^{-6}}$

$= 41.67\Omega$ $\quad \approx 62.5\Omega$

WHEN THE BIAS CURRENT IN EACH DIODE IS $\geq 10\mu A$, THE DIODE RESISTANCE WILL BE $\leq 2.5k\Omega$. TO LIMIT THE CURRENT SIGNAL TO A MAXIMUM OF 10% OF BIAS, THE CURRENT SIGNAL MUST BE $\leq 1\mu A$. THUS, THE SIGNAL VOLTAGE ACROSS THE "STARVED" DIODE WILL BE 2.5 mV WHICH IS APPROXIMATELY THE VALUE TO WHICH THE INPUT SIGNAL SWING SHOULD BE LIMITED.

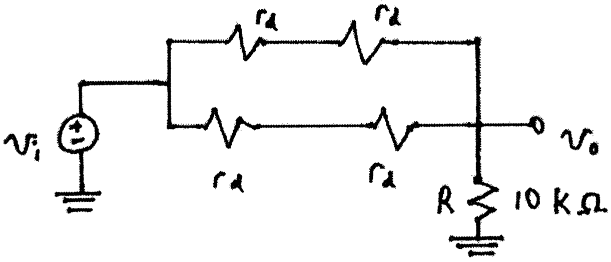
3.41

(a) $$\frac{v_o}{v_i} = \frac{R}{R + (2r_d /\!/ 2r_d)}$$

$$= \frac{R}{R + r_d}$$

WHERE $r_d = \frac{V_T}{I/2} = \frac{2V_T}{I}$

$= \frac{0.05\,V}{I}$



I (mA)	v_o/v_i (V/V)
0	0
10^{-3}	0.167
0.01	0.667
0.1	0.952
1.0	0.995
10	0.9995

(b) IF THE SIGNAL CURRENT IS TO BE LIMITED TO $\pm 10I$, THE CHANGE IN DIODE VOLTAGE Δv_D CAN BE FOUND FROM

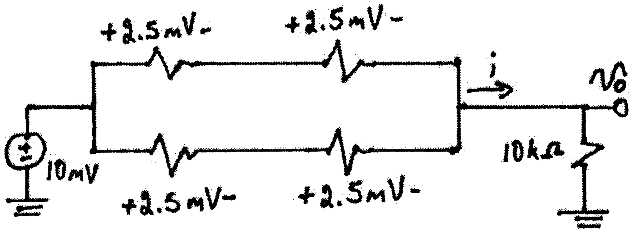
$$\frac{i_D}{I} = e^{\Delta v_D / nV_T} = 0.9 \text{ TO } 1.1$$

THUS, FOR $n = 1$

$\Delta v_D = -2.63\,mV$ TO $+2.38\,mV$

OR APPROXIMATELY $\pm 2.5\,mV$

(b cont.) For the diode current to remain within ±10% of their dc bias currents, the signal voltage across each diode must be limited to 2.5 mV. Now, if $v_{i\text{PEAK}} = 10\text{ mV}$ we can obtain the following situation



We see that $V_o = 5\text{ mV}$ and

$$i = \frac{5\text{ mV}}{10\text{ k}\Omega} = 0.5\ \mu\text{A}.$$

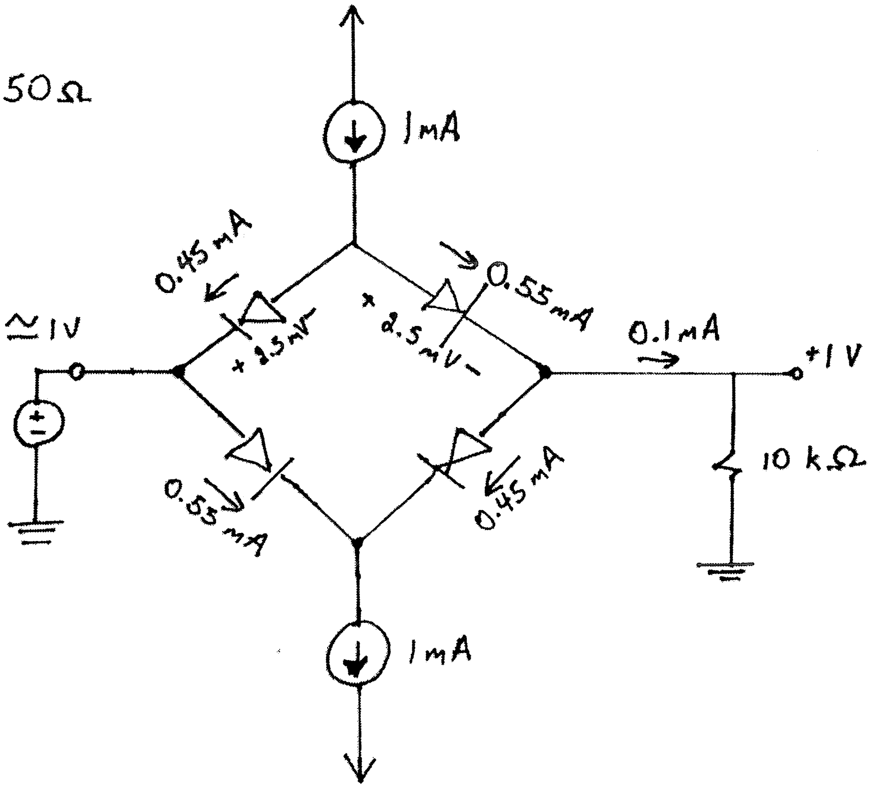
Thus, each diode is carrying a current signal of 0.25 mA. For this to be at most 10% of the dc current, the dc current in each diode must be at least 2.5 μA. It follows that the minimum value of I must be $\underline{\underline{5\ \mu\text{A}.}}$

(c) For $I = 1\text{ mA}$, $I_D = 0.5\text{ mA}$, and for maximum signal of 10%, $I_D = 0.05\text{ mA}$. Thus $i_D = 2i_d = 0.1\text{ mA}$ and the corresponding maximum v_o is $0.1\text{ mA} \times 10\text{ k}\Omega = 1\text{ V}$.

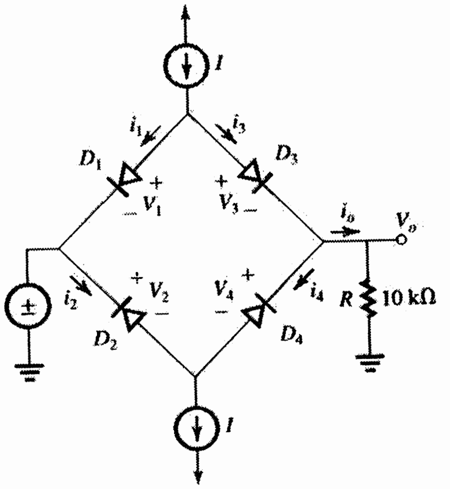
The corresponding peak input can be found by dividing v_o by the transmission factor of 0.995, thus

$$v_{i\,\text{MAX}} = \frac{1\text{V}}{0.995\text{V}} = \underline{\underline{1.005\text{ V}}}$$

See figure.
Each diode has $r_d = 50\,\Omega$



3.42



$I = 1$ mA

Each diode exhibits 0.7 V drop at 1 mA current.
using diode exponential model we have

$$v_2 - v_1 = V_T \ln\left(\frac{i_2}{i_1}\right)$$

and $v_1 = 0.7$ V, $i_1 = 1$ mA

$$\Rightarrow v = 0.7 + V_T \ln\left(\frac{i}{1}\right)$$

$$= 700 + 25\ln(i)$$

Calculation for different values of v_O

$v_O = 0$, $i_O = 0$, the current $I = 1$ mA, divides equally in D_3, D_4 side and D_1, D_2 side.

$$i_1 = i_2 = i_3 = i_4 = \frac{1}{2} = 0.5 \text{ mA}$$

$$v = 700 + 25 \ln(0.5) \simeq 683 \text{ mV}$$

$$v = v_1 = v_3 = 683 \text{ mV}$$

From circuit

$$v_I = -v_1 + v_3 + v_0 = -683 + 683 + 0 = 0 \text{ V}$$

For $v_O = 1$ V, $i_O = \dfrac{1}{10 \text{ K}} = 0.1$ mA

Because of symmetry of the circuit

$$i_3 = i_2 = \frac{I}{2} + \frac{i_O}{2} = 0.5 + 0.05 = 0.55 \text{ mA}$$

and $i_4 = i_1 = 0.45$

$$v_3 = v_2 = 700 + 25\ln\left(\frac{i_2}{1}\right) = 685 \text{ mV}$$

$$v_4 = v_1 = 700 + 25\ln(i_4) = 680 \text{ mV}$$

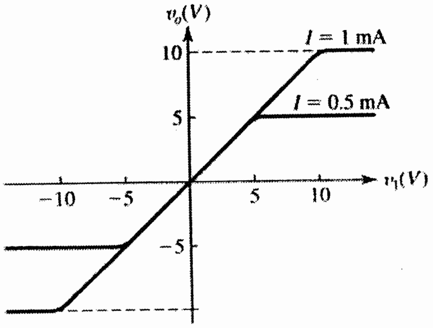
$v_O(v)$	i_O (mA)	$i_3 = i_2$ (mA)	$i_4 = i_1$ (mA)	$v_3 = v_2$ (mV)	$v_4 = v_1$ (mV)	$v_I = -v_1 + v_3 + v_0$ ln V
0	0	0.5	0.5	683	683	0
+1	0.1	0.55	0.45	685	680	1.005
+2	0.2	0.6	0.4	~687	677	2.010
+5	0.5	0.75	0.25	~693	665	5.028
+9	0.9	0.95	0.05	~699	~625	9.074
+9.9	0.99	0.995	0.005	~700	568	10.09
9.99	0.999	0.9995	0.0005	~700	510	10.18
10	1	1	0	700	0	10.7

$$v_I = -v_1 + v_2 + v_0 = -0.680$$
$$+ 0.685 + 1 = 1.005 \text{ V}$$

Similarly other values are calculated in the table for both positive and negative values of v_o
The largest values of v_o on positive and negative side are +10 V and −10 V respectively. This restriction is imposed by the current $I = 1$ mA
A similar table can be generated for the negative values. It is symmetrical.

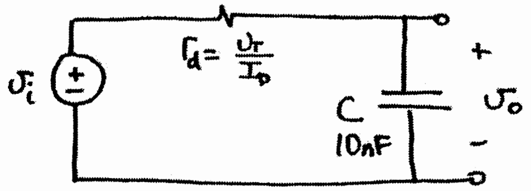
For $V_I > 10$, v_O will be saturated at 10 V and it is because $I = 1$ mA.

For I = 0.5 mA, will saturate at 0.5 mA × 10 K = 5 V



3.43

Opening the current source we get the following small-signal circuit: $(n=1)$



$$\frac{v_o}{v_i} = \frac{1/sC}{1/sC + r_d} = \frac{1}{1 + sCr_d}$$

$$\text{Phase Shift} = -\tan^{-1}\left(\frac{\omega C r_d}{1}\right)$$

$$= -\tan^{-1}\left(2\pi\, 10^5 \times 10\times10^{-9} \times 0.025/I\right)$$

For a phase shift of −45° we have

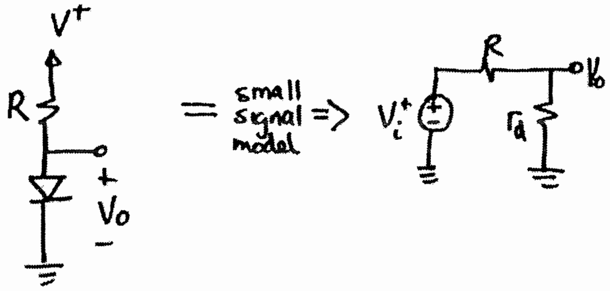
$$2\pi 10^5 \times 10(10^{-9}) \times \frac{0.025}{I} = 1$$

$$I = \underline{\underline{157\,\mu A}}$$

Range of phase shift for $I = 15.7\,\mu A$ to $1570\,\mu A$ is:

$$\underline{\underline{-84.3^\circ \text{ to } -5.71^\circ}}$$

3.44



(a) $$\frac{\Delta V_o}{\Delta V^+} = \frac{r_d}{r_d + R} = \frac{nV_T/I}{nV_T/I + R}$$

$$= \frac{nV_T}{nV_T + IR} \quad \text{where at no load } I = \frac{V^+ - 0.7}{R}$$

$$= \underline{\underline{\frac{nV_T}{nV_T + V^+ - 0.7}}} \quad \text{Q.E.D.}$$

(b) For m diodes in series use

$$I = \frac{V^+ - m\times 0.7}{R}$$

Thus:

$$\frac{\Delta V_o}{\Delta V^+} = \frac{m r_d}{m r_d + R} = \frac{m(nV_T)}{m(nV_T) + IR}$$

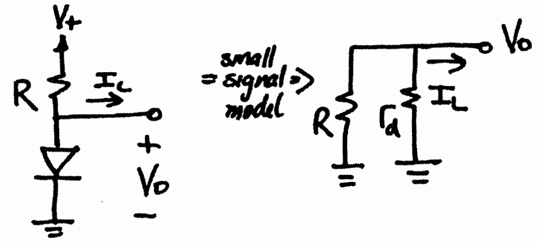
$$= \underline{\underline{\frac{m(nV_T)}{m(nV_T) + V^+ - 0.7m}}}$$

(c) Line Regulation for $V^+ = 10V$, $n=2$

i) $m=1$ $\quad \frac{\Delta V_o}{\Delta V^+} = \underline{\underline{5.35\,mV/V}}$

ii) $m=3$ $\quad \frac{\Delta V_o}{\Delta V^+} = \underline{\underline{18.63\,mV/V}}$

3.45



$$\Delta V_o = -I_L\,(R \| r_d)$$

$$\frac{\Delta V_o}{I_L} = \underline{\underline{-(R \| r_d)}} \qquad \text{Q.E.D.}$$

(b) Given at DC $I_D = \frac{V^+-0.7}{R}$

Also $r_d = \frac{nV_T}{I_D}$

We have:

$$\frac{\Delta V_o}{I_L} = -\frac{1}{\frac{1}{R}+\frac{1}{r_d}}$$

$$= -\frac{1}{\frac{I_D}{V^+-0.7}+\frac{I_D}{nV_T}}$$

$$= -\frac{nV_T}{I_D}\,\frac{1}{1+\frac{nV_T}{V^+-0.7}}$$

$$= \frac{-nV_T}{I_D}\,\frac{V^+-0.7}{V^+-0.7+nV_T} \qquad \text{Q.E.D.}$$

For $\frac{\Delta V_o}{I_L} \leqslant 5\,\frac{mV}{mA}$

$$-\frac{2\times0.025}{I_D}\times\frac{10-0.7}{10-0.7+0.05} \leqslant \frac{5\times10^{-3}}{10^{-3}}$$

$$I_D \geqslant 9.947\,mA \Rightarrow I_D = \underline{\underline{10\,mA}}$$

$$R = \frac{V^+-0.7}{I_D} = \frac{10-0.7}{10} = \underline{\underline{930\,\Omega}}$$

Thus the diode should be a 10mA diode.

(c) For m diodes

$$I_D = \frac{V^+-0.7m}{R} \quad \& \quad r_d = \frac{m(nV_T)}{I_D}$$

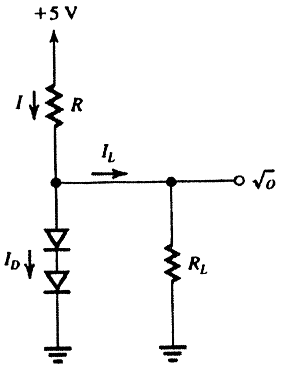
$$\frac{\Delta V_o}{I_L} = \frac{-1}{\frac{1}{R}+\frac{1}{r_d}}$$

$$= \frac{-1}{\frac{I_D}{V^+-0.7m}+\frac{I_D}{mnV_T}}$$

$$= -\frac{mnV_T}{I_D}\,\frac{1}{\frac{mnV_T}{V^+-0.7m}+1}$$

$$= \underline{\underline{-\frac{mnV_T}{I_D}\,\frac{V^+-0.7m}{V^+-0.7m+mnV_T}}}$$

3.46



Diode has 0.7 V drop at 10 mA current

$v_O = 1.5$ V when $R_L = 150\ \Omega$

$$I_D = I_S e^{V/V_T}$$

$$10 \times 10^{-3} = I_S e^{0.7/0.025}$$

$$\Rightarrow I_S = 6.91 \times 10^{-15}\ \text{A}$$

Voltage drop across each diode $= \dfrac{1.5}{2} = 0.75$ V

$$\therefore I_D = I_S e^{V/V_T} = 6.91 \times 10^{-15} \times e^{0.75/0.025}$$

$$= 73.9\ \text{mA}$$

$$I_L = 1.5/150 = 10\ \text{mA}$$

$$I = I_D + I_L = 73.9\ \text{mA} + 10\ \text{mA}$$

$$= 83.9\ \text{mA}$$

$$\therefore R = \frac{5 - 1.5}{83.9\ \text{mA}} = 41.7\ \Omega$$

Use small signal model to find voltage v_O when load resistor, R_L, has lower values

$$r_d = \frac{V_T}{I_D} = \frac{0.025}{73.9} = 0.34\ \Omega$$

When load is disconnected all the current I flows through the diode.

$$\therefore I_D = I = 83.9\ \text{mA}$$

$$v_D = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.025 \times \ln\left(\frac{83.9 \times 10^{-3}}{6.91 \times 10^{-15}}\right)$$

$v_D = 0.753$ V

So No load, $v_O = 2\ V_D = 2 \times 0.753 = 1.506$ V.

Increase in voltage $= 1.506 - 1.5 = 0.006$ V

Now load is changed

$R_L = 100\ \Omega$: $\quad I_L = \dfrac{1.5}{100} = 15$ mA

The diode current reduced by $= 15 - 10 = 5$ mA

$\therefore \Delta V_O = -5\ \text{mA} \times r_d = -1.7$ mV

$R_L = 75\ \Omega$: $\quad I_L = \dfrac{1.5}{75} = 20$ mA

Diode current reduced by $20 - 10 = 10$ mA

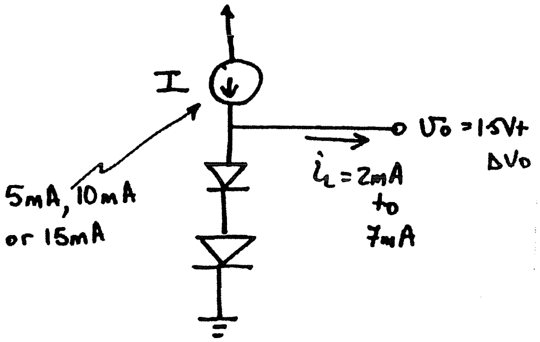
$\therefore \Delta v_O = -10\ r_d = -10 \times 0.34 = -3.4$ mV

$R_L = 50\ \Omega$: $\quad I_L = \dfrac{1.5}{50} = 30$ mA

Diode current reduced by $30 - 10 = 20$ mA

$\Delta V_O = -20\ r_d = 6.8$ mV

3.47



For a load current of 2 to 7mA, I must be greater than 7 mA. Thus the 5 mA source would not do.

We are left to choose between the 10 and 15 mA sources. The 15 mA source provides lower load regulation because the diodes will have more current flowing through them at all times

This is shown below:

Load Regulation if $I = 10\,mA$

use $\frac{i_{D2}}{i_{D1}} = e^{\Delta V/2\times nV_T}$

2 diodes

$\therefore e^{\Delta V/0.05\times 2} = 3/10$ to $8/10$

$\Delta V_o = -120\,mV$ to $-22.3\,mV$

$\therefore$ The peak to peak ripple is

$-120 - (-22.3) \approx -100\,mV$

Load Regulation $= \frac{\Delta V_o}{I_L} = \frac{-100}{5}$

$= -20\,\frac{mV}{mA}$

Load Regulation for $I = 15\,mA$.
Here the current through the diodes change from 8 to 13 mA corresponding to

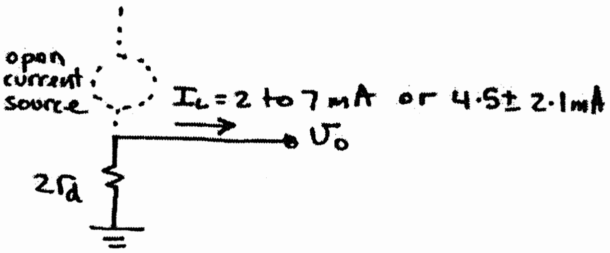
$\Delta V_o = 0.1 \ln(8/13)$

$= -49\,mV$

Load Regulation $= \frac{-49}{5} \approx -10\,\frac{mV}{mA}$.

The obvious disadvantage of using the 15 mA supply is the requirement of higher current and higher power dissipation.

Alternate solution of Line Regulation using the small signal model



Load Regulation $= \frac{\Delta V_o}{I_L} = -2r_d = -\frac{2nV_T}{I_D}$

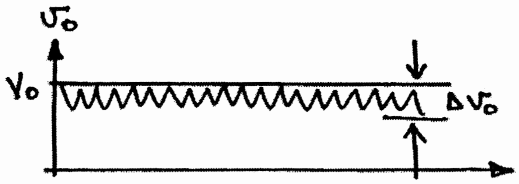
Where the bias current $I_D = 10 - 4.5$ for the 10 mA source.

$\Rightarrow \frac{\Delta V_o}{I_L} = \frac{-2\times 2\times 0.025}{10-4.5} = -18.2\,\frac{mV}{mA}$

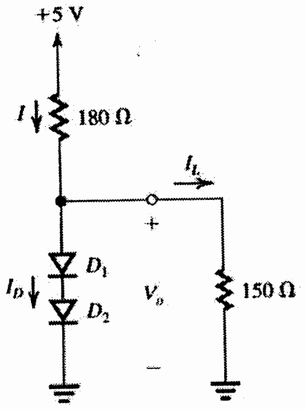
For 15 mA source $I_D = 15 - 4.5$

$\frac{\Delta V_o}{\Delta I_L} = \frac{-0.1}{15-4.5} = -9.5\,\frac{mV}{mA}$

Sketch of output:-



3.48



Both diodes are 0.7 V, 10 mA diodes

First find V_o with no load, i.e $I_L = 0$ and $I = I_L$.

Use iteration to find V_o and I_D

$$I_D = \frac{5 - 0.7 \times 2}{180\ \Omega} = 20\text{ mA}$$

$$V_2 - V_1 = 2.3 V_T \log\left(\frac{I_2}{I_1}\right)$$

$$V_2 = 0.7 + 2.3 \times 25 \times 10^{-3} \times \log\left(\frac{20}{10}\right)$$

$$= 0.717\text{ V}$$

$$I_D = \frac{5 - 0.717 \times 2}{180} = 23.79\text{ mA}$$

$$V_2 = 0.7 + 2.3 V_T \log\left(\frac{23.79}{10}\right)$$

$$= 0.7216\text{ V}$$

$$I_D = \frac{5 - 2 \times 0.7216}{180} = 19.76\text{ mA}$$

$$V_2 = 0.7 + 2.3 V_T \log\left(\frac{19.76}{10}\right)$$

$$= 0.717\text{ V}$$

$$I_D = \frac{5 - 2 \times 0.717}{180} = 19.81\text{ mA} \cong I$$

It is almost similar to earlier result, we stop iteration here

$V_D = 0.717$ V and $I_D = 19.81$ mA

So $V_O = 2 \times 0.717 = 1.434$ V

a. Load of 150 Ω is connected

$$I_L = \frac{0.717 \times 2}{150} = 9.56\text{ mA}$$

$$\therefore I_D = I_\Gamma\, 9.56 = 19.81 - 9.56 = 10.25\text{ mA}$$

So now

$$V_O = 2V_D = 2\left[0.7 + 2.3V_T \log\left(\frac{10.25}{10}\right)\right]$$

$$= 1.401\text{ V}$$

b. As found earlier, with no load $V_o = 1.434$ V

c. With 150 Ω load connected and V_o is lowered by 0.1 V of its nominal value.

$V_O = 1.401 - 0.1 = 1.301$ V

Voltage across each diode = 1.301/2 = 0.6505 V

$$I_D = 10 \times 10^{-3} e^{\Delta V / V_T} \text{ where}$$

$$\Delta V = \frac{1.401 - 1.301}{2} = 0.05\text{ V}$$

$$= 7.4\text{ mA}$$

$$I_L = \frac{1.301\text{ V}}{150\ \Omega} = 8.7\text{ mA}$$

$$\therefore I = I_D + I_L = 16.1\text{ mA}$$

∴ New value of 5V supply =

$180\ \Omega \times 16.1\text{ mA} + V_O \cong 4.2$ V

So the 5 V supply can be lowered to ~ 4.2 V

d. New value of the voltage supply = 5 + (5−4.2) = 5.8 V. Now do the problem again as done in the begining and in parts a and b.

$$I_D = \frac{5.8 - 2 \times 0.7}{180} = 24.4\text{ mA}$$

$$V_2 = V_1 + 2.3V_T \log\left(\frac{I_2}{I_1}\right)$$

$$= 0.7 + 2.3V_T \log\left(\frac{24.4}{10}\right)$$

$$= 0.722\text{ V}$$

$$I_D = \frac{5.8 - 2 \times 0.722}{180}$$

$$= 24.2\text{ mA}$$

Doing one more iteration, almost same value is obtained

∴ $V_D = 0.722$ V, $I_D = 24.2$ mA

Now when 150 Ω load is present

$$I_L = \frac{2 \times 0.722}{150} = 9.6\text{ mA}$$

So $I_D = 24.2 - 9.6 = 14.6$ mA

$$\therefore V_D = 0.7 + 2.3 \times V_T \log\left(\frac{14.6}{10}\right)$$

$$= 0.7095\text{ V}$$

$$V_O = 2\ V_D \cong 1.42\text{ V}$$

∴ Loaded output voltage = 1.42 V

e. Percentage change in output voltage

$$= \frac{1.42 - 1.301}{5.8 - 4.2} \times 100$$

$$\cong 7.4\%$$

3.49

(a) $V_Z = V_{Z0} + r_z I_{ZT}$

$10 = 9.6 + r_z \times 50\times10^{-3}$

$r_z = \underline{\underline{8\,\Omega}}$

Power rating:

$V_Z = V_{Z0} + r_z \times 2I_{ZT}$

$= 9.6 + 8\times100\times10^{-3}$

$= 10.4V$

$P = 10.4\times100\times10^{-3} = \underline{\underline{1.04W}}$

(b) $V_Z = V_{Z0} + r_z I_{ZT}$

$9.1 = V_{Z0} + 30\times10\times10^{-3}$

$V_{Z0} = \underline{\underline{8.8V}}$

$V_Z = 8.8 + 30\times20\times10^{-3} = 9.4V$

$P = 9.4\times20\times10^{-3} = \underline{\underline{188mW}}$

(c) $6.8 = 6.6 + 2\times I_{ZT}$

$I_{ZT} = \underline{\underline{100mA}}$

$V_Z = 6.6 + 2\times200\times10^{-3} = 7V$

$P = 7\times200\times10^{-3} = \underline{\underline{1.4W}}$

(d) $18 = 17.2 + r_z\times5\times10^{-3}$

$r_z = \underline{\underline{160\,\Omega}}$

$V_Z = 17.2 + 160\times10\times10^{-3} = 18.8V$

$P = 18.8\times10\times10^{-3} = \underline{\underline{188mW}}$

(e) $7.6 = V_{Z0} + 1.5\times200\times10^{-3}$

$V_{Z0} = \underline{\underline{7.2V}}$

$V_Z = 7.2 + 1.5\times400\times10^{-3} = 7.8V$

$P = 7.8\times400\times10^{-3} = \underline{\underline{3.12W}}$

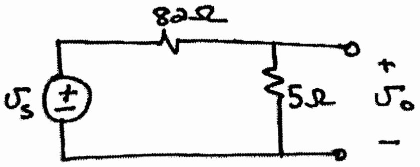
3.50

(a) Three 6.8 V zeners provide 3 × 6.8 = 20.4 V with 3 × 10 = 30 Ω resistance. Neglecting R, we have
Load Regulation = −30 mv/mA.

(b) For 5.1 V zeners we use 4 diodes to provide 20.4 V with 4 × 30 = 120 Ω resistance.
Load regulation = −120 mV/mA

3.51

Small signal model for line regulation:



$$\frac{\Delta v_o}{\Delta v_s} = \frac{5}{5+82}$$

$$\Delta v_o = \frac{5}{87}\times\Delta v_s$$

$$= \frac{5}{87}\times1.3$$

$$= \underline{\underline{74.7\text{ mV}}}$$

3.52

$r_z = 30\Omega$

$I_{ZK} = 0.5\,mA$

$V_Z = 7.5\ V$

$I_Z = 12\,mA$

$7.5 = V_{Z0} + 12 \times 30 \times 10^{-3}$

$\Rightarrow V_{Z0} = 7.14\ V$

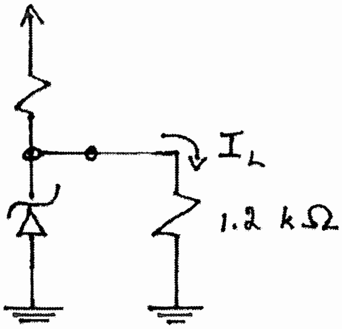
$I_z = \frac{7.5}{1.2} = 6.25\,mA$

SELECT $I = 10\,mA$

SO THAT $I_Z = 3.7\,mA$

WHICH IS $> I_{ZK}$

$R = \frac{10-7.5}{10} = \underline{\underline{250\ \Omega}}$



FOR $\Delta V^+ = \pm 1V$

$\Delta V_o = \pm 1 \times \frac{1.2 // 0.03}{0.250 + (1.2 // 0.03)}$

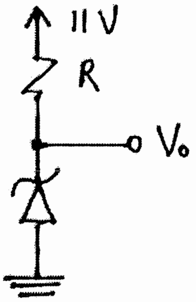
$= \pm\ 0.1\ V$

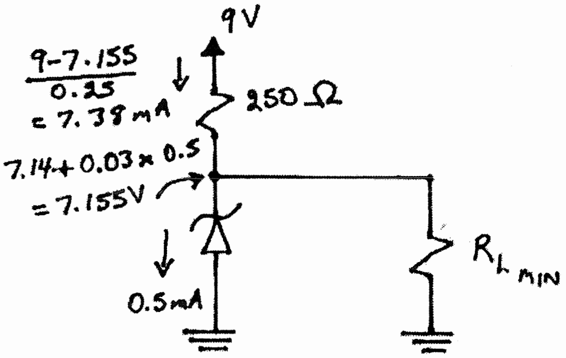
THUS $V_o = +7.4\ V$ TO $+7.6\ V$

WITH $V^+ = 11\ V$ AND $I_L = 0$

$V_o = V_{Z0} + \frac{11 - V_o}{0.25} \times 0.03$

$\Rightarrow V_o = \underline{\underline{7.55\ V}}$

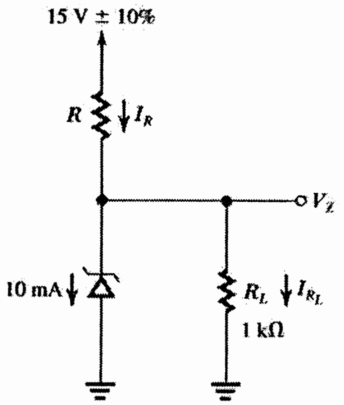



9V
$\frac{9-7.155}{0.25}$
= 7.38 mA
250 Ω
7.14 + 0.03 × 0.5
= 7.155V
0.5mA
$R_{L_{MIN}}$

$R_{L_{MIN}} = \frac{7.155}{7.38 - 0.5}$

$= \underline{\underline{1.04\ k\Omega}}$

3.53



$V_z = V_{zo} + r_z I_z$

$9.1 = V_{zo} + 30(0.009)$

$V_{zo} = 8.83\text{V}$

$V_Z = 8.83 + 30(0.01) = 9.13\text{ V}$

$I_{RL} = 9.13/1\text{ k}\Omega = 9.13\text{ mA}$

$I_R = 10 + 9.13 = 19.13\text{ mA}$

$$\therefore R = \frac{15 - 9.13}{19.13} = 306.8\ \Omega$$

$\cong 300\ \Omega$

$$V_Z = 8.83 + 30\left(\frac{15 - V_Z}{300} - \frac{V_Z}{1000}\right)$$

$$= 10.33 - \frac{V_Z}{10} - \frac{3}{100\ V_Z}$$

$V_Z = 9.14\text{ V}$

$$V_Z = 8.83 + 30\left(\frac{15 \pm 1.5 - V_Z}{300} - \frac{V_Z}{1000}\right)$$

$$= \frac{1}{1.13}[8.83 + 1.5 \pm 0.15] = 9.14 \pm 0.13\text{ V}$$

$\therefore \pm 0.13$ V variation in output voltage Halving the load current $\equiv R_L$ doubling

$$V_Z = 8.83 + 30\left(\frac{15 - V_Z}{300} - \frac{V_Z}{2000}\right)$$

$$= \frac{10.33}{1.115} = 9.26\text{ V}$$

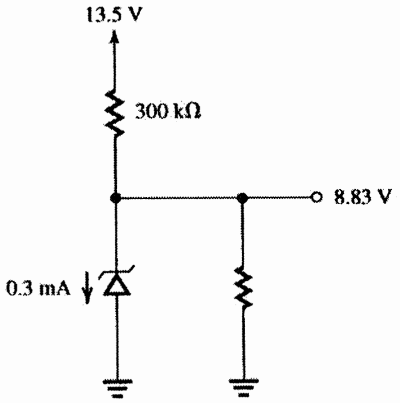
$\therefore 9.26 - 9.14 = 0.12$ V increase in output voltage.

At the edge of the breakdown region

$V_Z \cong V_{ZO} = 8.83\text{ V}\quad I_{ZK} = 0.3\text{ mA}$

$$R_L = \frac{8.83}{\frac{13.5 - 8.83}{300} - 0.0003}$$

$= 578\ \Omega$



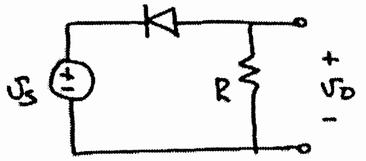
Lowest output voltage = 8.83 V

$$\text{Line Regulation} = \frac{r_Z}{R + r_Z} = \frac{30}{300 + 30}$$

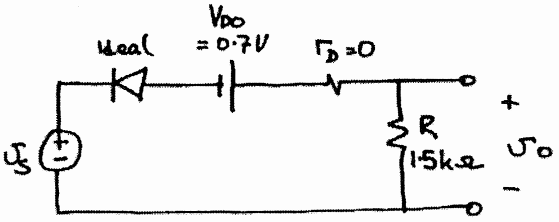
$$= 90\ \frac{\text{mV}}{\text{V}}$$

$$\text{Load Regulation} = -(r_Z \parallel R) = -29.1\ \frac{\text{mV}}{\text{mA}}$$

3.54

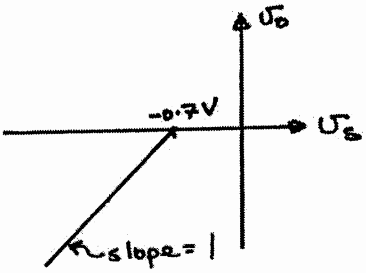


Using the constant voltage drop model:

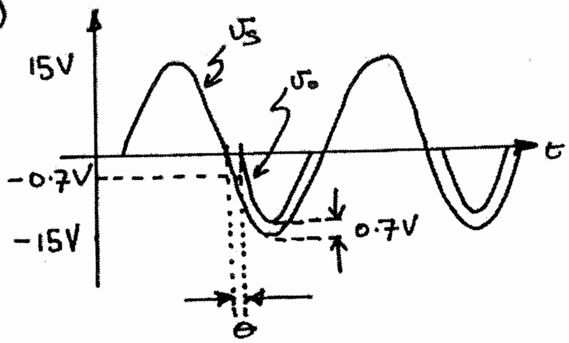


(a) $v_O = v_S + 0.7\text{ V}$, for $v_S \leq -0.7\text{V}$

$v_O = 0$, for $v_S \geq -0.7\text{V}$



(b)



(c) The diode conducts at an angle

$\theta = \sin^{-1}\frac{0.7}{15} = 2.67°$ & stops

at $\pi - \theta = 177.33°$

Thus the conduction angle is $\pi - 2\theta$

$= 174.66°$ or 3.05 rad.

$$v_{O,avg} = \frac{-1}{2\pi}\int_{\theta}^{\pi-\theta} 15\sin\phi - 0.7\, d\phi$$

$$= \frac{-1}{2\pi}\Big[-15\cos\phi - 0.7\phi\Big]_{\theta}^{\pi-\theta}$$

$$= \frac{-1}{2\pi}\Big[15\times 2\cos\theta - 0.7(\pi - 2\theta)\Big]$$

$$= \underline{\underline{-4.43\text{V}}}$$

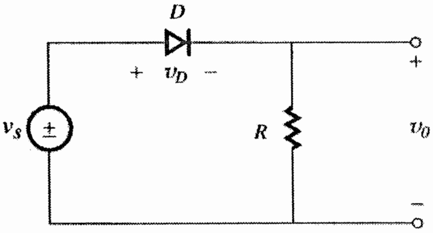
(d) Peak current in diode is:

$$\frac{15-0.7}{1.5\times10^{3}} = \underline{\underline{9.5\text{mA}}}$$

(e) PIV occurs when v_S is at its +ve peak and $v_O = 0$.

$\text{PIV} = \underline{\underline{15\text{V}}}$

3.55



$$i_D = I_S e^{v_D/V_T}$$

$$\frac{i_D}{i_D(1\text{ mA})} = e^{[v_D - v_D(\text{at 1mA})]/V_T}$$

$$v_D - v_D(\text{at 1mA}) = V_T \ln\frac{i_D}{1\text{ mA}}$$

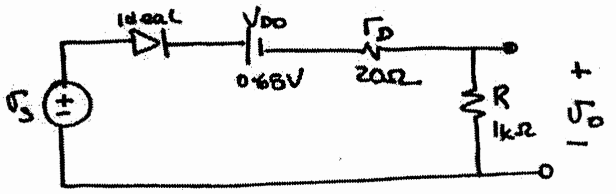
$$v_D = v_D(\text{at 1mA}) + V_T \ln\left[\frac{v_O/R}{10^{-3}}\right]$$

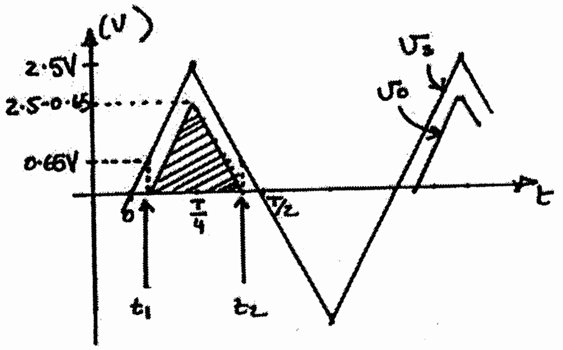
$$v_O = v_S - v_D$$

$$= v_S - v_D(\text{at 1mA}) - V_T\ln\left(\frac{v_O}{R}\right)$$

where R is in kΩ

3.56





find t_1 & t_2 by:

$$\frac{2.5}{T/4} = \frac{0.65}{t_1} \Rightarrow t_1 = 0.065T$$

$$t_2 = T/2 - 0.065T = 0.435T$$

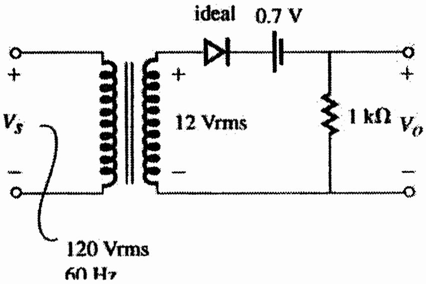
$$v_O\big|_{avg} = \frac{1}{T}\int_T \frac{R}{R+r_D}(v_S - 0.65)\,dt$$

$$= \frac{1}{T}\,\frac{R}{R+r_D}\left(\text{AREA OF SHADED } \blacktriangle\right)$$

$$= \frac{1}{T}\,\frac{R}{R+r_D}(2.5-0.65)\left(\frac{T}{4}-0.065T\right)$$

$$= \frac{1000}{1020}(0.342) = \underline{\underline{0.335\text{ V}}}$$

3.57



$\hat{V}_O = 12\sqrt{2} - 0.7 = 16.27$ V

Conduction begins at

$v_S = 12\sqrt{2}\sin\theta = 0.7$

$\theta = \sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right)$

$= 0.0412$ rad

Conduction ends at $\pi - \theta$

$\therefore$ Conduction angle $= \pi - 2\theta = 3.06$ rad

The diode conducts for

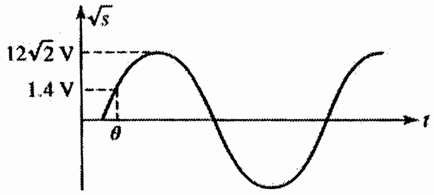
$\frac{3.06}{2\pi} \times 100 = 48.7\%$ of the cycle

$$V_{O,\text{avg}} = \frac{1}{2\pi}\int_{\theta}^{\pi-\theta}(12\sqrt{2}\sin\phi - 0.7)d\phi$$

$= 5.06$ V

$i_{D,\text{avg}} = \frac{v_{O,\text{avg}}}{R} = 5.06$ mA

$$\text{Peak voltage across } R = 12\sqrt{2} - 2V_D$$
$$= 12\sqrt{2} - 1.4$$
$$= 15.57 \text{ V}$$



$$\theta = \sin^{-1}\frac{1.4}{12\sqrt{2}} = 0.0826 \text{ rad}$$

Fraction of cycle that D_1 & D_2 conduct is

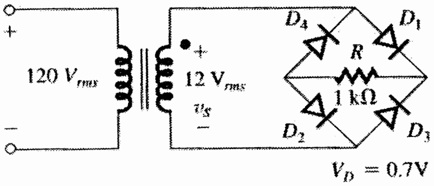
$$\frac{\pi - 2\theta}{2\pi} \times 100 = 47.4\%$$

Note D_3 & D_4 conduct in the other half cycle so that there is $2(47.4) = 94.8\%$ conduction interval.

$$v_{O,\text{avg}} = \frac{2}{2\pi}\int_{\theta}^{\pi-\theta}(12\sqrt{2}\sin\phi - 2V_D)d\phi$$
$$= \frac{1}{\pi}[-12\sqrt{2}\cos\phi - 1.4\phi]_{\theta}^{\pi-\theta}$$
$$= \frac{2(12\sqrt{2}\cos\theta)}{\pi} - \frac{1.4(\pi - 2\theta)}{\pi}$$
$$= 9.44 \text{ V}$$

$$i_{R,\text{avg}}\frac{v_{O,\text{avg}}}{R} = \frac{9.44}{1} = 9.44 \text{ mA}$$

3.58



$V_D = 0.7$V

3.59

$120\sqrt{2} \pm 10\% : 24\sqrt{2} \pm 10\%$

⇒ turns Ratio = 5:1

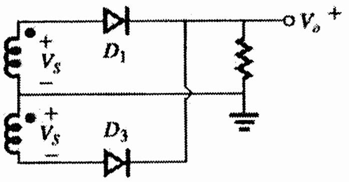
$$v_S = \frac{24\sqrt{2}}{2} \pm 10\%$$

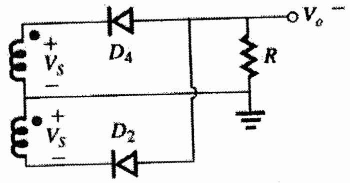
$$\text{PIV} = 2V_S|_{\max} - V_{DO}$$
$$= 2 \times \frac{24\sqrt{2}}{2} \times 1.1 - 0.7$$
$$= 36.6 \text{ V}$$

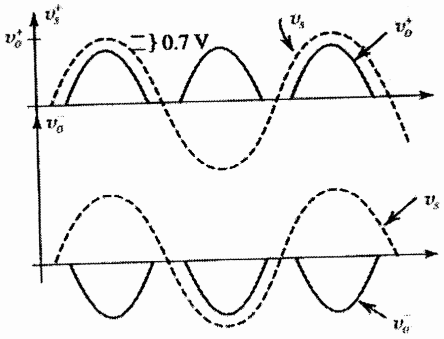
using a factor of 1.5 for safety we select a diode having a PIV rating of 55 V

3.60

The circuit is a full wave rectifier with centre tapped secondary winding. The circuit can be analyzed by looking at v_O^+ and v_O^- seperately.







$$v_{O,\,\text{avg}} = \frac{1}{2\pi}\int (V_S \sin\phi - 0.7)d\phi = 15$$

$$= \frac{2V_S}{\pi} - 0.7 = 15$$

assumed $V_S >> 0.7$ V

$$V_S = \frac{15 + 0.7}{2}\pi = 24.66 \text{ V}$$

Thus voltage across secondary winding

$$= 2V_S = 49.32 \text{ V}$$

Looking at D_4

$$\text{PIV} = V_S - V_O^-$$

$$= V_S + (V_S - 0.7)$$

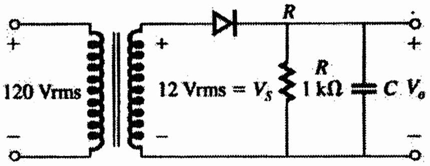
$$= 2V_S - 0.7$$

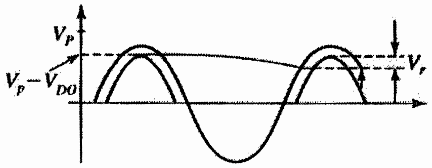
$$= 48.6 \text{ V}$$

If choosing a diode, allow a safety margin of

1.5PIV = 73 V

3.61





(i) $v_r \cong (V_P - V_{DO})\frac{T}{CR}$ Eq. (4.28)

$$0.1(V_P - V_{DO}) = (V_P - V_{DO})\frac{T}{CR}$$

$$C = \frac{1}{0.1 \times 60 \times 10^3} = 166.7\ \mu\text{F}$$

(ii) for

$$v_r = 0.01(V_P - V_{DO}) = \frac{(V_P - V_{DO})T}{CR}$$

(a)

(i) $v_{O,\,\text{avg}} = V_P - V_{DO} - \frac{1}{2}V_r$

$$= 12\sqrt{2} - 0.7 - \frac{1}{2}(12\sqrt{2} - 0.7)0.1$$

$$= (12\sqrt{2} - 0.7)\left(1 - \frac{0.1}{2}\right)$$

$$= 15.5 \text{ V}$$

ii) $v_{O,\,\text{avg}} = (12\sqrt{2} - 0.7)\left(1 - \frac{0.01}{2}\right)$

$$= 16.19 \text{ V}$$

(b)

i) we have the conduction angle =

$$\omega\Delta t \cong \sqrt{2V_r/(V_p - V_{DO})}$$

$$= \sqrt{\frac{2 \times 0.1(V_p - 0.7)}{(V_p - 0.7)}}$$

$$= \sqrt{0.2}$$

$$= 0.447 \text{ rad}$$

∴ Fraction of cycle for

$$\text{conduction} = \frac{0.447}{2\pi} \times 100$$

$$= 7.1\%$$

(ii) $\omega\Delta t \simeq \sqrt{2 \times 0.01 \frac{(V_P - 0.7)}{V_P - 0.7}} = 0.141$ rad

Fraction of cycle $= \frac{0.141}{2\pi} \times 100 = 2.25\%$

(c)(i)

$$i_{D,\text{avg}} = I_L\left(1 + \pi\sqrt{\frac{2(V_P - V_{DO})}{V_r}}\right)$$

$$= \frac{V_{O,\text{avg}}}{R}\left(1 + \pi\sqrt{\frac{2(V_P - V_{DO})}{0.1(V_P - V_{DO})}}\right)$$

$$= \frac{15.5}{10^3}\left(1 + \pi\sqrt{\frac{2}{0.1}}\right)$$

$$= 233 \text{ mA}$$

(ii) $i_{D,\text{avg}} = \frac{16.19}{10^3}(1 + \pi\sqrt{200})$

$$= 735 \text{ mA}$$

NB next user $I_L \cong V_P/R = \frac{V_P - V_{DO}}{R}$

but here are used $i_{D,\text{avg}} = \frac{V_P - V_{DO} - \frac{1}{2}V_r}{R}$

which is more accurate.

(d) (i) $i_{D,\text{peak}} = I_L\left(1 + 2\pi\sqrt{\frac{2(V_P - V_{DO})}{V_r}}\right)$

$$= \frac{15.42}{10^3}\left(1 + 2\pi\sqrt{\frac{2}{0.1}}\right)$$

$$= 449 \text{ mA}$$

(ii) $i_{D,\text{peak}} = \frac{16.19}{10^3}\left(1 + 2\pi\sqrt{\frac{2}{0.01}}\right)$

$$= 1455 \text{ mA}$$

3.62

I) $v_r = 0.1(V_P - V_{DO} \times 2) = \frac{V_P 2V_{DO}}{2fCR}$

discharge occurs only over $\frac{1}{2}T = \frac{1}{2f}$

$$C = \frac{(V_P - 2V_{DO})}{(V_P - 2V_{DO})}\,\frac{1}{2(0.1)fR} = 83.3\ \mu\text{F}$$

(ii) $C = \frac{1}{2(0.01)fR} = 833\ \mu\text{F}$

(b) (i) Fraction of cycle $= \frac{2\omega\Delta t}{2\pi} \times 100$

$= \sqrt{\frac{2(0.1)}{\pi}} \times 100 = 14.2\%$

(ii) Fraction of cycle

$= \sqrt{\frac{2(0.01)}{\pi}} \times 100 = 4.5\%$

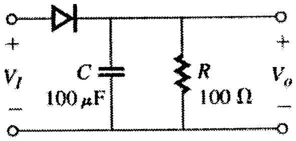
(c) i) $i_{D,\text{avg}} = \frac{14.79}{1}\left(1 + \pi\sqrt{\frac{1}{0.2}}\right) = 119\ \text{mA}$

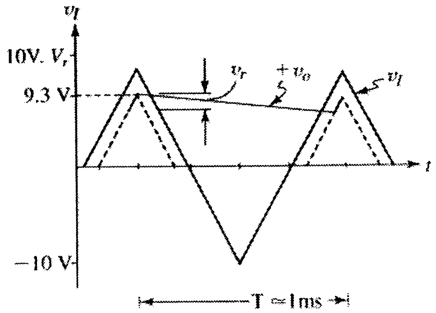
(ii) $i_{D,\text{avg}} = \frac{15.49}{1}(1 + \pi/\sqrt{0.02}) = 356\ \text{mA}$

(d) (i) $\hat{i}_D = \frac{14.79}{1}\left(1 + 2\pi\sqrt{\frac{1}{0.2}}\right) = 223\ \text{mA}$

(ii) $\hat{i}_D = \frac{15.49}{1}\left(1 + 2\pi\sqrt{\frac{1}{0.02}}\right) = 704\ \text{mA}$

3.63





During the diode's off interval, the capacitor dia-charges through the resistor R according to :

$v_o = 9.3\,e^{-t/RC} \cong 9.3(1 - T/CR)$

$\therefore v_r = 9.3 - 9.3(1 - T/CR)$

$= \frac{9.3T}{CR}$

$= \frac{9.3}{fCR}$ NB this is Eq(4.38)

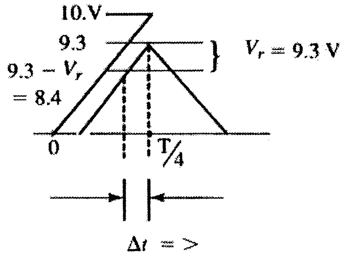
$= 0.93\text{V}$

$v_{o,\text{avg}} = V_D - V_{DO} - 1/2\ v_r$

$= 9.3 - \frac{1}{2}0.93$

$= 8.84\ V$

(b)



$\Delta t \Rightarrow \frac{10}{T/4} = \frac{0.93}{\Delta t}$

$\Delta t = 0.02325T$

$= 0.02325$ ms

(c) $\therefore$ Charge gained during conduction = Charge lost during discharge

$i_{c,\text{avg}}\,\Delta t = Cv_r$

$i_{c,\text{avg}} = \frac{Cv_r}{\Delta t} = \frac{100 \times 10^{-6} \times 0.93}{0.02325 \times 10^{-3}}$

$= 4.0\ \text{A}$

$i_{D,\text{avg}} \cong i_{c,\text{avg}} + i_{c,\text{avg}}\ \frac{v_{D,\text{avg}}}{R}$

$\cong 4.0 + \frac{8.84}{100} = 4.09\text{A}$

(d) $i_{c,\max} = C \left.\dfrac{\partial v_I}{\partial t}\right|_{\text{at onset of conduction}}$

$= C \dfrac{\partial v_I}{\partial t}$

$= 100 \times 10^{-6} \times 40 \times 10^{3}$

$= 4A$

$i_{D,\max} = i_{C,\max} + i_{L,\max}$

$= 4 + v_{o,\max} / 100$

$= 4 + 9.3 / 100$

$= 4.09\ A.$

Note that in this case $i_{D,avg} = i_{D,max}$ during to the linear input (i_c is constant and i_L is approximately constant).

3.64 let capacitor C be connected across each of the load resistors R. The two supplies, v_{O^+} and v_{O^-} are identical. Each is a full-wave rectifier similar to that based on the center-tapped-transformer circuit for each supply, the dc output is 15 V and the ripple is 1 V peak-to-peak. Thus $v_O = 15 \pm 1/2$ V. It follows that the peak value of v_s must be 15.5 + 0.7 = 16.2 V.

$\therefore$ Voltage across secondary =2(16.2)

= 32.4 V

RMS across secondary $= \dfrac{32.4}{\sqrt{2}} = 22.9\text{V rms}$

Turns Ratio $= \dfrac{120}{22.9} = 5.24{:}1$

Use Eq.(4.35) to find

$i_{D,\max} = I_L(1 + 2\pi\sqrt{V_P/2V_r})$

$= 0.2(1 + 2\pi\sqrt{15.5/2})$

$= 3.70\ A$

$v_r = \dfrac{V_P}{2fCR} = 1$

Eq (4.28)

DISCHARGE OCCURS OVER $T/2 = \dfrac{1}{2f}$

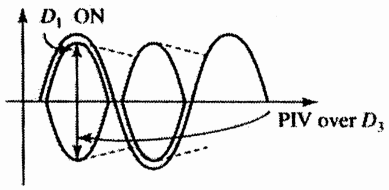
$\Rightarrow C = \dfrac{15.5}{2 \times 60 \times 75}$

where 200 mA $= \dfrac{15}{R}$

$R = \dfrac{15}{0.2} = 75\ \Omega$

$C = 1722\ \mu\text{F}$

Consider Ds when looking at PIV



PIV$= \hat{v}_O + \hat{v}_S$

= 15.5 + 16.2= 31.7 V.

Allowing for 50 % safety margin

PIV= 1.5 × 31.7= 47.6 V

use Eq(4.34) to find

$i_{D,\text{avg}} = I_L(1 + \pi\sqrt{V_P/2V_r})$

$= 0.2(1 + \pi\sqrt{15.5/2})$

$= 1.95\text{ A}$

3.65

$v_O = v_I(1 + R/R)$

$= 2v_I$ when the diode is conducting.

(a) $v_I = +\underline{1V}$ $v_O = \underline{2V}$ $v_A = \underline{1.7V}$ $v_- = v_I = \underline{1V}$

b)

$v_I = \underline{2V}$ $v_O = \underline{4V}$ $v_A = \underline{4.7V}$ $v_- = \underline{2V}$

c)

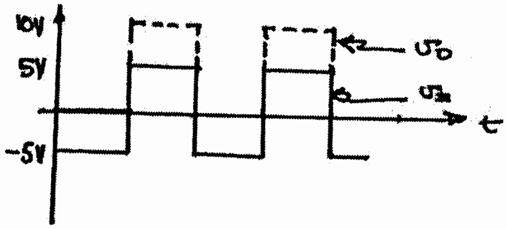
$v_I = -1V$ $v_A = \underline{-12V}$ ~ diode is cut off

$v_O = \underline{0V}$

$v_- = \underline{0V}$

d)

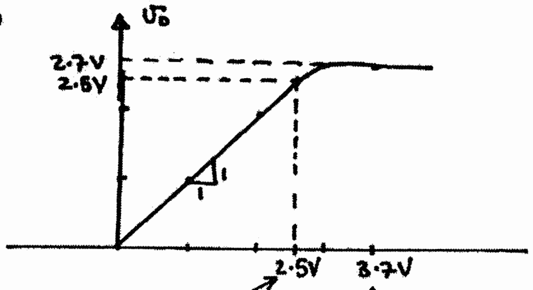
$v_I = -2\text{ V}$ $v_A = \underline{\underline{-12\text{V}}}$ $v_O = \underline{\underline{0\text{V}}}$ $v_- = \underline{\underline{0\text{V}}}$



$v_{O,\text{avg}} = \underline{\underline{5\text{V}}}$

3.66

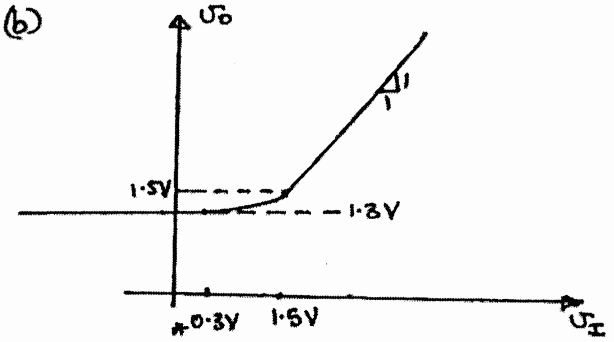
(a)



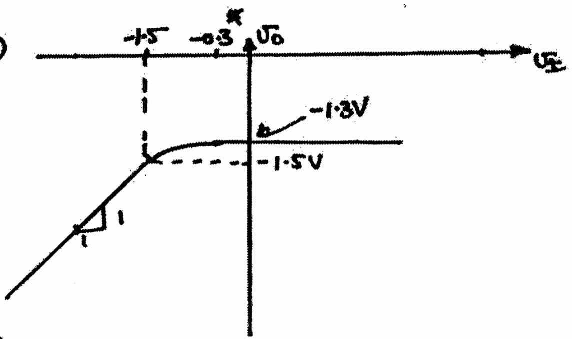
Diode starts to conduct at $v_D = 0.5\text{V}$

* assumed the diode conducts 1mA when $v_D = 0.7\text{V}$

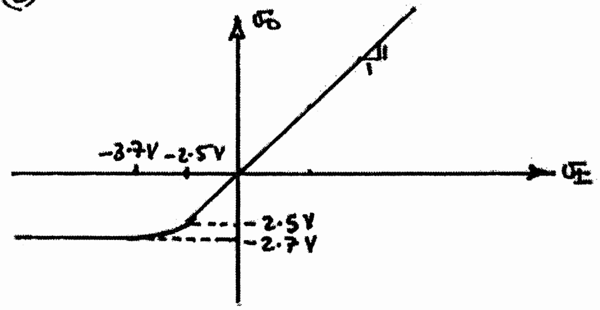
(b)



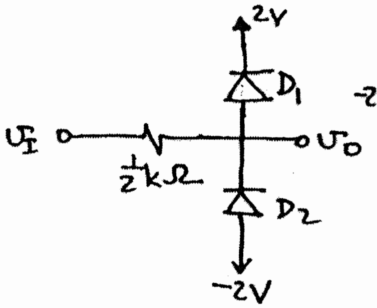
(c)



(d)

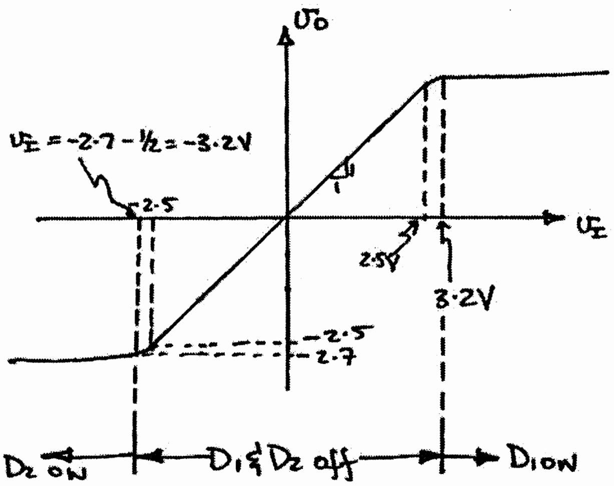


3.67

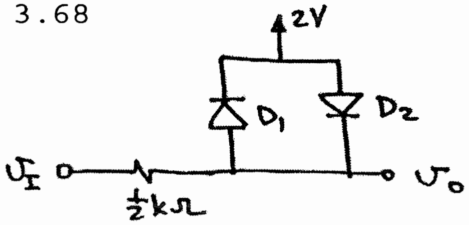


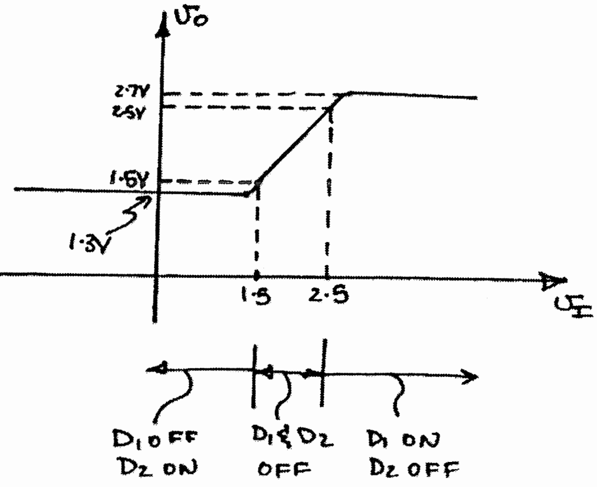
$-2.5 \leq v_I \leq 2.5$
~ both D_1 & D_2 off
– $v_O = v_I$

For $v_I \geq 2.5\text{V}$ ~ D_1 on
$v_{D1} = 0.7$ at $i_{D1} \geq 1\text{mA}$
$v_O = 2.7\text{V}$ at $v_I = 2.7 + \frac{1}{2} \times 1$
$= \underline{\underline{3.2\text{V}}}$



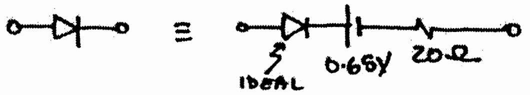
3.68


2V
D_1
D_2
v_I
½kΩ
v_O

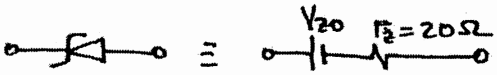


3.69

For each diode


≡
IDEAL
0.65V
20Ω

For the zener diode



$$8.2 = V_{Z0} + 10\times10^{-3} \times 20$$

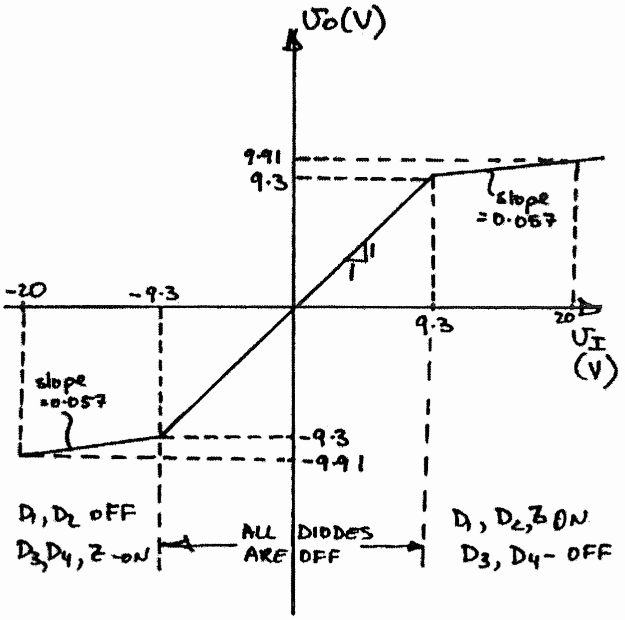
$$V_{Z0} = 8.0\text{V}$$

The limiter thresholds are

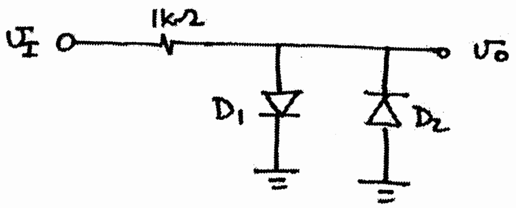
$$\pm(2\times0.65 + 8.0) = \pm 9.3\text{V}$$

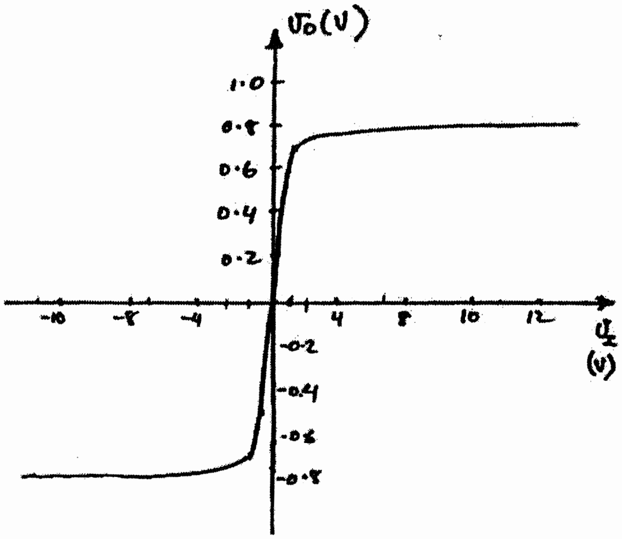
For $v_I > 9.3$ (as well as for $v_I < -9.3$)

$$\frac{\partial v_O}{\partial v_I} = \frac{r_{D1} + r_Z + r_{D2}}{1\text{k}\Omega + r_{D1} + r_Z + r_{D2}} = \frac{3(20)}{1\text{k}\Omega + 3(20)} = 0.057\frac{\text{V}}{\text{V}}$$



3.70





for D_1

Given $\frac{i_D}{1mA} = e^{\frac{v_O-0.7}{nV_T}}$

$$(v_O - 0.7) = nV_T \ln(i_D/1mA)$$

$$= 0.1 \log\left(\frac{i_D}{10^{-3}}\right) \leftarrow \therefore \text{can find } v_O \text{ from } i_D$$

$$\therefore i_D = 10^{-3} \times 10^{\frac{v_O-0.7}{0.1}}$$

$$= 10^{-3} \times 10^{10(v_O-0.7)}$$

& $v_I = v_O + i_D \times 10^3$

$$= v_O + 10^{10(v_O-0.7)}$$

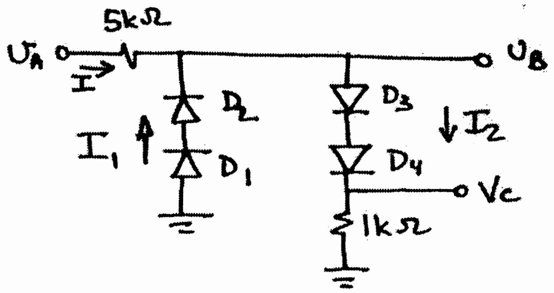
for D_2: $v_I = v_O - 10^{-10(v_O-0.7)}$

v_O(V)	v_I(V)	
0.5	0.510	D_1 ON
0.6	0.7	
0.7	1.7	
0.8	10.7	
0	0	
−0.5	−0.51	D_2 ON
−0.6	−0.7	
−0.7	−1.7	
−0.8	−10.7	

The limiter is fairly hard with a gain $K \cong 1$

$L_+ \cong \underline{\underline{0.8V}}$, $L_- \cong \underline{\underline{-0.8V}}$

3.71



$v_D = 0.7 + 0.1 \log \left({}^{i_D}/_{0.1} \right)$

(a) For $v_A > 0$ D_1, D_2 off $\Rightarrow I_1 = 0$

$I = I_2 = v_C / 1k\Omega \quad v_A = V_B + I_2 5$

(b) For $v_I < 0$ D_3, D_4 off $\Rightarrow v_C = 0$

$I = -I_1 \quad v_B = -(v_{D1} + v_{D2})$

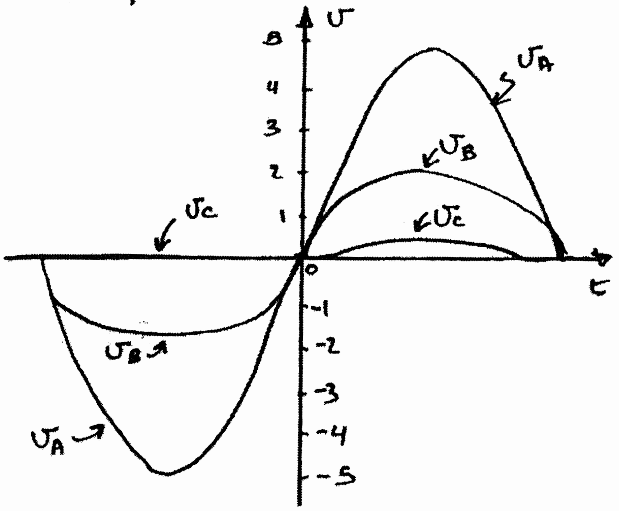
$v_A = -(V_B + 5I_1)$

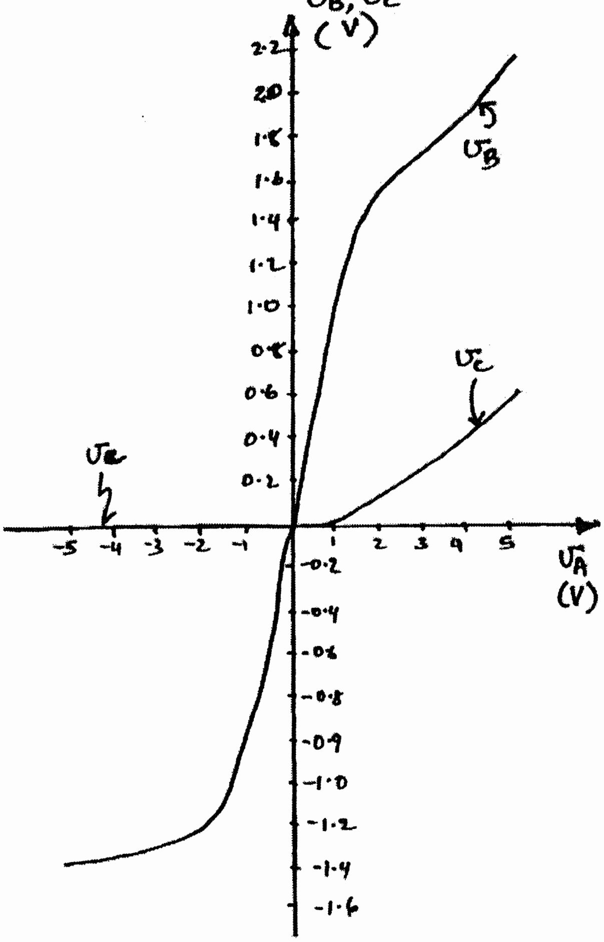
(a) List of points for $v_A > 0$

v_C (V)	I_2 (mA)	v_{D3}, v_{D4} (V)	$v_B = v_C + V_{D3} + V_{D4}$ (V)	v_A (V)
0.0001	0.0001	0.4	0.8	0.8
0.001	0.001	0.5	1.00	1.01
0.01	0.1	0.6	1.21	1.26
0.1	0.1	0.7	1.50	1.90
0.2	0.2	0.73	1.66	2.66
0.3	0.3	0.75	1.80	3.30
0.4	0.4	0.76	1.92	3.92
0.5	0.5	0.77	2.04	4.54
0.6	0.6	0.78	2.16	5.16

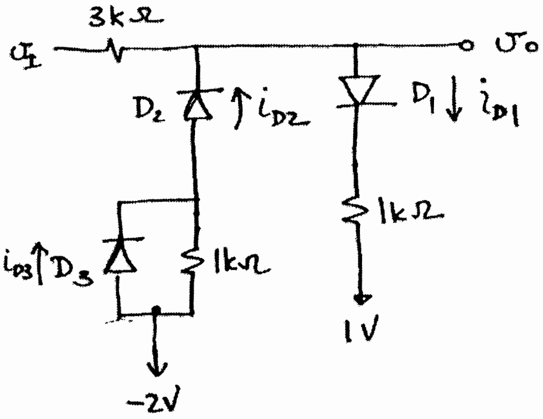
(b) List of Points for $V_A < 0$

I_1 (mA)	v_{D1}, v_{D2} (V)	v_B (V)	v_A (V)
0.0001	0.4	−0.80	−0.80
0.001	0.5	−1.00	−1.01
0.01	0.6	−1.20	−1.25
0.10	0.7	−1.40	−1.90
0.20	0.73	−1.46	−2.46
0.30	0.75	−1.50	−3.00
0.40	0.76	−1.52	−3.52
0.50	0.77	−1.54	−4.04
0.60	0.78	−1.56	−4.56
0.70	0.785	−1.57	−5.07





3.72



3kΩ
v_I
v_O
D_2
i_{D2}
D_1
i_{D1}
1kΩ
i_{D3}
D_3
1kΩ
1V
−2V

At currents $i_{D1} > 1mA$, $v_{D1} \simeq 0.7V$

Let $v_{D1} = 0.71V$ $v_I > 5.7V$

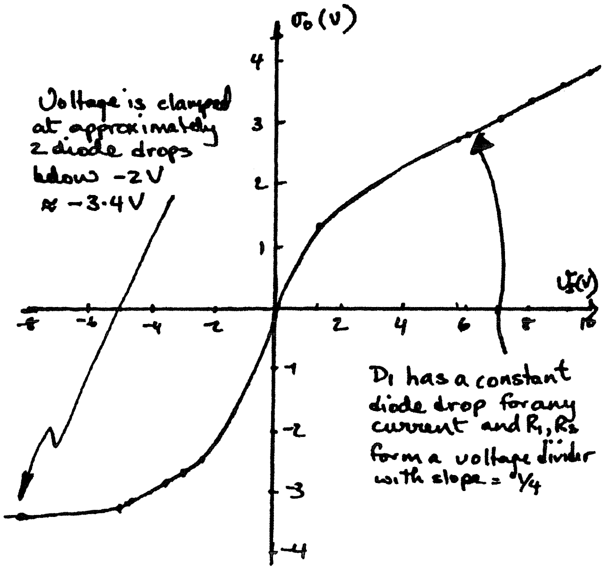
$$v_O = 1.71 + i_{D1} \times 1\,k\Omega$$

$$= 1.71 + \left(\frac{v_I - 1.71}{4}\right) \times 1$$

$$= \frac{v_I}{4} + 1.2825$$ NB slope = 1/4

∴ for $v_I > 5V$ slope $v_O/v_I \approx 1/4$

v_I (V)	v_O (V)
5.8	2.7325
6.0	2.7825
7.0	3.0325
8.0	3.2825
9.0	3.5325
10.0	3.7825



where points for $-8 \le v_I \le 6$ V are calculated as shown below:

$i_D = 1mA$ at $v_D = 0.7V$ $n=1$

$$i_D = I_S e^{0.7/0.025} = 10^{-3}$$

$$I_S = 6.914 \times 10^{-16} A.$$

∴ For Diodes use $\underline{i_D = 6.914 \times 10^{-16} e^{v_D/0.025}}$

D_1 conducting $i_{D2} = 0$

i_{D1} (A)	v_{D1} (V)	v_O (V)	$v_I = (4k)i_{D1} + v_{D1} + 1$ (V)
10^{-10}	0.297	1.297	1.297
10^{-6}	0.527	1.528	1.5313
10^{-5}	0.584	1.595	1.625
10^{-4}	0.64	1.742	2.042
10^{-3}	0.70	2.7	5.7
0.2×10^{-2}	0.74	6.74	12.74
10^{-2}	0.758	11.75	41.75

← even at small i_{D1} $v_O > 1V$, $v_O \approx v_I$ since $i_{D1} \approx 0$

For the D_2, D_3 arm conducting use the following equations:
Note $v_I < -2.5V$

Starting with a value for ① V_A we have

$$V_{D3} = V_A + 2$$

$$i_{D3} = I_S e^{v_{D3}/0.025} \quad ②$$

$$i_{D2} = i_{D3} + \frac{V_A + 2}{1} \quad ③$$

$$v_{D2} = 0.025 \ln\left(\frac{i_{D2}}{6.914 \times 10^{-16}}\right) \quad ④$$

$$v_O = v_A - v_{D2} \quad ⑤$$

$$v_I = v_O - i_{D2} \times 3\,k\Omega \quad ⑥$$

① v_A (V)	② i_{D3} (A)	③ i_{D2} (A)	④ v_{D2} (V)	⑤ v_O (V)	⑥ v_I (V)
-2.001	7×10^{-14}	10^{-6}	0.527	-2.528	-2.531 Ⓐ
-2.01	10^{-15}	10^{-5}	0.585	-2.595	-2.625
-2.10	3.8×10^{-14}	10^{-4}	0.642	-2.724	-3.024
-2.20	2×10^{-12}	0.2×10^{-3}	0.659	-2.859	-3.459
-2.5	33 μA	0.5×10^{-3}	0.682	-3.128	-4.628 Ⓑ
-2.6	18 μA	0.6×10^{-3}	0.687	-3.287	-5.087
-2.7	1mA	1.7×10^{-3}	0.713	-3.413	-8.516 Ⓒ
-2.71	1.5mA	2.2mA	0.720	-3.43	-10

Ⓐ for small i_{D2}, D_3 is off and D_2 is on
∴ i_2 flows through 1kΩ resistor

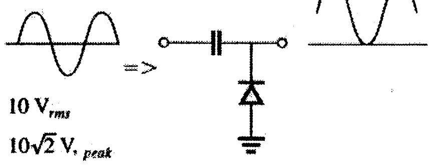
CONT.

Ⓑ 0.5 V drop across D_3 causes D_3 to start to conduct

Ⓒ $U_A = -2.7V$

The 0.7 voltage across D_3 clamps the voltage across R_3 so that D_3 control the current i_{D2}

3.73



Average (dc) value of output = $10\sqrt{2}$

= 14.14 V

4.1

Case	Mode
1	active
2	saturation
3	active
4	saturation
5	inverted active mode
6	active
7	cut-off
8	cut-off

4.2

$$i_c = I_s e^{v_{BE}/V_T}$$

For Device #1

$$0.2 \times 10^{-3} = I_{s1} e^{0.72/0.025}$$

$$I_{s1} = \underline{\underline{6.214 \times 10^{-17} A}}$$

For Device #2

$$12 \times 10^{-3} = I_{s2} e^{0.72/0.025}$$

$$I_{s2} = \underline{\underline{3.728 \times 10^{-15} A}}$$

Since $I_s \propto A$, the relative junction areas is:

$$\frac{A_2}{A_1} = \frac{I_{s2}}{I_{s1}} = \frac{i_{c2}}{i_{c1}} = \frac{12}{0.2} = \underline{\underline{60}}$$

4.3

$A_{E2} = 10^{-6} A_{E1} \Rightarrow$

$I_{S2} = 10^{-6} I_{S1}$

$i_{C1} = I_{S1} e^{v_{BE1}/V_T}$

$i_{C2} = I_{Sa} e^{v_{BE2}/V_T}$ & $i_{C1} = i_{C2} \Rightarrow$

$I_{S1} e^{v_{BE1}/V_T} = i_{C1} \approx i_{C2} = I_{S2} e^{v_{BE2}/V_T}$

$I_{S1} e^{v_{BE1}/V_T} = 10^{-6} I_{S1} e^{v_{BE2}/V_T}$

$10^6 = e^{(v_{BE2} - v_{BE1})/V_T}$

$v_{BE2} - v_{BE1} = V_T \ln(10^6) = 0.025 \ln(10^6)$

$= 0.345$

4.4

$i_{C1} = I_{S1} e^{v_{BE1}/V_T} = 10^{-12} e^{0.7/0.025} = 1.45 \text{ A}$

$i_{C2} = I_{S2} e^{v_{BE2}/V_T} = 10^{-18} e^{-0.7/0.025} = 1.45 \ \mu\text{A}$

If we set i_C^* to 1.45 µA in case *I* and

v_{BE} are allowed to vary

$1.45 \times 10^{-6} = 10^{-12} e^{v_{BE}/0.025}$

$v_{BE} = 0.354$

4.5

$i_{C,\text{old}} = I_{S,\text{old}}\, e^{v_{BE,\text{old}}/V_T}$

$v_{BE,\text{old}} = V_T \ln\left(\frac{i_{C,\text{old}}}{I_{S,\text{old}}}\right)$

$i_{C,\text{old}} = 1 \text{ mA}$; $I_{S,\text{old}} = 5 \times 10^{-15}\text{A}$

$V_T = 0.025$ Volts

$v_{BE,\text{old}} = 0.025 \ln\left(\frac{1 \times 10^{-3}}{5 \times 10^{-15}}\right) = 0.651$

$i_{C,\text{new}} = 1 \text{ mA}$; $I_{S,\text{new}} = 5 \times 10^{-18}\text{A}$

$V_T = 0.025$

$v_{BE,\text{new}} = 0.025 \ln\left(\frac{1 \times 10^{-3}}{5 \times 10^{-18}}\right) = 0.823$

4.6

$i_C = I_S e^{v_{BE}/V_T}$

$10 \times 10^{-3} = I_S\, e^{0.768/0.025} \Rightarrow I_S = 6.273 \times 10^{-16}\text{A}$

For

$v_{BE} = 0.7 \text{ V} \Rightarrow i_C = 6.273 \times 10^{-16} e^{0.7/0.025}$

$= 0.907 \text{ mA}$

For

$i_C = 10\ \mu\text{A} \Rightarrow 10 \times 10^{-6} = 6.273 \times 10^{-16}$
$e^{v_{BE}/0.025}$

$\therefore\ v_{BE} = 0.587\ \text{V}$

Alternate way – without calculating I_S

For $v_{BE} = 0.7\ \text{V}$

$$\frac{i_C}{10\ \text{mA}} = e^{\frac{0.7-0.76}{0.025}}$$

$\therefore\ i_C = 0.907\ \text{mA}$

For $i_C = 10\ \mu\text{A}$

$$\frac{10 \times 10^{-6}}{10 \times 10^{-3}} = e^{\frac{v_{BE}-0.76}{0.025}}$$

$v_{BE} = 0.587\ \text{V}$

4.7

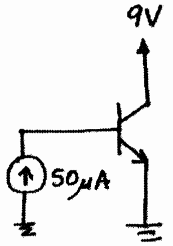
$i_c = \beta i_B$

$400 = \beta \times 7.5$

$\beta = \frac{400}{7.5} = \underline{\underline{53.3}}$

$\alpha = \frac{\beta}{\beta+1} = \frac{53.3}{54.3} = \underline{\underline{0.982}}$

4.8



$\beta = 60$ to 300

$I_c = \beta I_B$ ranges from

$= 60 \times 50\mu A$ to $300 \times 50\mu A$

$= \underline{\underline{3mA \text{ to } 15mA}}$

$I_E = I_c + I_B$ ranges from

$= \underline{\underline{3.05mA \text{ to } 15.05mA}}$

Max Power $= 9 \times I_{c\,max} = 9 \times 15$

$= \underline{\underline{135mW}}$

4.9

$i_C = I_S e^{v_{BE}/V_T}$

$i_B = \frac{i_C}{\beta}$

$i_E = \frac{\beta+1}{\beta} i_C$

$i_C = (5 \times 10^{-15}) e^{0.650/0.025} = 977\ \mu\text{A}$

i_C is constant and independent of β

i_B ranges from $\frac{i_C}{\beta} = \frac{977 \times 10^{-6}}{50} = 19.6\ \mu\text{A}$

to $\frac{i_C}{\beta} = \frac{977 \times 10^{-6}}{200} = 4.89\ \mu\text{A}$

i_E ranges from

$\frac{\beta+1}{\beta} i_C = \frac{51}{50} 977 \times 10^{-6} = 998\ \mu\text{A}$

to $\frac{\beta+1}{\beta} i_C = \frac{201}{200} 977 \times 10^{-6} = 983\ \mu\text{A}$

4.10

$i_E = 1\ \text{mA}$

Case I: $i_B = 50\ \mu\text{A}$

$i_C = i_E - i_B = 1 \times 10^{-3} - 50 \times 10^{-6} = 950\ \mu\text{A}$

$\beta = \frac{i_C}{i_B} = \frac{950 \times 10^{-6}}{50 \times 10^{-6}} = 19$

$\alpha = \frac{\beta}{\beta+1} = \frac{19}{20} = 0.95$

Case II: $i_B = 10\ \mu\text{A}$

$i_C = i_E - i_B = 1 \times 10^{-3} - 10 \times 10^{-6}\ \text{A}$

$= 990\ \mu\text{A}$

$\beta = \frac{i_C}{i_B} = \frac{990 \times 10^{-6}}{10 \times 10^{-6}} = 99$

$\alpha = \frac{\beta}{\beta+1} = \frac{99}{100} = 0.99$

Case III: $i_B = 25\ \mu A$

$$i_C = i_E - i_B = 1 \times 10^{-3}\ A - 25 \times 10^{-6}\ A$$
$$= 975\ \mu A$$

$$\beta = \frac{i_C}{i_B} = \frac{975 \times 10^{-6}}{25 \times 10^{-6}} = 39$$

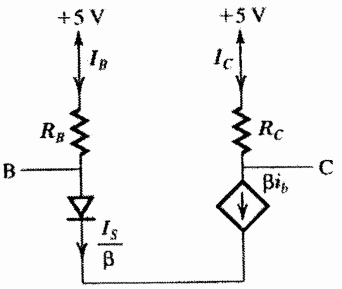
$$\alpha = \frac{\beta}{\beta+1} = \frac{39}{40} = 0.975$$

4.11

$$I_B = \frac{I_S}{\beta} e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln\left[\frac{\beta I_B}{I_S}\right]$$

$$V_{BE} = 25 \ln\left[\frac{10^{-3}}{5 \times 10^{-15}}\right] = 650\ \text{mV}$$

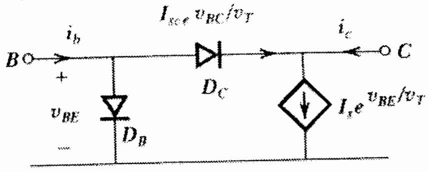
$$I_B = I_C/125 = 1000/125 = 8\ \mu A$$



$$R_B = \frac{V_{BB} - V_{BE}}{I_B} = \frac{5 - 0.65}{0.008} = 544\ k\Omega$$

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{5 - 1}{1} = 4\ k\Omega$$

4.12



$$I_C = 0 \text{ when } I_{SC} e^{V_{BC}/V_T} = I_S e^{V_{BE}/V_T}$$

$$\frac{I_{SC}}{I_S} = e^{(V_{BE} - V_{BC})/V_T} = 100$$

$$V_{CE} = V_{BE} - V_{BC} = V_T \ln\frac{I_{SC}}{I_S}$$
$$= 25 \ln 100 = 115\ \text{mV}$$

For $V_{ce} = 0.4\ V \quad V_{BC} = 0.7 - 0.4 = 0.3\ V$

$$i_{BC} = I_{SC}\, e^{0.3/V_T} = 10^{-13} e^{12} = 0.0168\ \mu A$$

For $V_{CE} = 0.3\ V \quad V_{BC} = 0.7 - 0.3 = 0.4\ V$

$$i_{BC} = I_{SC}\, e^{0.4/V_T} = 10^{-13} e^{16} = 0.089\ \mu A$$

For $V_{CE} = 0.1\ V \quad V_{BC} = 0.7 - 0.1 = 0.6\ V$

$$i_{BC} = I_{SC}\, e^{0.6/V_T} = 10^{-13} e^{24} = 2.65\ \text{mA}$$

For $V_{BE} = 0.7\ V$

$$i_{BE} = \frac{I_S}{\beta} e^{0.7/V_T} = \frac{10^{-15}}{100} e^{28} \Rightarrow 14.5\ \mu A$$

$$i_{CE} = I_{SC}\, e^{0.7/V_T} = 10^{-15} e^{28} = 1.45\ \text{mA}$$

For $V_{CE} = 0.4\ V \qquad V_{BC} = 0.3\ V$

$$i_B = i_{BE} + i_{BC} = 14.5 + 0.02 = 14.52\ \mu A$$
$$i_C = i_{CE} - i_{BC} = 1.45 - 0 = 1.45\ \text{mA}$$
$$i_C / i_B = 1.45\ \text{mA} / 14.52\ \mu A = 100$$

For $V_{CE} = 0.3\ V \qquad V_{BC} = 0.4\ V$

$$i_b = 14.5 + 0.089 = 145.89\ \mu A$$

$$i_C = 1.45 - \frac{0.089}{1000} = 1.45\ \text{mA}$$

$$i_C/i_b \approx 1.45\ \text{mA}/146\ \mu A = 9 \cdot 9$$

For $V_{CE} = 0.1\ V \qquad V_{BC} = 0.6\ V$

$$i_b = 14.5 + 2.65 = 4.1\ \text{mA}$$
$$i_C = 1.45 - 2.65 = -1.2\ \text{mA}$$

V_{CE} too low for model

4.13

given: $i_C = I_S\, \epsilon^{V_{BE}/V_T} - I_{SC}\, e^{V_{BC}/V_T}$

and $i_C = \dfrac{I_S}{\beta} e^{V_{BE}/V_T} + I_{SC}\, e^{V_{BC}/V_T}$

and $\beta_{\text{forced}} = \left.\dfrac{i_C}{i_B}\right|_{\text{Sat}^n} \leq \beta$

$$\beta_{\text{forced}} = \beta \cdot \frac{I_S\, e^{(V_{CEsat} + V_{BC})/V_T} - I_{SC}\, e^{V_{BC}/V_T}}{I_S\, e^{(V_{CEsat} + V_{BC})/V_T} + I_{SC}\, e^{V_{BC}/V_T}}$$

$$= \beta \cdot \frac{I_S}{I_S} \frac{e^{V_{BC}/V_T}}{e^{V_{BC}/V_T}} \frac{[e^{V_{CEsat}/V_T} - I_{SC}/I_S]}{[e^{V_{CEsat}/V_T} + \beta I_{SC}/I_S]}$$

$$\therefore\ e^{V_{CEsat}} = \frac{-\beta \frac{I_{SC}}{I_S} - \beta \frac{I_{SC}}{I_S} \times \beta_{\text{forced}}}{\beta - \beta_{\text{forced}}}$$

$$= \frac{I_{SC}}{I_S}\left[\frac{\beta + \beta\ \beta_{\text{forced}}}{\beta - \beta\ \beta_{\text{forced}}}\right]$$

$$= \frac{I_{SC}}{I_S}\left[\frac{1 + \beta_{\text{forced}}}{1 - \beta_{\text{forced}}/\beta}\right] \qquad \text{QED}$$

For $\beta_{\text{forced}} = 50$

$$V_{CEsat} = 25 \ln\left[100 \cdot \frac{1 + 50}{1 - 50/100}\right]$$

$$= 25\ \ln[10200] = 230.8\text{ mV}$$

For $\beta_{\text{forced}} = 10$

$$V_{CEsat} = 25\ \ln\left[100 \cdot \frac{1+10}{1-10/100}\right]$$

$$= 25\ \ln[122.2] = 177.7\text{ mV}$$

For $\beta_{\text{forced}} = 5$

$$V_{CEsat} = 25\ \ln\left[100 \cdot \frac{1+5}{1-5/100}\right]$$

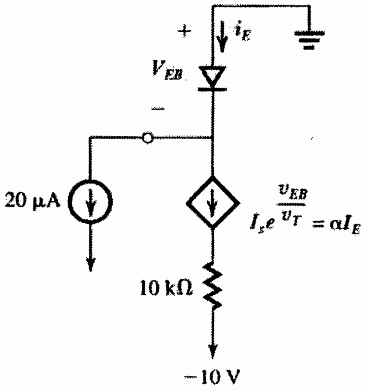
$$= 25\ \ln[631.6] = 161.2\text{ mV}$$

For $\beta_{\text{forced}} = 1$

$$V_{CEsat} = 25\ \ln\left[100 \cdot \frac{1+1}{1-1/100}\right]$$

$$= 25\ \ln[202] = 132.7\text{ mV}$$

4.14



$\beta = 40$

$\alpha_F = \frac{40}{41}$

$I_S = 10^{-13}\text{ A}$

$i_E = \frac{I_S}{\alpha} e^{V_{EB}/V_T} = I_S\, e^{V_{EB}/V_T} + 0.02 \times 10^{-3}\text{ A}$

$I_S\ e^{V_{EB}/V_T}\left(\frac{1}{\alpha} - 1\right) = 0.02 \times 10^{-3}\text{ A}$

$10^{-13}\ e^{V_{EB}/0.025}\left(\frac{41}{40} - 1\right) = 0.02 \times 10^{-3}\text{ A}$

$V_{EB} = 0.570\text{ V} \Rightarrow V_B = -0.570\text{ V}$

$i_E = \frac{I_S}{\alpha} e^{V_{EB}/V_T} = \frac{10^{-13}}{40} e^{\frac{0.57}{0.025}}$

$= 0.82\text{ mA}$

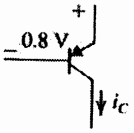
$i_C = \alpha i_E \Rightarrow$

$V_C = -10 + \alpha i_E \times 10$

$= -10 + \frac{40}{41} \times 0.82 \times 10$

$= -2\text{ V}$

4.15



$\because\ i_C = I_S\, e^{V_{EB}/V_T}$

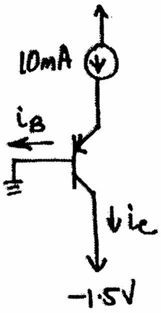
Use $\frac{i_C}{1\text{A}} = e^{\frac{v_{EB} - 0.8}{0.025}}$

to calculate v_{EB} for a particular i_C

For $i_C = 10\text{ mA}$ $\quad v_{EB} = 0.685\text{ V}$

For $i_C = 5\text{ A}$ $\quad v_{EB} = 0.840\text{ V}$

4.16



$\beta = 10$

$i_c = \alpha\, i_E = \frac{10}{11} \times 10 = \underline{\underline{9.09\text{mA}}}$

$i_B = i_E - i_c = \underline{\underline{0.91\text{ mA}}}$

$i_c = I_s\, e^{V_{EB}/V_T}$

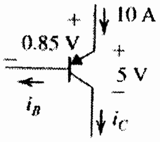
$9.09 \times 10^{-3} = 10^{-16}\, e^{V_{EB}/0.025}$

$V_E = V_{EB} = \underline{\underline{0.803\text{V}}}$

For $\beta = 1000$

$i_c = \frac{\beta}{\beta + 1}\, i_E = \frac{1000}{1001} \times 10 = \underline{\underline{9.99\text{mA}}}$

4.17



for $\beta = 15$

$i_E = (\beta + 1) i_B$

$10 = (\beta + 1) i_B$

$$i_B = \frac{10}{16} = 0.625 \text{ A}$$

Calculating I_{SI}

$$i_C = \frac{\beta}{\beta+1} i_E = I_{S1} e^{v_{EB}/v_T}$$

$$\frac{15}{16} \times 10 = I_{S1} e^{0.85/0.025}$$

$$I_{S1} = 1.608 \times 10^{-14} \text{ A}$$

Compare this to

$$I_{S2} = i_C e^{-v_{EB}/v_T}$$

$$= 10^{-3} e^{-0.7/0.025}$$

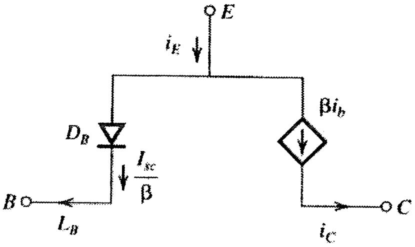
$$= 6.914 \times 10^{-16}$$

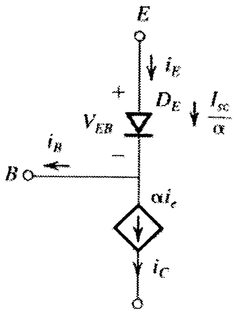
$\because I_S \alpha$ area

$$\frac{\text{Area1}}{\text{Area2}} = \frac{I_{S1}}{I_{S2}} = \frac{1.608 \times 10^{-14}}{6.914 \times 10^{-16}}$$

= 23.3 times larger

4.18

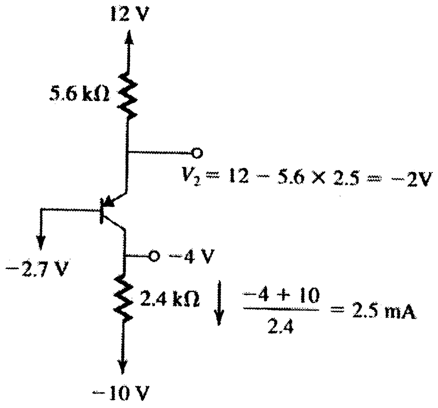

E
i_E
D_B
$\frac{I_{sc}}{\beta}$
βi_b
B
L_B
C
i_C


E
i_E
+
V_{EB}
D_E
$\frac{I_{sc}}{\alpha}$
i_B
B
−
αi_e
i_C

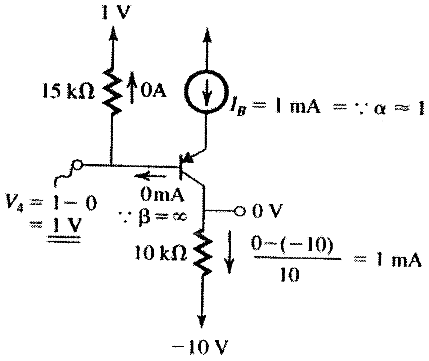
4.19

(a) $I_1 = \frac{10.7 - 0.7}{10} = 1$ mA

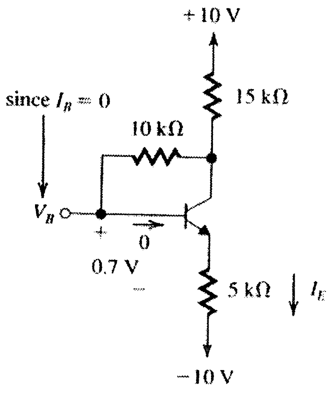
(b)


12 V
5.6 kΩ
$V_2 = 12 - 5.6 \times 2.5 = -2V$
−2.7 V
−4 V
2.4 kΩ
$\frac{-4+10}{2.4} = 2.5$ mA
−10 V

(c)


1 V
15 kΩ
0A
$I_B = 1$ mA $= \because \alpha \approx 1$
$V_4 = 1 - 0 = \underline{1 \text{ V}}$
0mA
$\because \beta = \infty$
0 V
10 kΩ
$\frac{0-(-10)}{10} = 1$ mA
−10 V

(d)


+10 V
15 kΩ
since $I_B = 0$
10 kΩ
V_B
0
+
0.7 V
−
5 kΩ
I_E
−10 V

$I_E = I_C$

$$\frac{V_B - 0.7 + 10}{5} = \frac{10 - V_E}{15}$$

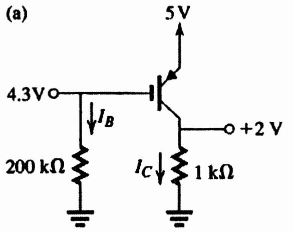
$15V_6 + 139.5 = 50 - 5V_6$

$V_6 = -4.475\text{ V}$

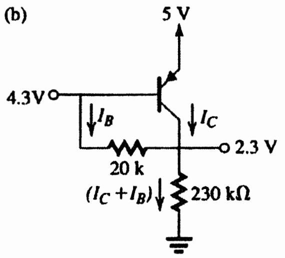
$$I_5 = \frac{V_6 - 0.7 + 10}{5}$$

$$= \frac{-4.475 - 0.7 + 10}{5} = 0.965\text{ mA}$$

4.20


(a)
5 V
4.3V
I_B
+2 V
200 kΩ
I_C
1 kΩ

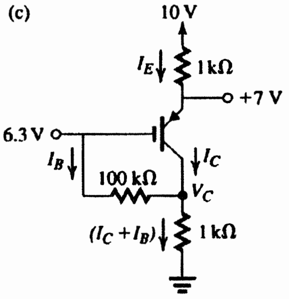
$$\frac{I_c}{I_B} = \beta = \frac{\left(\frac{2}{1\text{ K}}\right)}{\left(\frac{4.3}{200\text{ K}}\right)} = \frac{2\text{ m}}{0.0215\text{ m}} = 93$$


(b)
5 V
4.3V
I_B
I_C
2.3 V
20 k
$(I_C + I_B)$
230 kΩ

$$(I_c + I_B) = \frac{2.3}{230} = 10\text{ mA}$$

$$I_B = \left(\frac{4.3 - 2.3}{20\text{ K}}\right) = 0.1\text{ mA}$$

$$\frac{I_c}{I_B} = \left(\frac{10\text{ m} - 0.1\text{ m}}{0.1\text{ m}}\right) = \beta = 99$$


(c)
10 V
I_E
1 kΩ
+7 V
6.3 V
I_B
I_C
100 kΩ
V_C
$(I_C + I_B)$
1 kΩ

$$I_E = \left(\frac{10 - 7}{1\text{ K}}\right) = 3\text{ mA}$$

$I_E = I_c + I_B = 3\text{ mA}$

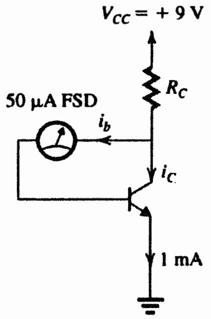
$V_c = 3\text{ m}(1\text{ K}) = 3\text{ V}$

$$I_B = \frac{6.3 - 3}{100\text{ K}} = 33\ \mu\text{A}$$

$$\frac{I_E}{I_B} = \beta + 1 = \frac{3\text{ m}}{33\ \mu} = 90.9$$

$\beta = 89.9$

4.21


$V_{CC} = +9$ V
R_C
50 μA FSD
i_b
i_C
1 mA

For F.S.D $i_b = 50\ \mu\text{A}$

$i_C = 1000 - 50 = 950\ \mu\text{A}$

Since $R_m = 0\ \Omega$ $V_{CE} = V_{BE} = 0.7\text{ V}$

∴ active mode

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{9 - 0.7}{1\text{mA}} = 8.3\text{ k}\Omega$$

$$I_C = \beta I_B \therefore\ \beta = \frac{950}{50} = 19$$

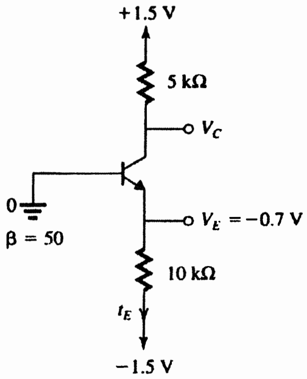
For FSD/5: $i_b = 10\ \mu\text{A},\ i_C = 990\ \mu\text{A}$

$\Rightarrow \beta = 99$

For FSD/10: $i_b = 5\ \mu\text{A},\ i_C = 995\ \mu\text{A}$

$\Rightarrow \beta = 199$

4.22


+1.5 V
5 kΩ
V_C
0
$\beta = 50$
$V_E = -0.7$ V
10 kΩ
I_E
−1.5 V

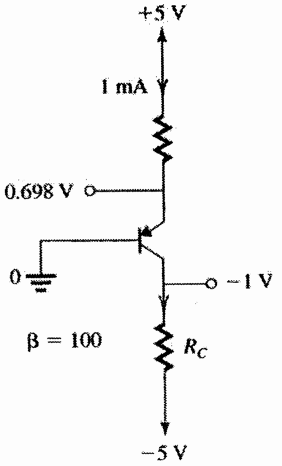
$$I_E = \frac{V_E - V_{EE}}{R_E} = \frac{0.8}{10\ \text{K}} = 80\ \mu\text{A}$$

$$I_C = \frac{\beta}{\beta + 1} I_E = \frac{50}{51} \times 80 = 78\ \mu\text{A}$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{80}{51} = 1.6\ \mu\text{A}$$

$$V_c = V_{cc} - I_c R_C = 1.5\ \text{V} - 0.078 \times 5\ \text{V}$$
$$= 1.11\ \text{V}$$

4.23



$$V_{BE(1\text{mA})} - V_{BE(0.1\text{mA})}$$
$$= 25\ln\left[\frac{1}{0.1}\right]$$

$$\therefore\ V_{BE(1\text{mA})} = 640\ \text{mV} + 57.9\ \text{mV}$$
$$= 698\ \text{mV}$$

$$I_C = \frac{100}{101} I_E = 0.99\ \text{mA}$$

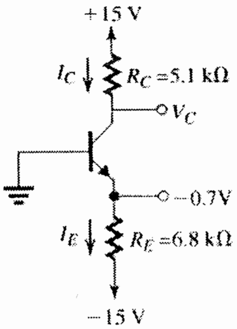
$$R_C = \frac{-1 - (-5)}{0.99} = 4.04\ \text{k}\Omega$$

$$R_E = \frac{5 - 0.698}{1} = 4.3\ \text{k}\Omega$$

V_C can be raised untill $\approx +0.4$ V

$$R_C = \frac{5 + 0.4}{0.99} = 5.45\ \text{k}\Omega$$

4.24



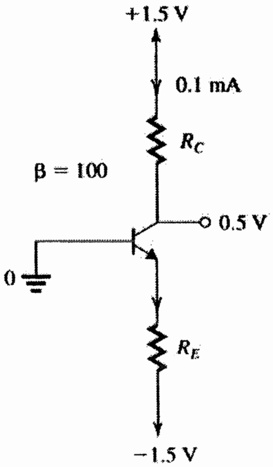
$\alpha \approx 1$

$$I_E \approx I_c = \frac{(-0.7 - (-15))}{6.8\ \text{k}\Omega} = 2.1\ \text{mA}$$

$$V_c = 15 - 5.1\ \text{K}(2.1\ \text{m})$$

$$V_c = 4.3\ \text{V}$$

4.25



$$\Delta V_{BE} = V_T \ln\left[\frac{I_{C7}}{I_{C1}}\right]$$
$$= 25 \ln [0.1] = -57.6\ \text{mA}$$

$$V_{BE(0.1)} = 742\ \text{mV}$$

$$R_C = \frac{1.5 - 0.5}{0.1} = 10\ \text{k}\Omega$$

$$V_E = -0.742\ \text{V}$$

$$R_E = \frac{-0.742 + 1.5}{\frac{\beta + 1}{\beta} I_C}$$
$$= \frac{100}{101} \cdot \frac{0.758}{0.1} = 7.5\ \text{k}\Omega$$

4.26

(a) $V_B = \underline{0V}$

$V_E = V_B - 0.7 = \underline{-0.7V}$

$$I_E = \frac{-0.7 + 3}{2.2} = \underline{1.05\,\text{mA}}$$

$I_c = \frac{30}{31} I_E = \underline{\underline{1.02\,mA}}$

$V_c = 3 - 1.02 \times 2.2 = \underline{\underline{0.756\,V}}$

$I_B = \frac{I_c}{\beta} = \frac{1.02}{30} = \underline{\underline{0.034\,mA}}$

(b) $V_B = \underline{0\,V}$

$V_E = V_B + 0.7 = \underline{0.7\,V}$

$I_E = \frac{3 - V_E}{1} = \frac{3 - 0.7}{1} = \underline{\underline{2.3\,mA}}$

$I_c = \alpha I_E = \frac{30}{31} \times 2.3 = \underline{\underline{2.23\,mA}}$

$V_c = -3 + 1 \times I_c = -3 + 2.23$

$= \underline{\underline{-0.77\,V}}$

$I_B = \frac{I_c}{\beta} = \frac{2.23}{30} = \underline{\underline{0.0743\,mA}}$

(c) $V_B = \underline{3\,V}$

$V_E = V_B + 0.7 = \underline{\underline{3.7\,V}}$

$I_E = \frac{9 - V_E}{1.1} = \frac{9 - 3.7}{1.1} = \underline{\underline{4.82\,mA}}$

$I_c = \alpha I_E = \frac{30}{31} \times 4.82 = \underline{\underline{4.66\,mA}}$

$V_c = I_c \times 0.56 = \underline{\underline{2.62\,V}}$

$I_B = \frac{I_c}{\beta} = \frac{4.66}{30} = \underline{\underline{0.155\,mA}}$

(d) $V_B = \underline{3\,V}$

$V_E = 3 - 0.7 = \underline{\underline{2.3\,V}}$

$I_E = V_E / 0.47 = 2.3 / 0.47 = \underline{\underline{4.89\,mA}}$

$I_c = \alpha I_E = \frac{30}{31} \times 4.89 = 4.73\,mA$

$V_c = 9 - 1 \times I_c = 9 - 4.73 = \underline{\underline{4.22\,V}}$

$I_B = I_c / \beta = \frac{4.73}{30} = \underline{\underline{0.158\,mA}}$

4.27

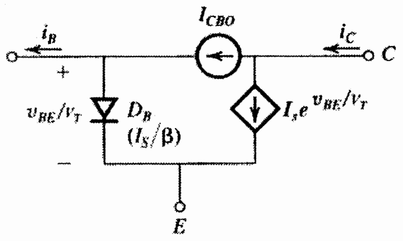
I_{CBO} doubles for every 10°C rise in temperature.

Thus if $I_{CBO} = 20\,nA$ at 25°C

At 85°C $I_{CBO} = 2^{\frac{85-25}{10}} \times 20$

$= \underline{\underline{1280\,nA}}$

4.28



$$i_B = \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}} - I_{CBO} \qquad (1)$$

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} + I_{CBO} \qquad (2)$$

$$i_E = I_S\left(1 + \frac{1}{\beta}\right) e^{v_{BE}/V_T} \qquad (3)$$

for β opencircuited. $i_B = 0$ and (1) gives

$$\frac{I_S}{\beta} e^{v_{BE}/V_T} = I_{CBO} \Rightarrow e^{v_{BE}/V_T} = \frac{\beta I_{CBO}}{I_S}$$

substitute into (2) & (3) $\Rightarrow$

$$i_C = (\beta + 1) I_{CBO}$$

$$i_E = (\beta + 1) I_{CBO}$$

4.29

GIVEN $i_E = 0.5mA$, $V_{EB} = 0.692V$ } AT 20°C

(a) The junction temperature rises to 50°C

$V_{EB} = 0.692 - 2\times10^{-3}(50-20)$

$= \underline{\underline{0.632V}}$

(b) The Base-Emitter Voltage is fixed $V_{EB} = 0.7V$ at ALL TEMPERATURES

At 20°C ~ $i_E = 0.5mA$ at $V_{EB} = 0.692V$

Thus for $V_{EB} = 0.7V$ we have

$$\frac{i_E}{0.5\times10^{-3}} = e^{\frac{0.7-0.692}{0.025}}$$

$i_E = \underline{\underline{0.689mA}}$

Now if $T = 50°C$ & $V_{EB} = 0.7V$

from (a) we see that at 50°C,

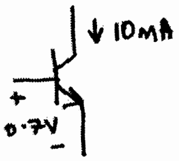
$I_E = 0.5mA$, $V_{EB} = 0.632V$

Therefore for $V_{EB} = 0.7V$

$$\frac{i_E}{0.5\times10^{-3}} = e^{\frac{0.7-0.632}{0.025}}$$

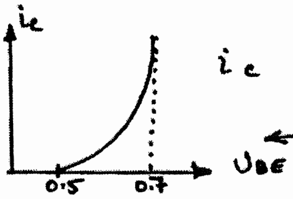
$i_E = \underline{7.59mA}$

4.30



$$\frac{i_c}{10mA} = e^{\frac{v_{BE}-0.7}{0.025}}$$

$$= e^{\frac{0.5-0.}{0.025}}$$



$i_c = \underline{\underline{3.35\,\mu A}}$

Notice the current drops significantly at $V_{BE} = 0.5V$

4.31

V_{BE} changes by –2 mV/°C for a particular current. Given that at 25° C $V_{BE} = 0.7$ V and $i_C = 10$ mA

Thus

@ −25° C $V_{BE} = 0.7 - 2\times10^{-3}(-50)$

$= 0.8$ V and $i_C = 10$ mA

@ 125° C $V_{EB} = 0.7 - 2\times10^{-3}(100)$

$= 0.5$ V and $i_C = 10$ mA

4.32

$r_o = \frac{1}{3\times10^{-5}} = \underline{\underline{33.3k\Omega}}$

$V_A = r_o I_c = 33.3\times10^{3} \times 3\times10^{-3}$

$= \underline{\underline{100V}}$

$r_o = \frac{V_A}{I_c} = \frac{100}{30} = \underline{\underline{3.3k\Omega}}$

4.33

$r_o = V_A / I_C = 200 / I_C$

$@ I_C = 1mA \quad r_o = \underline{\underline{200 k\Omega}}$

$@ I_C = 100\mu A \quad r_o = \frac{200}{0.1} = \underline{\underline{2.0 M\Omega}}$

4.34

$V_{BE} = 0.72\text{ V} \cdot i_C = 1.8\text{ mA} \quad V_{CE} = 2\text{ V}$

$i_C = 2.4\text{ mA} \quad V_{CE} = 14\text{ V}$

$$r_o = \frac{\Delta V_{CE}}{\Delta i_C} = \frac{14-2}{2.4-1.8} = 20\text{ k}\Omega$$

Near saturation $V_{CE} = 0.3\text{V}$

$$\therefore \frac{\Delta V_{CE}}{\Delta i_C} = \frac{0.3-2}{i_C - 1.8} = 20\text{ k}\Omega$$

$i_C = 1.72\text{ mA}$

Calculating V_{CE} for $i_C = 2.0\text{ mA}$

$$\frac{\Delta V_{CE}}{\Delta i_C} = r_o$$

$$\frac{V_{CE}-2}{2-1.8} = 20 \Rightarrow V_{CE} = 6\text{ V}$$

Take the ratio of currents to find the early voltage (with Eq 5.36)

$$\frac{2.4}{1.8} = \underbrace{e^{\frac{V_{BE}-V_{BE}}{V_T}}}_{=1}\left(\frac{1+14/V_A}{1+2/V_A}\right)$$

$$2.4 + \frac{4.8}{V_A} = 1.8 + \frac{25.2}{V_A}$$

$V_A = 34\text{ V}$

$$r_o = \frac{V_A}{I_C'}$$

where I_C' is the current near saturation ↔ active boundary. As calculated above $I_C = 1.72\text{ mA}$

$$r_o = \frac{34\text{ V}}{1.72\text{ mA}} = 19.8\text{ k}\Omega$$ compared to the above calculation of 20 kΩ.

4.35

Large signal or DC β:

$h_{FE} = \frac{i_C}{i_B} = \frac{1.2mA}{8\mu A} = \underline{\underline{150}}$

Small signal $h_{fe} = \frac{0.1mA}{0.8\mu A} = \underline{\underline{125}}$

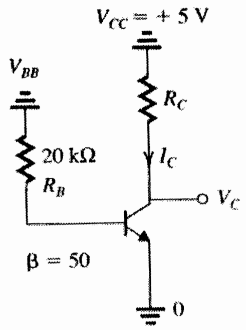
$r_o = \frac{V_A}{I_C} = \frac{100V}{1.2mA} = 83.3k\Omega$

$\Delta i_C = h_{fe}\Delta i_B + \frac{\Delta V_{CE}}{r_o}$

$= 125 \times 2\mu A + 2/83.3k\Omega = 0.274mA$

$\therefore i_C = 1.2mA + \Delta i_C = \underline{\underline{1.474mA}}$

4.36



(a) active region

$$I_C = \frac{V_{CC}-V_C}{R_C}$$

$$= \frac{5-1}{1\text{ K}} = 4\text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{4}{50} = 0.08\text{ mA}$$

$$V_{BB} = 0.7 + \frac{20 \times 4}{50}$$

$$= +2.3\text{ V}$$

(b) edge of saturation $v_C = 0.3$ V

$I_C = \frac{5-0.3}{1} = 4.7$ mA

$I_B = I_C/\beta = 4.7/50 = 0.094$ mA

$V_{BB} = 0.094 \times 20 + 0.7 = 2.58$ V

(c) deep saturation $v_C = 0.2$ V $\beta_F = 10$

$I_C = (5-0.2)/1 = 4.8$ mA

$I_B = I_C/\beta_{\text{forced}} = 4.8/10 = 0.48$ mA

$V_{BB} = 0.48 \times 20 + 0.7 = +10.3$ V

4.37

Assume active:

$V_E = 3$ V, $V_B = 2.3$ (Assume $V_{BE} = 0.7$ V)

$I_B = \frac{2.3}{10\text{ K}} = 2.3$ mA

$I_c = 2.3\text{ m}(50) = 115$ mA

$V_c = 115\text{ m}(1\text{ K}) = 115$ V, $V_c < V_B$ (not true!)

saturation. Use $V_{\text{ECSAT}} = 0.2$ V

$+3 - V_{\text{ECSAT}} - V_c = 0$

$V_c = 3 - 0.2 = 2.8$ V

$V_B = 2.3$

$V_E = 3$ V

$V_c > V_B < V_E$ ∴ SATURATED

$$\beta_{\text{forced}} = \frac{I_{\text{CSAT}}}{I_B} = \frac{\left(\frac{2.8}{1\text{ K}}\right)}{\left(\frac{3}{10\text{ K}}\right)} = \frac{2.8\text{ m}}{0.3\text{ m}} = 9.33$$

Transistor will operate at edge of saturation when $V_c = V_B = 2.8$ V

$$\therefore R_B = \frac{V_B}{I_B} = \frac{2.8}{3\text{ m}} = 9.3\text{ k}\Omega$$

4.38

(a) $V_B = 2V$

$V_E = 2 - 0.7 = \underline{\underline{1.3V}}$

$I_E = \frac{V_E}{1} = 1.3mA$

$I_c \cong 1.3$ mA

$V_c = 5 - 1.3 = \underline{\underline{3.7V}}$

(b) $V_B = 1V$

$V_E = 1 - 0.7 = \underline{\underline{0.3V}}$

$I_E \cong I_c = 0.3mA$

$V_c = 5 - 0.3$

$= \underline{\underline{4.7V}}$

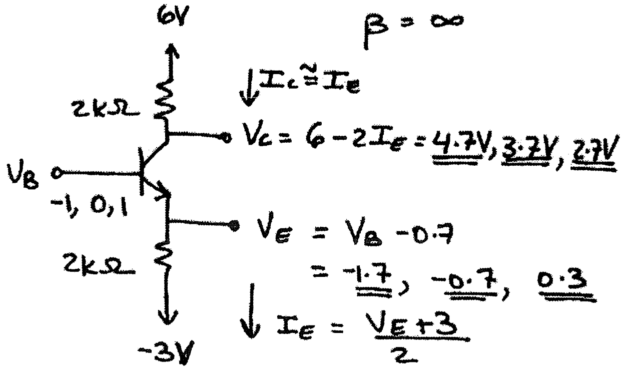
(c) $V_B = 0V$ - cutoff

$V_E = \underline{\underline{0V}}$

$I_E = 0A$

$V_c = \underline{\underline{5V}}$

4.39



$V_c = 6 - 2I_E = \underline{\underline{4.7V}}, \underline{\underline{3.7V}}, \underline{\underline{2.7V}}$

$V_E = V_B - 0.7$

$= \underline{\underline{-1.7}}, \underline{\underline{-0.7}}, \underline{\underline{0.3}}$

$I_E = \frac{V_E + 3}{2}$

$= 0.65, 1.15, 1.65mA$

- Want V_B when $I_E = \frac{1}{10} \times 1.15mA$

$= 0.115mA$

$V_E = -3 + 0.115 \times 2 = -2.77$ V

$V_B = V_E + 0.7 = \underline{\underline{-2.07V}}$

-Want V_B at the edge of conduction

At the edge of conduction assume $V_{BE} = 0.5$ V

$\therefore V_B - 0.5 - 2I_E + 3 = 0 \leftarrow I_E = 0$ at edge of conduction

$V_B = \underline{\underline{-2.6V}}$

$V_E = V_B - 0.5 = \underline{-3V}$

$I_C \cong 0A \quad \therefore V_C = \underline{6V}$

At saturation assume $V_{CE} = 0.2V$

$V_{CB} = -0.5V$

$\therefore \; I_E = \dfrac{V_B - 0.7 + 3}{2} \cong I_C = \dfrac{6 - (V_B - 0.5)}{2}$

$\therefore \; V_B + 2.3 = 6.5 - V_B$

$V_B = \underline{\underline{2.1V}}$

$V_E = 2.1 - 0.7 = \underline{\underline{1.4V}} \qquad V_C = V_B - 0.5 = \underline{\underline{1.6V}}$

-Want V_B at $\beta_{forced} = 2$, $V_{CE} = 0.2V$

$V_{CB} = -0.5V$

$\beta_{forced} = \dfrac{I_{csat}}{I_B} = 2$

$I_E = I_B + I_{csat} = \dfrac{I_{csat}}{2} + I_{csat}$

$= 3/2 \, I_{csat}$

$V_E = V_B - 0.7 = -3 + 2I_E$

$= -3 + 3I_{csat}$

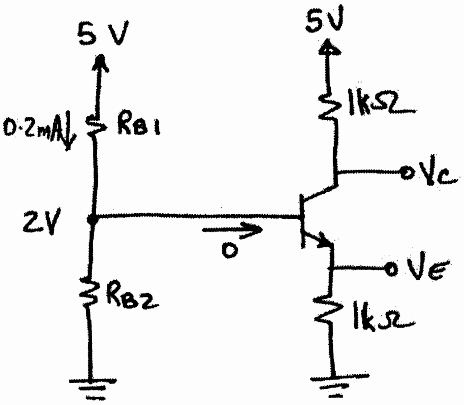
$I_{csat} = \dfrac{2.3 + V_B}{3}$

$I_C = \dfrac{V_{CC} - (V_B - 0.5)}{2} = I_{csat}$

$6.5 - V_B = \frac{2}{3}(2.3) + \frac{2}{3}V_B$

$V_B = \dfrac{6.5 - \frac{2(2.3)}{3}}{1\frac{2}{3}} = \underline{\underline{2.98V}}$

4.40



For $\beta = \infty$

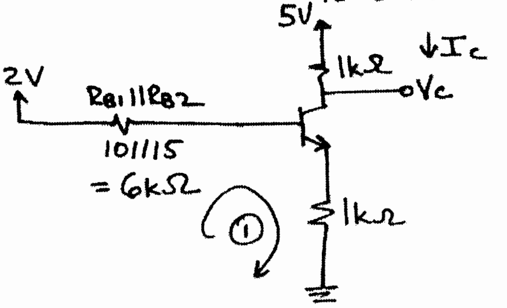
$\dfrac{5}{R_{B1} + R_{B2}} = 0.2 \quad \& \quad \dfrac{R_{B2}}{R_{B1} + R_{B2}} 5 = 2$

$R_{B1} + R_{B2} = 25k\Omega \nearrow$ sub

$\Rightarrow \quad \dfrac{R_{B2}}{25} \times 5 = 2$

$R_{B2} = \underline{\underline{10k\Omega}} \quad R_{B1} = \underline{\underline{15k\Omega}}$

Now for $\beta = 100$, use thevenin's to obtain:



Loop ① $\; 2 - 6\left(\dfrac{I_E}{\beta + 1}\right) - 0.7 - I_E(1) = 0$

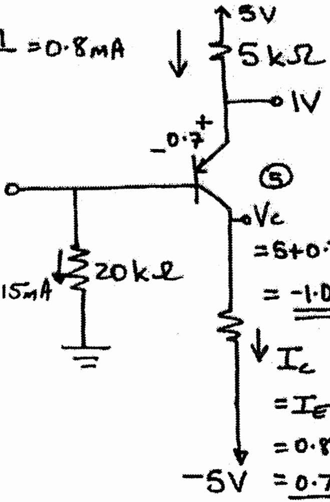
101

$I_E = 1.29mA$

$I_C = \frac{100}{101} I_E = \frac{100}{101} \times 1.29 = \underline{\underline{1.28mA}}$

$V_C = 5 - 1.28(1) = \underline{\underline{3.72V}}$

4.41


5V
5kΩ
1V
+
_0.7
V_C
20kΩ
I_C
−5V

① $I_E = \frac{5-1}{5} = 0.8mA$

② $V_B = 1 - 0.7 = 0.3V$

③ $I_B = \frac{0.3}{20} = 0.015mA$

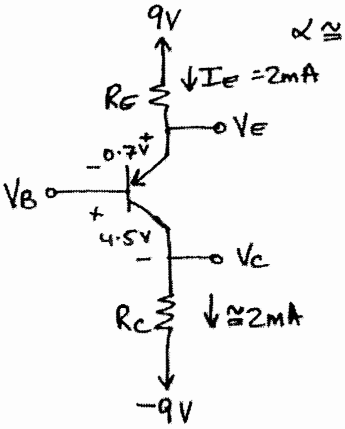
⑤ $V_C = 5 + 0.785(5) = \underline{\underline{-1.075V}}$

④ $I_C = I_E - I_B = 0.8 - 0.015 = \underline{\underline{0.785\,mA}}$

⑥ $\beta = \frac{I_C}{I_B} = \frac{0.785}{0.015} = \underline{\underline{52.3}}$

⑦ $\alpha = \frac{I_C}{I_E} = \frac{0.785}{0.8} = \underline{\underline{0.98}}$

4.42


9V
R_E
$I_E = 2mA$
V_E
_0.7V +
V_B
+
4.5V
−
V_C
R_C
≅2mA
−9V

$\alpha \cong 1$

$9 - 2R_E - 0.7 - 4.5 - 2R_C + 9 = 0$

$6.4 - R_E - R_C = 0$

LET $V_B = 0V$

$R_E = \frac{9-0.7}{2} = \underline{\underline{4.15k\Omega}}$

$R_C = \frac{-4.5+9}{2} = \underline{\underline{2.25k\Omega}}$

Using 5% resistor values

$R_E = 3.9k\Omega \quad R_C = 2.2k\Omega$

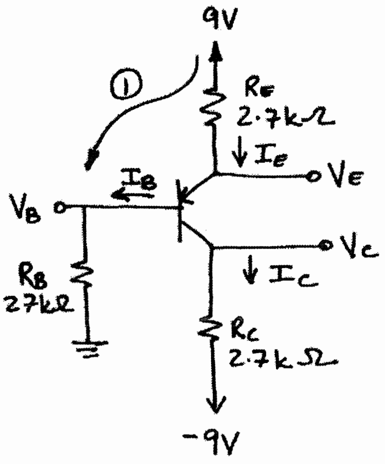
$I_E = \frac{9-0.7}{3.9} = \underline{\underline{2.12mA}}$

$V_C = -9 + 2.12 \times 2.2 = -4.3V$

$\therefore V_{BC} = \underline{\underline{4.3V}}$

4.43

$\beta = 30$


9V
①
R_E
2.7kΩ
I_E
I_B
V_E
V_B
V_C
I_C
R_B
27kΩ
R_C
2.7kΩ
−9V

Loop ① $9 - 2.7I_E - 0.7 - \frac{I_E}{31} R_B = 0$

$I_E = 2.3243\,mA$

$V_B = R_B \times I_E/31 = \underline{\underline{2.02V}}$

$V_E = 9 - 2.7I_E = \underline{\underline{2.72V}}$

$V_C = -9 + \frac{30}{31} I_E(2.7) = \underline{\underline{-2.93V}}$

FOR $R_B = 270k\Omega$

Loop ① $9 - 2.7I_E - 0.7 - \frac{R_B}{31} I_E = 0$

$I_E = 0.7274\,mA$

$$V_B = R_B \times \frac{I_E}{31} = \underline{\underline{6.34V}}$$

$$V_E = 9 - 2.7I_E = \underline{\underline{7.04V}}$$

$$V_C = \frac{30}{31} I_E(2.7) - 9 = \underline{\underline{-7.10V}}$$

To return the voltages to the ones first calculated we have

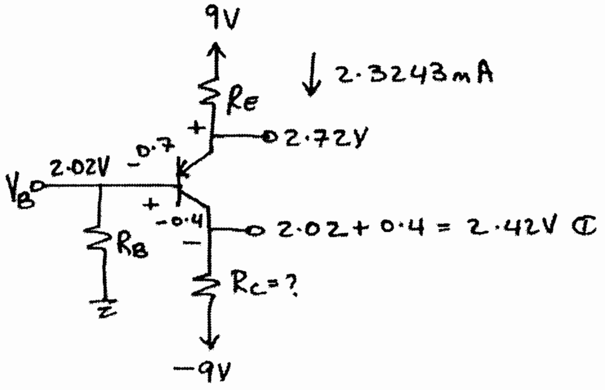
Loop ① $\sim I_E = 2.3243\ mA$

$$9 - 2.7I_E - 0.7 - \frac{270}{\beta+1} I_E = 0$$

$$\beta = \underline{\underline{309}}$$

4.44

Using the values from the first part of P5.76 and for the edge of saturation $V_{BC} > -0.4V$



9V
R_E
↓ 2.3243mA
+
2.72V
0.7
–
2.02V
V_B
+
-0.4
–
2.02 + 0.4 = 2.42V ©
R_B
R_C = ?
−9V

CIRCUIT AT THE
EDGE OF SATURATION

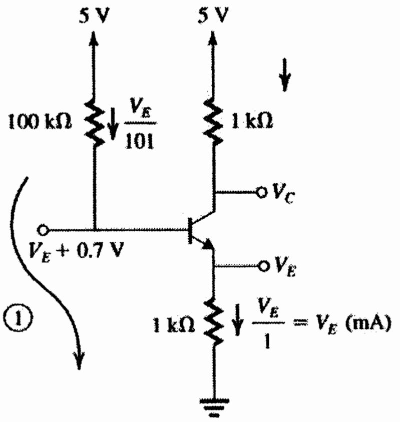
$$I_C = \frac{30}{31} I_E = \frac{30}{31} \times 2.3243$$

$$R_C = \frac{2.42 + 9}{30/31 \times 2.3243} = \underline{\underline{5.08\,k\Omega}}$$

4.45

$\beta = 100$

(a) $R_B = 100\ k\Omega$ – ∵ R_B is large assume active mode.



5 V
5 V
100 kΩ
$\frac{V_E}{101}$
1 kΩ
V_C
$V_E + 0.7$ V
V_E
①
1 kΩ
$\frac{V_E}{1} = V_E$ (mA)

$$\frac{100}{101} I_E = \frac{100}{101} V_E \text{ (mA)}$$

Loop (1)

$$5 - \frac{V_E}{101} \times 100 - 0.7 - V_E \times 1 = 0$$

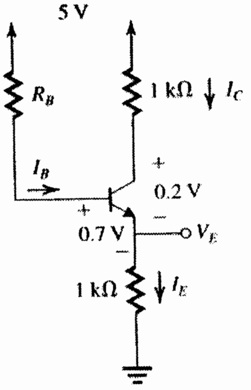
$V_E = 2.16$ V

$V_B = V_E + 0.7 = 2.86$ V

$$V_C = 5 - 1 \times \frac{100}{101} V_E = 2.86 \text{ V}$$

Thus the BJT is in active mode as assumed.

(b) $R_B = 10\ k\Omega$ - assume saturation



5 V
R_B
1 kΩ
I_C
I_B
+
0.2 V
–
+
0.7 V
–
V_E
1 kΩ
I_E

$$I_B = \frac{5 - (V_E + 0.7)}{R_B}$$

$$I_C = \frac{5 - (V_E + 0.2)}{1}$$

$$I_E = \frac{V_E}{1} = I_B + I_C$$

$$\therefore V_E = \frac{4.3 - V_E}{10} + 4.8 - V_E$$

$10V_E + V_E + 10V_E = 4.3 + 48$

$V_E = 2.49$ V

$V_C = 2.49 + 0.2 = 2.69$ V

$V_B = V_E + 0.7 = 3.19$ V

Check : $I_C = \frac{5 - 2.69}{1} = 2.31$ mA

$I_B = \frac{5 - 3.19}{10} = 0.181$ mA

$\frac{I_C}{I_B} = \frac{2.31}{0.181} = 12.76 < 100$

Hence, we are in saturation as assumed!

(c) $R_B = 1$ kΩ - expect saturation, use circuit in (b)

$I_B = \frac{5 - (V_E + 0.7)}{R_B} = \frac{4.3 - V_E}{1}$

$I_C = \frac{5 - (V_E + 0.2)}{1} = \frac{4.8 - V_E}{1}$

$I_E = I_B + I_C = V_E$

$4.3 - V_E + 4.8 - V_E = V_E$

$V_E = 3$ V

$V_B = 3.7$ V

$V_C = 3.2$ V

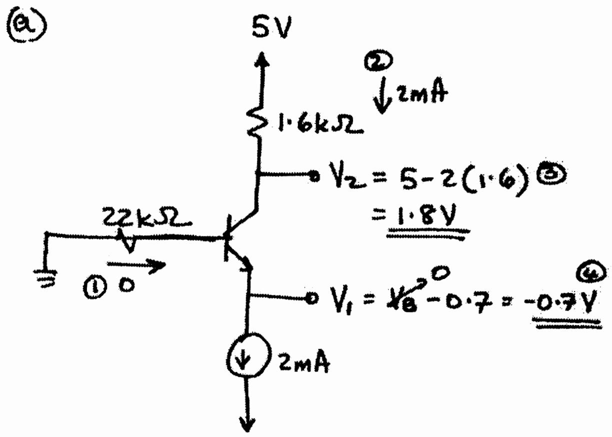
Check $I_B = 4.3 - 3 = 1.3$ mA

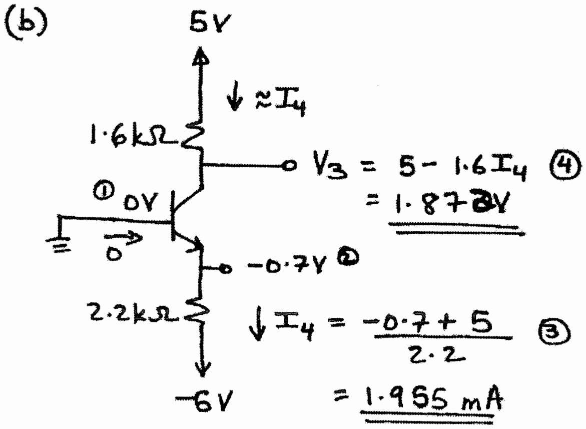
$I_C = 4.8 - 3 = 1.8$ mA

$\frac{I_C}{I_B} = \frac{1.8}{1.3} = 1.4 < 100$

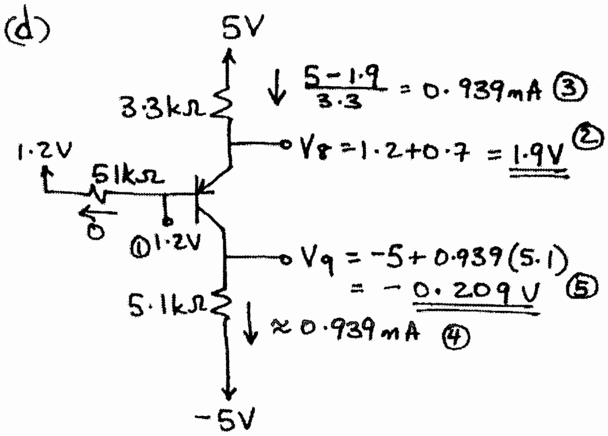
∴ Saturation as assumed

4.46

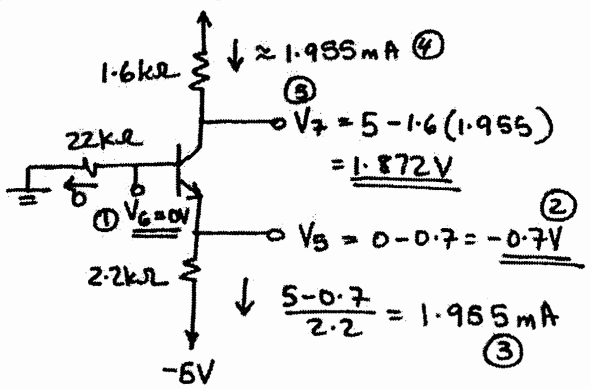

(a)
5V
② ↓2mA
1.6kΩ
$V_2 = 5 - 2(1.6)$ ③
$= 1.8$V
22kΩ
① 0
$V_1 = \cancel{V_B}^{\,0} - 0.7 = -0.7$V ④
2mA


(b)
5V
↓ ≈I_4
1.6kΩ
$V_3 = 5 - 1.6 I_4$ ④
$= 1.87$V
① 0V
0
−0.7V ②
2.2kΩ
↓ $I_4 = \frac{-0.7 + 5}{2.2}$ ③
$= 1.955$ mA
−6V

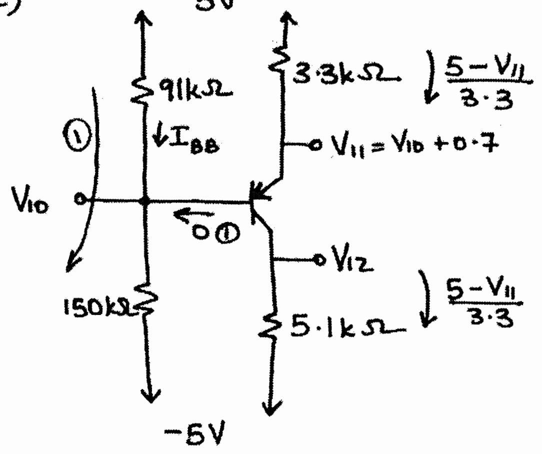
see below for part (c)


(d)
5V
↓ $\frac{5 - 1.9}{3.3} = 0.939$mA ③
3.3kΩ
$V_8 = 1.2 + 0.7 = 1.9$V ②
1.2V
51kΩ
0
① 1.2V
$V_9 = -5 + 0.939(5.1)$
$= -0.209$ V ⑤
5.1kΩ
↓ ≈0.939mA ④
−5V

(c)


5V
1.6kΩ
≈ 1.955mA ④
⑤
$V_7 = 5 - 1.6(1.955) = 1.872V$
22kΩ
0
① $V_6 = 0V$
②
$V_5 = 0 - 0.7 = -0.7V$
2.2kΩ
$\frac{5-0.7}{2.2} = 1.955mA$
③
−5V

(e)


5V
91kΩ
I_{BB}
①
V_{10}
0 ①
150kΩ
3.3kΩ
$\frac{5-V_{11}}{3.3}$
$V_{11} = V_{10} + 0.7$
V_{12}
5.1kΩ
$\frac{5-V_{11}}{3.3}$
−5V

Loop ①

$$5 - 91I_{BB} - 150I_{BB} + 5 = 0$$

$$I_{BB} = \frac{10}{91+150}$$

$$V_{10} = -5 + 150I_{BB}$$

$$= -5 + \frac{150}{91+150} \times 10$$

$$= \underline{\underline{1.224V}}$$

$$V_{11} = V_{10} + 0.7 = \underline{\underline{1.924V}}$$

$$\therefore I_C \approx I_E = \frac{5-V_{11}}{3.3}$$

$$V_{12} = -5 + \left(\frac{5-V_{11}}{3.3}\right)5.1 = \underline{\underline{-0.246V}}$$

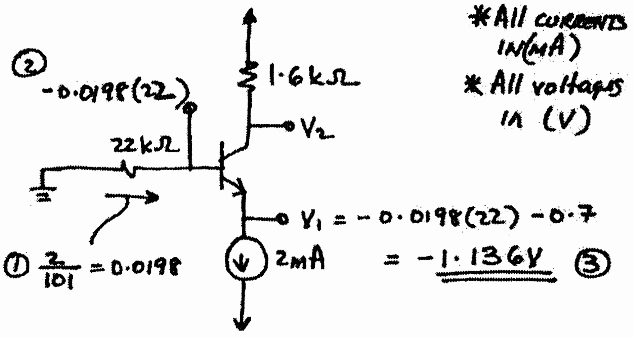
4.47

(a)

$\underline{\beta = 100}$

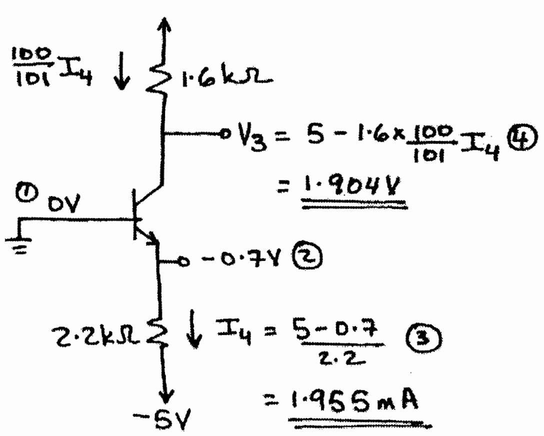
*All currents in (mA)

*All voltages in (V)

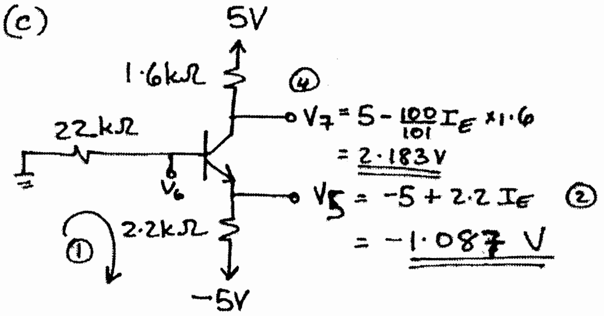

②
$-0.0198(22)$
5V
1.6kΩ
V_2
22kΩ
① $\frac{2}{101} = 0.0198$
2mA
$V_1 = -0.0198(22) - 0.7$
$= \underline{\underline{-1.136V}}$ ③

④ $V_2 = 5 - 2\left(\frac{100}{101}\right)1.6 = \underline{\underline{1.832V}}$

(b)


5V
$\frac{100}{101}I_4$
1.6kΩ
$V_3 = 5 - 1.6 \times \frac{100}{101}I_4$ ④
$= \underline{\underline{1.904V}}$
① 0V
−0.7V ②
2.2kΩ
$I_4 = \frac{5-0.7}{2.2}$ ③
$= \underline{\underline{1.955mA}}$
−5V

(c)


5V
1.6kΩ
④
$V_7 = 5 - \frac{100}{101}I_E \times 1.6$
$= \underline{\underline{2.183V}}$
22kΩ
V_6
$V_5 = -5 + 2.2I_E$ ②
$= \underline{\underline{-1.087V}}$
①
2.2kΩ
−5V

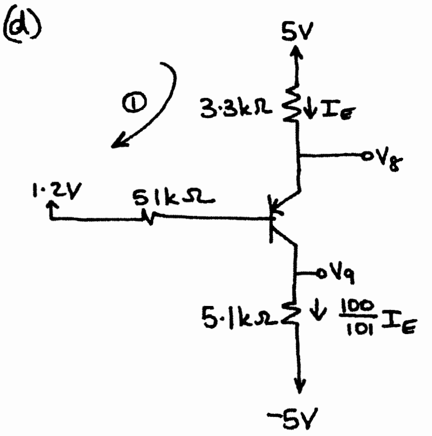
Loop ① $0 - \frac{I_E}{101}22 - 0.7 - 2.2I_E + 5 = 0$

$I_E = 1.778mA$

③ $V_6 = V_5 + 0.7 = \underline{\underline{-0.387V}}$

CONT

(d)


①
5V
3.3kΩ
I_E
V_8
1.2V
51kΩ
V_9
5.1kΩ
$\frac{100}{101}I_E$
−5V

Loop ①

$$5 - 3.3I_E - 0.7 - \frac{I_E}{101}51 - 1.2 = 0$$

$$I_E = 0.8147\,mA$$

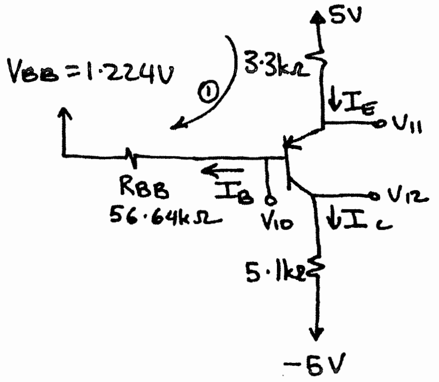
$$V_8 = 5 - 3.3I_E = \underline{\underline{2.3114\,V}}$$

$$V_9 = -5 + 5.1 \times \frac{100}{101}I_E = \underline{\underline{-0.8862V}}$$

(e) Use Thévenin's theorem to simplify the bias network:

$$V_{BB} = -5 + \frac{150}{150+91} \times 10 = 1.224\,V$$

$$R_{BB} = 150 // 91 = 56.64\,k\Omega$$


5V
$V_{BB} = 1.224V$
①
3.3kΩ
I_E
V_{11}
R_{BB}
56.64kΩ
I_B
V_{10}
I_C
V_{12}
5.1kΩ
−5V

Loop ①

$$5 - 3.3I_E - 0.7 - \frac{I_E}{101}R_{BB} - 1.224 = 0$$

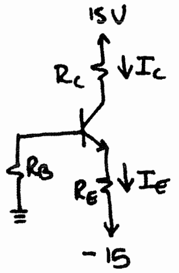
$$I_E = 0.7967\,mA$$

$$V_{11} = 5 - 3.3I_E = \underline{\underline{2.371V}}$$

$$V_{12} = \frac{100}{101}I_E \times 5.1 - 5 = \underline{\underline{-0.977V}}$$

$$V_{10} = V_{11} - 0.7 = \underline{\underline{1.67V}}$$

4.48


15V
R_C
I_C
R_B
R_E
I_E
−15

Nominal $\beta = 100$.

Thus,

nominal $\alpha = \frac{100}{101} = 0.99$

nominal $I_E = 1mA$

nominal $I_C = 0.99mA$

nominal $V_C = 5V$

Thus, $R_C = \frac{15-5}{0.99} = 10.1k\Omega \xRightarrow{use} \underline{10k\Omega}$

$$I_E = 1 = \frac{15-0.7}{R_E + \frac{R_B}{\beta+1}}$$

$$= \frac{14.3}{R_E + \frac{R_B}{101}}$$

$$\Rightarrow R_E + \frac{R_B}{101} = 14.3 \quad (1)$$

As β varies from 50 to 150, need to limit the variation of I_E to ±10% of 1mA. One can reason that the maximum variation in I_E occurs for $\beta = 50$ (as opposed to $\beta = 150$). To see this more that when β decreases from 100 to 50 the base current doubles while a change in β from

CONT.

100 to 150 causes the base current to decrease to 2/3 its nominal value. Thus our decision will be based on imposing the 10% limit for $\beta = 50$.

$$0.9 = \frac{14.3}{R_E + \frac{R_B}{\beta+1}} = \frac{14.3}{R_E + \frac{R_B}{51}}$$

$$R_E + \frac{R_B}{51} = 15.89 \qquad (2)$$

$$(2) - (1) \Rightarrow \quad R_B\left(\frac{1}{51} - \frac{1}{101}\right) = 1.59$$

$$\Rightarrow R_B = 163.8\,k\Omega \overset{use}{\Longrightarrow} \underline{\underline{164\,k\Omega}}$$

Sub into (1) gives

$$R_E = 12.7\,k\Omega \overset{use}{\Longrightarrow} \underline{\underline{13\,k\Omega}}$$

To find the expected range of I_C & V_C corresponding to β variation from 50 to 150 we use

$$I_C = \alpha \frac{14.3}{R_E + \frac{R_B}{\beta+1}}$$

for $\beta = 50$ $\quad I_C = \frac{50}{51}\,\frac{14.3}{13 + \frac{164}{51}} = \underline{\underline{0.864\,mA}}$

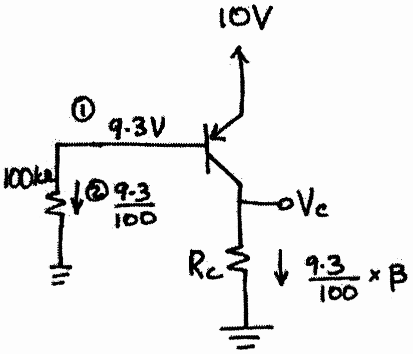
$$V_C = 15 - 0.864 \times 10 = \underline{\underline{6.36\,V}}$$

for $\beta = 150$ $\quad I_C = \frac{150}{151} \times \frac{14.3}{13 + \frac{164}{151}}$

$$= \underline{\underline{1.008\,mA}}$$

$$V_C = 15 - 1.008 \times 10 = \underline{\underline{4.92\,V}}$$

4.49



For $V_C = 5\,V = \frac{9.3}{100} \times \beta \times R_C \qquad \beta = 50$

$$R_C = \frac{500}{9.3 \times 50} = \underline{\underline{1.08\,k\Omega}}$$

For $\beta = 100$

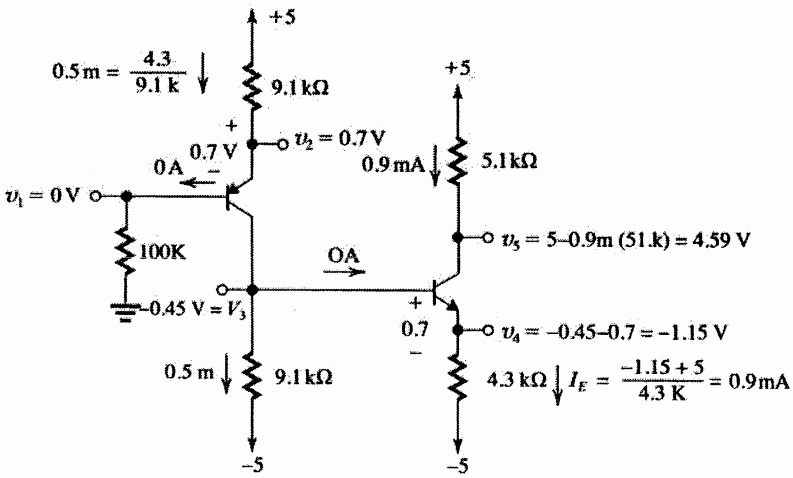
$$V_C = \frac{9.3}{100} \times \beta \times R_C = \frac{9.3}{100} \times 100 \times 1.08$$

$$= \underline{\underline{10.04\,V}} \leftarrow V_{BC} = 9.3 - 10.04 = -0.74$$

Since $V_{BC} < -0.4\,V$ the transistor $\underline{\underline{\text{saturates}}}$!

4.50

(a) $\beta = \infty$


+5
0.5 m = 4.3 / 9.1 k
9.1 kΩ
+5
+
0.7 V
−
v₂ = 0.7 V
0 A
0.9 mA
5.1 kΩ
v₁ = 0 V
v₅ = 5–0.9m (51.k) = 4.59 V
100K
0A
−0.45 V = V₃
+
0.7
−
v₄ = −0.45–0.7 = −1.15 V
0.5 m
9.1 kΩ
4.3 kΩ
I_E = (−1.15 + 5) / 4.3 K = 0.9mA
−5
−5

$$+5 - I_{E1}(9.1\text{ K}) - 0.7 - I_{B1}(100\text{ K}) = 0$$

$$I_{B1} = \frac{I_{E1}}{\beta + 1}$$

$$4.3 = I_{E1}\left(9.1\text{ K} + \frac{100\text{ K}}{101}\right)$$

$$I_{E1} = \frac{4.3}{10,090} = .43\text{ mA}$$

$$V_2 = 5 - 9.1\text{ K}(.43\text{ m}) = 1.36\text{ V}$$

$$V_1 = 1.36 - 0.7 = .66\text{ V}$$

$$I_{C1} = \alpha I_{E1} = .426\text{ m}$$

$$-5 + 9.1\text{ K}(I_{C1} + I_{B2}) - 0.7 - I_{E2}(4.3\text{ K}) + 5 = 0$$

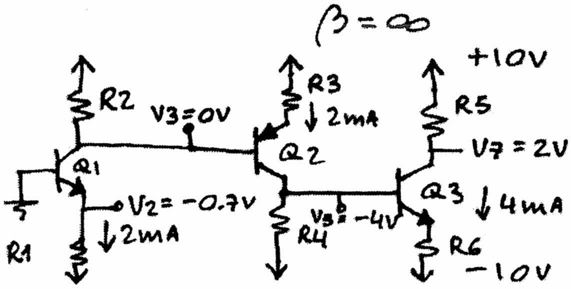
$$9.1\text{ K}(.426\text{ m}) + \frac{9.1\text{ K } I_{E2}}{101} - 0.7 - I_{E2}(4.3\text{ K}) = 0$$

$$I_{E2} = \frac{3.2}{4210} = .75\text{ mA}$$

$$V_4 = -5 + I_{E2}(4.3\text{ K}) = -1.8\text{ V}$$

$$V_3 = V_4 + 0.7 = -1.08\text{ V}$$

4.50


β = ∞
+10V
R2
V3=0V
R3
↓2mA
R5
V7 = 2V
Q1
Q2
Q3
↓4mA
V2= −0.7V
V5= −4V
R4
↓2mA
R1
R6
−10V

$R_1 = \frac{9.3}{2} = \underline{\underline{4.7K\Omega}}$

$R_2 = \frac{10}{2} = 5 \rightarrow \underline{\underline{5.1K\Omega}}$

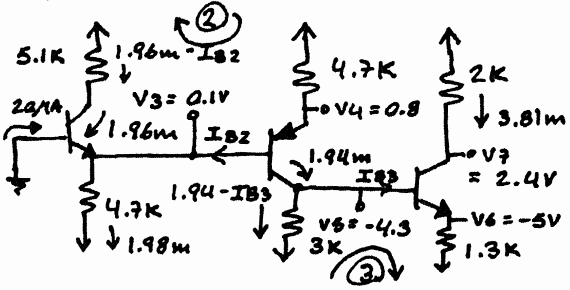
$R_3 = \frac{9.3}{2} = \underline{\underline{4.7K\Omega}}$

$R_4 = \frac{6}{2} = \underline{\underline{3K\Omega}}$

$R_5 = \frac{8}{4} = \underline{\underline{2K\Omega}}$

$R_6 = \frac{10-4.7}{4} = \underline{\underline{1.3K\Omega}}$

$\beta = 100$


②
5.1K
1.96m - IB2
V3 = 0.1V
20μA
1.96m
IB2
4.7K
V4 = 0.8
2K
3.81m
V7 = 2.4V
1.94m
IB3
1.94 - IB3
4.7K
1.98m
V5 = -4.3
3K
V6 = -5V
1.3K
③

② $(1.96 - I_{B2}) \times 5.1$
$= (\beta+1) I_{B2} \times 4.7 + 0.7$

$I_{B2} = 0.0194\,mA$

$I_{E2} = 1.96\,mA$

$V_3 = \underline{\underline{0.1V}}$ $V_4 = \underline{\underline{0.8V}}$

③ $(1.94 - I_{B3}) \times 3$
$= 0.7 + 1.3 \times (\beta+1) \cdot I_{B3}$

$I_{B3} = 0.038\,mA$

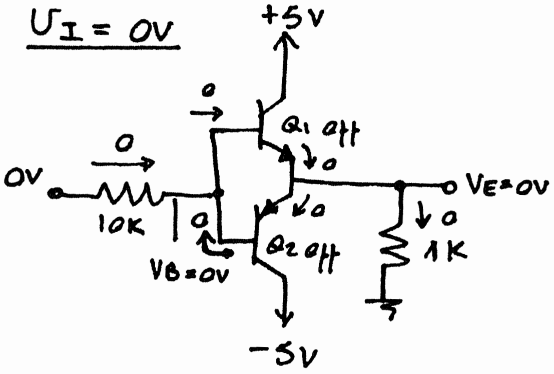
$I_{E3} = 3.85\,mA$

$V_5 = \underline{\underline{-4.3V}}$ $V_6 = \underline{\underline{-5V}}$

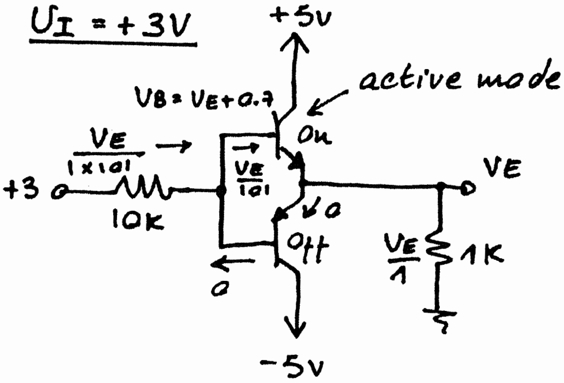
$V_7 = \underline{\underline{2.4V}}$

4.51

$\underline{v_I = 0V}$


+5V
0
Q1 off
0
0V
0
10K
VE = 0V
0
0
VB = 0V
Q2 off
0
1K
-5V

$\underline{v_I = +3V}$

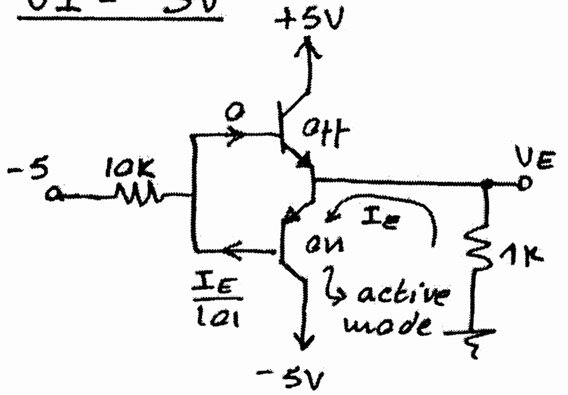

+5v
active mode
VB = VE + 0.7
On
VE / 1×101
VE / 101
+3
10K
VE
0
Off
VE / 1
1K
0
-5v

$3 = \frac{V_E}{101} \times 10 + 0.7 + V_E$

$\Rightarrow V_E = \underline{\underline{2.09V}}$

$V_B = \underline{\underline{2.79V}}$

$\underline{v_I = -5V}$

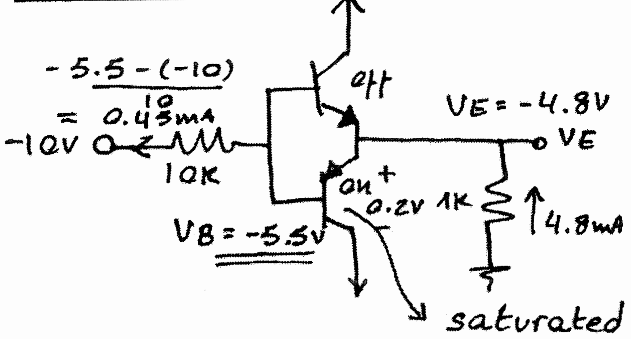

+5V
0
off
-5
10K
VE
I_E
on
1K
$\frac{I_E}{101}$
active mode
-5V

$$I_E = \frac{5-0.7}{1+10/101} = 3.91\text{mA}$$

$$V_E = -\underline{\underline{3.91}}V$$

$$V_B = -\underline{\underline{4.61}}V$$

$\underline{v_I = -10V}$


+5V
$\frac{-5.5-(-10)}{10}$
= 0.45mA
off
VE = -4.8V
-10V
10K
VE
on
+
0.2V
1K
4.8mA
$V_B = \underline{\underline{-5.5V}}$
saturated

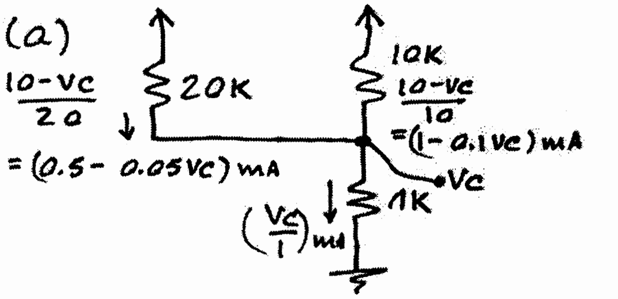
$$\frac{I_C}{I_B} = \frac{4.35}{0.45} = 9.7 < 100$$

thus, Q_2 is saturated as assumed

$$V_E = -\underline{\underline{4.8V}} \quad V_B = \underline{\underline{-5.5V}}$$

4.52

(a)


$\frac{10-V_C}{20}$
= (0.5 - 0.05VC) mA
20K
10K
$\frac{10-V_C}{10}$
= (1 - 0.1VC) mA
VC
1K
$\left(\frac{V_C}{1}\right)$ mA

$$(0.5 - 0.005V_C) + (1 - 0.1V_C) = V_C$$

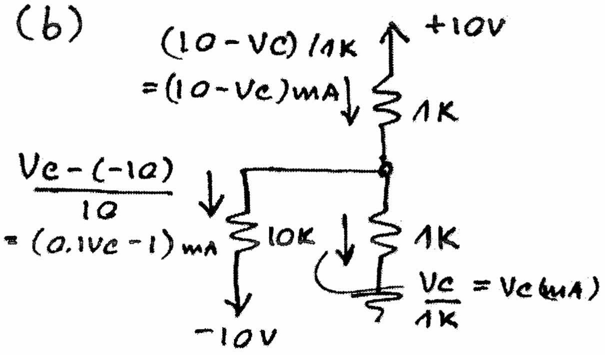
$$V_C = \underline{\underline{1.3V}}$$

$$I_C = \frac{10-1.3}{10} = 0.87\text{mA}$$

$$I_B = \frac{10-1.3}{20} = 0.435\text{mA}$$

thus $\beta_{\text{forced}} = \frac{0.87}{0.435} = \underline{\underline{2}}$

(b)


(10 - VC) / 1K
= (10 - VC) mA
+10V
1K
$\frac{V_C-(-10)}{10}$
= (0.1VC - 1) mA
10K
1K
$\frac{V_C}{1K}$ = VC (mA)
-10V

$$10 - V_C = (0.1V_C + 1) + (V_C)$$

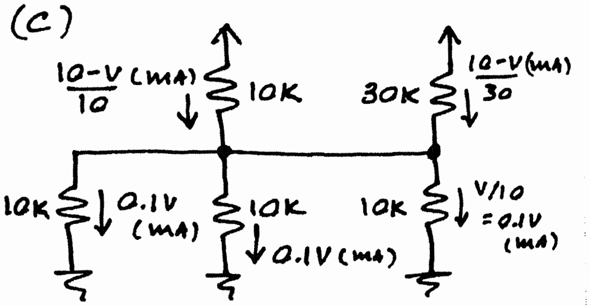
$$\Rightarrow V_C = +\underline{\underline{4.29}}V$$

$$I_C = 4.29\text{mA}$$

$$I_B = \frac{4.29+10}{10} = 1.43\text{mA}$$

$$\beta_{\text{forced}} = \frac{4.29}{1.43} = \underline{\underline{3}}$$

(c)



Node equation:

$$\frac{10-V}{10} + \frac{10-V}{30} = 0.1V + 0.1V + 0.1V$$

$$30 - 3V + 10 - V = 9V$$

$$40 = 13V$$

$$\Rightarrow V = \underline{\underline{3.08V}}$$

thus, $V_{C3} \simeq V_{C4} \simeq 3.08V$

$$I_{B3} = 0.1V = 0.308 \text{ mA}$$

$$I_{E3} = \frac{10-3.08}{10} \simeq 0.692 \text{ mA}$$

$$I_{C3} = 0.692 - 0.308 = 0.384 \text{ mA}$$

$$\beta_{3\,forced} = \frac{0.384}{0.308} = \underline{\underline{1.25}}$$

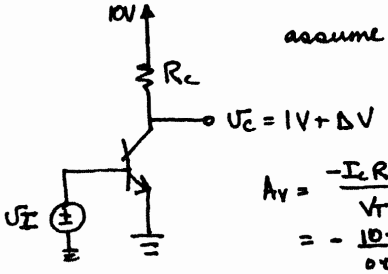
$$I_{C4} = \frac{10-3.08}{30} = 0.231 \text{ mA}$$

$$I_{E4} = 0.1V = 0.308 \text{ mA}$$

$$I_{B4} = 0.308 - 0.231 = 0.077 \text{ mA}$$

$$\beta_{4\,forced} = \frac{0.231}{0.077} = \underline{\underline{3}}$$

4.53



assume $v_{CEsat} = 0.2V$

$$A_v = \frac{-I_C R_C}{V_T}$$

$$= -\frac{10-1}{0.025} = \underline{\underline{-360 \frac{V}{V}}}$$

At the verge saturation $v_{CEsat} = 0.3V$

$$\therefore v_C = 1 + \Delta V = 0.3.$$

$$\Delta v = \underline{\underline{-0.7V}}$$

$$\therefore v_O = 0.3V \quad i_C = \frac{10-0.3}{R_C}$$

$$\frac{i_{C2}}{i_{C1}} = \frac{9.7/R_C}{(10-1)/R_C} = e^{\Delta v / v_T}$$

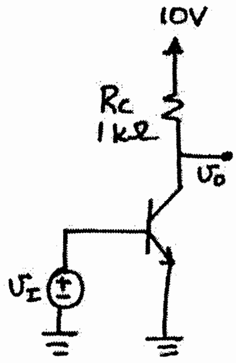
$\therefore$ Maximum Input signal

$$\Delta v = 0.025 \ln \frac{9.7}{9} = \underline{\underline{1.87 \text{mV}}}$$

If we assume linear operation right to saturation we can use the gain A_v to calculate the maximum input swing. Thus for an output swing $\Delta v_o = 0.8$ we have

$$\Delta v_i = \frac{-\Delta v_o}{A_v} = \frac{-0.7}{-360} = \underline{\underline{1.94 \text{mV}}}$$

4.54


10V
R_C
$1k\Omega$
v_O
v_I

$V_{CE} = 10 - 10 I_C$ ← DC collector voltage

$$A_V = -\frac{I_C R_C}{V_T}$$

$$= -\frac{I_C \times 10}{0.025}$$

$$= -400 I_C \quad (1)$$

-Assuming the output voltage $v_O = 0.3V$ is the lowest v_{CE} to stay out of saturation.

$$\therefore v_O = 0.3 = 10 - i_C R_C$$

$$= 10 - I_C R_C + \Delta v_O$$

$$\Delta v_O = -10 + 0.3 + I_C \times 1 \quad (2)$$

-Max output voltage before the transistor is cut off

$$V_{CE} + \Delta v_O = V_{CC}$$

$$\Delta v_O = V_{CC} - V_{CE}$$

$$= 10 - 10 + 10 I_C$$

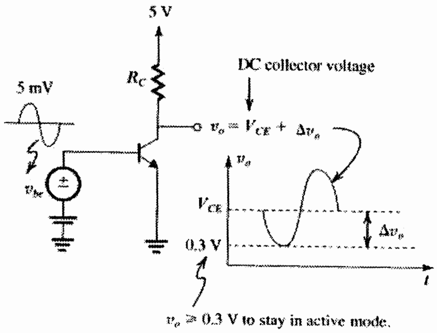
$$= 10 I_C \quad (3)$$

Use (1) to calculate the gain and (2), (3) to calculate the output limits in order to stay in active mode for a particular bias current I_C.

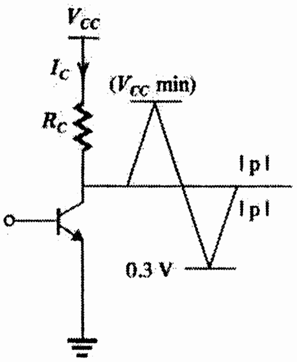
I_C (mA)	A_V (V/V)	ΔV_O (V)
1	−40	−8. to 1
2	−80	−7. to 2
5	−200	−4.7 to 5
8	−320	−1.7 to 8
9	−360	−0.7 to 9

4.55

Since we are assuming linear operation we don't have to go to $i_C = I_S e^{V_{BE}/V_T}$ equation.


5 V
R_C
5 mV
v_{be}
DC collector voltage
$v_o = V_{CE} + \Delta v_o$
v_o
V_{CE}
Δv_o
0.3 V
t
$v_o \geq 0.3$ V to stay in active mode.

$$A_V = -\frac{I_C R_C}{v_T} = -\frac{V_{CC} - V_{CE}}{v_T}$$


V_{CC}
I_C
R_C
(V_{CC} min)
|p|
|p|
0.3 V

$V_{CC} = 0.3 + |P| + I_C R_C$

$|A_V| = (-)g_m R_C = \frac{I_C R_C}{V_T} \geq \frac{p}{V_T}$

$\therefore V_{CC}$ min

4.56

On the verge of Satuation

$V_{CE} - \Delta v_O = 0.3$ V

for linear operation $\Delta v_O = A_V v_{be}$

$V_{CE} - |A_V v_{be}| = 0.3$

$(5 - I_C R_C) - A_V \times 5 \times 10^{-3} = 0.3$

$5 - |A_V V_T| - |A_V \times 5 \times 10^{-3}| = 0.3$

$|A_V(0.025 + 0.005)| = 5 - 0.3$

$|A_V| = 156.67$ Note A_v is negative.

$\therefore A_V = -156.67$ V / V

Now we can find the dc collector voltage. Refer to the sketch of the output voltage, we see that

$|\Delta v_O| = |(A_V \times 0.005)|$

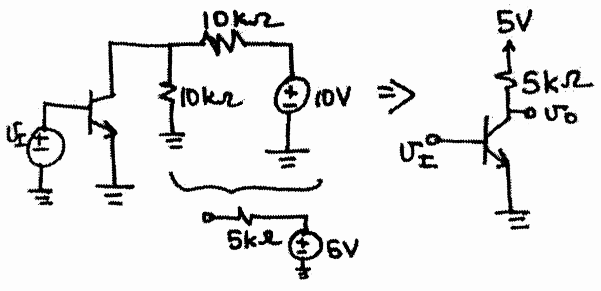
$\therefore V_{CE} = 0.3 + |A_V| 0.005 = 1.08$ V

$= V_{CEsat} + \text{P} + |A_V| V_T$

$I_C R_C = |A_V| V_T$

	A_v(V/V)	P(V)	$A_v V_T$	$V_{cc} = \lvert A_V \rvert_T +$ P + 0.3
(a)	−20	0.2	0.5	1.0 → 1.0 V
(b)	−50	0.5	1.25	2.05 → 2.5 V
(c)	−100	0.5	2.5	3.3 → 3.5 V
(d)	−100	1.0	2.5	3.8 → 4.0 V
(e)	−200	1.0	5.0	6.3 → 6.5 V
(f)	−500	1.0	12.5	13.8 → 14 V
(g)	−500	2.0	12.5	14.8 → 15 V

4.57

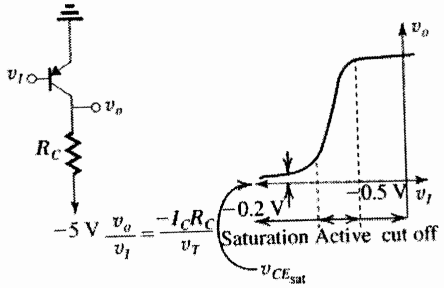

10kΩ
10kΩ
10V
v_I
⇒
5kΩ
6V
5V
5kΩ
v_o
v_I

$$\frac{v_o}{v_I} = -\frac{I_C R_C}{V_T} = \frac{-0.5 \times 5}{0.026}$$

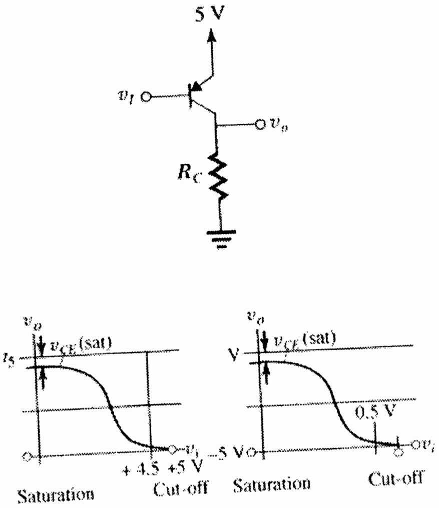
$$= \underline{\underline{-100 \text{ V/V}}}$$

4.58

(a)


v_I
v_o
R_C
−5 V
$\frac{v_o}{v_I} = \frac{-I_C R_C}{v_T}$
v_o
0.2 V
0.5 V
v_I
Saturation
Active
cut off
$v_{CE_{sat}}$

(b)


5 V
v_I
v_o
R_C
v_o
v_{CE}(sat)
I_5
v_I
+4.5 +5 V
Saturation
Cut-off
v_o
v_{CE}(sat)
V
0.5 V
−5 V
v_I
Saturation
Cut-off

4.59

Including the Early effect we note that:

$$i_C = I_S e^{v_{BE}/V_T}\left(1 + v_{CE}/V_A\right)$$

Also, note $I_C = I_S e^{v_{BE}/V_T}$ Eq (5.38b) is the value of the collector current with the Early voltage neglected.

Starting with the voltage at the collector we have:

$$v_O = V_{CC} - i_C R_C$$

$$= V_{CC} - R_C I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

Take derivative to get gain Av

$$A_v = \frac{\partial v_O}{\partial v_I}$$

$$= -R_C I_S\left[\frac{e^{v_{BE}/V_T}}{V_T}\left(1+\frac{v_{CE}}{V_A}\right) + \frac{e^{v_{BE}/V_T}}{V_A}\frac{\partial v_{CE}}{\partial v_I}\right]$$

$$A_v = -\frac{R_C I_S}{V_T} e^{v_{BE}/V_T}\left[1 + v_{CE}/V_A + \frac{V_T}{V_A}\frac{\partial v_{CE}}{\partial v_I}\right]$$

$$= \frac{-R_C I_C}{V_T}\left[1 + \frac{v_{CE}}{V_A} + \frac{V_T}{V_A}A_v\right]$$

$$-A_v\left[\frac{1}{\frac{R_C I_C}{V_T}} + \frac{V_T}{V_A}\right] = 1 + \frac{v_{CE}}{V_A} = \frac{V_A + V_{CE}}{V_A}$$

$$-A_v\left[\frac{V_A + R_C I_C}{\frac{R_C I_C V_A}{V_T}}\right] = \frac{V_A + V_{CE}}{V_A}$$

$$-A_v \Big/ \frac{R_C I_C}{V_T} = \frac{V_A}{V_A + R_C I_C} \times \frac{V_A + v_{CE}}{V_A}$$

$$= \frac{V_A + V_{CE}}{V_A + R_C I_C}$$ ÷ top + bottom by $V_A + V_{CE}$

$$= \frac{1}{\frac{V_A}{V_A + V_{CE}} + \frac{R_C I_C}{V_A + V_{CE}}}$$

This term is ≅ 1 ↑

∵ $V_A >> v_{CE}$

$$\therefore A_v \cong \left[\frac{-R_C I_C/V_T}{\left(1 + \frac{R_C I_C}{V_A + v_{CE}}\right)}\right]$$

Q.E.D.

For $V_{CC} = 6V$ $v_{CE} = 2.5V$ $V_A = 100V$

Ignoring the Early Voltage:

$$A_v = \frac{-I_C R_C}{V_T} = \frac{V_{CC} - V_{CE}}{V_T} = \frac{5 - 2.5}{0.025} = \underline{\underline{100 \frac{V}{V}}}$$

With the Early voltage

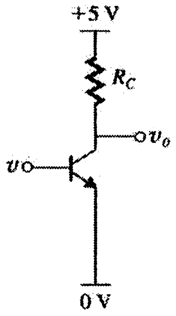
$$A_v \cong \frac{-I_C R_C/V_T}{1 + \frac{R_C I_C}{V_A + v_{CE}}}$$

But $v_{CE} = 0.5V$ & $\frac{I_C R_C}{V_T} = 100$ as shown above.

$$\therefore A_v = \frac{-100}{1 + \frac{2.5}{100 + 2.5}}$$

$$= \underline{\underline{-97.7 V/V}}$$

4.60



For $V_O = 2\text{V}, R_C = 1\ \text{k}\Omega$

$$I_C = \frac{5-2}{1} = 3\ \text{mA}$$

$$A_V = \frac{-I_C R_C}{V_T} = -120\ \text{V/V}$$

$$\Delta V_O = -120 \times 5 = -600\ \text{mV}$$

$$\Delta V_{BE} = V_T \ln[I_2/I_1]$$

$$\frac{I_2}{I_1} = \epsilon^{\Delta v_{BE}/v_T} = \epsilon^{5/25}$$

(a) $I_2 = I_1 e^{5/25} = 3 \times 1.22 = 3.66\ \text{mA}$

$\Delta V_O = (I_2 - I_1)R_C = 0.66 \times 1 = 0.660\ \text{V}$

$A_V = -660/5 = -132\ \text{V/V}$

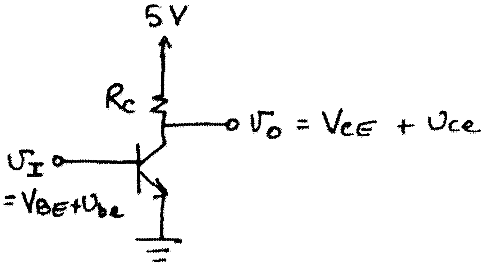
(b) $I_3 = I_1 e^{-5/25} = 3 \times 0.82 = 2.46\ \text{mA}$

$\Delta V_O = (I_3 - I_1)R_C = 0.544\ \text{V}$

$A_V = -544/5 = -109\ \text{V/V}$

ΔV_{BE}	ΔV_O (exp)	ΔV_O (linear)
+5 mV	−660 mV	−600 mV
−5 mV	+544 mV	+600 mV

4.61



(a) For maximum gain you would bias at the largest current since $A_v = -I_c R_c / V_T$. This also means you would bias at the edge of saturation

$$A_v = -\frac{V_{CC} - V_{CEsat}}{V_T} = -\frac{5-0.3}{0.025} = \underline{\underline{-188\ \text{V/V}}}$$

However any signal swing at the output would automatically drive it into saturation.

(b) For $A_v = -100\ \text{V/V}$

$$A_v = \frac{V_{CC} - V_{CE}}{V_T} = \frac{5 - V_{CE}}{V_T} = 100$$

$$V_{CE} = \underline{\underline{2.5\text{V}}}$$

(c) For a dc collector current of 0.5mA

$$R_C = \frac{5-2.5}{0.5} = \underline{\underline{5\ \text{k}\Omega}}$$

(d) $I_S = 10^{-15}\text{A} \Rightarrow$

$$I_C = I_S e^{V_{BE}/V_T}$$

$$0.5 \times 10^{-3} = 10^{-15} e^{V_{BE}/0.025}$$

$$V_{BE} = \underline{\underline{0.673\text{V}}}$$

(e) If we assume linear operation we can use A_v to find the output change for $v_{be} = 5\text{mV}$

$$v_{ce} = A_v v_{be} = -100 \times 0.005$$
$$= -0.5\text{ V} \sim \text{peak sine wave.}$$

∴ the output is a $\underline{\underline{0.5\text{V } p \text{ sine wave}}}$

(f) for $v_{ce} = 0.5$

$$i_c = \frac{0.5}{5} = \underline{\underline{0.1\text{mA peak}}}$$

This current is superimposed on I_C.

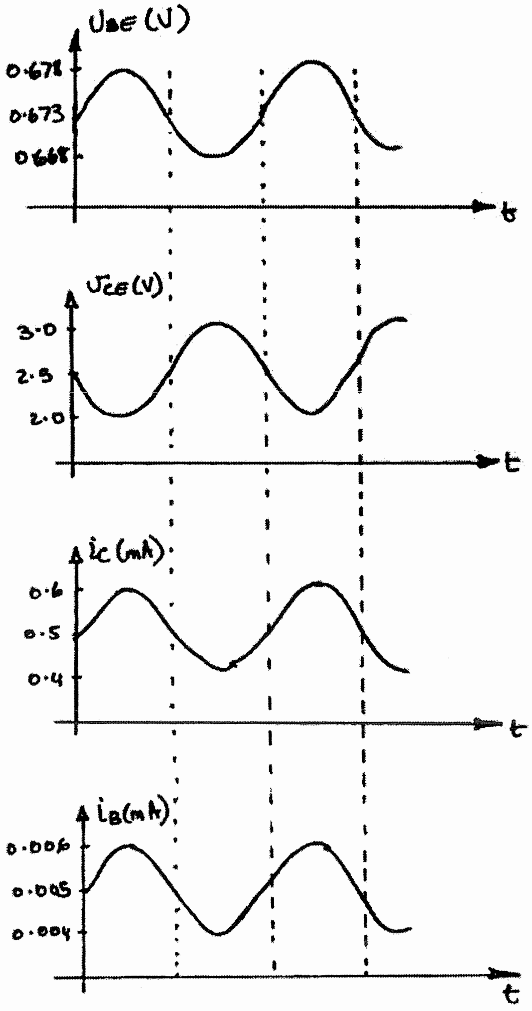
(g) $I_B = I_C/\beta = \frac{0.5}{100} = \underline{\underline{0.005\text{mA}}}$

$$i_b = \frac{i_c}{\beta} = \frac{0.1}{100} = \underline{\underline{0.001\text{ mA } p}}$$

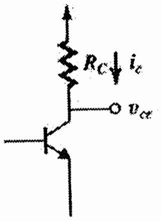
(h) $r_{in} = \frac{v_{be}}{i_b} = \frac{0.005}{0.001 \times 10^{-3}}$

$$= \underline{\underline{5\text{k}\Omega}}$$

(i) See sketches that follow:



4.62



$$A_V = \frac{v_{ce}}{v_{be}} = \frac{-I_C R_C}{V_T}$$

But $V_{ce} = -i_c R_C$

$$\therefore -\frac{i_C R_C}{v_{be}} = -\frac{I_C R_C}{V_T}$$

Now $g_m = \frac{\text{Output current}}{\text{Input voltage}} = \frac{i_C}{v_{be}}$

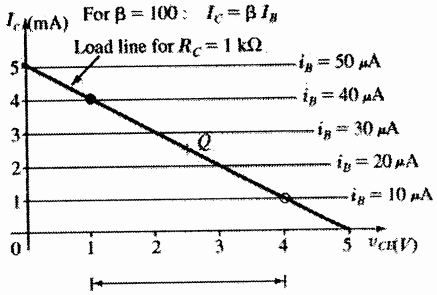
$$\therefore g_m R_C = \frac{I_C R_C}{V_T}$$

$$g_m = I_C / V_T$$

$$g_m = \frac{1\text{ mA}}{25\text{ mV}} = 40\text{ ms}$$

for $I_C = 1$ mA

4.63



Peak-to-peak V_c swing = 4 − 1 = 3 V

For Q point at $V_{CC}/2 = 2.5$ V

$V_{CE} = 2.5$ V : $I_C = 2.5$ mA

$I_B = 25\ \mu A$

$$I_B = \frac{V_{BB} - 0.7}{R_B} = 25\ \mu A$$

$$\Rightarrow V_{BB} = I_B R_B + 0.7 = 2.5 + 0.7 = 3.2\text{ V}$$

4.63

(a) Using the exponential characteristic :

$$i_C = I_{ce}\, e^{V_{be}/V_T} - I_C$$

giving $\frac{i_C}{I_C} = e^{V_{be}/V_T} - 1$

(b) Using small-signal approximation :

$$i_C = g_m V_{be} = \frac{I_C}{V_T} \cdot V_{be}$$

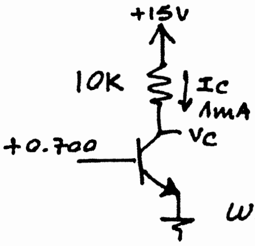
Thus, $\frac{i_C}{I_C} = \frac{V_{be}}{V_T}$

See table below

For signals at ± 5 mV, the error introduced by the small-signal approximation is 10 %

The error increases to above 20% for signals at ± 10 mV.

v_{be} (mV)	i_c/I_c Expan.	i_c/I_c small signal.	% Error
+1	+0.041	+0.040	−2
−1	−0.03P	−0.040	+2
+2	+0.083	+0.080	−4
−2	−0.077	−0.080	+4
+5	+0.221	+0.200	−9.5
−5	−0.181	−0.200	+10.3
+8	+0.377	+0.320	−15.2
−8	−0.274	−0.320	+16.8
+10	+0.492	+0.400	−18.7
−10	−0.330	−0.400	+21.3
+12	+0.616	+0.480	−22.1
−12	−0.381	−0.480	+25.9



$V_C = 15 - 1 \times 10$
$= 5V$

$v_{BE} = 0.7 + v_{be}$
where $v_{be} = 0.005V$

$i_C \cong I_C\left(1 + \frac{v_{be}}{V_T}\right)$ Eq. (5.83)

$i_C \simeq I_C + i_c$ where:

$i_c = \frac{1m \times 0.005}{25m} = 0.2m$

$i_C = 1mA + 0.2mA$

$v_C = V_{CC} - i_C R_C$ Eq. (5.101)

$\Rightarrow V_C - \underbrace{i_c R_C}_{0.2m \times 10K}$

$v_C = 5V - 2V$

$\text{gain} = \frac{-2V}{0.005V} = -400\,V/V$

while $-g_m \cdot R_C = -\frac{1m}{25m} \cdot 10K = -400\,V/V$

4.65

$g_m = \frac{I_C}{V_T} = \frac{1.2mA}{25mV} = \underline{\underline{48\frac{mA}{V}}}$

$r_\pi = \frac{\beta}{g_m} = \frac{120}{48 \times 10^{-3}} = \underline{\underline{2.5K\Omega}}$

$r_e = \frac{r_\pi}{\beta + 1} = \frac{2500}{121} = 20.6\Omega$

For a bias current of 120μA i.e. 10 times lower:

$g_m = \frac{48}{10} = 4.8\,mA/V$

$r_\pi = 10 \times 2.5 = 25K\Omega$

$r_e = 10 \times 20.6 = 206\Omega$

4.66

$I_C = 2mA \Rightarrow g_m = \frac{2mA}{25mV}$

$g_m = 80mA/V$

$r_e = \frac{V_T}{I_E}$, $I_E = I_C\frac{(\beta+1)}{\beta}$

$I_E = 2mA \times \frac{51}{50} = 2.04mA$

$r_e = \frac{25m}{2.04m} = 12.25\Omega$

$r_\pi = \frac{\beta}{g_m} = \frac{50}{80 \times 10^{-3}} = 625\Omega$

gain : $-g_m \times R_C$

For $R_C = 5K\Omega$ and $\hat{v}_{be} = 5mV$

$\hat{v}_o = -80m \times 5K \times 5mV$
$= -2V$

4.67

$g_m = 50\frac{mA}{V} = \frac{I_C}{V_T}$

$\Rightarrow I_C = g_m \times V_T = 50m \times 25m$

$= 1.25mA$

$r_\pi = 2K = \frac{\beta}{g_m} \Rightarrow \beta = 2K \times 50m$

$\underline{\underline{\beta = 100}} \longrightarrow \alpha = \frac{100}{101} = 0.99$

$I_E = \frac{I_C}{\alpha} = \frac{1.25mA}{0.99} = \underline{\underline{1.26mA}}$

4.68

g_m varies from: $1.2 \times 60 = 72\frac{mA}{V}$

to $0.8 \times 60 = 48\frac{mA}{V}$

β varies from 50 to 200

$R_{in}|_{base} = r_\pi = \beta / g_m$

Largest value: $r_\pi = \frac{\beta_{max}}{g_{m\,min}} = \frac{200}{48m}$

$= \underline{4.2K\Omega}$

Smallest value: $r_\pi = \frac{\beta_{min}}{g_{m\,max}} = \frac{50}{72m}$

$= \underline{\underline{694\Omega}}$

4.69

$V_C = 2V \Rightarrow I_C = \frac{V_{CC} - V_C}{R_C}$

$I_C = \frac{5-2}{3K} = 1mA$

$g_m = \frac{I_C}{V_T} = \frac{1m}{25m} = 40\frac{mA}{V}$

$i_C(t) = I_C + g_m v_{be}(t)$

$= 1m + 40 \cdot 10^{-3} \times 0.005 \sin\omega t$

$= \underline{\underline{1 + 0.2 \sin\omega t, \ mA}}$

$V_C(t) = 5 - R_C i_C(t)$

$= \underline{\underline{2 - 0.6 \sin\omega t, V}}$

$i_B(t) = i_C(t) / \beta$

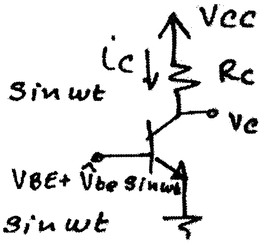
$= \frac{1 + 0.2 \cdot \sin\omega t}{100}, mA$

$= \underline{\underline{10 + 2 \sin\omega t, \ \mu A}}$

Voltage gain $= \frac{-0.6}{0.005} = \underline{\underline{-120}}\ V/V$

4.70

$i_C = I_C + g_m \hat{V}_{be} \sin\omega t$



$V_C = V_{CC} - I_C R_C - g_m \hat{V}_{be} R_C \sin\omega t$

To maintain BJT in active region, $V_C \geq V_{BE}$, thus

$V_{CC} - I_C R_C - g_m R_C \hat{V}_{be} \geq V_{BE} + \hat{V}_{be}$

To obtain the largest possible output signal we design such that this constraint is satisfied with the equality sign, that is:

$V_{CC} - R_C I_C - g_m R_C \hat{V}_{be} = V_{BE} + \hat{V}_{be}$

substituting $g_m = \frac{I_C}{V_T}$, gives.

$V_{CC} - R_C I_C - R_C I_C \frac{\hat{V}_{be}}{V_T} = V_{BE} + \hat{V}_{be}$

$\Rightarrow R_C I_C \left(1 + \frac{\hat{V}_{be}}{V_T}\right) = V_{CC} - V_{BE} - \hat{V}_{be}$

CONT.

$$R_C I_C = \frac{(V_{CC} - V_{BE} - \hat{V}_{be})}{\left(1 + \frac{\hat{V}_{be}}{V_T}\right)} \quad \text{Q.E.D.}$$

$$\text{Voltage gain} = -g_m \cdot R_C = -\frac{I_C}{V_T} \cdot R_C = -\frac{V_{CC} - V_{BE} - \hat{V}_{be}}{V_T + \hat{V}_{be}}$$

For $V_{CC} = 5V$, $V_{BE} = 0.7V$ and $\hat{V}_{be} = 5mV$

$$R_C I_C = \frac{5 - 0.7 - 0.005}{1 + \frac{0.005}{0.025}} = 3.6V$$

Thus,

$$V_C = 5 - 3.6 = +1.4V$$

Amplitude of output signal is

$$= 1.4 - (V_{BE} + \hat{V}_{be}) = 1.4 - 0.7 - 0.005 = 0.695\ V$$

$$\text{Voltage gain} = -\frac{0.695}{0.005} = -139\ \frac{V}{V}$$

Check

$$\text{Voltage gain} = -\frac{(5 - 0.7 - 0.005)}{0.025 + 0.005} = -143\ V/V$$

The difference is caused by decimal rounding-up of $R_C I_C$.

Otherwise:

$$\text{Voltage gain} = -\frac{0.716}{0.005} = -143\ V/V$$

4.71

	a	b	c	d	e	t	s
α	1.000	0.990	0.98	1	0.890	0.90	0.841
β	∞	100	50	∞	100	9	16
I_C (mA)	1.00	0.89	1.00	1.00	0.48	4.5	17.5
I_E (mA)	1.00	1.00	1.02	1.00	0.25	5	18.6
I_B (mA)	0	0.010	0.020	0	0.002	0.5	1.10
g_m (mA/V)	40	39.6	40	40	0.01	180	700
$r_e(\Omega)$	25	25	24.5	25	100	5	1.34
$r_\pi(\Omega)$	00	2.5 k	1.255	00	10.1 k	50	227

4.72

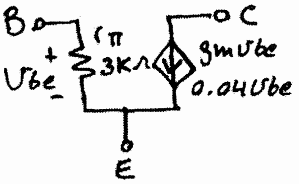
$I_C = 1mA$, $\beta = 120$ $\alpha = 0.992$

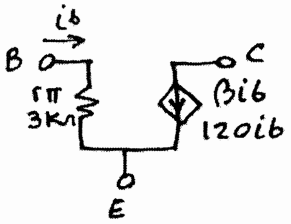
$$g_m = \frac{I_C}{V_T} = \frac{1}{25} = 40\frac{mA}{V}$$

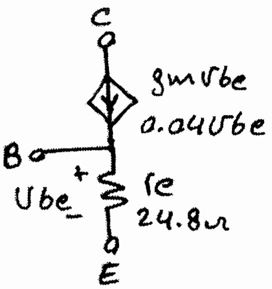
$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40 \times 10^{-3}} = 3K\Omega$$

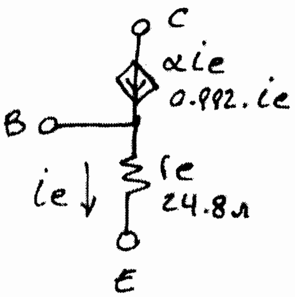
$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{0.992}{40 \times 10^{-3}} = 24.8\Omega$$

The four equivalent circuit models are:


B
C
+
v_{be}
−
r_π
3KΩ
$g_m v_{be}$
$0.04 v_{be}$
E


i_b
B
C
r_π
3KΩ
βi_b
$120 i_b$
E


C
$g_m v_{be}$
$0.04 v_{be}$
B
+
v_{be}
−
r_e
24.8Ω
E


C
αi_e
$0.992 \cdot i_e$
B
i_e
r_e
24.8Ω
E

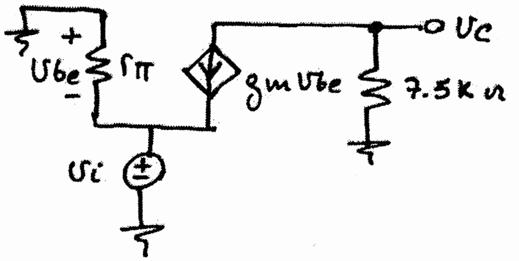
4.73

β very high $\longrightarrow \alpha = 1$

$I_C = I_E = 0.5\,mA$

$V_C = 5 - 7.5 \times 0.5 = +\underline{\underline{1.25V}}$

$$g_m = \frac{I_C}{V_T} = \frac{0.5m}{25m} = \underline{\underline{20\frac{mA}{V}}}$$



+
v_{be}
−
r_π
$g_m v_{be}$
7.5K Ω
v_c
v_i
±

Observe that $v_{be} = -v_i$ the output voltage v_c is found from:

$v_c = -g_m v_{be} \times 7.5K$

Thus the voltage gain is

$$\frac{v_c}{v_i} = g_m \times 7.5K$$

$$= 20 \times 7.5 = \underline{\underline{150}}\ V/V$$

4.74

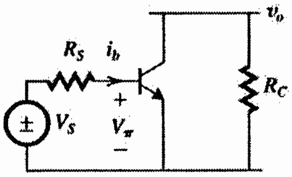
$$\frac{v_c}{v_{be}} = -g_m R_C \Rightarrow v_{be} = \frac{1}{50 \times 2}$$

$$= \underline{\underline{10\,mV}}\ pk\text{-}to\text{-}pk$$

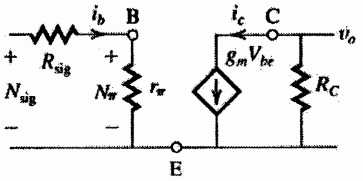
$$i_b = \frac{v_{be}}{r_\pi} = \frac{10 \times 10^{-3}}{\beta / g_m} = \frac{0.01}{100/0.05}$$

$i_b = \underline{\underline{0.005}}\,mA$ pk-to-pk

4.75



v_o
R_S
i_b
R_C
±
V_S
+
V_π
−



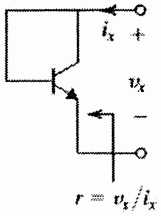
i_b
B
i_c
C
v_o
+
R_{sig}
+
N_{sig}
N_π
r_π
$g_m V_{be}$
R_C
−
−
E

$$\frac{v_O}{v_{sig}} = \frac{v_1}{v_1} = \frac{r_\pi}{r_\pi + R_{sig}}(-)g_m R_C$$

$$= \frac{-r_\pi g_m}{r_\pi + R_{sig}} R_c$$

$$= \frac{-\beta R_C}{r_\pi + R_{sig}}$$

4.76



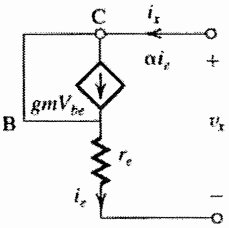
Apply V_x

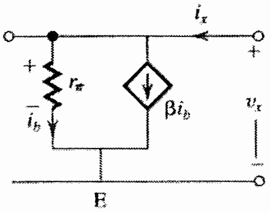
then $v_B = V_x$

$i_x = i_b + i_C$

$v_x = (i_b + i_c)r_e$

$= i_x$ re


C
i_x
αi_e
+
gmV_{be}
B
v_x
r_e
i_e
−


i_x
+
+
r_π
−
i_b
βi_b
v_x
−
E

$\therefore r = \frac{v_x}{i_e} = r_e$

(or)

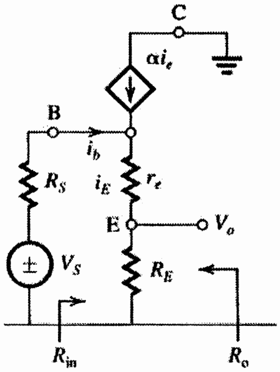
$i_x = \beta i_b + i_b$

$= (\beta + 1)i_b$

$= (\beta + 1)\frac{v_x}{r_\pi}$

$r = \frac{v_x}{i_x} = \frac{r_\pi}{\beta + 1} = r_e$

4.77


C
αi_e
B
i_b
R_S
i_E
r_e
E
V_o
V_S
R_E
R_{in}
R_o

Neglecting r_o

$R_{IN} = \frac{v_{be}}{i_b}$

$= \frac{i_e(r_e + R_E)}{i_e/(\beta + 1)}$

$= (\beta + 1)(r_e + R_E)$

$v_O = -\alpha\, i_e R_E$

$i_e = \frac{v_{be}}{r_e + R_E}$

$\therefore \frac{v_O}{v_{be}} = \alpha\frac{R_E}{r_e + R_E}$

$A_V = \frac{v_O}{v_{be}} = -\frac{\alpha}{r_e}\frac{R_E}{1 + R_E/r_e} = -\frac{g_m R_C}{1 + g_m R_E}$

4.78

$\beta = 200 \rightarrow \alpha = 0.995$

$I_C - \alpha I_E = 0.995 \times 10\text{ mA} = 9.95\text{ mA}$

$V_C = 9.95\text{ mV} \times 100 = 0.995\text{ V}$

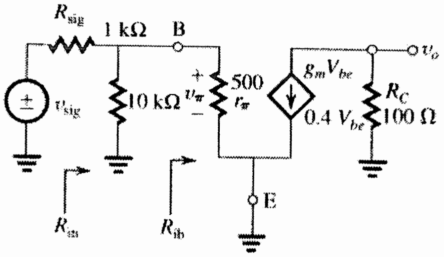
$I_B \cong \frac{10\text{ m}}{200} = 0.05\text{ mA}$

$V_B = 1.5 - 10\text{ k}\Omega \times 0.05\text{ mA}$

$= 1\text{ V}$

$\Rightarrow V_{BC} = +0.005$

→ Active region


R_{sig}
1 kΩ
B
v_o
+
500
r_π
v_π
−
g_mV_{be}
v_{sig}
10 kΩ
0.4 V_{be}
R_C
100 Ω
R_{in}
R_{ib}
E

4.79

$$g_m = \frac{I_C}{V_T} = \frac{9.95}{25\text{ m}} = 0.4\text{ A/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{0.4} = 500\ \Omega$$

$$R_{ib} = r_\pi = 500\ \Omega$$

$$R_{in} = 10\text{ k}\Omega \parallel r_\pi = 476\ \Omega$$

$$v_{be} = v_{sig} \times \frac{R_{in}}{R_{sig} + R_{in}} = v_{sig} \times 0.32$$

also :

$$v_O = -g_m v_{be} \cdot R_C$$

$$= -g_m R_C \times 0.32\ v_{sig}$$

$$= -0.4 \times 100 \times 0.32\ v_{sig}$$

$$= -12.8\ v_{sig}$$

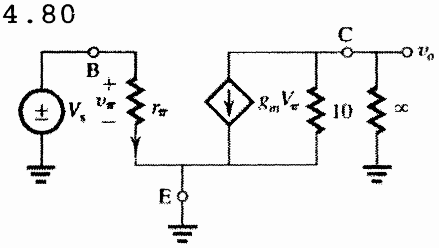
$$\Rightarrow \text{gain } \frac{v_O}{v_S} = -12.8 \simeq -13\ \frac{v}{V}$$

If $v_O = \pm 0.4$ V

$$\hat{v}_S = \frac{\hat{v}_O}{13} = 30\text{ mV}$$

$$\hat{v}_{be} = 0.32 \times 30\text{ m} = 9.8\text{ mA}$$

4.80



$$V_S = V_\pi \Rightarrow \frac{V_O}{V_S} = -g_m r_o$$

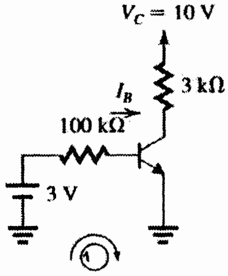
$$\text{but: } r_o = \frac{V_A}{I_C} = \frac{V_A}{V_T \cdot g_m}$$

$$\Rightarrow \frac{V_O}{V_S} = -\frac{V_A}{V_T}$$

$$\text{if } V_A = 25\text{ V} \Rightarrow \frac{V_O}{V_S} = -1000\ \frac{\text{V}}{\text{V}}$$

$$\text{if } V_A = 250\text{ V} \Rightarrow \frac{V_O}{V_S} = -10{,}000\ \frac{\text{V}}{\text{V}}$$

4.81



DC Analysis:

$$(1)\ I_B = \frac{3 - 0.7}{100}$$

$$I_B = 0.023\text{ mA}$$

Saturation begins to occur when $V_C \leq 0.7$ V

$$\therefore I_C \geq \frac{10 - 0.7}{3} = 3.1\text{mA}$$

$$I_C = \beta I_B \to \beta \geq \frac{3.1}{0.023} = 135$$

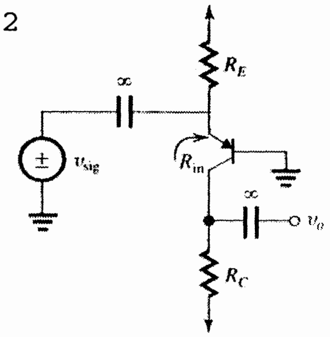
$\beta = 25$:

$$r_e = \frac{V_T}{I_E} = \frac{V_T}{(\beta + 1)I_B} = \frac{25 \times 10^{-3}}{26 \times 0.023 \times 10^{-3}}$$

$$r_e = 41.8\ \Omega$$

$$g_m = \frac{\alpha}{r_e} = \frac{25/26}{41.8} = 23\ \frac{\text{mA}}{\text{V}}$$

4.82



$$R_{in} = r_e \parallel R_E$$

$$\simeq r_e$$

$$= 75\ \Omega$$

$$I_E = \frac{25\text{ mV}}{75\ \Omega} = (0.33\text{ mA})$$

$$R_E = \frac{10 - 0.7}{0.33} = 28\text{ k}\Omega$$

$$n = 2.8$$

$$R_C = 14\text{ k}\Omega$$

$$\frac{V_O}{V_i} = \frac{\alpha R_C}{r_e} = \frac{14}{0.075} = 187\text{ V/V}$$

4.83

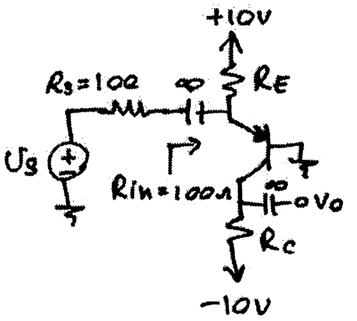
$R_{in} = R_E \parallel r_e \quad r_e \simeq 100\Omega$

thus, $\frac{V_T}{I_E} = 100 \rightarrow I_E = 0.25\,mA$

$V_E = 0.7V$

$R_E = \frac{10-0.7}{1m}$

$= \underline{9.3\,K\Omega}$



Selection of a value for Rc:

The voltage gain is directly proportional to Rc,

$$\frac{U_o}{U_s} = \frac{U_e}{U_s} \cdot \frac{U_o}{U_e}$$

$$= \frac{R_{in}}{R_s + R_{in}} \cdot \frac{\alpha R_c}{r_e}$$

$$\simeq \frac{100}{100+100} \cdot \frac{R_c}{0.1}$$

$= 5R_c$, R_c in $K\Omega$.

For an emitter-base signal as large as 10mV, the signal at the collector will be $g_m R_c \times 0.010$ volts. Thus the maximum collector voltage in the positive direction will be:

$$U_c|_{max} = V_c + 0.01\, g_m \cdot R_c$$

$$= -10 + I_c R_c + 0.01 \times \frac{1}{0.1} \times R_c$$

$$= -10 + 0.25 R_c + 0.1 R_c$$

$$= -10 + 0.35 R_c$$

To prevent saturation, $U_c|_{max} \leq V_B$ which is 0V.

Thus to obtain maximum gain while allowing an emitter-base signal as large as 10mV and at the same time keeping the transistor in the active mode we select Rc from:

$-10 + 0.35 R_c = 0$

$\Rightarrow R_c = \underline{28.6\,K\Omega}$

Voltage gain $= \frac{U_o}{U_s} = 5R_c = 143\,V/V$

4.84

For large β, the DC base current will be ~0

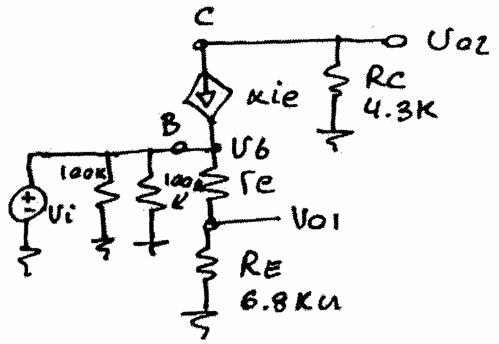
Thus the DC voltage at the base can be found directly using the voltage-divider rule

$$V_B = 15 \cdot \frac{100}{100+100} = 7.5\,V$$

4: $V_{BE} = 0.7$

$V_E = 7.5 - 0.7 = 6.8\,V$

$\rightarrow I_E = \frac{6.8\,V}{6.8\,K\Omega} = \underline{\underline{1\,mA}}$



$v_b = v_i$

$\rightarrow \frac{v_{o1}}{v_i} = \frac{R_E}{R_E + r_e}$ Q.E.D.

Also,

$$i_e = \frac{v_b}{r_e + R_E} = \frac{v_i}{r_e + R_E}$$

and,

$$v_{o2} = -\alpha i_e . R_C = -\frac{\alpha R_C v_i}{r_e + R_E}$$

Thus,

$\frac{v_{o2}}{v_i} = \frac{-\alpha R_C}{R_E + r_e}$ Q.E.D.

Substituting $r_e = \frac{V_T}{I_E} = 25\,\Omega$

and $R_E = 6.8\,K\Omega$, $R_C = 4.3\,K\Omega$

and $\alpha \simeq 1$ gives

$$\frac{v_{o1}}{v_i} = \frac{6.8}{0.025+6.8} = \underline{\underline{0.996\ V/V}}$$

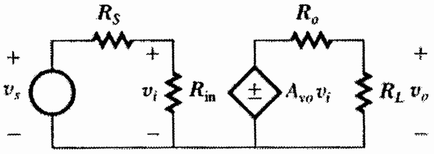
$$\frac{v_{o2}}{v_i} = -\frac{4.3}{6.8+0.025} = \underline{\underline{0.63\ V/V}}$$

If the node labeled v_{o2} is connected to ground:

$R_E = 0$

$$\frac{v_{o2}}{v_i} = -\frac{\alpha R_C}{r_e}$$

4.85



Given : R_s = 100 KΩ A_L = 2 kΩ & R_L = 1 kΩ

Find : R_{in}, A_{VO}, R_L

a) $|V_i(e)| \geq 0.9|v_S(t)|$

$$\vartheta_{i(t)} = \frac{R_{IN}}{R_{IN}+R_S} v_S(t)$$

$$\left|\frac{R_{IN}}{R_{IN}+R_S} v_S(e)\right| \geq 0.9|v_S(t)|$$

$$\frac{R_{in}}{R_{IN}+R_S} \geq 0.9$$

b) $v_O(t) = \frac{R_L}{R_L+R_O} A_{Vo}\, v_i(t)$

$$v_O'(t) = \frac{R_i^1}{R_i^1+R_O} A_{Vo}\, v_i(t)$$

$$|v_O'(t)| \geq 0.9|v_O(t)|$$

$$\frac{R_L'}{R_L'+R_O} \geq 0.9\frac{R_L}{R_L+R_O} \Rightarrow$$

$$R_O \leq \frac{R_L R_L'}{9R_L - 10R_L'} = \frac{(10^3)(2\times10^3)}{9(2\times10^3) - 10(1\times10^3)}$$

$= 250\,\Omega$

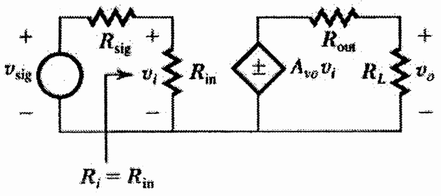
c)Taking the limiting values for R_{in} & R_o

$$10 = A_V\left(\frac{R_{IN}}{R_{IN} + R_S}\right)\left(\frac{R_L}{R_O + R_L}\right)$$

$$A_V\left(\frac{900 \times 10^3}{900 \times 10^3 + 100 \times 10^3}\right)\left(\frac{2 \times 10^3}{250 + 2 \times 10^3}\right)$$

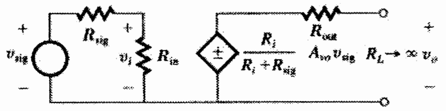
$$A_{VO} = 12.5$$

4.86


+
v_{sig}
−
R_{sig}
+
v_i
−
R_{in}
±
$A_{vo}v_i$
R_{out}
R_L
+
v_o
−
$R_i = R_{in}$

$$v_i = \frac{R_{in}}{R_{in} + R_{Sig}}v_{sig} = \frac{R_i}{R_i + R_{sig}}v_{sig}$$

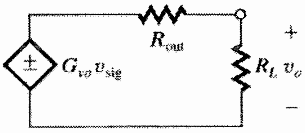
Setting $R_L \to \infty$ and substitution for v_i


+
v_{sig}
−
R_{sig}
+
v_i
−
R_{in}
±
$\frac{R_i}{R_i + R_{sig}}$
R_{out}
$A_{vo}v_{sig}$
$R_L \to \infty$
+
v_o
−

$$v_O = \frac{R_i}{R_i + R_{sig}}A_{VO}\ v_{sig} \Rightarrow G_{VO}$$

$$= v_O / v_{sig} = \frac{R_i}{R_i + R_{sig}}Av_O$$

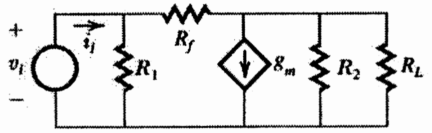
Connecting the load


±
$G_{vo}v_{sig}$
R_{out}
+
R_L
v_o
−

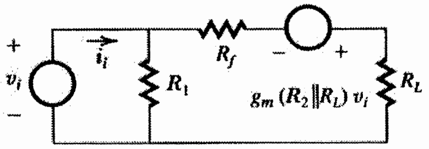
$$v_O = \frac{R_L}{R_L + R_{out}}G_{VO}\ v_{sig} \Rightarrow G_V = v_O/v_{sig}$$

$$= \frac{R_L}{R_L + R_{out}}G_{VO}$$

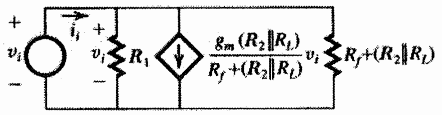
4.87


+
v_i
−
i_i
R_1
R_f
g_m
R_2
R_L

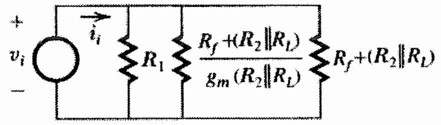
R_L and R_L are in Parallel. Also do a source transformation


+
v_i
−
i_i
R_1
R_f
−
+
$g_m(R_2\|R_L)v_i$
R_L

Combine Rf and $R_o \| R_L$ and do another source transformation


+
v_i
−
i_i
+
v_i
−
R_1
$\frac{g_m(R_2\|R_L)}{R_f+(R_2\|R_L)}v_i$
$R_f+(R_2\|R_L)$

The dependent current source is equivalent to a resistor


+
v_i
−
i_i
R_1
$\frac{R_f+(R_2\|R_L)}{g_m(R_2\|R_L)}$
$R_f+(R_2\|R_L)$

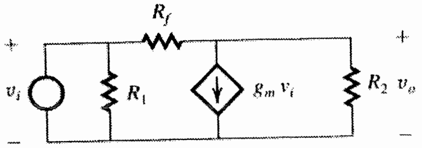
$$R_{in} = v_i / i_1 = R_1 \| \frac{R_f + (R_2 \| R_L)}{g_m(R_2 \| R_L)} \| (R_f + [R_2 \| R_L)]$$

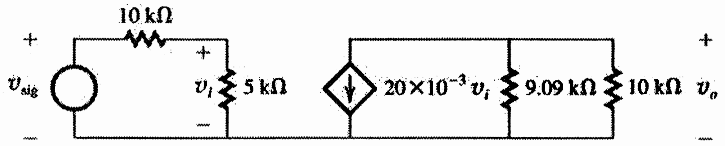
Consider the form

$$(R / e) \| R: \frac{\text{RR}}{\text{A / a} + R} = \frac{R}{1 + a}$$

$$R_{in} = R_1 \| \left[\frac{R_f + (R_2 \| R_L)}{1 + g_m(R_2 + R_L)}\right]$$

The circuit for A_{vo} is


R_f
+
v_i
−
R_1
$g_m v_i$
R_2
+
v_o
−

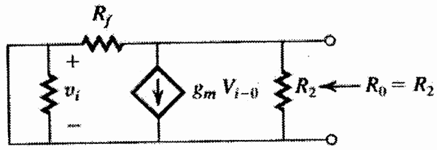


$$\frac{v_O - v_i}{R_f} + g_m v_i + \frac{1}{R_2}v_O$$

$$\left[\frac{1}{R_f} + \frac{1}{R_2}\right]V_O = \left(\frac{1}{R_f} - g_m\right)v_i$$

$$A_{VO} = v_O / v_i = \frac{1 - g_m R_f}{1 + R_f / R_2}$$

The circuit for R_o



for values given
$R_{in} = 99.90, A_{vo} = -9.9989, R_o = 100$
The dependence on R_f is

$$R_{in} = 100\frac{1100 R_f + 10^5}{1100 R_f + 1.21\times10^6}$$

$$A_{VO} = -10\left(\frac{R_f - 10}{R_f + 100}\right)$$

If R_f decreases the gain becomes sensitive to R_f
If $R_f \to \infty$ $R_{in} = 100, A_{vo} = -10$
with R_f

$$G_{VO} = \frac{R_{IN}}{R_{IN} + R_{avg}}A_{VO} = \frac{-99.9}{99.9 + 100}(-9.9989)$$

$$= -4.997 \text{ V/V}$$

Without R_f

$$G_{VO} = \left(\frac{100}{100 + 100}\right)(-10) = -5$$

4.88

$R_C = 10\text{ k}\Omega, V_A = 50\text{ V}, \beta = 100, I_C = 0.5\text{ mA}$

$$g_m = \frac{I_C}{V_T} = \frac{0.5\times10^{-3}}{0.025} = 20\times10^{-3}\text{ S}$$

$$r_O = \frac{V_A}{I_A} = \frac{50}{0.5\times10^{-3}} = 100\text{ k}\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20\times10^{-3}} = 5\times10^3\ \Omega$$

$$R_O = R_C \| r_O = (10\times10^3 \| 100\times10^3)$$

$$= 9.09\text{ k}\Omega$$

$$R_{in} = r_\pi = 5\times10^3$$

The circuit is now (see figure above)

$$A_v = \frac{v_O}{v_i} = -g_m(R_O \| R_L)$$

$$= -(20\times10^{-3})(9.09\times10^3 \| 10\times10^3)$$

$A_V = -95.23$

$$G_V = v_O / v_{sig} = \frac{R_{in}}{R_{in} + R_{sig}}$$

$$A_V = \left(\frac{5\times10^3}{5\times10^3 + 10\times10^3}\right)(-95.2) = -31.74$$

max signal v_{sig} is

$$\max\frac{|v_O(t)|}{|G_V|} = \frac{5\times10^{-3}}{31.74} = 157.5\ \mu\text{V}$$

4.89

$$|G_V| = \beta\frac{R_C \| R_L \| r_O}{R_{sig} + r_\pi}$$

IF $r_O \to \infty$ then $R_C \| R_L \| r_O \to R_C \| R_L$

Let $R_L{}' = R_C \| R_L$

$$|G_V| = \beta\frac{R_L{}'}{R_{sig} + r_\pi}$$

$$|G_V| = \frac{R_L{}'}{\frac{R_{sig}}{\beta} + \frac{r_\pi}{\beta}}$$

But $r_\pi/\beta = 1/g_m$

$$|G_V| = \frac{R_L{}'}{\frac{R_{sig}}{\beta} + \frac{r_1}{g_m}}$$

$R_L{}' = 10\text{ k}\Omega$; $R_{sig} = 10\text{ k}\Omega$; $\beta = 100$;
$I_C = 1\text{ mA}$

$g_m = I_c/V_T$, so

$$|G_V| = \frac{R_L'}{\frac{R_{\text{sig}}}{\beta} + \frac{V_T}{I_C}}$$

a) $$|G_V| = \frac{10^4}{\frac{10^4}{100} + \frac{0.025}{10^{-3}}} = 80 \text{ V/V}$$

b) If β ranges from 50 → 150

For $\beta = 50$:

$$|G_V| = \frac{10^4}{\frac{10^4}{50} + \frac{0.025}{10^{-3}}} = 44.44 \text{ V/V}$$

For $\beta = 150$:

$$|G_V| = \frac{10^4}{\frac{10^4}{150} + \frac{0.025}{10^{-3}}} = 109.09 \text{ V/V}$$

c) What is β range if $G_v \le |G_v| \le 96$

at $|G_v| = 64$:

$$\frac{10^4}{\frac{10^4}{\beta} + \frac{0.025}{10^{-3}}} = 64 \Rightarrow \beta = 76.19$$

at $|G_v| = 96$:

$$\frac{10^4}{\frac{10^4}{\beta} + \frac{0.025}{10^{-3}}} = 96 \Rightarrow \beta = 126.32$$

d) Suppose the nominal G_v is $G_{v-\text{nom}}$ and I_c is variable

$\beta = 50 \Rightarrow G_V = 0.8\ G_{V-\text{nom}}$

$\beta = 150 \Rightarrow G_V = 1.2\ G_{V-\text{nom}}$

Then

$$\frac{10^4}{\frac{10^4}{50} + \frac{0.025}{I_C}} = 0.8\ G_{V-\text{nom}}$$

$$\frac{10^4}{\frac{10^4}{150} + \frac{0.025}{I_C}} = 1.2\ G_{V-\text{nom}}$$

Take ratio

$$\frac{\frac{10^4}{150} + \frac{0.025}{I_C}}{\frac{10^4}{50} + \frac{0.025}{I_C}} = \frac{0.8}{1.2} \Rightarrow I_C = 0.125 \text{ mA}$$

$$\frac{10^4}{\frac{10^4}{\beta_{\text{nom}}} + \frac{0.025}{I_C}} = G_{V-\text{nom}}$$

$G_{V-\text{nom}} = 31.25\ \beta_{\text{nom}} = 83.33$

4.90

$$|G_V| = \beta\frac{R_C \| R_L \| r_O}{R_{\text{sig}} + r_\pi} = \beta\frac{(R_C \| R_L) \| r_O}{R_{\text{sig}} + r_\pi}$$

$$r_O = \frac{V_A}{I_C}$$

$$|G_V| = \frac{(R_C \| R_L) \| \frac{V_A}{I_C}}{\frac{R_{\text{sig}}}{\beta} + \frac{r_\pi}{\beta}};$$

$$\frac{r_\pi}{\beta} = \frac{1}{g_m} = \frac{V_T}{I_C}$$

thus, $$|G_V| = \frac{(R_C \| R_L) \| \frac{V_A}{I_C}}{\frac{R_{\text{sig}}}{\beta} + \frac{V_T}{I_C}}$$

$R_C \| R_L = 10\ \Omega$, $R_{\text{sig}} = 10\text{ k}\Omega$, $V_A = 25$ V, and $V_T = 0.025$ V

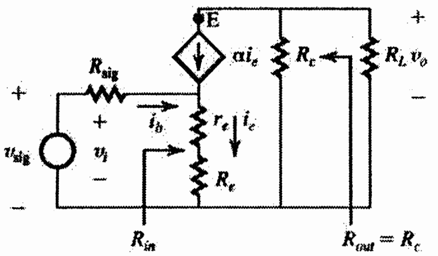
$$|G_V| = \frac{(10^4) \| 25/I_C}{\frac{10^4}{100} + \frac{0.025}{I_C}}$$

$$= \frac{25 \times 10^6 I_C}{(10^4 I_C + 25)(10^4 I_C + 2.5)}$$

I_c (ref)	$\lvert G_v \rvert$
0.1	27.47
0.2	41.15
0.5	55.56
1.0	57.14
1.25	55.55

The values of I_c that result in $|G_v| = 50$ are:
1 × 0.925 mA and 0.324 mA.
The 0.324 mA would be preferred since a lower power is required.

4.91



$$i_e = v_i/(r_e + R_e)$$

$$i_b = i_e - \alpha i_e = (1-\alpha)i_e = (1-\alpha)\frac{v_i}{r_e + R_e}$$

$$\alpha = \frac{\beta}{\beta+1} = \frac{100}{100+1} = 0.99$$

$$i_b = \frac{1}{(\beta+1)}\frac{v_i}{r_e + R_e}$$

$$R_{in} = v_i/i_b = (\beta+1)(r_e + R_e)$$

$$r_e = \frac{V_T}{I_E} = \frac{V_T}{I_C/\alpha} = \alpha\frac{V_T}{I_C}$$

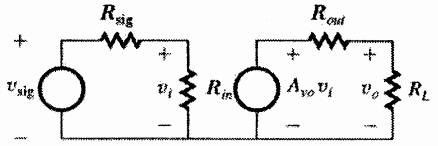
$$= (0.99)\left(\frac{0.025}{0.5 \times 10^{-3}}\right) = 49.5\ \Omega$$

$$R_{in} = (100+1)(49.5 + 150) = 20150\ \Omega$$

$$A_{VO} = -\alpha i_e R_C = -\alpha R_C \frac{1}{r_e + R_e}$$

$$A_{VO} = -(0.99)(10 \times 10^3)/(49.5 + 150) = -49.62$$

now model becomes



$$v_O = \frac{R_L}{R_L + R_{out}}A_{VO}\,v_i$$

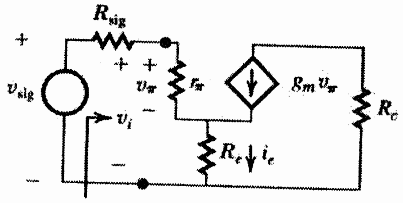
$$v_O = \frac{R_L}{R_L + R_{out}}A_{VO}\frac{R_{in}}{R_{in} + R_{sig}}v_{sig}$$

$$G_V = v_O/v_{sig}$$

$$= \frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 10\ \text{k}\Omega}(-49.62)\frac{.20150\ \Omega}{20150\ \Omega + 10000\ \Omega}$$

$$= -16.582$$

4.92



$$R_{in} = (\beta + 1)(r_e + R_e)$$

$$R_{in} = (\beta + 1)(r_e + R_e)$$

$$r_e = \frac{V_T}{I_C}$$

$R_{in} = (\beta+1)\left(\frac{V_T}{I_C} + R_e\right)$ multiply both sides by I_C and rearrange :

$$-(\beta+1)R_e I_C + R_{in}I_C = (\beta+1)V_T$$

Given $\beta = 100$; $R_{in} = 20\ \text{k}\Omega$; $V_T = 0.025$ V

Equation becomes

$-101\,R_e I_c + (2 \times 10^4)\,I_c = (101)(0.025) = 2.525$ (Eq A)

Our unknowns are I_c & R_e This is one equation.

$$i_e = v_\pi/r_\pi + g_m v_\pi = (1/r_\pi + g_m)v_\pi$$

$$= \left(\frac{1}{\beta} + 1\right)g_m v_\pi$$

$$= \left(\frac{1}{\beta} + 1\right)\frac{I_C}{V_T}v_\pi$$

$$v_{sig} = R_e i_e + v_\pi + R_{sig}\frac{v_\pi}{r_\pi}$$

$$= R_e\left(\frac{1}{\beta} + 1\right)\frac{I_C}{V_T}v_\pi + v_\pi + \frac{R_{sig}}{\beta}\frac{I_C}{V_T}v_\pi$$

$$v_{sig} - v_{sig} = \left[\frac{1}{\beta} + 1\right]\frac{v_\pi}{V_T}R_e I_C + \frac{R_{sig}v_\pi}{\beta V_T}I_C$$

$$0.1 - 0.005 = \left[\frac{1}{100} + 1\right]\left[\frac{5 \times 10^{-3}}{0.025}\right] \bullet$$

$$R_e I_C + \frac{(5000)(5 \times 10^{-3})}{(100)(0.025)}I_C$$

$0.005 = 0.202\,R_e I_c + 10\,I_c$ (Eq B)

Equations A and B can be solved simultaneously

$I_C = 1.25$ mA

$$R_e I_C = 0.00064$$

$$\Rightarrow R_e = 0.22264 / 1.25 \times 10^{-3}$$

$$= 178.11$$

$$G_V = \frac{v_O}{v_{\text{sig}}} = \frac{v_O}{v_\pi} \cdot \frac{v_\pi}{v_{\text{sig}}}$$

$$v_O / v_\pi = -R_C g_m = -R_C \frac{I_C}{V_T}$$

$$= -(5 \times 10^3)\left(\frac{1.25 \times 10^{-3}}{0.025}\right) = -250$$

$$G_V = (-250)\left(\frac{5 \times 10^{-3}}{0.1}\right) = -12.5$$

4.93

$$|G_V| = \frac{\beta R_C}{R_{\text{sig}} + (\beta + 1)(r_e + R_e)}$$

$$r_e = \frac{V_T}{I_E}$$

$$|G_V| = \frac{\beta R_C}{R_{\text{sig}} + (\beta + 1)(V_T / I_E + R_e)}$$

$R_{\text{sig}} = 10\ \text{k}\Omega$; $R_c = 10\ \text{k}\Omega$; $\beta = 100$;

$V_T = 0.025$ V;

$I = 1$ mA

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

$I_E = I_c/\alpha = 1.01 \times 10^{-3}$A

If $R_e = 0$

$$|G_V| = \frac{(100)(10 \times 10^3)}{10 \times 10^3 + (101)[0.025 / (1.01 \times 10^{-3})]} = 80$$

Suppose $|G_v|$ has a nominal value $G_{v\text{-num}}$ and $0.8\ G_{v\text{-nom}}$ corresponds to $\beta = 50$. Let R_e be a variable (note that $\alpha = 0.98$):

$$\frac{\beta R_C}{R_{\text{sig}} + (\beta + 1)[0.025 / (1.02 \times 10^{-3}) + R_e]}$$

$$= 0.8 G_{V-\text{nom}}$$

$$\frac{(50)(10^4)}{10^4 + (51)(0.025 / 1.02 \times 10^{-3} + R_e)} = 0.8 G_{V-\text{nom}}$$

at $\beta = 150$ $G_v = 1.2\ G_{v\text{-nom}}$

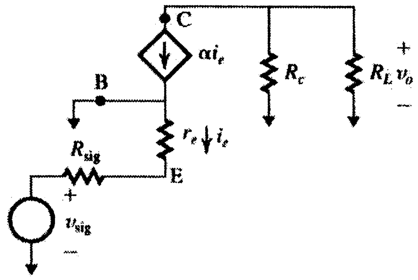
$$\frac{(150)(10^4)}{10^4 + (151)(0.025 / 1.01 \times 10^{-3} + R_e)} = 1.2 G_{V-\text{nom}}$$

These two equations can be solved simultaneously for R_e & $G_{v\text{-nom}}$

$R_e = 179.3$ V

$G_{v\text{-nom}} = -30.625$

4.94



$$v_{be}(t) = r_e i_e$$

$$v_O(t) = -\alpha i_e(R_C \| R_L)$$

$$v_{be}(t) = -r_e \frac{v_O(t)}{\alpha(R_C \| R_L)}$$

$$|v_O(t)| = \frac{\alpha(R_C \| R_L)}{r_e} |v_{be}(t)|$$

$$= \frac{\alpha(R_C \| R_L)}{V_T} I_E |v_{be}(t)|$$

Suppose $\alpha \approx 1$

$$|v_O(t)| = \frac{(10\ \text{k}\Omega) \| (10\ \text{k}\Omega)}{0.025}(0.25\ \text{mA})(10 \times 10^{-3})$$

$$|v_O(t)| = 0.5\ \text{V}$$

$$G_V = v_O(t) / v_{\text{sig}(t)} = \alpha \frac{R_C \| R_L}{R_{\text{sig}} + r_e} = \alpha \frac{R_C \| R_L}{R_{\text{sig}} + V_T / I_E}$$

$$G_V = \frac{(10\ \text{k}\Omega) \| (10\ \text{k}\Omega)}{1\ \text{k}\Omega + 0.025 / 10^{-3}} \text{ Since } \alpha \approx 1$$

$$= 4.88\ \text{V/V}$$

$$|v_{\text{sig}}(t)| = |v_O(t)| / G_V$$

$$|v_{\text{sig}}(t)| = 0.5 / 4.88 = 0.1025\ \text{V}$$

4.95

$$|v_O(t)|_{\max} = (0.5\ \text{V})$$

$$|i_e(t)|_{\max} = \frac{|v_O(t)|_{\max}}{R_L} = \frac{0.5}{2 \times 10^3} = 250\mu\text{A}$$

$$r_e = \frac{|v_{be}(t)|_{\max}}{|i_C(t)|_{\max}} = \frac{5 \times 10^{-3}}{250\ \mu\text{A}} = 20\Omega$$

$$r_e = \frac{V_T}{I_E} \Rightarrow I_E = \frac{V_T}{r_e} = \frac{0.025}{20} = 1.2\text{mA}$$

$$|i_E(t)|_{\max} = I_E + |i_C(t)|_{\max} = 1.5\ \text{mA}$$

$$|i_E(t)|_{\max} = I_E - |i_C(t)|_{\max} = 1\ \text{mA}$$

Suppose $\beta = 100$

$$G_V = \frac{(\beta + 1) R_L}{(\beta + 1) R_L + (\beta + 1) r_e + R_{\text{sig}}}$$

$$= \frac{(101)(2 \times 10^3)}{(101)(2 \times 10^3) + (101)(20) + 200 \times 10^3} = 0.499$$

$(G_V = v_O(t)/v_{sig}(t)) \Rightarrow$

$$v_{sig}(t) = \frac{v_O(t)}{G_V} = \frac{0.5}{0.499}$$

$|V_{sig}|_{max} = 1.00$ Volt

4.96

$I_c = 1$mA; $\beta = 100$; $R_{sig} = 20$ kΩ;
$R_L = 1000$ Ω

$$I_E = \frac{\beta+1}{\beta} I_C = \frac{101}{100} 10^{-3} = 1.01\text{mA}$$

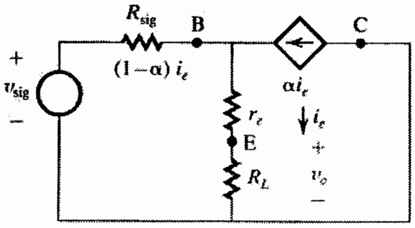
$$r_e = \frac{V_T}{I_E} = \frac{0.025}{1.01 \times 10^{-3}} = 24.752\Omega$$

$$R_{in} = (\beta+1)(r_e + R_L) = (101)(24.752 + 1000)$$
$$= 103.5 \text{ k}\Omega$$

we have:

$$v_O/v_{sig} = G_V = \frac{(\beta+1)R_L}{(\beta+1)R_L + (\beta+1)r_e + R_{sig}}$$

$$= \frac{(101)(1000)}{(101)(1000) + (101)(24.752) + 20 \times 10^3} = 0.8178$$



$$i_C(t) = v_O(t)/R_L = \frac{G_V V_{sig}}{R_L}$$

$V_{be}(t) = r_e i_e(t) = (r_e/R_L) G_V V_{sig}(t) \Rightarrow$

$v_{be}(t)/v_{sig}(t) = (r_e/R_L)G_V$

$= (24.752/1000)(0.8178) = 0.02024$

$v_b(t) = v_O(t) + v_{be}(t) \Rightarrow$

$v_b(t)/v_{sig}(t) = G_V + (r_e/R_L)G_V = (1 + r_e/R_L)G_V$

$v_b(t)/v_{sig}(t) = (1 + 24.752/1000)(0.8178)$

$= 0.838056$

b) $v_{be}(t)/v_{sig}(t) = 0.02024$

$\Rightarrow |v_{sig}(t)|_{max}$

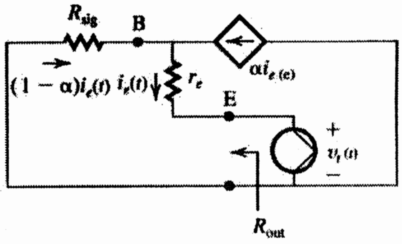
$= |v_{be}(t)|_{max}/0.02024$

$|v_{sig}(t)|_{max} = 10 \times 10^{-3}/0.02024 = 0.494 V_{old}$

$|v_o(t)|_{max} = G_V |v_{sig}(t)|_{max} = (0.494)(0.8178)$

$= 0.404$ V

c) If R_L is removed $i_e = 0$, therefore,
$v_e = v_{sig}$, Thus
$G_{vo} = 1$,.
Now for R_{out}



$$R_{out} = -\frac{v_t(t)}{i_C(t)}$$

$$i_e(t) = \frac{v_b(t) - v_e(t)}{r_e} = \frac{v_b(t) - v_t(t)}{r_e}$$

$v_b(t) = -i_e(t)(1-\alpha)R_{sig} \Rightarrow$

$$i_e(t) = \frac{-i_e(t)(1-\alpha)R_{sig} - v_t(t)}{r_e};$$

$r_e i_e(t) = -i_e(1-\alpha)R_{sig} - v_t(t)$

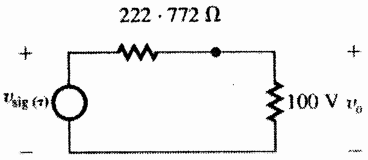
$$i_e(t) = \frac{-v_t}{r_e + (1-\alpha)R_{sig}}$$

Substituting into R_{out} expression

$$R_{out} = r_e + (1-\alpha)\, R_{sig} = r_e + \frac{1}{\beta+1} R_{sig}$$

$$= 24.752 + \frac{20 \times 10^3}{101} = 222.772$$

now



$$v_O(t)/v_{sig}(t) = \frac{1000}{1000 + 222.772} = 0.8178$$

This agrees with G_v

4.97

$I_C = 0.25$ mA ; $R_{sig} = 10$ kΩ ; $R_L = 1$ kΩ ;
$V_T = 0.025$

$$G_V = \frac{(\beta+1)\,R_L}{(\beta+1)\,R_L + (\beta+1)r_e + R_{sig}}$$

$$r_e = \frac{V_T}{I_E} = \frac{\beta\ V_T}{(\beta+1)I_C}$$

$$G_V = \frac{(\beta+1)\,R_L}{(\beta+1)\,R_L + \beta V_T/I_C + R_{sig}}$$

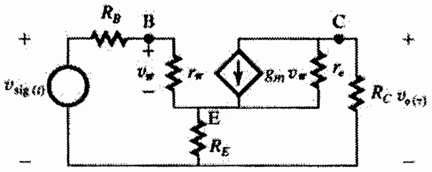
$$R_{out} = r_e + R_{sig}/(\beta+1)$$

$$= \frac{\beta\ V_t}{(\beta+1)\,I_C} + \frac{R_{sig}}{\beta+1}$$

for $\beta = 100$ $\quad\beta = 50$ $\quad\beta = 150$
$G_v = 0.8347$ $\quad G_v = 0.7727$ $\quad G_v = 0.85$
$R_{out} = 199.01\ \Omega$ $\quad R_{out} = 298.0\ \Omega$ $\quad R_{out} = 166.0\Omega$

4.98

Part a) Nodal equations:



Part a)

$$\frac{v_e}{R_E} + \frac{v_e - v_{sig}}{R_B + r_\pi} - g_m V_\pi + \frac{v_e - v_c}{r_o} = 0$$

$$g_m v_\pi + \frac{v_c - v_e}{r_o} + \frac{v_e}{R_C} = 0$$

$$\frac{v_\pi}{r_\pi} + \frac{v_e + v_\pi - v_{sig}}{R_B} = 0$$

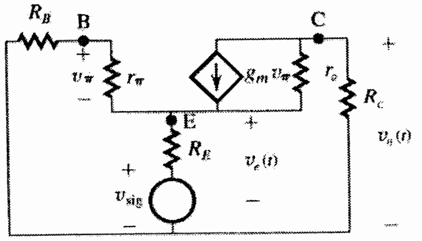
Solving:

$$\frac{v_e(t)}{v_{sig}(t)} = \frac{(g_m r_o r_\pi - R_E)R_C}{(r_\pi R_C + R_E r_\pi + r_o r_\pi + g_m R_E r_o r_\pi + R_C R_B + R_E r_o + r_o R_B + R_E R_C + R_E R_B)}$$

$$\frac{v_e(t)}{v_{sig}(t)} = \frac{R_E(g_m r_o r_\pi + R_C + r_o)}{(r_\pi R_C + R_E r_\pi + r_o r_\pi + g_m R_E r_o r_\pi + R_C R_B + R_E r_o + r_o R_B + R_E R_C + R_E R_B)}$$

$$r_o = \frac{|V_A|}{I_C}$$

Part b) Nodal equations:



$$\frac{v_e - v_{sig}}{R_E} + \frac{v_e}{R_B + r_\pi} - g_m v_\pi + \frac{v_e - v_C}{r_o} = 0$$

$$g_m v_\pi + \frac{v_c - v_e}{r_o} + \frac{v_e}{R_C} = 0$$

$$\frac{v_\pi}{r_\pi} + \frac{v_e + v_\pi}{R_B} = 0$$

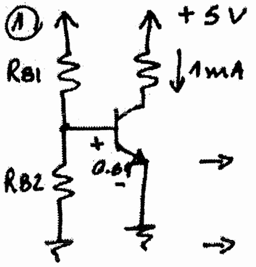
Solutions

$$\frac{V_e(t)}{V_{sig}(t)} = \frac{R_C(g_m r_o r_\pi + R_B + r_\pi)}{(r_\pi R_C + R_E r_\pi + r_o r_\pi + g_m R_E r_o r_\pi + R_C R_B + R_E r_o + r_o R_B + R_E R_C + R_E R_B)}$$

$$\frac{V_e(t)}{V_{sig}(t)} = \frac{(R_C + r_o)(R_B + r_\pi)}{(r_\pi R_C + R_E r_\pi + r_o r_\pi + g_m R_E r_o r_\pi + R_C R_B + R_E r_o + r_o R_B + R_E R_C + R_E R_B)}$$

4.99

①



$$\frac{5 \times R_{B2}}{R_{B1} + R_{B2}} = 0.690$$

$\rightarrow 5R_{B2} = 0.69R_{B1} + 0.69R_{B2}$

$\rightarrow 4.31R_{B2} = 0.69R_{B1}$

$$\Rightarrow \frac{R_{B1}}{R_{B2}} = \underline{\underline{6.24}}$$

② Since $V_{BE} = \dfrac{5\,R_{B2}}{R_{B1} + R_{B2}}$

If both R_{B2} & R_{B1} are at 0.99 or 1.01 of their nominal value → V_{BE} will not be affected.

We must consider the cases when one resistor is at 0.99 and the other at 1.01 of their nominal value.

If: $R_{B2}' = 1.01\,R_{B2}$
$\quad R_{B1}' = 0.99\,R_{B1}$
$\Rightarrow V_{BE} = 0.702\text{V}$

If: $R_{B2}' = 0.99\,R_{B2}$
$\quad R_{B1}' = 1.01\,R_{B1}$
$\Rightarrow V_{BE} = 0.678\text{V}$

thus V_{BE} ranges from 0.678V to 0.702V

CONT.

For Ic: $I_c = I_s e^{V_{be}/V_T}$

for $V_{be} = 0.690 \rightarrow I_c = 1mA$

$\Rightarrow I_s = 1.032 \times 10^{-15}$

for $V_{BE} = 0.678 \rightarrow I_c = 0.619m$

$V_{BE} = 0.702 \rightarrow I_c = 1.62m$

Ic ranges from 0.618 mA to 1.62 mA.

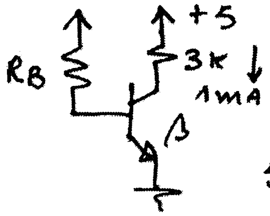
③ If $R_c = 3K\Omega$

$V_{CE} = 5 - 3K \times 0.62m = 3.14V$

$V_{CE} = 5 - 3K \times 1.62m = 0.14V$

This circuit is too sensitive to parameter variations as shown here for a 1% resistor tolerance.

4.100



$R_B = ?$ if $\beta = 100$

$I_B \times \beta = I_c$

$\frac{5-0.7}{R_B} = \frac{1m}{100}$

$\rightarrow R_B = \underline{\underline{430K\Omega}}$

$V_{CE} = 5V - 3K \cdot 1mA = 2V$

If $\beta = 50$: $I_c = \frac{5-0.7}{430k} \times 50$

$I_c = 0.50mA$

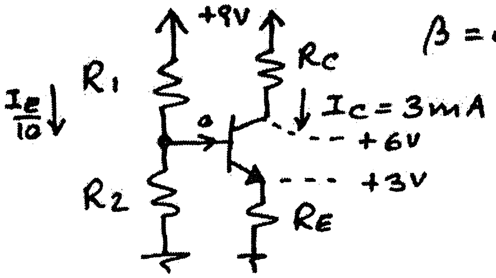
$\Rightarrow V_{CE} = 5 - 3K \times 0.5m = +3.5V$

If $\beta = 150$: $I_c = 1.5mA$

$V_{CE} = 0.5V$

This design is too sensitive to variations of β.

4.101



$\beta = \infty$

$R_c = \frac{3V}{3mA} = 1K\Omega$

$R_E = \frac{3V}{3mA} = 1K\Omega$

$V_b = 0.7 + 3 = 3.7V$

$R_1 = \frac{9 - 3.7}{I_E/10} = 17.7K\Omega$

$9V = (R_1 + R_2)\frac{I_E}{10} \rightarrow R_2 = 12.3K$

Choose suitable 5% resistors

$R_1 = 17.7K \rightarrow 18K\Omega$

$R_2 = 12.3K \rightarrow 13K\Omega$

$R_1 = R_2 = 1K$

$V_{BB} = \frac{9 \times 13}{18 + 13} = 3.77V$

For these values of R and $\beta = 90$: $R_B = 18 \parallel 13 = 7.55K\Omega$

$I_E = \frac{3.77 - 0.7}{1K + \frac{7.55K}{91}} = 2.83mA$

$\alpha = 0.989 \Rightarrow I_c = 2.80mA$

If R_E is reduced by $\sim \frac{7.55K}{91}$

$\rightarrow R_E = 910\Omega$

$\Rightarrow I_E = 3.09mA$

$I_c = 3.05mA$.

4.102

For $\beta = \infty, I_B = 0, I_E = 0.6\text{ mA}$

$$R_c = \frac{3\text{ V}}{0.6\text{ m}} = 5\text{ k}\Omega = R_E$$

$V_b = 0.7 + 3 = 3.7$

$$R_1 = \frac{9-3.7}{I_E/2} = \frac{10.6}{.6\text{ m}} = 17.7\text{ k}\Omega$$

$$9 = (R_2 + R_1)\frac{I_E}{2} \Rightarrow R_2 = \frac{18}{I_E} - R_1 = 12.3\text{ k}\Omega$$

Suitable 5%. Resistors: $R_1 = 17.4\text{ k}\Omega$

$R_2 = 12.1\text{ k}\Omega$

$\beta = 90$:

$$R_B = (17.4\text{ K}) \| (12.1\text{ K}) = \frac{17.4\text{ K}(12.1\text{ K})}{29500} = 7.137\ \Omega$$

$$V_{BB} = \frac{9(12.1\text{ K})}{12.1\text{ K} + 17.4\text{ K}} = 3.7\text{ V}$$

$$I_E = \frac{3.7-0.7}{5\text{ K} + \frac{7137}{(90+1)}} = \frac{3}{5\text{ K} + 78.4} = .6\text{ mA}$$

4.103

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta+1}}$$

(a) For $\beta = 100$, varying between 50 and 150 the maximum deviation in I_E (from the nominal value obtained for $\beta = 100$) occurs at the low end of β values ($\beta = 50$). Thus, to keep I_E within ± 5% of nominal we must impose the constraint

$$I_E(\beta=50) \geqslant 0.95\, I_E(\beta=100)$$

or, $$\frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{51}} \geqslant 0.95\, \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{101}}$$

or, $$R_E + \frac{R_B}{101} \geqslant 0.95\left(R_E + \frac{R_B}{51}\right)$$

$$0.05 R_E \geqslant R_B\left(\frac{0.95}{51} - \frac{1}{101}\right)$$

$$\Rightarrow \frac{R_B}{R_E} \leqslant 5.73$$

Thus, the largest ratio of R_B/R_E is 5.73

(b) $I_E . R_E = V_{cc}/3$

$$\rightarrow \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta+1}} . R_E = \frac{V_{cc}}{3}$$

$$\frac{V_{BB} - 0.7}{1 + \frac{R_B}{R_E} . \frac{1}{\beta+1}} = \frac{V_{cc}}{3}$$

$$V_{BB} = \frac{1}{3} V_{cc}\left(1 + \frac{5.73}{101}\right) + 0.7$$

$$\Rightarrow \underline{\underline{V_{BB} = 0.35\, V_{cc} + 0.7}}$$

(c) $V_{cc} = 10\text{V}$

$V_{BB} = 0.35 \times 10 + 0.7 = 4.2\text{V}$

$$\rightarrow \frac{R_2}{R_1 + R_2} \times 10 = 4.2$$

$$\frac{R_2}{R_1 + R_2} = 0.42 \quad ①$$

$$I_E . R_E = \frac{1}{3} V_{cc}$$

CONT.

$2 \times R_E = \frac{1}{3} \times 10$

$\Rightarrow R_E = 1.67\,K\Omega$

$R_B = 5.73 \times 1.67 = 9.55\,K\Omega$

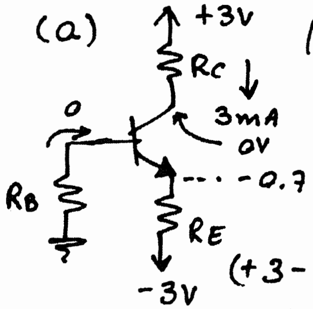
$\frac{R_1 \cdot R_2}{R_1 + R_2} = 9.55$

Substituting from ① gives

$R_1 = \frac{9.55}{0.42} = 22.7\,K\Omega$

4.104

(a)



$\beta = \infty, I_B = 0$

$I_E = I_C$

$3V = R_C \times 3mA$

$\rightarrow R_C = 1K\Omega$

$(+3 - 0.7) = R_E \times 3mA$

$\rightarrow R_E = 767\Omega$

(b) $\beta = 90 \quad \frac{V_{RE}}{10} = V_{RB}$

$I_B \cdot R_B = \frac{I_E \cdot R_E}{10}$

$\rightarrow \frac{I_E}{(\beta+1)} \cdot R_B = \frac{I_E \cdot R_E}{10}$

$\rightarrow R_B = \frac{(\beta+1)}{10} R_E$ ①

also, $0 = V_{RB} + 0.7 + V_{RE} - 3$

$2.3 = \frac{V_{RE}}{10} + V_{RE}$

$\rightarrow V_{RE} = \frac{2.3}{1.1} = 2.09V$

$2.09 = I_E \times R_E$ ②

but: $I_E = \frac{I_C}{\alpha} = \frac{3mA}{0.989} = 3.033m$

Substituting in ②:

$R_E = 689\Omega$

from ①:

$R_B = 6269\Omega$

(c) Standard 5% values:

$R_C = 1K\Omega$

$R_E = 689\Omega \rightarrow 680\Omega$

$R_B = 6269\Omega \rightarrow 6.2K\Omega$

(d) $\beta = \infty: \quad I_B = 0$

$I_C = I_E$

$V_B = 0$

$V_E = -0.7$

$I_E = \frac{3 - 0.7}{R_E} = \frac{3 - 0.7}{680} = 3.38mA$

$V_C = 3 - 3.38m \times 1K = -0.38V$

For $\beta = 90$:

$I_E = \frac{2.3}{680 + \frac{6.2K}{91}} = 3.07mA$

$I_C = \alpha I_E = 3.04mA$

$V_B = \frac{R_B \cdot I_E}{(\beta+1)} = -0.209$

$V_E = -0.209 - 0.7 = 0.909\,V$

$V_C = 3 - I_C \cdot R_C$

$= 3 - 3.04 \times 1 = -0.04V$

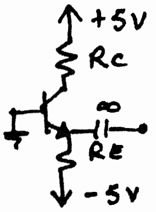
4.105

$V_E = -0.7V$

To obtain $I_E = 1mA$

$R_E = \frac{-0.7 - (-5)}{1}$

$= 4.3K\Omega$



CONT.

To maximize gain while allowing ±1V signal at collector, design for a dc collector voltage of +1V. Thus,

$$R_C = \frac{5-1}{I_C} \simeq \frac{4}{1} = \underline{\underline{4\,K\Omega}} \; (\text{for } \alpha = 1)$$

For 100°C rise in temperature, V_{BE} decreases by $2 \times 100 = 200\,mV$ and thus I_E increases by $\frac{0.2V}{R_E}$

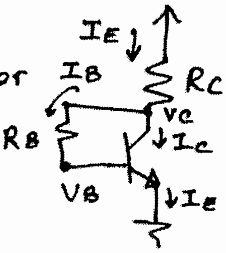
$$= \frac{0.2V}{4.3K\Omega} = 0.047\,mA$$

i.e. an increase of $\underline{\underline{4.7\%}}$

The change in β from 50 to 150 causes α to change from 0.980 to 0.993 which implies an increase in collector current of $\underline{1.3\%}$

Thus the overall increase in I_C is $\underline{\underline{6\%}}$

4.106



To allow a collector voltage swing of ±1V, we design for:

$$V_C = V_B + 1$$
$$= 0.7 + 1 = 1.7V$$

$I_E = 0.5\,mA$

$$\rightarrow R_C = \frac{5-1.7}{0.5} = \underline{\underline{6.6\,K\Omega}}$$

For $\beta = 100$:

$$I_B = \frac{I_E}{\beta+1} = \frac{0.5}{101} \simeq 5\,\mu A$$

$$I_B \cdot R_B = 1\,V$$

$$R_B = \frac{1V}{5\mu A} = \frac{1}{5}\,M\Omega = \underline{\underline{200\,K\Omega}}$$

Now, if the BJT used has $\beta = 50$, the emitter current resulting can be found from Eq (5.74)

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{\beta+1}}$$

$$= \frac{5-0.7}{6.6 + \frac{200}{51}} = \underline{\underline{0.41\,mA}}$$

and $I_B = \frac{0.41}{51} \simeq 8\,\mu A$

Thus the collector will be higher than the base by $8 \times 0.2 = 1.6V$, allowing for a $\underline{\underline{\pm 1.6V}}$ signal swing at the collector.

For $\beta = 150$:

$$I_E = \frac{5-0.7}{6.6 + \frac{200}{151}} = 0.54\,mA$$

$$I_B = \frac{0.54}{151} = 36\,\mu A$$

Thus the collector voltage will be higher than that of the base by

$$3.6 \times 0.2 = 0.72V$$

allowing for only $\underline{\underline{\pm 0.72V}}$ signal swing.

4.107

$I_B = I_C/\beta = 3\,mA/90 = 0.033\,m$

$V_C = R_B . I_B + 0.7$

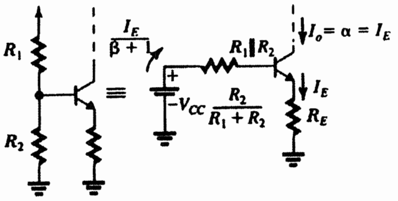
$V_C = 1.5\,V \rightarrow R_B = \underline{\underline{24.2\,K\Omega}}$

$I_E = \dfrac{I_C}{\alpha} = 3.03\,mA$

$I = I_C - I_B \equiv I_E$

$I = \underline{\underline{3.03}}\,mA$

4.108



R_1
R_2
$\equiv$
$\frac{I_E}{\beta+1}$
$R_1 \| R_2$
$I_o = \alpha = I_E$
+
$-V_{CC}\frac{R_2}{R_1+R_2}$
I_E
R_E

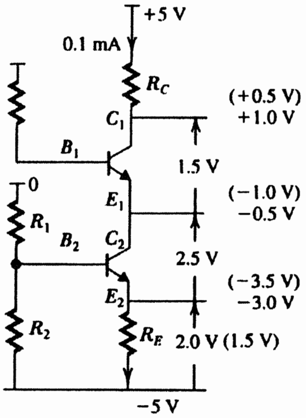
$$V_{CC}\cdot\frac{R_2}{R_1+R_2} = \frac{I_E}{\beta+1}(R_1 \| R_2) + V_{BE} + I_E R_E$$

$$\Rightarrow I_E = \frac{V_{CC}\dfrac{R_2}{R_1+R_2} - V_{BE}}{R_E + \dfrac{(R_1 \| R_2)}{\beta+1}}$$

Thus,

$$I_O = \alpha I_E = \frac{\alpha\cdot\left[\dfrac{V_{CC}R_2}{R_1+R_2} - V_{BE}\right]}{R_E + \dfrac{(R_1 \| R_2)}{\beta+1}}$$

4.109



+5 V
0.1 mA
R_C
C_1
B_1
0
E_1
R_1
B_2
C_2
E_2
R_2
R_E
(+0.5 V)
+1.0 V
1.5 V
(−1.0 V)
−0.5 V
2.5 V
(−3.5 V)
−3.0 V
2.0 V (1.5 V)
−5 V

The constraints imposed cannot be met

$V_{E1} < -0.7$ V for Q_1 active.

Change V_{RE} to 1.5 V then

$V_{E2} = -3.5$ V

$V_{C2} = V_{E2} + 2.5 = -1.0$ V

$V_{C1} = V_{C2} + 1.5 = +0.5$ V

For $\beta = \infty$

$R_{E2} = 1.5\text{ V}/0.1\text{ mA} = 15\text{ k}\Omega$

$V_{B2} = -3.5 + 0.7 = -2.8$ V

Then $\dfrac{V_{R1}}{V_{R2}} = \dfrac{2.8}{2.2} = \dfrac{R_1}{R_2}$

$V_{B1} = 0\ (I_{B1} = 0)$

$V_{E1} = -0.7$ V

$V_{C2} = V_{E1} + 1.5 = +0.8$ V

$R_{C2} = \dfrac{V_{CC}}{0.1\text{ mA}}\cdot 42\text{ k}\Omega$

For I_{E2} ($\beta = 50$) within 5% I_{E2} ($\beta = \infty$)

For $\beta = 50$

$$I_E = \frac{1.5}{R_E + (R_1 \| R_2)/51}$$

$\beta = \infty$

$$I_E = \frac{1.5}{R_E}$$

Need $\frac{R_1 \parallel R_2}{51} \leq \frac{5}{100} R_E$

$\therefore R_1 \parallel R_2 \leq 51 R_E / 2 = 38.25\ \text{k}\Omega$

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{R_1}{1 + R_1/R_2} = \frac{R_1}{1 + 28/22} < 38.25\ \text{k}\Omega$$

$\therefore R_1 < 86.9\ \text{k}\Omega$ use $82\ \text{k}\Omega$

$R_2 < 68.3\ \text{k}\Omega$ use $68\ \text{k}\Omega$

$R_1 \parallel R_2 = 37\ \text{k}\Omega < 38.25\ \text{k}\Omega$

For $\beta = \infty$ and 5% values

$$V_{B2} = \frac{-5 \times R_1}{R_1 + R_2} = -2.73\ \text{V}$$

$V_{E2} = 2.27 + 0.7 = -1.57\ \text{V}$

$I_{E2} = 1.57/15 = 0.1046\ \text{mA}$

For $\beta = 50$ V determine R_B

$$I_{E2} = \frac{2.27 - 0.7}{37/51 + 15} = 0.0998\ \text{mA}$$

$I_{C2} = 0.98 \times I_{E2} = 0.098\ \text{mA}$

$I_{C1} = 0.98 \times I_{C2} = 0.096\ \text{mA}$

$I_{B1} = I_{C2}/50$

$V_{RE} = 0.099 \times 15 = 1.47\ \text{V}$

$V_{E2} = -5 + V_{RE} = -3.53\ \text{V}$

For $V_{CE2} = 2.5\ \text{V}$

$V_{E1} = V_{C2} = V_{E2} + V_{CE2} = -1.03\ \text{V}$

$V_{B1} = V_{E1} + 0.7 = -0.33\ \text{V}$

$$R_B = V_{B1} \times \frac{\beta}{I_{C1}} = 173.7\ \text{k}\Omega \text{ use } 180\ \text{k}\Omega$$

For $\beta = 50$

$I_{C1} = 0.096\ \text{mA}$

$$V_{B1} = -\frac{0.09}{50} \times 180 = -0.35\ \text{V}$$

$V_{E1} = V_{B1} - 0.7 = -1.05\ \text{V}$

$V_{C1} = 5 - 0.096 \times 43 = 0.872\ \text{V}$

$V_{CE1} = 1.9\ \text{V}$

For $\beta = 100$

$$I_{E2} = \frac{1.57}{37/101 + 15} = 0.102\ \text{mA}$$

$I_{C1} = 0.99 \times 0.99 \times I_{E2} = 0.10\ \text{mA}$

$V_{C1} = 0.7\ \text{V}$

$$V_{B1} = -\frac{0.10}{101} \times 180 = -0.878\ \text{V}$$

$V_{E1} = V_{B1} - 0.7 = -0.878\ \text{V}$

$V_{CE1} = 0.7 + 0.878 = 1.578\ \text{V}$

For $\beta = 200$

$$I_{E2} = \frac{1.57}{37/201 - 15} = 0.103\ \text{mA}$$

$I_{C1} = 0.995 \times 0.995 \times I_{E2} = 0.102\ \text{mA}$

$V_{C1} = 0.615\ \text{V}$

$$V_{B1} = -\frac{0.102}{201} \times 180 = -0.091\ \text{V}$$

$V_{E1} = V_{B1} - 0.7 = -0.791\ \text{V}$

$V_{CE1} = 1.45\ \text{V}$

4.110

$$I_o = 2\text{mA} = \alpha \times \frac{5 - 0.7}{R} \simeq \frac{4.3}{R}$$

$$\Rightarrow R = \underline{\underline{2.15\text{K}\Omega}}$$

V_C min $= \underline{\underline{0\text{V}}}$ (In actual practice, V_C min $\simeq 0.4$V)

4.111

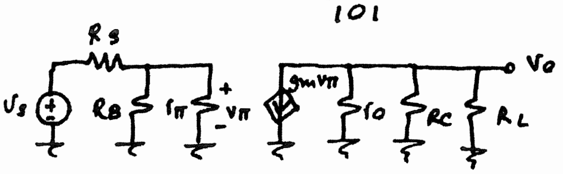
$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_B/(\beta+1)}$$

where, $V_{BB} = V_{CC} \cdot \frac{R_2}{R_1 + R_2}$

$$= 9 \cdot \frac{15}{27+15} = 3.21\text{V}$$

$R_B = R_1 // R_2 = 15//27 = 9.64\ \text{K}\Omega$

Thus, $I_E = \frac{3.21 - 0.7}{1.2 + \frac{9.64}{101}} = \underline{\underline{1.94\text{mA}}}$



$$g_m = \frac{I_C}{V_T} = \frac{0.99 \times 1.94}{0.025} = 76.8 \frac{\text{mA}}{\text{V}}$$

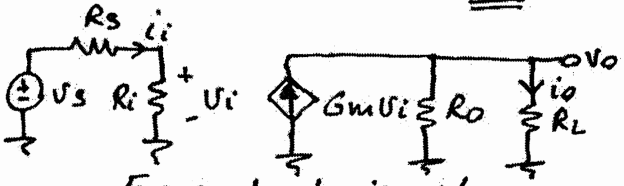
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{76.8} = 1.3\text{K}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.99 \times 1.94} = 52.1\ \text{K}\Omega$$

$R_i = R_B // r_\pi = 9.64 // 1.3 = \underline{\underline{1.15\text{K}\Omega}}$

$G_m = -g_m = -\underline{\underline{76.8}} \frac{\text{mA}}{\text{V}}$

$R_o = R_C \| r_o = 2.2 \| 52.1 = \underline{\underline{2.11}}$ kΩ



Equivalent circuit

$A_v \equiv \frac{v_o}{v_s} = \frac{v_i}{v_s}\cdot\frac{v_o}{v_i}$

$= \frac{R_i}{R_s+R_i} \cdot \frac{G_m (R_o \| R_L)\, v_i}{v_i}$

$= \frac{-1.15}{10+1.15} \times 76.8 \times (2.11 \| 2)$

$= \underline{\underline{-8.13}}$ V/V

$A_i = \frac{i_o}{i_i} = \frac{v_o \cdot R_L}{v_s/(R_s+R_i)}$

$\rightarrow A_i = \frac{v_o}{v_s} \cdot \frac{R_s+R_i}{R_L}$

$= -8.13 \times \frac{(10+1.15)}{2}$

$= -\underline{\underline{45.3}}$ A/A

4.112

$V_{CC} = 9$V $\quad V_{BB} = \frac{1}{3} V_{CC} = 3$V

Neglecting the base current,

$R_1 + R_2 = \frac{9}{0.2} = 45$ kΩ

$\frac{R_2}{R_1+R_2} = \frac{1}{3}$

$\Rightarrow R_2 = \underline{\underline{15\text{ k}\Omega}}, \quad R_1 = \underline{\underline{30\text{ k}\Omega}}$

$R_B = R_1 \| R_2 = \frac{30 \times 15}{45} = 10$ kΩ

$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta+1}}$

$2 = \frac{3-0.7}{R_E + 10/101} \Rightarrow R_E = \underline{\underline{1.05\text{ k}\Omega}}$

Use $R_E = \underline{\underline{1\text{ k}\Omega}}$

The resulting I_E will be

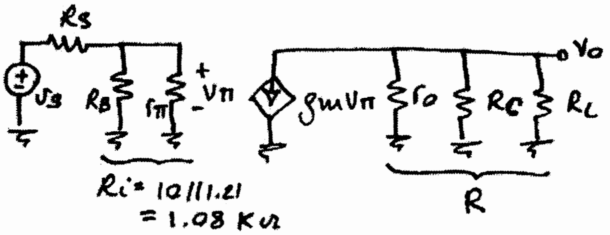
$I_E = \frac{3-0.7}{1+10/101} = 2.09$ mA

$I_C = \alpha I_E = 0.99 \times 2.09 = 2.07$ mA

$g_m = \frac{I_C}{V_T} = \frac{2.07}{0.025} = 82.9 \frac{\text{mA}}{\text{V}}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{82.9} = 1.21$ kΩ

$r_o = \frac{V_A}{I_C} = \frac{100}{2.07} = 48.3$ kΩ.



$\frac{v_o}{v_s} = \frac{v_\pi}{v_s} \cdot \frac{v_o}{v_\pi} = \frac{R_i}{R_s+R_i} \cdot \frac{-g_m v_\pi R}{v_\pi}$

$= \frac{-1.08}{10+1.08} \times 82.9 \times R$

To obtain $\frac{v_o}{v_s} = -8 \frac{V}{V}$ we use:

$R = \frac{8 \times 11.08}{1.08 \times 82.9} = 0.99$ kΩ

Now $R = r_o \| R_C \| R_L$

$0.99 = 48.3 \| R_C \| 2$

$\Rightarrow R_C = 2.04$ kΩ

use $R_C = \underline{\underline{2\text{ k}\Omega}}$

Check: $V_C = 9 - 2.07 \times 2 = 4.86$V while $V_B \simeq 3$V. Thus in active mode as assumed.

4.113

$V_{BB} = 9 \cdot \frac{47}{82+47} = 3.28\,V$

$R_B = 47 \| 82 = 29.88\,K\Omega$

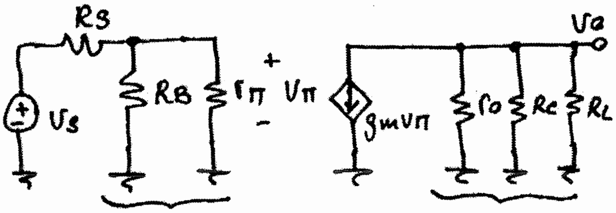
$I_E = \frac{3.28 - 0.7}{3.6 + \frac{29.88}{101}} = 0.66\,mA$

$I_C = 0.99 \times 0.66 = 0.65\,mA$

$g_m = \frac{0.65}{0.025} = 26\,\frac{mA}{V}$

$r_\pi = \frac{100}{26} = 3.85\,K\Omega$

$r_o = \frac{100}{0.66} = 151.5\,K\Omega$



$R_i = 29.88 \| 3.85 = 3.41\,K\Omega$

$151.5 \| 6.8 \| 2 = 1.53\,K\Omega$

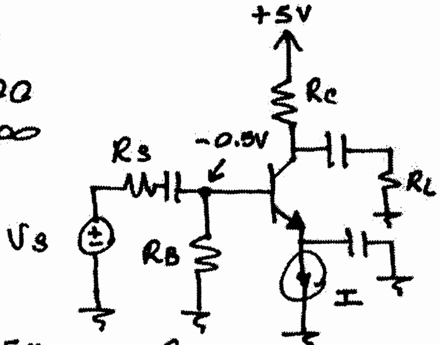
$A_v = \frac{v_o}{v_s} = \frac{3.41}{10+3.41} \times -26 \times 1.53$

$= \underline{\underline{-10.1\,V/V}}$ Which is about 25% higher than in the original design. The improvement is not as large as might have been expected because although R_i increases, g_m decreases by about the same factor. Indeed most of the improvement is due to the increase in R_C and hence in the effective load resistance.

4.114

$\beta = 100$

$r_o = \infty$



$R_{in} = 5K\Omega$, $R_{in} = R_B \| r_\pi$

$\Rightarrow 5K = \frac{R_B \cdot r_\pi}{R_B + r_\pi}$

$5K r_\pi = R_B (r_\pi - 5K)$

but: $r_\pi = \frac{V_T}{I_B}$ and $R_B \cdot I_B = 0.5$

$\rightarrow 5K \cdot \frac{V_T}{\cancel{I_B}} = \frac{0.5}{\cancel{I_B}} (r_\pi - 5K)$

thus, $r_\pi = 5250\Omega$

then $R_B = 105K$

choose $R_B = \underline{\underline{100K\Omega}}$

and $I_B = 4.76\,\mu A$

$I_E = (\beta+1) I_B = 101 \times 4.76\,\mu A$

$I_E = 0.48\,mA$

$I = I_E \rightarrow I \simeq \underline{\underline{0.5\,mA}}$

To avoid saturation:

$V_C - V_B \geq -0.5$

$V_C = 5V - R_C [I_C + g_m v_{be}]$

$I_C = I \cdot \alpha = 0.5m \times 100/101$

$= 0.49\,mA$

$g_m = \frac{V_T}{I_C} = \frac{25m}{0.49m} \simeq 50\,\frac{mA}{V}$

$v_{be} = 0.005V$

$\rightarrow V_C = 5 - R_C [0.49m + 50m \times 5m]$

$= 5 - 0.74 \times 10^{-3} \times R_C$

Then:

$V_C - V_B = (5 - 0.74m\,R_C) - (-0.5 + V_{be})$

$= 5.495 - 0.74m\,R_C \geq -0.5$

$R_C \leq \underline{\underline{8.1\,K\Omega}}.$

Base-to-Collector open circuit gain:

$\frac{V_c}{V_b} = -g_m R_C = -50m \times 8.1K$

$= -\underline{\underline{405}}\,V/V$

For $R_S = 10K$, $R_L = 10K$

$\frac{V_o}{V_b} = -g_m (R_C // R_L)$

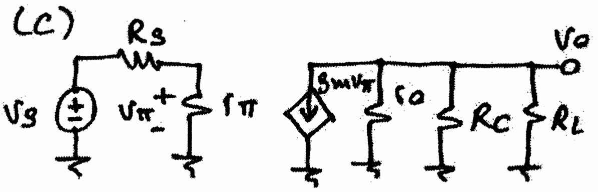
$= -50m \times 4.47K$

$= -223\,V/V$

$\frac{V_c}{V_s} = \frac{V_b}{V_s} \cdot \frac{V_o}{V_b} = \frac{5}{5+10} \times -223$

$= \underline{\underline{-74.3}}\,V/V$

(c)



$R_L = 10K\Omega$, $R_S = 2.5K$

$r_o = 200K\Omega$

$g_m = \frac{I_C}{V_T} \simeq \frac{0.5m}{25m} = 20\frac{mA}{V}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5K\Omega$

$A_v = \frac{V_o}{V_s} = \frac{V_\pi}{V_s} \times \frac{V_o}{V_\pi}$

$= \frac{r_\pi}{r_\pi + R_S} \times -g_m (r_o // R_C // R_L)$

$= -\frac{5}{5+2.5} \times 20 (200 // 20 // 10)$

$= -\underline{\underline{86}}\,V/V$

4.115

$I_E = 0.5mA$

(a) $I_E = \frac{15 - 0.7}{R_E + \frac{R_S}{\beta + 1}}$

$0.5 = \frac{14.3}{R_E + \frac{2.5}{100}}$

$\Rightarrow R_E = \underline{\underline{28.57}}\,K\Omega.$

(b) $V_C = 15 - R_C \cdot I_C$

$5 = 15 - R_C \times 0.99 \times 0.5m$

$\Rightarrow R_C = 20.2K\Omega$

$\simeq \underline{\underline{20K\Omega}}$

4.116

(a) For each transistor

$V_{BB} = 15 \times \frac{47}{100+47} = 4.8V$

$R_B = R_1 // R_2 = 100 // 47 = 32K\Omega$

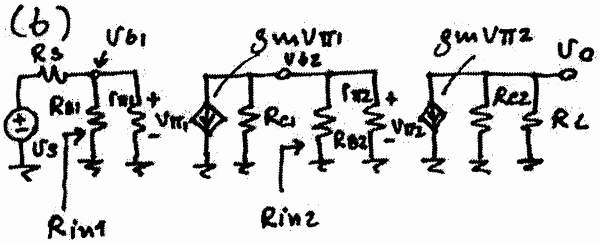
$I_E = \frac{4.8 - 0.7}{3.9 + \frac{32}{101}} = 0.97\,mA$

$I_C = 0.99 \times 0.97 = \underline{\underline{0.96}}\,mA$

$V_C = V_{CC} - I_C \times R_C$

$= 15 - 0.96 \times 6.8 = \underline{\underline{8.5}}\,V$

(b)



$R_{B1} = R_{B2} = R_B = 32\,K\Omega$

$g_{m1} = g_{m2} = \dfrac{0.96}{0.025} = 38.4\,\dfrac{mA}{V}$

$r_{\pi1} = r_{\pi2} = \dfrac{100}{38.4} = 2.6\,K\Omega$

$R_{C1} = R_{C2} = 6.8\,K\Omega$

$r_{o1} = r_{o2} = \infty$

(c) $R_{in1} = R_{B1} \,//\, r_{\pi1}$

$= 32 \,//\, 2.6 = \underline{\underline{2.4\,K\Omega}}$

$\dfrac{v_{b1}}{v_s} = \dfrac{R_{in1}}{R_s + R_{in1}}$

$= \dfrac{2.4}{5 + 2.4} = \underline{\underline{0.32}}\ V/V$

(d) $R_{in2} = R_{B2} \,//\, r_{\pi2}$

$= 32 \,//\, 2.6 = \underline{\underline{2.4}}\,K\Omega$

$v_{b2} = -g_{m1} v_{\pi1}\,(R_{C1} \,//\, R_{in2})$

$= -38.4\, v_{b1}\,(6.8 \,//\, 2.4)$

$\dfrac{v_{b2}}{v_{b1}} = -\underline{\underline{68.1}}\ V/V$

(e) $v_o = -g_{m2} v_{\pi2}\,(R_{C2} \,//\, R_L)$

$= -38.4\, v_{b2}\,(6.8 \,//\, 2)$

$\dfrac{v_o}{v_{b2}} = -\underline{\underline{59.3}}\ V/V$

(f) $\dfrac{v_o}{v_s} = \dfrac{v_{b1}}{v_s} \times \dfrac{v_{b2}}{v_{b1}} \times \dfrac{v_o}{v_{b2}}$

$= 0.32 \times -68.1 \times -59.3$

$= \underline{\underline{1292}}\ V/V$

4.119

$R_{in} = (\beta+1)(r_e + 250)$

$\beta = 100 \quad r_e = \dfrac{V_T}{I_e} = \dfrac{0.025}{0.1} = 250\Omega$

$R_{in} = 101 \times (250 + 250)$

$= \underline{\underline{50.5\,K\Omega}}$

$\dfrac{v_b}{v_s} = \dfrac{R_{in}}{R_s + R_{in}} = \dfrac{50.5}{20 + 50.5}$

$= 0.72\ V/V$

$\dfrac{v_o}{v_b} = \dfrac{-\alpha\,(20 \,//\, 20)}{(r_e + R_E)}$

$= -\dfrac{0.99 \times 10}{0.250 + 0.250} = -\underline{\underline{19.8}}\ V/V$

Thus, $\dfrac{v_o}{v_s} = 0.72 \times -19.8 = -\underline{\underline{14.2}}\ V/V$

For $v_{be} = 5mV$, $v_e = 5mV$ also (since $R_e = r_e = 250\Omega$)

Thus,

$v_b = 5 + 5 = 10mV$

$v_s = \dfrac{10m}{0.72} = \underline{\underline{13.88}}\,mV$

$v_o = 13.88 \times 14.2 = \underline{\underline{197.2}}\,mV$

4.120

(a) $I_C = 0.99 \times 0.5$ mA

$= 0.495$ mA

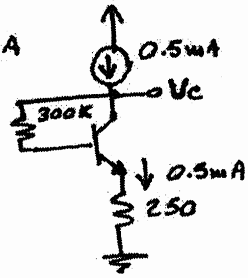
$V_C = I_E R_E + V_{BE} \ldots$

$+ I_B R_B$

$= 0.5 \times 0.175 + 0.7$

$+ 0.005 \times 300$

$= \underline{\underline{2.28\text{V}}}$

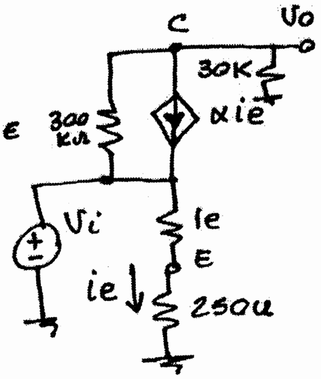


(b) $i_e = \frac{v_i}{r_e + R_E}$

$r_e = \frac{V_T}{I_E} = 50\Omega$

$\rightarrow i_e = \frac{v_i}{50 + 250}$

$i_e = \frac{v_i}{300}$



Node equation at c:

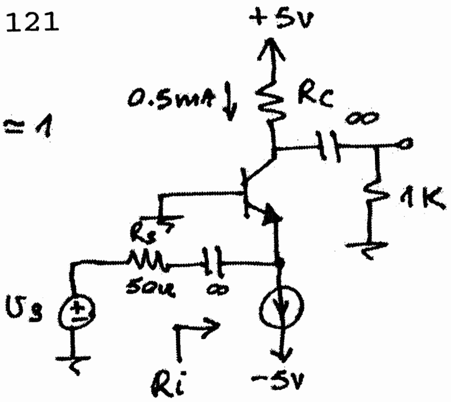
$$\frac{v_o - v_i}{300K} + \alpha i_e + \frac{v_o}{30K} = 0$$

$$\frac{v_o - v_i}{300K} + \frac{\alpha v_i}{(250+50)} + \frac{v_o}{30K} = 0$$

$$\Longrightarrow \frac{v_o}{v_i} = \underline{\underline{-90}}\ \text{V/V}$$

4.121

$\alpha \simeq 1$



$R_i = \frac{V_T}{I} = 50\Omega \Rightarrow I = \underline{\underline{0.5\text{mA}}}$

$V_C = 5 - 0.5 \cdot R_C$

$V_{Cmin} = V_C - 0.01\, g_m\, (R_C \| 1K)$

To prevent saturation V_{Cmin} $= 0$

$\rightarrow 0 = V_C - 0.01 \times 20\ (R_C \| 1)$

$= 5 - 0.5\ \frac{R_C}{R_C + 1}$

$5R_C + 5 - 0.5R_C^2 - 0.5R_C - 0.2R_C = 0$

$0.5R_C^2 - 4.3R_C + 5 = 0$

$$R_C = \frac{4.3 + \sqrt{4.3^2 + 10}}{1}$$

$= 9.64\text{K}\Omega$

Select $R_C = \underline{\underline{9.1\text{K}\Omega}}$

$V_C = 0.45$V

$$\frac{v_o}{v_s} = \frac{R_i}{R_s + R_i}\, g_m\, (R_C \| 1)$$

$$= \frac{50}{50+50} \times 20 \times (9.1 \| 1)$$

$$= \underline{\underline{9}}\ \text{V/V}$$

For $v_{be\,max} = 10\text{ mV}$

$v_{s\,max} = 20\text{ mV}$

$V_{c\,max} = 180\text{ mV}$

Thus the collector voltage swings from

$(0.45 - 0.18)\text{V}$ to $(0.45 + 0.18)\text{V}$

i.e from 0.27V to 0.63V

4.122

$$R_i = r_e = \frac{V_T}{I_E} = \frac{V_T}{0.5} = 50\Omega$$

To find the voltage gain v_o/v_s first note that

$$\frac{v_e}{v_s} = \frac{R_i}{R_s + R_i} = \frac{50}{50+50} = 0.5$$

Then,

$$\frac{v_c}{v_e} = \frac{\alpha \times (\text{Total resistance at c})}{r_e}$$

$$\simeq \frac{1 \times (100\text{k}\Omega // 1\text{k}\Omega)}{50\Omega}$$

$$= 19.8\text{ V/V}$$

Thus, $\frac{v_o}{v_s} = 19.8 \times 0.5 = 9.9\text{ V/V}$

4.123

(a) $I_E = \frac{9 - 0.7}{1 + 100/(\beta+1)}$

for $\beta = 40$, $I_E = \frac{8.3}{1 + \frac{100}{41}} = 2.41\text{ mA}$

$V_E = 1 \times 2.41 = 2.41\text{ V}$

$V_B = 2.41 + 0.7 = 3.11\text{ V}$

For $\beta = 200$, $I_E = \frac{8.3}{1 + \frac{100}{201}} = 5.54\text{ mA}$

$V_E = +5.54\text{ V}$

$V_B = +6.24\text{ V}$

(b) $R_i = 100\text{k}\Omega // (\beta+1)[r_e + (1//1)]$

$= 100 // (\beta+1)[r_e + 0.5]$

For $\beta = 40$, $I_E = 2.41\text{ mA}$

$\rightarrow r_e = 10.37\ \Omega$

thus $R_i = 100 // 41 \times (0.01037 + 0.5)$

$= 100 // 21$

$= 17.30\ \Omega$

For $\beta = 200$, $I_E = 5.54\text{ mA}$

$\rightarrow r_e = 4.51\ \Omega$

thus $R_i = 100 // 201(0.0045 + 0.5)$

$= 100 // 101.4$

$= 50.3\text{ k}\Omega$

(c) $\frac{v_o}{v_s} = \frac{v_b}{v_s} \cdot \frac{v_o}{v_b}$

$= \frac{R_i}{R_s + R_i} \cdot \frac{(1//1)}{(1//1) + r_e}$

For $\beta = 40$,

$$\frac{v_o}{v_s} = \frac{17.3}{10 + 17.3} \times \frac{0.5}{0.5 + 0.01037}$$

$= 0.621\text{ V/V}$

For $\beta = 200$,

$$\frac{v_o}{v_s} = \frac{50.3}{10 + 50.3} \cdot \frac{0.5}{0.5 + 0.0045}$$

$= 0.827\text{ V/V}$

4.124

$$I_E = \frac{5-0.7}{3.3 + \frac{100}{101}} = \underline{\underline{1.00}}\text{ mA}$$

$$r_e = \frac{25}{1.00} = 25\,\Omega$$

$$R_i = (\beta+1)[r_e + (3.3 \| 1)]$$

$$= \underline{\underline{80.0}}\text{ K}\Omega$$

$$\frac{v_o}{v_s} = \frac{v_b}{v_s}\cdot\frac{v_o}{v_b} = \frac{R_i}{R_s+R_i}\,\frac{(3.3 \| 1)}{r_e + (3.3 \| 1)}$$

Thus,

$$\frac{v_o}{v_s} = \frac{80}{100+80} \times \frac{(3.3 \| 1)}{0.025 + (3.3 \| 1)}$$

$$= \underline{\underline{0.430}}\text{ V/V}$$

$$\frac{i_o}{i_i} = \frac{v_o/R_L}{v_s/(R_s+R_i)}$$

$$= \frac{v_o}{v_s} \cdot \frac{(R_s+R_i)}{R_L}$$

$$= 0.43 \times \frac{(100+80)}{1}$$

$$= \underline{\underline{77.4}}\text{ A/A}$$

$$R_{out} = 3.3 \,\|\, \left[r_e + \frac{100}{\beta+1}\right]$$

$$= 3.3 \,\|\, \left[0.025 + \frac{100}{101}\right]$$

$$= \underline{\underline{0.776}}\text{ K}\Omega$$

4.125

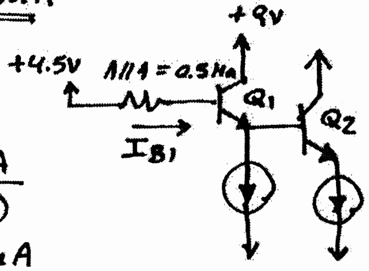
(a) $I_{E2} = \underline{\underline{5\text{ mA}}}$

$\beta_1 = 50, \quad \beta_2 = 100$

$$I_{E1} = 50\mu + I_{B2}$$

$$= 50 + \frac{I_{E2}}{\beta_2+1} = 50 + \frac{5000}{101}$$

$$\simeq \underline{\underline{0.1\text{ mA}}}$$



$$I_{B1} = \frac{0.1\text{ mA}}{(50+1)}$$

$$= 1.96\,\mu\text{A}$$

$$V_{B1} = 4.5 - 0.5 \times 1.96 = \underline{\underline{3.52}}\text{ V}$$

$$V_{B2} = 3.52 - 0.7 = \underline{\underline{2.82}}\text{ V}$$

(b) Refer to Fig. P.5.148

$$\frac{v_o}{v_{b2}} = \frac{R_L}{R_L + r_{e2}}$$

$R_L = 1\text{ K}\Omega \quad r_{e2} = \frac{25}{5} = 5\,\Omega$

$$\frac{v_o}{v_{b2}} = \frac{1}{1 + 0.005} = \underline{\underline{0.995}}\text{ V/V}$$

$$R_{ib2} = (\beta_2+1)(r_{e2} + R_L)$$

$$= (101) \times (1.005)$$

$$= \underline{\underline{101.5}}\text{ K}\Omega$$

(c) $\dfrac{v_{e1}}{v_{b1}} = \dfrac{R_{ib2}}{R_{ib2} + r_{e1}}$

$$r_{e1} = \frac{V_T}{100\,\mu\text{A}} = 250\,\Omega$$

$$\rightarrow \frac{v_{e1}}{v_{b1}} = \frac{101.5}{101.5 + 0.25} = \underline{\underline{0.997}}\,\frac{\text{V}}{\text{V}}$$

$$R_i = 1\text{ M}\Omega \,\|\, 1\text{ M}\Omega \,\|\, (\beta_1+1)(r_{e1} + R_{ib2})$$

$$= 1 \,\|\, 1 \,\|\, 51 \times (0.25 + 101.5)\text{ K}\Omega$$

$$= 1 \| 1 \| 5.2\text{ M}\Omega$$

$$= 0.499\text{ M}\Omega = \underline{\underline{499}}\text{ K}\Omega$$

(d) $\dfrac{v_{b1}}{v_s} = \dfrac{R_i}{R_s + R_i} = \dfrac{499}{100 + 499} = 0.833$ V/V

(e) $\frac{v_o}{v_s} = \frac{v_{b1}}{v_s} \cdot \frac{v_{e1}}{v_{b1}} \cdot \frac{v_o}{v_{e1}}$

$= 0.833 \times 0.997 \times 0.995$

$= \underline{\underline{0.826}}$ V/V

5.1

The capacitance per unit area is : $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$

$\varepsilon_{ox} = 3.45 \times 10^{-11}\ F/m$

$t_{ox} = 5nm \Rightarrow C_{ox} = \frac{3.45 \times 10^{-11}}{5 \times 10^{-9}} = 6.9\ fF/\mu m^2$

$t_{ox} = 40nm \Rightarrow C_{ox} = 0.86\ fF/\mu m^2$

For 1PF capacitance, we require an area A:

$A = \frac{10^{-12}}{6.9 \times 10^{-15}} = 145\ \mu m^2$ for $t_{ox} = 5nm$

$A = \frac{10^{-12}}{0.86 \times 10^{-15}} = 1163\ \mu m^2$ for $t_{ox} = 20nm$

For a square plate capacitor of 10PF:

$A = 10 \times 145 = 1450\ \mu m^2$ or $38 \times 38\ \mu m^2$ square for $t_{ox} = 5nm$

$A = 10 \times 1163 = 11630\ \mu m^2$ or $108 \times 108\ \mu m^2$ square for $t_{ox} = 20nm$

5.2

With V_{DS} small, compared to V_{ov},

$$r_{DS} = \frac{1}{(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{OV})}$$

(a) V_{ov} is doubled → r_{DS} is halved. factor = 0.5

(b) W is doubled → r_{DS} is halved. factor = 0.5

(c) W and L are doubled → r_{DS} is unchanged. factor = 1.0

(d) If oxide thickness t_{ox} is halved, and

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}},$$

C_{ox} is doubled. If W and L are also halved, r_{DS} is halved, factor = 0.5

5.3

The transistor size will be minimized if W/L is minimized, since W/L appears in the equations that must be satisifyed, we can minimize (W/L). Clearly we want to minimize L by using the smallest feture size.

$L = 0.18\ \mu m$

$$r_{DS} = \frac{1}{k_n'(W/L)(v_{GS} - V_t)}$$

$$r_{DS} = \frac{1}{k_n'(W/L)v_{OV}}$$

Two conditions need to met for v_{OV} and r_{DS}

Condition 1:

$$r_{DS,1} = \frac{1}{400 \times 10^{-6}(W/L)v_{OV,1}}$$

$$= 200 \Rightarrow (W/L)v_{OV,1} = 12.5$$

Condition 2:

$$r_{DS,2} = \frac{1}{400 \times 10^{-6}(W/L)v_{OV,2}}$$

$$= 1000 \Rightarrow (W/L)v_{OV,2} = 2.5$$

If condition 1 is met, condition 2 will be met since the over-voltage can always be reduced to satisfy this requirement. For condition 1, we want to decrease W/L as much as possible (so long as it is greater than or equal to 1), while still meeting all of the other constraints.

This requires our using the largest possible $v_{GS,1}$ voltage. $v_{GS,1} = 1.8$ Volts, so $v_{OV,1} = 1.4$ Volts that

$$W/L = \frac{12.5}{v_{OV,1}} = \frac{12.5}{1.4} \cong 8.93$$

Condition 2 now can be used to find $v_{GS,2}$

$$v_{OV,2} = \frac{12.5}{W/L} = \frac{2.5}{12.5/1.4} = 0.28$$

$$\Rightarrow v_{GS,2} = 0.68\text{Volts} \Rightarrow 0.68 \le v_{GS} \le 1.8$$

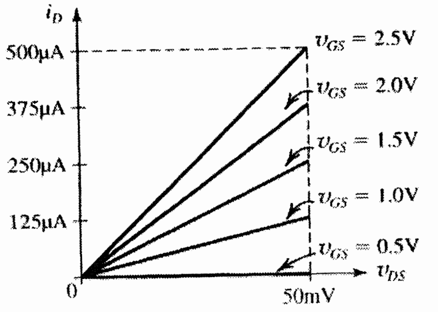
5.4

$k_n = 5\ \text{mA}/\text{V}^2 \quad V_t = 0.5\ \text{V}$

Small v_{DS}

$$i_D = k_n\ (v_{GS} - V_t)v_{DS} = k_n\ v_{ov}\ v_{DS}$$

$$g_{DS} = \frac{1}{r_{DS}} = k_n v_{OV}$$



(V)	(V)	(mS)	(Ω)
V_{GS}	V_{OV}	g_{DS}	r_{DS}
0.5	0	0	∞
1.0	0.5	2.5	400
1.5	1.0	5.0	200
2.0	1.5	7.5	133
2.5	2.0	10	100

5.5

$V_{DS\,sat} = V_{ov}$

$V_{ov} = V_{GS} - V_t = 2.5 - 1 = 1.5\text{ V}$

$\Rightarrow V_{DS\,sat} = 1.5\text{ V}$

In saturation:

$$i_D = \frac{1}{2}K'_n\left(\frac{W}{L}\right)V_{OV}^2 = \frac{1}{2}K_nV_{OV}^2$$

$$i_D = \frac{1}{2}\times\frac{1\text{mA}}{V^2}\times(1.5\text{ V})^2$$

$$i_D = (1.125\text{ mA})$$

5.6

a) $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-11}}{15\times10^{-9}} = 2.3\text{ fF}/\mu m^2$

$K'_n = \mu_n C_{ox} = 550\times10^{-4}\times2.3\times10^{-3} = 126.5\ \mu A/V^2$

b) $i_D = \frac{1}{2}K'_n\frac{W}{L}(V_{GS}-V_t)^2 \Rightarrow 100 = \frac{1}{2}\times126.5\times\frac{16}{0.8}(V_{GS}-0.7)^2$

$V_{GS} - 0.7 = 0.28 \Rightarrow V_{OV} = 0.28V$

$V_{GS} = 0.98V$

$V_{DS\,min} = V_{GS} - V_t = 0.28V$

c) for small v_{DS}: (triode region) $i_D \simeq k'_n\frac{W}{L}V_{OV}\cdot V_{DS}$

$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{K'_n\frac{W}{L}V_{OV}} = \frac{1}{126.5\times10^{-6}\times\frac{16}{0.8}V_{OV}} = 1000$

$\Rightarrow V_{OV} = 0.4V$

$V_{GS} = V_{OV} + V_t = 0.4 + 0.7 = 1.1V$

5.7

p–Channel

$V_{tp} = -0.7\text{ V}.$

(a) $|v_{OV}| = 0.5\text{ V}.$

$v_{GS} = -1.2\text{ V}. = v_G$

(b) for $v_{GD} = V_{tp}$, $v_{DS} = v_{GS} - v_{DS}$

$= (-1.2) - (-0.5) = -0.7\text{ V}.$

$v_{DS} = v_D \le -0.7\text{ V}.$

(c) $i_D = 1$ mA in saturation mode

$$\therefore k_p = \frac{2i_D}{(v_{GS} - v_{tp})^2} = 8\text{ mA/V}^2$$

For $v_D = -10$ mV, ohmic mode

$$i_D = k_p\left(v_{GS} - V_{tp} - \frac{1}{2}v_{DS}\right)(v_{DS})$$

$= 39.6\ \mu\text{A}$

For $v_D = -2$ V, sat mode, $i_D = 1$ mA

5.8

$i_D = \frac{1}{2}k'_n\frac{W}{L}(V_{GS} - V_t)^2 \quad k'_n = \mu_n C_{ox}$

for equal drain currents :

$$\mu_n C_{ox}\frac{W_n}{L} = \mu_p C_{ox}\frac{W_p}{L} = \frac{W_p}{W_n} = \frac{\mu_n}{\mu_p}$$

$$= \frac{1}{0.4} = 2.5$$

5.9

For small $V_{DS} = i_D \simeq k'_n\frac{W}{L_1}(V_{GS} - V_t)\ V_{DS}$

$$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{k'_n\frac{W}{L}(V_{GS} - V_t)}$$

$$= \frac{1}{50\times10^{-6}\times20\times(5 - 0.8)}$$

$r_{DS} = 238\ \Omega \quad V_{DS} = r_{DS}\times i_D = 238\text{ mV}$

for the same performance of a p-channel device :

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5 \Rightarrow \frac{W_p}{L} = \frac{W_n}{L}\times2.5 =$$

$$20\times2.5 \Rightarrow \frac{W_p}{L} = 50$$

5.10

$k_n' = \mu_n C_{ox} = \mu_n \frac{\epsilon_{ox}}{t_{ox}} = 650 \times 10^{-4} \times \frac{3.45 \times 10^{-11}}{20 \times 10^{-9}} = 112.1\,\mu A/V^2$

a) triode region: $v_{DS} < V_{GS} - V_t$

$i_D = k_n' \frac{W}{L}\left[(v_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$

$i_D = 112.1 \times 10^{-6} \times 10 \times \left[(5 - 0.8) \times 1 - \frac{1}{2} \times 1^2\right] = 4.15\,mA$

b) edge of saturation region: $V_{DS} = V_{GS} - V_t$

$i_D = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2 = \frac{1}{2} \times 112.1 \times 10^{-6} \times 10 \times (1.2)^2 = 0.8\,mA$

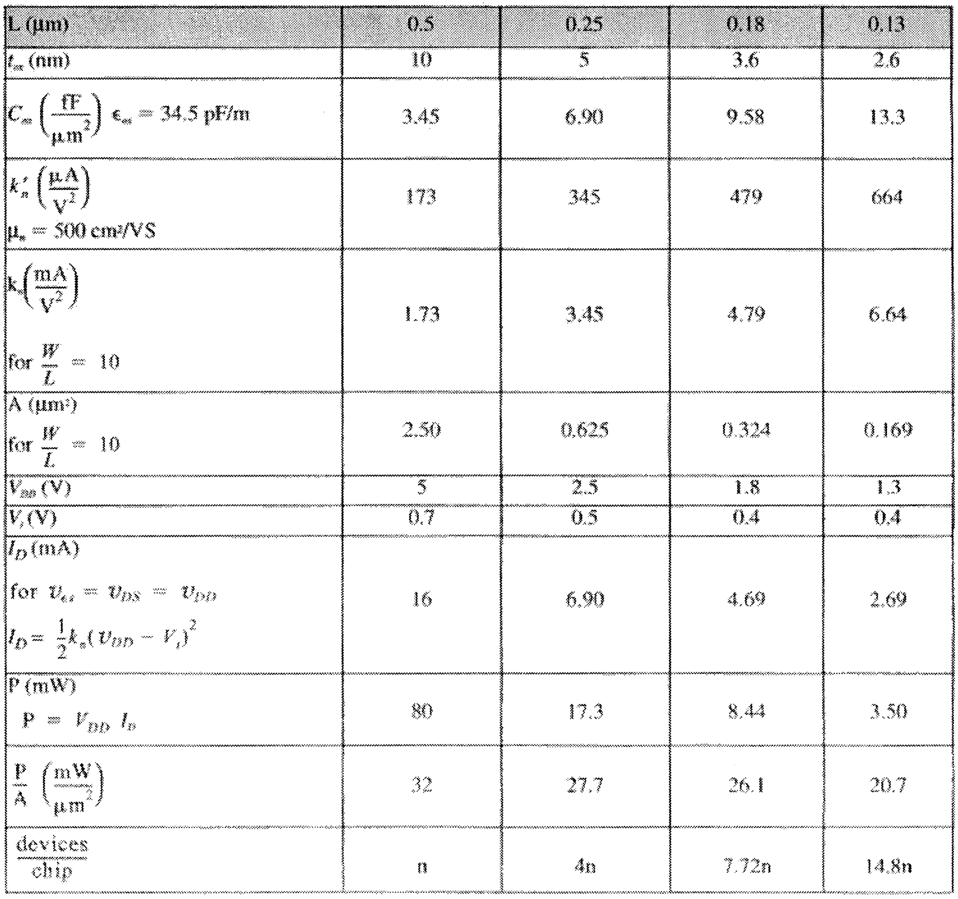
c) triode region: $v_{DS} < V_{GS} - V_t$

$i_D = 112.1 \times 10^{-6} \times 10\left[(5 - 0.8) \times 0.2 - \frac{1}{2} \times 0.2^2\right] = 0.92\,mA$

d) Saturation region: $v_{DS} > V_{GS} - V_t$

$i_D = \frac{1}{2} \times 112.1 \times 10^{-6} \times 10 \times (5 - 0.8)^2 = 9.9\,mA$

5.11



L (μm)	0.5	0.25	0.18	0.13
t_{ox} (nm)	10	5	3.6	2.6
$C_{ox}\left(\frac{fF}{\mu m^2}\right)$ $\epsilon_{ox} = 34.5$ pF/m	3.45	6.90	9.58	13.3
$k_n'\left(\frac{\mu A}{V^2}\right)$ $\mu_n = 500$ cm²/VS	173	345	479	664
$k_n\left(\frac{mA}{V^2}\right)$ for $\frac{W}{L} = 10$	1.73	3.45	4.79	6.64
A (μm²) for $\frac{W}{L} = 10$	2.50	0.625	0.324	0.169
V_{DD} (V)	5	2.5	1.8	1.3
V_t (V)	0.7	0.5	0.4	0.4
I_D (mA) for $v_{GS} = v_{DS} = v_{DD}$ $I_D = \frac{1}{2}k_n(v_{DD} - V_t)^2$	16	6.90	4.69	2.69
P (mW) $P = V_{DD} I_D$	80	17.3	8.44	3.50
$\frac{P}{A}\left(\frac{mW}{\mu m^2}\right)$	32	27.7	26.1	20.7
$\frac{\text{devices}}{\text{chip}}$	n	4n	7.72n	14.8n

$$i_D = 191.7 \times 10^{-6} \times 10\left[(5 - 0.7) \times 0.2 - \frac{1}{2}(0.2)^2\right]$$

$$= 1.61 \text{ mA}$$

(d) saturation region: $V_{DS} > V_{GS} - V_t$

$$i_D = \frac{1}{2} \times 191.7 \times 10^{-6} \times 10 \times (5 - 0.7)^2$$

$$= 17.7 \text{ mA}$$

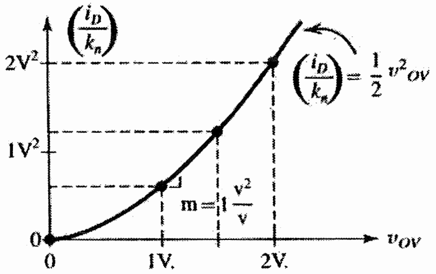
5.12

Sat mode, $\lambda = 0$

$$\left(\frac{i_D}{k_n}\right) = \frac{1}{2}v_{OV}^2$$

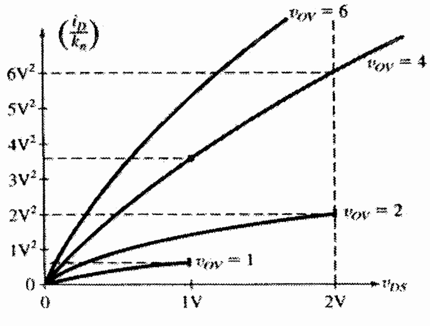
Slope at $v_{OV} = 1$ V.

$$m = 1\frac{\text{V}^2}{\text{V}}$$



Ohmic mode, $\lambda = 0$

$$\left(\frac{i_D}{k_n}\right) = v_{OV}v_{DS} - \frac{1}{2}v_{DS}^2$$



$$\left.\frac{\partial \frac{i_D}{k_n}}{\partial v_{DS}}\right|_{v_{DS}=0} = v_{OV}$$

For pmos, change

$v_{DS} \to v_{SD}$

$v_{OV} \to v_{SG} - |V_{tp}|$

5.13

$$i_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS} - V_t)^2 \Rightarrow 0.2 \times 10^{-3} = \frac{1}{2} \times 0.1 \times 10^{-3}(V_{GS} - 1)^2$$

$V_{GS} - 1 = 2 \Rightarrow V_{GS} = 3\text{V}$

$V_{DSmin} = V_{GS} - V_t = 3 - 1 = 2\text{V}$

For $i_D = 0.8\text{mA}$: $0.8 = \frac{1}{2} \times 0.1(V_{GS} - 1)^2$

$V_{GS} - 1 = 4 \Rightarrow V_{GS} = 5\text{V}$

$V_{DSmin} = V_{GS} - V_t = 5 - 1 = 4\text{V}$

5.14

$V_{GS} = V_{DS}$ indicates operation in saturation mode: $i_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS} - V_t)^2$

$$\left.\begin{aligned} 4 &= \frac{1}{2}k_n' \frac{W}{L}(5 - V_t)^2 \\ 1 &= \frac{1}{2}k_n' \frac{W}{L}(3 - V_t)^2 \end{aligned}\right\} \Rightarrow 4 = \frac{(5 - V_t)^2}{(3 - V_t)^2} \Rightarrow$$

$(5 - V_t) = 2(3 - V_t) \Rightarrow V_t = 1\text{V}$, $k_n' \frac{W}{L} = 0.5 \text{ mA/V}^2$

5.15

$$i_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS} - V_t)^2 \Rightarrow 0.8 = \frac{1}{2} \times 50 \times 10^{-3} \frac{W}{L}(5 - 1)^2$$

$\frac{W}{L} = 2 \Rightarrow W = 2 \times 2 = 4\,\mu\text{m}$

5.16

For the channel to remain continous:

$V_{DS} \leq V_{GS} - V_t \Rightarrow V_{DSmax} = 1.5 - 0.8 = 0.7\text{V}$

5.17

$$r_{DS} = \left[k_n' \frac{W}{L} V_{OV}\right]^{-1}$$

$$= \frac{1}{50 \times \frac{100}{5}(V_{GS} - 1)} \text{M}\Omega$$

$$r_{DS} = \frac{1}{V_{GS} - 1} \text{k}\Omega$$

$V_{GS} = 1.1V \Rightarrow r_{DS} = 10 \text{ k}\Omega$

$V_{GS} = 11V \Rightarrow r_{DS} = 100\ \Omega$

$\Rightarrow 100\Omega \leq r_{DS} \leq 10 \text{ k}\Omega$

a) $r_{DS} \propto \frac{1}{W}$ so if W is halved, r_{DS} is doubled:

$200\ \Omega \leq r_{DS} \leq 20\ \text{k}\Omega$

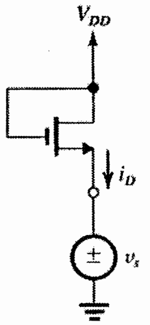
b) $r_{DS} \propto L$ so if L is halved, r_{DS} is also halved:

$50\ \Omega \leq r_{DS} \leq 5\ \text{k}\Omega$

c) $r_{DS} \propto \frac{L}{W}$ so if both W and L are halved, $\frac{W}{L}$ stays unchanged and so does r_{DS}.

$100\ \Omega \leq r_{DS} \leq 10\ \text{k}\Omega$

5.18



$v_{GD} = 0 \Rightarrow$ saturation

$$i_D = \frac{1}{2}k_n(v_{GS} - V_t)^2$$

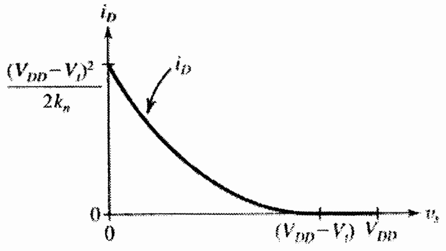
$$v_{GS} = V_{DD} - v_S$$

$$\therefore\ i_D = \frac{1}{2}k_n\ [(V_{DD} - V_t) - v_S]^2$$

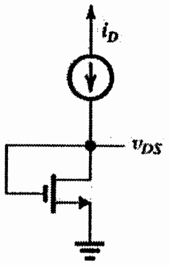
$$i_D = \frac{1}{2}k_n[(V_{DD} - V_t)^2 - 2(V_{DD} - V_t)v_S + v_S^2]$$

$$0 \leq v_S \leq (V_{DD} - V_t)$$

$$i_D = 0,\ v_S \geq (V_{DD} - V_t)$$

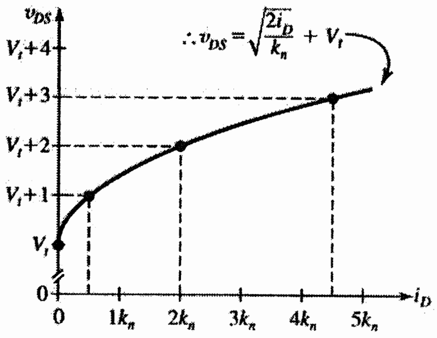


5.19



$$v_{DS} = v_{GS}$$

$$i_D = \frac{1}{2}k_n(v_{DS} - V_t)^2$$



5.20

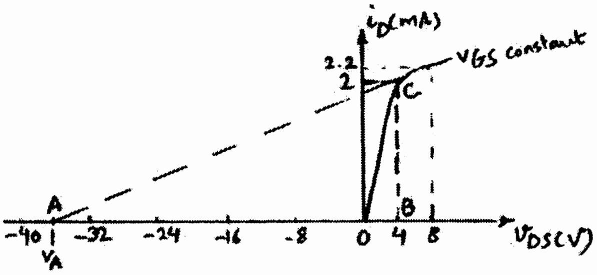
$V_{DS} = V_D - V_S \qquad V_{GS} = V_G - V_S$

$V_{OV} = V_{GS} - V_t = V_{GS} - 1.0$ According to Table 5.1, three regions are possible.

Case	V_S	V_G	V_D	V_{GS}	V_{OV}	V_{DS}	Region of Operation
a	+1.0	+1.0	+2.0	0	−1.0	+1.0	cut-off
b	+1.0	+2.5	+2.0	+1.5	+0.5	+1.0	sat.
c	+1.0	+2.5	+1.5	+1.5	+0.5	+0.5	sat.
d	+1.0	+1.5	0	+0.5	−0.5	−1.0	sat.
e	0	+2.5	1.0	+2.5	+1.5	+1.0	triode.
f	+1.0	+1.0	+1.0	0	−1.0	0	cut-off.
g	−1.0	0	0	+1.0	0	+1.0	sat.
h	−1.5	0	0	+1.5	+0.5	+1.5	sat.
i	−1.0	0	+1.0	+1.0	0	+2.0	sat.
j	+0.5	+2.0	+0.5	+1.5	+0.5	0	triode.

* with V_{DS} negative, drain and source are reversed to show the device is in the saturation region.

5.21



$$r_o = \left.\frac{\Delta v_{DS}}{\Delta i_D}\right|_{v_{GS}\,\text{Const}} = \frac{8-4}{2.2-2} = 20\,\text{k}\Omega$$

To calculate V_A, consider the ABC triangle:

$V_A + 4 = 2\,\text{mA} \times r_o = 2 \times 20 = 40\text{V} \Rightarrow V_A = 36\text{V}$

$\lambda = \dfrac{1}{V_A} = 0.028\ \text{V}^{-1}$

5.22

$\lambda = 0.02\ v^{-1} \Rightarrow v_A = 50\text{V}$ for

$L = 1\ \mu\text{m}$

$V_A = v'_A L \Rightarrow v'_A = 50\text{V}$

for $L = 3\ \mu\text{m}$: $V_A = 50 \times 3 = 150\text{V}$

$$r_o = \frac{V_A}{I_D} = \frac{150}{0.08} = 1875\ \text{k}\Omega$$

$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} \Rightarrow \Delta i_D$$

$$= \frac{\Delta v_{DS}}{r_o} = \frac{5-1}{1875} = 2.13\ \mu\text{A}$$

for V_{DS} raised from 1 V to 5 V, i_D increases from 80 μA to 82.13 μA.

$\dfrac{\Delta i_D}{i_D} = 2.7\,\%$ change in i_D.

In order to reduce $\dfrac{\Delta i_D}{i_D}$ by a factor of 2, Δi_D has to be halved, or equivalently r_o has to be doubled. In order to double r_o, V_A has to be doubled and this can be done by doubling the length. L $= 2 \times 3 = 6$ μm

5.23

original

$$r_O = \left[\lambda \frac{1}{2} k'_n \frac{W}{L}(V_{GS} - V_t)^2\right]^{-1}$$

$$= \left[\frac{1}{2}\lambda k'_n \frac{W}{L}(V_{ov})^2\right]^{-1}$$

$$\text{new } r_O = \left[\frac{1}{2}\lambda k'_n \frac{4W}{4L}\left(\frac{1}{2}V_{ov}\right)^2\right]^{-1} = 4r_O$$

Note that quadrupling *W* and *L* had no effect, but decreasing the overdrive voltage by half increased the output resistance by a factor of 4.

5.24

MOS	1	2	3	4
λ (V^{-1})	0.02	0.01	0.1	0.005
V_A (V)	50	100	10	200
I_D (mA)	5	3.33	0.1	0.2
r_o (kΩ)	10	30	100	1000

$r_o = \dfrac{V_A}{I_D}$, $\lambda = \dfrac{1}{V_A}$

5.25

$v_{GS} = -3\text{V}$ $v_{SG} = 3\text{V}$ $V_t = -1\text{V}$

$v_{DS} = -4\text{V}$ $v_{SD} = 4\text{V}$ $V_A = -50\text{V}$

$\lambda = -0.02\text{V}^{-1}$

$$i_D = \frac{1}{2}k'_p \frac{W}{L}(V_{GS} - V_t)^2(1 + \lambda V_{DS})$$

$$3 = \frac{1}{2}k'_p \frac{W}{L}(-3 + 1)^2(1 + 0.02 \times 4)$$

$$= 2.16 k'_p \frac{W}{L}$$

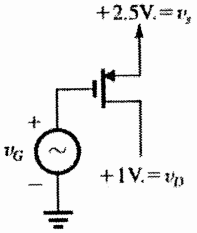
$$k'_p \frac{W}{L} = 1.39\text{mA}/\text{V}^2$$

5.26

	V_s	V_G	V_D	V_{SG}	$\lvert V_{ov}\rvert$	V_{SD}	Region of Operation
a	+2	+2	0	OV.	OV.	2V.	cutoff
b	+2	+1	0	+1V.	OV.	2V.	cutoff/sat
c	+2	0	0	+2V.	1V.	2V.	Sat
d	+2	0	+1	+2V.	1V.	1V.	Sat/ohmic
e	+2	0	+1.5	+2V.	1V.	0.5V	ohmic
f	+2	0	+2	+2V.	1V.	OV.	ohmic

pmos $V_{tp} = -1\text{V}$,

5.27



pmos

$V_{tp} = -0.5\text{V}$.

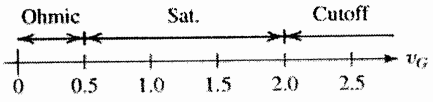
$v_{SD} = 1.5\text{V}$.

$v_{GS} \geq V_{tp} \Rightarrow$ Cutoff

$\therefore\ v_G \geq 2.0\text{V}. \Rightarrow$ Cutoff

$v_{GD} \leq V_{tp} \Rightarrow$ ohmic

$\therefore v_{GD} \leq\ +0.5\text{V} \Rightarrow$ ohmic



5.28

$$\frac{\Delta i_D}{I_D} = \frac{\left.\frac{\partial i_D}{\partial k_n'}\right|_{ID} \frac{dk_n'}{dT}\Delta T + \left.\frac{\partial i_D}{\partial V_t}\right|_{I_D} \frac{dV_t}{dT}\Delta T}{\left.\left[\frac{1}{2}k_n'\frac{W}{L}(v_{ES} - V_t)^2\right]\right|_{I_D}}$$

(a) $\dfrac{\Delta i_D}{I_D} = \dfrac{1}{k_n'}\dfrac{dk_n'}{dT}\Delta T + \dfrac{-2}{(V_{GS} - V_t)}\dfrac{dV_t}{dT}\Delta T$

(b)

$$\frac{\left(\frac{\Delta i_D}{I_D}\right)}{\Delta \text{T}} = -\frac{0.002}{\text{C}^\circ} = \frac{1}{k_n'}\frac{dk_n'}{dT} - \left(\frac{2}{4\text{V}}\right)\left(\frac{-2\text{mV}}{\text{C}^\circ}\right)$$

for $V_t = +1$ V, $V_{GS} = 5$ V, $V_{OV} = 4$ V,

$$\therefore\ \frac{\left(\frac{dk_n'}{dT}\right)}{k_n'} = -0.003/\text{C}^\circ\ (-0.3\%\text{C}^\circ)$$

5.29

a) $I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow 2 = \frac{1}{2} k_n' \frac{W}{L} (3-1)^2$

$\Rightarrow k_n' \frac{W}{L} = 1\ mA/V^2$

$V_1 = V_{DS} = 3V$

b) $V_2 = V_S = V_D - V_{DS} = 1 - 3 = -2^V$

c) $V_3 = V_S = V_D - V_{DS} = 0 - (-3) = 3^V$

d) $V_4 = V_D = V_S + V_{DS} = 5 + (-3) = 2^V$

In order to calculate R_{max} that can be inserted in series with the drain, V_{DS} has to be equal to $V_{GS} - V_t$, so that the device is operating on the edge of saturation: $|V_{DS}| = 3 - 1 = 2^V$. Note that since i_D is the same, V_{GS} stays the same.

a) $R_{D\,max} = \frac{3-2}{2^{mA}} = 0.5 K\Omega$

b) $V_2 = -2^V \Rightarrow V_D = -2 + 2 = 0 \Rightarrow R_{D\,max} = \frac{1}{2} = 0.5^{K\Omega}$

Note that V_2 is fixed through $V_{GS} = 3V$.

c) $V_{GS} = -3V \Rightarrow V_S = V_3 = 3^V$. Now for V_{DS} to be $-2V$, V_D has to be 1^V.

$R_{D\,Max} = \frac{1V}{2^{mA}} = 0.5 K\Omega$

d) $V_{GS} = -3V \Rightarrow V_G = V_4 = 2^V$. Adding the resistor between V_4 and drain means that V_D has to be $5 - 2 = 3^V$ and this leaves 1^V voltage drop on the resistor $\therefore R_{D\,max} = \frac{1}{2} = 0.5^{K\Omega}$

In order to calculate the Largest Resistor added to the gates, note that since the gate doesn't draw any current, the value of the resistor is immaterial.

Now we calculate $R_{S\,max}$, assuming that the voltage drop accross the current source is at least 2^V:

a) $V_1 = 8^V$ then $V_{GS} = 3^V \Rightarrow V_S = 8 - 3 = 5^V$

$R_{S\,Max} = \frac{5}{2} = 2.5 K\Omega$

b) $V_2 = -9 + 2 = -7^V$, $V_S = 1 - |V_{GS}| = -2V$

$R_{S\,max} = \frac{-2-(-7)}{2} = 2.5 K\Omega$

c) $V_3 = 10 - 2 = 8^V$, $V_S = 0 + |V_{GS}| = 3V$

$R_{S\,max} = \frac{8-3}{2} = 2.5 K\Omega$

d) $V_4 = -5 + 2 = -3^V$, $V_S = -3 + |V_{GS}| = 0^V$

$R_{S\,max} = \frac{5}{2} = 2.5 K\Omega$

5.30

$I_D = \frac{V_{DD} - V_D}{R_D} \Rightarrow \frac{5-0}{R_D} = 1^{mA} \Rightarrow \underline{R_D = 5K\Omega}$

$V_D = V_G \Rightarrow$ Saturation

therefore: $i_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$

$1 = \frac{1}{2} \times 60 \times 10^{-3} \times \frac{100}{3} (V_{GS} - 1)^2$

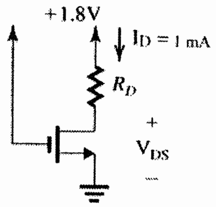
$\Rightarrow V_{GS} = 2V \quad \Rightarrow V_S = -2^V$

$R_S = \frac{-2-(-5)}{1} = \underline{3K\Omega}$

5.31

$I_D = 1$ mA, $V_t = 0.5$ V, $V_{DD} = 1.8$ V.

To operate at the edge of saturation, V_{DS} must equal V_{OV}



$V_{GS} = V_G - V_S = 1.8 - 0 = 1.8$ V

$V_{OV} = V_{GS} - V_t = 1.8 - 0.5 = +1.3$ V

with $V_{DS} = V_{OV} = 1.3$ V,

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{1.8 - 1.3}{1\ \text{mA}} = 500\ \Omega$$

5.32

$R = \frac{3.5}{0.115} = 3.04\,\mathrm{K}\Omega$

$\underset{\mathrm{mA}}{0.115} = \frac{1}{2} \times 60 \times 10^{-3} \times \frac{W}{0.8}\left(-1.5-(-0.7)\right)^2 \Rightarrow W = 4.8\,\mu\mathrm{m}$

5.33

$V_{GS1} = 1.5V$, $120\,\mu A = \frac{1}{2} \times 120 \times \frac{W_1}{1}(1.5-1)^2$

$\Rightarrow W_1 = 8\,\mu m$

$V_{GS2} = 3.5 - 1.5 = 2^V$, $120 = \frac{1}{2} \times 120\,\frac{W_2}{1}(2-1)^2$

$\Rightarrow W_2 = 2\,\mu m$

$R = \frac{5-3.5}{0.120} = 12.5\,\mathrm{K}\Omega$

5.34

$V_{GS1} = 1.5\ \mathrm{V}$

$$120\ \mu\mathrm{A} = \frac{1}{2} \times 120 \times \frac{W_1}{1}(1.5-1)^{\frac{1}{2}}$$

$W_1 = 8\ \mu\mathrm{m}$

$V_{GS2} = 2\ \mathrm{V}$

$$120\ \mu\mathrm{A} = \frac{1}{2} \times \frac{W_2{}^2}{1}(2-1)$$

$W_2 = 2\ \mu\mathrm{m}$

$V_{GS3} = 1.5\mathrm{V}$

$W_3 = 8\ \mu\mathrm{m}$

5.35

$V_I = V_{GS} = 5V$, $V_O = V_{DS} = 0.05V$

$r_{DS} = 50\Omega = \frac{V_{DS}}{I_D} \Rightarrow I_D = \frac{0.05}{50} = 0.001^A = 1\,mA$

$R = \frac{V_{DD} - V_O}{I_D} = \frac{5 - 0.05}{1} = \underline{4.95\,\mathrm{K}\Omega}$

$V_{DS} < V_{GS} - V_t \Rightarrow$ triode region

$I_D = K_n' \frac{W}{L}\left[(V_{GS} - V_t)V_{DS} - \frac{V_{DS}^2}{2}\right]$

$1 = 100 \times 10^{-3}\,\frac{W}{L}\left[(5-1) \times 0.05 - \frac{0.05^2}{2}\right] \Rightarrow \underline{\frac{W}{L} = 50}$

5.36

In circuit a: $V_2 = 10 - 4 \times 2 = 2\ \mathrm{V}$

assume saturation:

$I_D = 2 = \frac{1}{2} \times 1 \times (V_{GS} - 2)^2$

$= V_{GS} = 4\mathrm{V}$

$\Rightarrow V_1 = -4\mathrm{V}$, $V_{DS} = 6\mathrm{V} > V_{GS} - V_t$

so our assumption was correct.

In circuit b:

$I_D = 1 = \frac{1}{2} \times 1 \times (V_{GS} - 2)^2 \Rightarrow$

$V_{GS} = 3.41\ \mathrm{V}$ $V_3 = 3.41\ \mathrm{V}$

In circuit c:

$I_D = 2\ \mathrm{mA} \Rightarrow V_{GS} = -4\ \mathrm{V} \Rightarrow V_S$

$= 4\ \mathrm{V} = V_4$

$V_5 = -10 \times 2.5 \times 2 = -5\ \mathrm{V}$

In circuit d:

$I_D = 2\ \mathrm{mA} \Rightarrow V_{GS} = -4\ \mathrm{V} \Rightarrow V_6 = 6\ \mathrm{V}$

$\Rightarrow V_7 = V_6 - 4 = 2\ \mathrm{V}$

If we replace the current source with a resistor in each of those circuits:

In circuit a:

$$R = \frac{-4-(-10)}{2} \simeq 3.01\ \mathrm{k}\Omega$$

(by looking at the table for 1% resistors)

now recalculate I_D: $I_D = \frac{1}{2} \times 1 \times (V_{GS} - V_t)^2$

$V_{GS} - V_t = 0 - (-10 + 3.01 I_D) - 2$

$= 8 - 3.01 I_D \Rightarrow$

$2 I_D = (8 - 3.01 I_D)^2 \Rightarrow I_D$

$= 1.99\ \mathrm{mA} \Rightarrow V_2 = 2.04\ \mathrm{V}$

$V_1 = -4.01 \text{ V}$

In circuit b:

$$R = \frac{10 - 3.41}{1} = 6.59 \text{ k} \simeq 6.65 \text{ k}\Omega$$

then

$$V_{GS} = 10 - 6.65I$$

$$= \frac{1}{2} \times 1(10 - 6.65I - 2)^2 \Rightarrow I = 0.99 \text{ mA}$$

$$V_3 = 10 - 6.65 \times 0.99 = 3.41 \text{ V}$$

In circuit c:

$$R = \frac{10 - 4}{2} \simeq 3.01 \text{ k}\Omega,$$

$$V_{GS} = -(10 + 3.01I)$$

$$I = \frac{1}{2} \times 1 \times (-10 + 3.01I + 2)^2$$

$$I_D = 1.99 \text{ mA}$$

$$V_4 = 10 - 3.01 \times 1.99 = 4.01 \text{ V}$$

$$V_5 = -10 + 2.5 \text{ k} \times 1.99 = -5.03 \text{ V}$$

In circuit d:

$R = \frac{2}{2} = 1\text{k}$ so V_7 is still 2 V.

5.37

a) $V_{GS} = -V_1$. $10\mu A = \frac{1}{2} \times 0.4 \times 10^3 (V_{GS} - 1)^2 \Rightarrow$

$V_{GS} = 1.22^V \Rightarrow \underline{V_1 = -1.22^V}$

b) $100\mu A = \frac{1}{2} \times 0.4 \times 10^3 \times (V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.71^V, \underline{V_2 = -1.71^V}$

c) $1 = \frac{1}{2} \times 0.4 \times (V_{GS} - 1)^2 \Rightarrow V_{GS} = 3.23^V \Rightarrow \underline{V_3 = -3.23V}$

d) $10 = \frac{1}{2} \times 0.4 \times 10^3 (V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.22^V \Rightarrow \underline{V_4 = 1.22^V}$

e) $1 = \frac{1}{2} \times 0.4 (V_{GS} - V_t)^2 \Rightarrow V_{GS} = 3.24^V \Rightarrow \underline{V_5 = 3.24^V}$

f) $I = \frac{1}{2} \times 0.4 \times (5 - 100I - 1)^2 \Rightarrow I = 0.045\text{mA}, 0.036\text{mA}$

$V_6 = 5 - 100 \times 0.036 = \underline{1.4^V}$

g) $I = \frac{1}{2} \times 0.4 \times (5 - 1 \times I - 1)^2 \Rightarrow I = 1.38\text{mA}$

$V_7 = \underline{5 - 1.38 \times 1 = 3.62^V}$

h) $I = \frac{1}{2} \times 0.4 \times (5 - 100 - I)^2 \Rightarrow I = \cancel{0.045^{mA}}, 0.036^{mA}$

$V_8 = -5 + 100 \times 0.036 = \underline{-1.4^V}$

Note that $I = 0.045^{mA}$ in circuits h and f is not acceprable, because it results in $V_{GS} < V_t$ that is not physitally possible.

5.38

a) $V_{GS2} = -V_2$, $I = \frac{V_2 - (-5)}{1K} = \frac{1}{2} \times 2 \times (-V_2 - 1)^2$

$\Rightarrow V_2 + 5 = V_2^2 + 2V_2 + 1 \Rightarrow V_2^2 + V_2 - 4 = 0 \Rightarrow X\, V_2 = 1.55^V$

$V_2 = -2.56^V$

$V_2 = 1.55^V$ is not acceptable because it results in $V_{GS} < 0$ that is not possible for an NMos.

Therefore $\underline{V_2 = -2.56V}$

$i_{D1} = i_{D2} \Rightarrow \frac{V_2 - (-5)}{1K} = \frac{1}{2} \times 2(5 - V_1 - 1)^2 \Rightarrow$

$2.44 = (4 - V_1)^2 \Rightarrow 4 - V_1 = \pm 1.56^V \Rightarrow V_1 = 2.44^V$

$V_1 = 5.56^V X$

The second answer results in $V_{GS} = 5 - 5.56 < 0$ which is not acceptable. Therefore $\underline{V_1 = 2.44^V}$

b) $\frac{10 - V_3}{1K} = \frac{V_5}{1K} = i_D \Rightarrow 10 - V_3 = V_5$ ①

$i_{D1} = \frac{V_5}{1k} = \frac{1}{2} \times 2 \times (V_3 - V_4 - 1)^2 \Rightarrow V_5 = (V_3 - V_4 - 1)^2$ ②

$i_{D2} = V_5/_{1K} = \frac{1}{2} \times 2 \times (V_4 - V_5 - 1)^2 \Rightarrow V_5 = (V_4 - V_5 - 1)^2$ ③

②,③ $\Rightarrow V_3 - V_4 - 1 = V_4 - V_5 - 1 \Rightarrow V_5 = 2V_4 - V_3$ ④

①,④ $\Rightarrow 2V_4 - V_3 = 10 - V_3 \Rightarrow \underline{V_4 = 5^V}$

③ $\Rightarrow V_5 = (4 - V_5)^2 \Rightarrow V_5^2 - 9V_5 + 16 = 0 \Rightarrow V_5 = 6.55^V X$

$V_5 = 2.45^V$

$V_5 = 6.55$ results in $i_D = 6.55\text{mA}$, $V_3 = 4.45^V$ and thisisnot physicly possible. So $\underline{V_5 = 2.45^V}$

$V_3 = 10 - 2.45 = \underline{7.55V}$

5.39

The PMOS transistor operates in saturation region if

$V_{SD} \geq V_{SG} - |V_t|$

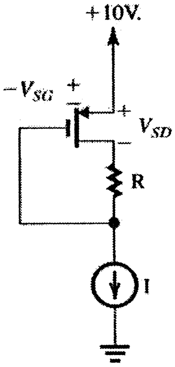
or

$V_{SD} \geq V_{SG} - 1$

Also, $V_{SD} + IR = V_{SG} \Rightarrow V_{SD}$

$= V_{SG} - IR$

$\Rightarrow IR \leq |V_t|$ for PMOS to be in saturation.



a) $R = 0 \Rightarrow IR = 0 < |V_t|$

saturation:

$I = 100 = \frac{1}{2} \times 8 \times 2.5$

$\times (V_{SG} - |V_t|^2)$

$V_{SG} - 1 = \pm 1 \Rightarrow V_{SG} = 2\ \text{V}$

$= V_{SD}$

b)

$R = 10\ \text{k}\Omega = IR = 10 \times 0.1 = 1\ \text{V} \Rightarrow$ saturation

$V_{SG} = 2\ \text{V} \Rightarrow V_{SD} = 2 - 1 = 1\ \text{V}$

c) $R = 30\ \text{k}\Omega \Rightarrow \text{IR} = 30 \times 0.1$

$= 3\ \text{V} \Rightarrow$ triode region

$100 = 8 \times 25$

$\left[(V_{SG} - |V_t|)V_{SD} - \frac{1}{2}V_{SD}^2\right]$

$0.5 = \left[(V_{SG} - 1)(V_{SG} - 3) - \frac{1}{2}(V_{SG} - 3)^2\right]$

$0.5 = 0.5V_{SG}^2 - V_{SG} - 1.5$

$\Rightarrow V_{SG}^2 - 2V_{SG} - 4 = 0$

$V_{SG} = 3.24\text{V}, -1.2\ \text{V}$ X

$V_{SD} = 3.24 - 3 = 0.24\ \text{V}$

$100\ \text{k}\Omega \Rightarrow \text{IR} = 100 \times 0.1$

$= 10\ \text{V} \Rightarrow$ triode region

$100 = 8 \times 25$ X

5.40

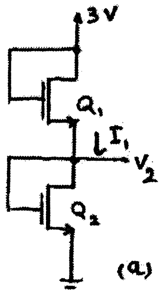
a) Q_2, Q_1 operating in Saturation: $i_{D1} = i_{D2}$

$\Rightarrow V_{GS1} = V_{GS2}$

$3V = V_{GS1} + V_{GS2} \Rightarrow V_{GS1} = V_{GS2} = 1.5^V$

$\underline{V_2 = 1.5^V}$

$I_1 = \frac{1}{2} \times 20 \times \frac{30}{10}(1.5-1)^2 = \underline{7.5\mu A}$



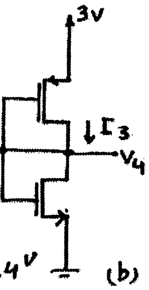
b) Both transistors have $V_D = V_G$ and therefore they are operating in Saturation: $i_{D_1} = i_{D2}$

$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_4-1)^2 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(3-V_4-1)^2$

$2.5(V_4-1)^2 = (2-V_4)^2$

$1.58(V_4-1) = (2-V_4) \Rightarrow V_4 = 1.39\,V \simeq \underline{1.4^V}$

$I_3 = \frac{1}{2} \times 20 \times \frac{30}{10}(1.39-1)^2 = \underline{4.6\mu A}$



c) $\frac{W_1}{L_1} = \frac{75}{10} = 7.5$

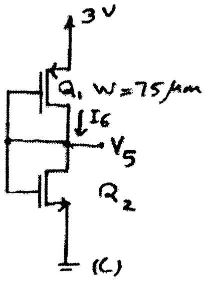
$\frac{W_2}{L_2} = \frac{30}{10} = 3$ $\qquad \frac{W_1/L_1}{W_2/L_2} = 2.5$

$i_{D_1} = i_{D2}$

Since $\mu_n C_{ox}\frac{W_2}{L_2} = \mu_p C_{ox}\frac{W_1}{L_1}$

$\Rightarrow V_{GS_1} = V_{GS2} = \frac{3}{2} = \underline{1.5V = V_5}$

$I_6 = \frac{1}{2} \times 20 \times \frac{30}{10}(1.5-1)^2 = \underline{7.5\mu A}$



5.41

Since $V_{G1} = V_{D1}$ then Q_1 is in saturation. We assume that Q_2 is also in saturation, then because $I_{D1} = I_{D2}$, V_{GS1} would be equal to V_{GS2}.

$$V_{GS1} = V_{GS2} = \frac{5}{2} = 2.5 \text{ V}$$

$$I_1 = \frac{1}{2} \times 50 \times \frac{10}{1}(2.5 - 1)^2 = 562.5 \ \mu\text{A}$$

$V_{GS3} = V_{GS1} = 2.5$ V. Since Q_3 and Q_4 have the same drain current, then

$V_{GS3} = V_{GS4} = 2.5$ V. This is based on the assumption that Q_3 & Q_4 are saturated:

$$V_{GS3} = V_{GS1} \Rightarrow I_2 = I_{GS3} = I_{GS1}$$

$$= 562.5 \ \mu\text{A}$$

$$V_2 = 5 - 2.5 = 2.5 \text{ V}$$

Now if Q_3 and Q_4 have $W = 100 \ \mu\text{m}$ then:

$$I_2 = \frac{1}{2} \times 50 \times \frac{100}{1}(2.5 - 1)^2 = 5.625 \text{ mA}$$

or

$$\frac{I_{Q3}}{I_{Q1}} = \frac{W_3}{W_1} = \frac{100}{10} \Rightarrow I_{Q3} =$$

$$10 \times 562.5 \ \mu\text{A} = 5.625 \text{ mA}$$

5.42

Part a

Find the R_D corresponding to point B, which is the saturation-triode boundary with

$V_{DS,B} = 0.5$ Volts

Also on the boundary

$$i_{D,B} = \frac{K'_n \frac{W}{L} v^2_{DS,B}}{2}$$

$$5 \ \frac{(0.25x10^{-3})(40)(0.5)^2}{2} \ 5 \ 1.25 \text{ mA}$$

$$R_D \ 5 \ \frac{2.5 - 0.5}{1.25x10^{-3}} \ 5 \ 1600 \ \Omega$$

Part b

Find v_{GS} corresponding to point B.

$V_{DS,B} = 0.5 \Rightarrow V_{GS,B} = V_{GDS} + V_{DS,B} = V_l + V_{DS,B} = 0.5 + 0.5 = 1.0$ Volts

Part c

Find $V_{DS,c}$ corresponding to point C, where $v_{GS,C} = 2.5$ Volts and the transistor is in the triode region$v$

$$V_{DS,C,} + R_D\left[k'_n\frac{W}{L}\left((v_{GS,C} - V_t)v_{DS,C} - \frac{v^2_{DS,C}}{2}\right)\right]$$

$$= V_{DD} \Rightarrow V_{DS,C} + 1600$$

$$\left[(0.25 \ x \ 10^{-3})40((2.5 - 0.5) - 0.5v_{DS,C} - \frac{v^2_{DS,C}}{2}\right]$$

The roots of this equation are 0.07720 & 4.04778

Clearly the $v_{DS} \approx 0.07720$ is the choice because the other one is above V_{DD}.

The current, i_D, corresponding to point C, $i_{D,C}$, is

$$i_{D,C} = \frac{V_{DD} - V_{DS,C}}{R_D}$$

$$= \frac{2.5 - 0.07720}{1600} = 1.514\text{mA}$$

An equivalent resistor value can now be calculated at point C

$$R_{\text{equivalent}} = \frac{V_{DS,C}}{i_{DS,C}} = \frac{0.07720}{1.514 \times 10^{-3}} = 50.98\,\Omega$$

This can be compared to the value of r_{DS}, which is really derived for $v_{DS} = 0$.

$$r_{DS} = \frac{1}{k'_n\frac{W}{L}(V_{GS} - V_t)}$$

$$= \frac{1}{(0.25 \times 10^{-3})(40)(2.5 - 0.5)} = 50$$

The value is close to the equivalent resistor value, but they are not exactly equal.

Part d

$V_{GS} = 0.8$, so the transistor is in saturation.

Find V_{DS}.

$$V_{DS} + R_D \left[k'_n\frac{W}{L}\frac{(v_{GS} - V_t)^2}{2}\right] = V_{DD} \Rightarrow V_{DS}$$

$$+ \ 1600\left[\frac{(0.25x \ 10^{-3})40(0.8 - 0.5)^2}{2}\right]$$

$$= 2.5$$

$$V_{DS} \approx 1.78\text{Volts}$$

The voltage gain is

$$A_v = -k'_n\frac{W}{L}(V_{GS} - V_t)R_D = -(0.25x \ 10^{-3})$$

$$(40)(0.8 - 0.5)(1600) = -4.8$$

5.43

a)

Point A : $V_{IA} = V_t = 1^V$, $V_{OA} = V_{DD} = 5^V$

For $V_i < V_t$, the transistor is not on, $V_{GS} < V_t$. Point A is when $V_{GS} = V_t$ and the transistor turns on. As V_i increases, the i_D increases and v_o decreases. v_o decreases to the point that it is below V_I by V_t volts. At this point, B, the MOSFET enters the triode region: $V_{OB} = V_{IB} - V_t$ or $V_{DS} = V_{GS} - V_t$. So at point B: $I = \frac{V_{DD} - V_{OB}}{R}$

$$I = \frac{V_{DD} - (V_{GS} - V_t)}{R} = \frac{1}{2} \times k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

$$\frac{5 - V_{GS} + 1}{24} = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2 \Rightarrow 12V_{GS}^2 - 23V_{GS} + 6 = 0$$

$$V_{GS} = 1.61V \Rightarrow V_I = 1.61^V \qquad V_O = 1.61 - 1 = 0.61^V$$

Point B: $\underline{V_{OB} = 0.61^V}$ $\quad \underline{V_{IB} = 1.61V}$

b) $I_Q = \frac{1}{2} \times 1 \times 0.5^2 = \underline{0.125 mA}$

$V_{OQ} = 5 - 24 \times 0.125 = \underline{2^V}$

$V_{IQ} = V_{GS} = V_{OV} + V_t = 0.5 + 1 = \underline{1.5^V}$

Now to calculate the incremental gain A_v at this bias point, From equation 4.41, we have: $A_v = -2\,V_{RD}/V_{OV} = \frac{-2(V_{DD} - V_{OQ})}{V_{OV}}$

$$A_v = \frac{-2(5-2)}{0.5} = -12 V/V$$

c) $V_{IQ} = 1.5^V$, $V_t = 1^V$, $V_{IB} = 1.61^V$. Thus the largest amplitude of a sine wave that can be applied to the input while the transistor remain in saturation is: $1.61 - 1.5 = 0.11V$

The amplitude of the output voltage signal that results is approximately equal to $V_{OQ} - V_{OB} = 2 - 0.61 = 1.39^V$. The gain implied by this amplitudes is:

$$\text{gain} = \frac{-1.39}{0.11} = 12.64 V/V$$

This gain is 5.3% different from the incremental gain calculated in part (b). This difference is due to the fact that the segment of the voltage transfer curve considered here is not perfectly linear.

5.44

$R_D = 20 k\Omega$, $V_{RD} = 2^V \Rightarrow I_D = 0.1 mA$

$$A_v = -\frac{2 V_{RD}}{V_{OV}} \Rightarrow -10 = -\frac{2 \times 2}{V_{OV}} \Rightarrow V_{OV} = 0.4^V$$

$V_{GS} = 1.2V \Rightarrow V_t = 1.2 - 0.4 = \underline{0.8V}$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow 0.1 = \frac{1}{2} \times 50 \times 10^{-3} \frac{W}{L} 0.4^2$$

$\Rightarrow \underline{\frac{W}{L} = 25}$

5.55 the maximum gain achievable is:

$$|A_{v\max}| = \frac{V_{DD}}{(V_{OV}/2)} = \frac{5}{(0.2/2)} = 50\ V/V$$

the gain is maximum when V_{OV} is minimum (= 0.2 V) and when the drop across R_D (= $I_D R_D$) is largest possible, which occurs when we operate closest to point B

At B: $V_{DS}| = V_{GS}|_B - V_t = V_{OV}$

$V_{DS}|_B = 0.2$

to allow for ±0.5 V swing

$V_{DS} = 0.2 + 0.5 = 0.7$ V

$$\rightarrow |A_V| = \frac{(5 - 0.7)}{0.2/2} = 43 \text{ V/V}$$

$\Delta V_i \times 43 = \Delta V_O$

$$\Delta V_i = \frac{\pm 0.5}{43} = \pm 11.6 \text{ mV}$$

c) If $I_D = 100\ \mu$A,

$k_n' = 100\ \mu\text{A/V}^2 \Rightarrow \frac{W}{L} = ?$

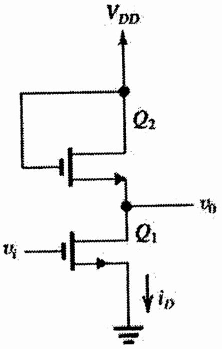
In saturation: $I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{k_n' V_{OV}^2}$

$$\frac{W}{L} = \frac{2 \times 100\ \mu}{100\ \mu \times (0.2)^2} = 50$$

(d) $V_{DD} - I_D R_D = 0.7$

$5 - 100\ \mu \cdot R_D = 0.7 \Rightarrow R_D = 43\ \text{k}\Omega$

5.46



given $V_{t1} = V_{t2} = V_t$

for Q_2 $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2[V_{DD} - V_o - V_t]^2$

for Q_1 $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_1[V_i - V_t]^2$

for $V_t \leq V_i \leq V_o + V_t$

equate i_{D1} and i_{D2}

$$\left(\frac{W}{L}\right)_2[V_{DD} - V_o - V_t]^2 = \left(\frac{W}{L}\right)_1[V_i - V_t]^2$$

$$[V_{DD} - V_o - V_t] = \sqrt{\frac{(W/L)_1}{(W/L)_2}} \cdot [V_i - V_t]$$

$$V_o = V_{DD} - V_t + V_t\sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

$$-V_i\sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

for $\sqrt{\frac{(W/L)_1}{(W/L)_2}} = \sqrt{\frac{\left(\frac{50}{0.5}\right)}{\left(\frac{5}{0.5}\right)}} = \sqrt{10}$

$A_v = -\sqrt{10} = -3.16$

5.47

$I_D = \frac{1}{2}k_n'\frac{W}{L}V_{ov}^2 \Rightarrow I_D = \frac{1}{2} \times 2 \times 1^2 = 1\text{ mA}$

$i_D = \frac{1}{2} \times 2 \times (1+0.1)^2 = 1.21\text{ mA}$ $(v_{gs} = 0.1\text{V})$

$i_D = 1.21 - 1 = 0.21\text{ mA}$

If $v_{gs} = -0.1^V \Rightarrow i_D = \frac{1}{2} \times 2(1-0.1)^2 = 0.81\text{ mA}$

$i_D = 0.81 - 1 = -0.19\text{ mA}$

For positive increment: $g_m = \frac{\Delta i_D}{\Delta v_{gs}} = \frac{0.21}{0.1} = 2.1\text{ mA/V}$

For negative increment: $g_m = \frac{0.19}{0.1} = 1.9\text{ mA/V}$

An estimate of $g_m = \frac{2.1+1.9}{2} = \underline{2\text{ mA/V}}$

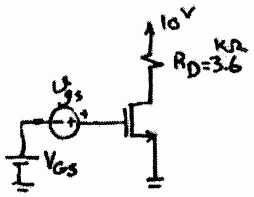
$g_m = k_n'\frac{W}{L}V_{ov} = 2 \times 1 = \underline{2\text{ mA/V}}$. same as estimated

5.48

a) $I_D = \frac{1}{2} \times 1 \times (4-2)^2 = \underline{2\text{ mA}}$

$V_D = V_{DD} - R_D I_D = 10 - 2 \times 3.6$

$\underline{V_D = 2.8\text{V}}$



b) $g_m = k_n'\frac{W}{L}V_{ov} = 1 \times (4-2) = \underline{2\text{ mA/V}}$

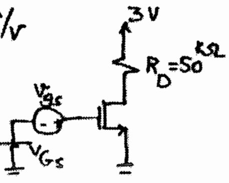
c) $A_v = \frac{v_d}{v_{gs}} = -g_m R_D = -2 \times 3.6 = \underline{-7.2\text{ V/V}}$

d) $r_o \simeq \frac{1}{\lambda I_D} = \frac{1}{0.01 \times 2} = \underline{50\text{ k}\Omega}$

$A_v = \frac{v_d}{v_{gs}} = -g_m(R_D \| r_o) = -2(3.6 \| 50) = \underline{-6.7\text{ V/V}}$

5.49

$g_m R_D = 5 \Rightarrow g_m = \frac{5}{50} = 0.1\text{ mA/V}$



For 0.5V output signal and a gain of 5V/V, $v_{gs} = \frac{0.5}{5} = 0.1^V$

So we can write $V_{DS} - 0.5 \geq V_{GS} + 0.1 - V_t$

or $V_{DS} \geq V_{GS} + 0.6 - 0.8 \Rightarrow \underline{V_{DS} \geq V_{GS} - 0.2}$

Also, from the other side: $V_{DS} + 0.5 \leq V_{DD}$

or $V_{DS} \leq 3 - 0.5 \Rightarrow \underline{V_{DS} \leq 2.5\text{V}}$

We design the circuit for lowest possible V_{DS} that guarantees the device operation in saturation: $V_{DS} = V_{GS} - 0.2$

$V_{DS} = V_{DD} - R_D I_D \Rightarrow V_{GS} - 0.2 = 3 - 50 \times I_D$

$\Rightarrow I_D = \dfrac{3.2 - V_{GS}}{50}$

Also, from eq. 4.71: $g_m = \dfrac{2 I_D}{V_{GS} - V_t} = 0.1$

$0.1 = \dfrac{2}{V_{GS} - 0.8} \times \dfrac{3.2 - V_{GS}}{50}$

$\Rightarrow V_{GS} = 1.49^V$, $I_D = \underline{0.034\,mA}$

$V_{DS} = 1.49 - 0.2 = 1.29^V$ $\qquad V_{OV} = 1.49 - 0.8 = \underline{0.69V}$

$\dfrac{W}{L} = \dfrac{I_D}{\frac{1}{2} k'_n V_{OV}^2} = \dfrac{0.034 \times 10^3}{\frac{1}{2} \times 100 \times 0.69^2} = 1.43$

$\underline{\dfrac{W}{L} = 1.43}$

5.50

$$\left.\begin{array}{l} A_v = -g_m R_D \;, \\ g_m = \dfrac{2 I_D}{V_{OV}} \quad \text{eq. 4.71} \end{array}\right\} \Rightarrow A_v = -\frac{2 R_D I_D}{V_{OV}} = \frac{-2(V_{DD} - V_D)}{V_{OV}} \quad ①$$

Minimum v_{DS} for edge of saturation:

$v_{DS} \geq v_{GS} - V_t$ or $v_{DS\,min} = v_{GS\,max} - V_t$

$V_{DS} - |A_v| \hat{v}_i = V_{GS} + \hat{v}_i - V_t$

IF we replace A_v with ①:

$$V_D - \frac{2(V_{DD} - V_D)}{V_{OV}} \hat{v}_i = V_{OV} + \hat{v}_i$$

$$\Rightarrow V_D \left(1 + \frac{2\hat{v}_i}{V_{OV}}\right) = V_{OV} + \hat{v}_i + \frac{2 V_{DD}}{V_{OV}} \hat{v}_i$$

$$V_D = \frac{V_{OV} + \hat{v}_i + 2 V_{DD} (\hat{v}_i / V_{OV})}{1 + 2(\hat{v}_i / V_{OV})}$$

$V_{DD} = 3V$, $\hat{v}_i = 20\,mV$ $\quad m = 10 = \dfrac{V_{OV}}{\hat{v}_i} \Rightarrow \underline{V_{OV} = 0.2^V}$

$V_D = \dfrac{0.2 + 0.02 + 2 \times 3 \times 10^{-1}}{1 + 2 \times 0.1} = \underline{0.68^V}$

$A_v = \dfrac{-2(3 - 0.68)}{0.2} = \underline{-23.2\ V/V}$

IF $I_D = 100\,\mu A = 0.1\,mA$:

$A_v = -\dfrac{2 R_D I_D}{V_{OV}} \Rightarrow 23.2 = \dfrac{2 \times R_D \times 0.1}{0.2} \Rightarrow$

$\underline{R_D = 23.2\,k\Omega}$

$I_D = \dfrac{1}{2} k'_n \dfrac{W}{L} V_{OV}^2 \Rightarrow 0.1 = \dfrac{1}{2} \times 100 \times 10^{-3} \dfrac{W}{L} 0.2^2$

$\Rightarrow \underline{\dfrac{W}{L} = 50}$

5.51

$$\text{Given } u_n = 500\text{cm}^2 / Vs$$

$$\mu_p = 250\text{cm}^2 / Vs \quad C_{ox} = 0.4\frac{\text{fF}}{\mu\text{m}^2}$$

$$k'_n = \mu_n C_{ox} = 20\mu\text{A} / \text{V}^2$$

$$k'_p = 10\mu\text{A} / \text{V}^2$$

Use equations

$$(5.55) g_m = k'\frac{W}{L}V_{ov}$$

$$(5.56) g_m = \sqrt{2\text{k}'\frac{W}{L}I_D}$$

$$(5.57) \; g_m = \frac{2I_D}{V_{ov}}$$

case type	I_D (mA)	$\lvert V_{GS} \rvert$	$\lvert V_t \rvert$	$\lvert V_{ov} \rvert$	W (μm)	L (μm)	$\frac{W}{L}$	$k'\frac{W}{L}$ (mA/V²)	gm(ms)
a(N)	1	3	2	1	100	1	100	2	2
b(N)	1	1.2	0.7	0.5	50	0.125	400	8	4
c(N)	10	–	–	2	250	1	250	5	10
d(N)	0.5	–	–	0.5	–	–	200	4	2
e(N)	0.1	–	–	1.41	10	2	5	0.1	0.141
f(N)	0.1	1.8	0.8	1	40	4	10	0.2	0.2
g(P)	1	–	–	2	–	–	25	*	* See comment
h(P)	1	3	1	2	–	–	50	0.5⁺	1
i(P)	10	–	–	1	4000	2	2000	20	20
j(P)	10	–	–	4	–	–	125	1.25	5
k(P)	0.05	–	–	1	30	3	10	0.1	0.1
l(P)	0.1	–	–	5	–	–	0.8	0.008×	0.04

Note - the circled entries are the givens.

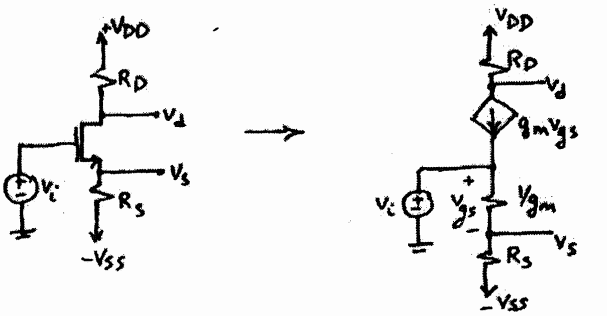
5.52

$$g_m = \sqrt{2k_n' \frac{W}{L} I_D} \Rightarrow \frac{W}{L} = \frac{g_m^2}{2k_n I_D}$$

$$\frac{W}{1} = \frac{1}{2\times 50\times 10^{-3}\times 0.5} \Rightarrow \underline{W = 20\,\mu m}$$

$$g_m = \frac{2I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2\times 0.5}{1} = 1 \Rightarrow V_{GS} = 1 + V_t = \underline{1.7^V}$$

5.53



$$\frac{v_s}{v_i} = \frac{R_S}{R_S + \frac{1}{g_m}} = \underline{\frac{R_S g_m}{R_S g_m + 1}}$$

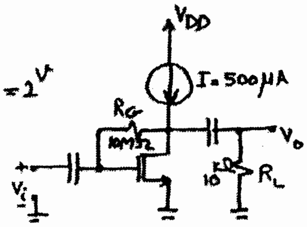
$$\frac{v_d}{v_i} = \frac{-g_m v_{gs} R_D}{v_i} = -g_m R_D \frac{1/g_m}{1/g_m + R_S} = \underline{\frac{-g_m R_D}{1 + g_m R_S}}$$

5.54

$$r_o \simeq \frac{V_A}{I_D} = \frac{50}{0.5} = 100\,k\Omega$$

$$g_m = \frac{2I_D}{V_{GS} - V_t}, \quad V_{GS} = V_{DS} = 2^V$$

$$g_m = \frac{2\times 0.5}{2 - 0.9} = 0.91\,mA/V$$



$$\frac{v_o}{v_i} = -g_m (r_o \| R_L) = -0.91(100^k \| 10^k) = \underline{-8.3\,V/V}$$

For I = 1mA or twice the current:

$$\frac{I_{D1}}{I_{D2}} = \frac{(V_{GS_1} - V_t)^2}{(V_{GS_2} - V_t)^2} \Rightarrow V_{GS2} = V_t + \sqrt{2}\,(V_{GS1} - V_t)$$

5.55

$$\text{NMOS:}\quad g_m = \sqrt{2k_n' \frac{W}{L} I_D} = \sqrt{2\times 90\times 10^{-3}\times \frac{20}{2}\times 0.1} = \underline{0.42\frac{mA}{V}}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{8\times 2}{0.1} = \underline{160\,k\Omega}$$

$$\chi = \frac{\gamma}{2\sqrt{2\phi_f + |V_{SB}|}} = \frac{0.5}{2\sqrt{2\times 0.34 + 1}} = 0.2$$

$$g_{mb} = \chi g_m = 0.2\times 0.42 = 0.084\,mA/V$$

$$g_m = \frac{2I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2\times 0.1}{0.42} = 0.48V$$

$$\text{PMOS:}\quad g_m = \sqrt{2\times 30\times 10^{-3}\times \frac{20}{2}\times 0.1} = \underline{0.24\,mA/V}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{12\times 2}{0.1} = \underline{240\,k\Omega}$$

$$\chi = 0.2 \Rightarrow g_{mb} = 0.2\times 0.24 = \underline{0.048}\,mA/V$$

$$V_{ov} = \frac{2\times 0.1}{0.24} = \underline{0.83V}$$

5.56

$V_t = 1V,\ k_n' = \frac{W}{L} = 2\ mA/V^2$

(a) dc analysis $V_G = \frac{5}{15}\ 15\ V_t = 5V$, assume

$I_D = 1$ mA

$V_S = 3$ V, $V_{GS} = 2$ V, $V_{OV} = 1$V.

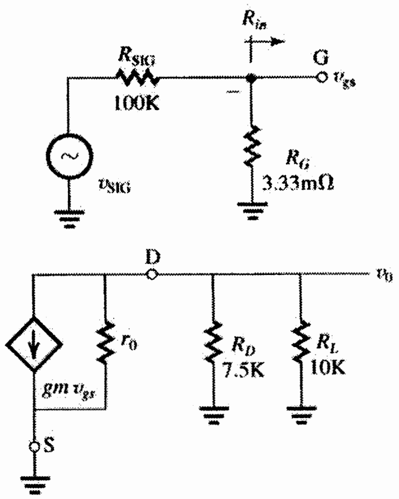
$I_D = \frac{1}{2}k' V_{OV}^2 = 1$ mA (check)

$V_D = V_{DD} - I_D R_D = 7.5$V.

(b) $r_0 = \frac{V_A}{I_D} = \frac{100\ V.}{1\ mA} = 100\ k\Omega$

$g_m = \sqrt{2k_n I_D} = 2$ mS

(c)


R_{in}
R_{SIG}
100K
G
v_{gs}
v_{SIG}
R_G
3.33mΩ
D
v_0
r_0
R_D
7.5K
R_L
10K
$gm\, v_{gs}$
S

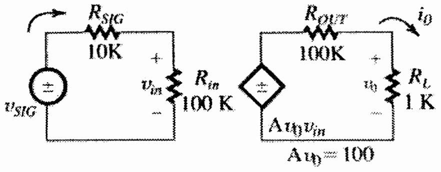
(d) $R_{in} = R_G = 3.33\ \text{M}\Omega$

$$\frac{v_{gs}}{v_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} = 0.97$$

$$\frac{v_O}{v_{gs}} = -g_m(r_O \| R_D \| R_L) = -8.2$$

$$\frac{v_O}{v_{sig}} = -8.0$$

5.57

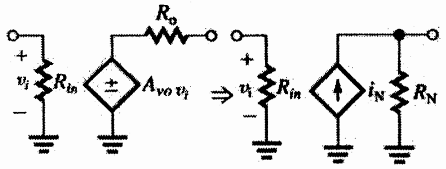

R_{SIG}
10K
v_{SIG}
v_{in}
R_{in}
100 K
R_{OUT}
100K
$A_{v0} v_{in}$
$A_{v0} = 100$
v_0
R_L
1 K
i_0

$$G_v = \frac{v_O}{v_{SIG}} = \frac{R_{in}}{R_{SIG} + R_{in}} A_{v_O} \frac{R_L}{R_{out} + R_L}$$

$$= 82.6$$

$$A_i = \frac{i_O}{i_i} = \frac{v_O}{v_{SIG}} \quad \frac{R_{SIG} + R_{in}}{R_L} = 9090$$

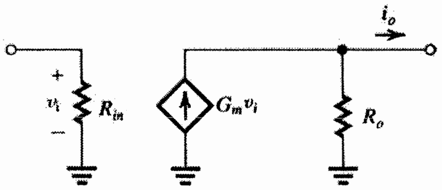
5.58


R_o
v_i
R_{in}
$A_{vo} v_i$
⇒
v_i
R_{in}
i_N
R_N

Where i_N = Norton's current source = $\dfrac{A_{VO} V_i}{R_O}$

and $R_N = R_o$ this is equivalent to Fig. P5.82

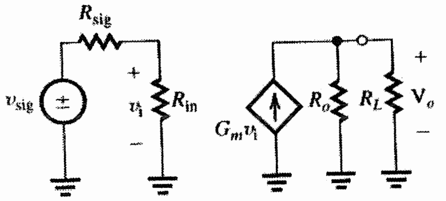
where $G_m = \dfrac{A_{VO}}{R_O}$


i_o
v_i
R_{in}
$G_m v_i$
R_o

If the output is shorted, $i_o = G_m V_i$ or

$G_m = \left.\dfrac{i_O}{V_i}\right|_{R_L = O}$ with a signal source and

load connected,


R_{sig}
v_{sig}
v_i
R_{in}
$G_m v_i$
R_o
R_L
V_o

by voltage division, $V_i = \dfrac{V_{sig} R_{in}}{R_{in} + R_{sig}}$

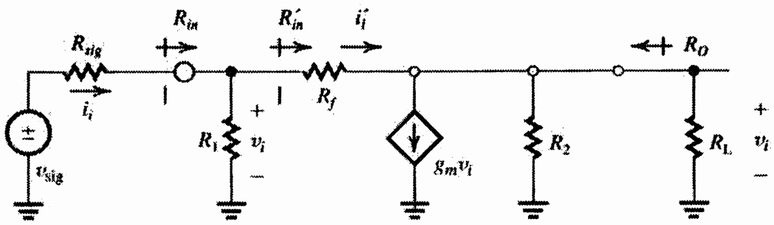
since $V_o = G_m V_i\ (R_O \| R_L)$, substitution for V_i

yields

$$V_O = \frac{V_{sig} R_{in}}{R_{in} + R_{sig}} \cdot G_m(R_O \| R_L), \text{ so that}$$

$$G_V = \frac{V_O}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} G_m(R_O \| R_L)$$

5.59



note $R_{in} = R_1 \parallel R_{in}'$

$R_L' \equiv R_2 \parallel R_L$

$$v_O = v_i = \frac{R_L'}{R_f + R_L'} - g_m v_i (R_f \parallel R_L')$$

$$= v_i \left[\frac{R_L' - g_m R_f R_L'}{R_f + R_L'}\right] = v_i \frac{R_L'(1 - g_m R_f)}{R_L' + R_f}$$

$$A_{vO} = \left.\frac{v_O}{v_i}\right| = \frac{R_2(1 - g_m R_f)}{R_2 + R_f} = -g_m R_2 \frac{\left(1 - \frac{1}{g_m R_f}\right)}{1 + \frac{R_2}{R_f}}$$

$$i_i' = \frac{v_i - v_o}{R_f} = \frac{v_i}{R_f}\left[1 - \frac{R_L'(1 - g_m R_f)}{R_L' + R_f}\right]$$

$R_L' = R_2 \;(R_L \to \infty)$

$$\frac{i_i'}{v_i} = \frac{1}{R_{in}'} = \frac{1}{R_f}\frac{R_f + R_L' - R_L' + g_m R_f R_L'}{R_f + R_L'}$$

$$= \frac{1 + g_m R_L'}{R_f + R_L'}$$

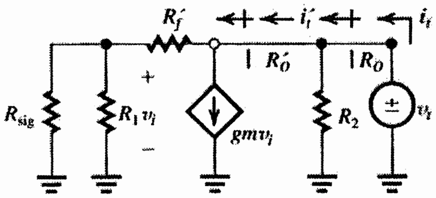
$$R_{in}' = \frac{R_f + R_L'}{1 + g_m R_L'} = \frac{R_f + R_2 \parallel R_L}{1 + g_m(R_2 \parallel R_L)}$$

$$R_{in} = R_1 \parallel R_{in}' = R_1 \parallel \frac{R_f + R_2 \parallel R_L}{1 + g_m(R_2 \parallel R_L)}$$

Output resistance assuming $R_{sig} = 0$

$R_O = R_2 \parallel R_f \cong R_2$

Output resistance including R_{sig}



$R_1' \equiv R_1 \parallel R_{sig}$

$$i_t' = \frac{v_t}{R_1' + R_f} + g_m v_t \frac{R_1'}{R_1' + R_f}$$

$$v_t = \frac{1 + g_m R_L'}{R_1' + R_f}$$

$$R_O' = \frac{R_1' + R_f}{1 + g_m R_L'}$$

$$R_O = R_2 \parallel R_O' = R_2 \parallel \frac{(R_1 \parallel R_{sig}) + R_f}{1 + g_m(R_1 \parallel R_{sig})}$$

Evaluate for

$R_1 = 100\ \text{k}\Omega,\ R_f = 1\ \text{m}\Omega,\ g_m = 100\ \text{mA/V}$

$R_2 = 100\ \Omega,\ R_L = 1\ \text{k}\Omega\ (R_{sig}\ \text{assumed}\ \phi)$

$R_{in} = 49.8\ \text{k}\Omega$

$A_{vO} = -10.0$

$R_O = 100\ \Omega$

R_{in} is out in half by R_f.

Given $R_{sig} = 100\ \text{k}\Omega,\ R_f \to \infty$, and

$R_f = 1\ \text{m}\Omega$

$$G_V = \frac{v_O}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}\left(-g_m R_L' \frac{1 - \frac{1}{g_m R_f}}{1 + \frac{R_L'}{R_f}}\right)$$

$G_V = -4.55 (R_f \to \infty),\ G_V = -3.02$

$(R_f = 1\ \text{m}\Omega)$

5.60

R_{in} = depends on biasing

$$A_{vo} = -g_m(r_o \parallel R_D)$$

$$= -0.4\frac{\text{mA}}{v}(50\ \text{k}\Omega \parallel 6\ \text{k}\Omega)$$

$$= -2.14\text{V/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10v}{0.2\ \text{mA}} = 50\ \text{k}\Omega$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = 0.4\ \text{mA/V}$$

$$R_o = r_o \| R_D = 50\ \text{k}\Omega \| 6\ \text{k}\Omega = 5.36\ \text{k}\Omega$$

WITH R_L=10 kΩ and assuming losses due to source impedance are negligible

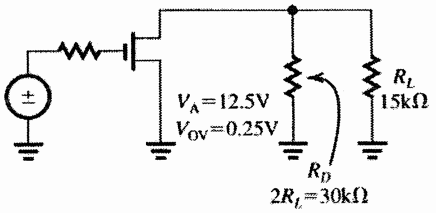
$$G_v = A_v = -g_m(r_o \| R_D \| R_L)$$

$$= -0.4\frac{\text{mA}}{v}(5.36\ \text{k}\Omega \| 10\ \text{k}\Omega) = -1.40\text{V/V}$$

For a 0.2V peak output, the input must be

$$\frac{0.2\text{V}}{1.4} = 0.143\text{V peak}$$

5.61



a) $g_m r_o$ = ? $g_m = \frac{2I_D}{V_{ov}}$ and $r_o = \frac{V_A}{I_D}$

$g_m r_o$ =

$$\frac{2I_D}{V_{ov}} \cdot \frac{V_A}{I_D} = \frac{2V_A}{V_{ov}} = \frac{2 \cdot 12.5}{0.25} = 100$$

b) If Gv = −10V/V,

$R_L = 15\ \text{k}\Omega, R_D = 2R_L = 30\ \text{k}\Omega$.

what is gm?

$$Gv = Av = -g_m(R_D \| \infty \| R_L)$$

$$-10\text{V/V} = -g_m(30\text{K} \| 15\text{K}) = -g_m \cdot 10\text{K}$$

$$gm = \frac{1\ \text{mA}}{\text{V}}$$

therefore:

$$I_D = \frac{V_{ov}}{2} \cdot g_m = \frac{0.25\text{V}\ 1\text{mA/V}}{2} = 0.125\ \text{mA}.$$

c) If $R_D = R_L$

$$\Rightarrow Gv = -g_m \cdot \frac{R_L}{2} = \frac{-1\ \text{mA}}{V} \cdot 7.5\ \text{K}$$

$$G_v = -7.5\text{V/V}$$

5.62

$$Gv = Av = -g_m(R_D \| R_L \| r_O)$$

If $R_D \| R_L = \infty \Rightarrow Gv = -g_m r_O$

since $g_m = \frac{2I_D}{V_{ov}}$ and $r_O = \frac{V_A}{I_D}$

$$Gv = \frac{-2I_D}{V_{ov}} \cdot \frac{V_A}{I_D} = \frac{-2V_A}{V_{ov}}$$

5.63

$$g_m = 5\ \text{mS}$$

$$i_d = g_m v_{gs} = \frac{g_m}{1 + g_m R_s} v_g$$

$$\frac{g_m}{1 + g_m R_s} = 1\ \text{mS}$$

$$\therefore R_s = \frac{4}{g_m} = 800\Omega$$

5.64

$$Rs = 1\ \text{k}\Omega$$

$$\frac{-g_m R'_L}{1 + g_m R_s} = -15$$

$$-g_m R'_L = -30$$

$$\therefore g_m = \frac{1}{R_s} = 1\text{ms}$$

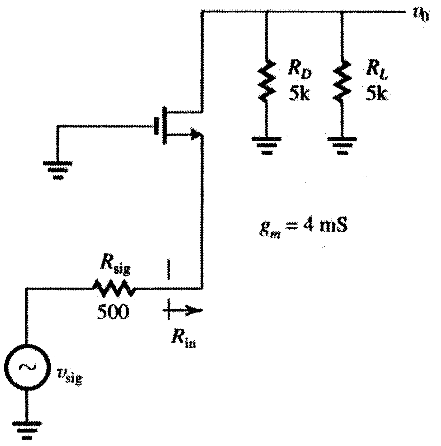
for $A_v = -10$, let $R_s = \frac{2}{g_m} = 2\ \text{k}\Omega$

5.65

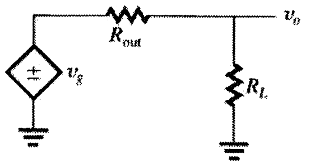
$$R_{in} = \frac{1}{g_m} = 250\Omega$$

$$Gv = \frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} g_m(R_D \| R_L) = +3.3$$

$g_m = \sqrt{2k_n I_D}$, so for $\frac{1}{g_m} = R_{sig}$, g_m must decrease to 1/2, and I_D must decrease to 1/4



5.66



$1\text{K} < R_L < 3\text{K}$

$R_{L,\,nom} = 2\text{K}$

for $R_{L,\,min}$

$$\frac{R_{L,\,min}}{R_{L,\,min} + R_{out}} \geq (0.80)\frac{R_{L,\,nom}}{R_{L,\,nom} + R_{out}}$$

$$\frac{1\text{K}}{1\text{K} + R_{out}} \geq \frac{1.6\text{K}}{2\text{K} + R_{out}}$$

$2\text{K}^2 + 1\text{K}R_{out} \geq 1.6\text{K}^2 + 1.6\text{K}R_{out}$

$400 \geq 0.6R_{out}$

$R_{out} \leq 667\Omega$

for $R_{L,\,max}$

$$\frac{R_{L,\,max}}{R_{L,\,max} + R_{out}} \leq (1.20)\frac{R_{L,\,nom}}{R_{L,\,nom} + R_{out}}$$

$$\frac{3\text{K}}{3\text{K} + R_{out}} \leq \frac{2.4\text{K}}{2\text{K} + R_{out}}$$

$R_{out} \leq 2\ \text{k}\Omega$

Therefore $R_{L,\,min}$ is the ruling case and

$R_{out} \leq 667\Omega$

$$g_m = \sqrt{2k_n I_D} \geq \frac{1}{667\Omega}$$

$k_n = 16\text{mA}/\text{V}^2$

$\therefore\ I_D \geq 70\ \mu\text{A}$

$$V_{ov} = \frac{2I_D}{g_m} = 0.093\text{V}.$$

5.67

Source Follower

$|v_{gs}| \leq 50\text{mV}$

$|v_o| \leq 0.5\text{V}$

$R_L = 2\text{k}\Omega$

$$v_o = g_m v_{gs} R_L \Rightarrow g_m \geq \frac{500\text{mV}}{50\text{mV}}\frac{1}{2\text{k}\Omega} = 5\text{mS}$$

For low distortion, keep

$|v_{gs}| < 0.2V_{ov} \Rightarrow V_{ov} = 0.25\text{V}.$

$$\therefore\ I_D \geq \frac{g_m V_{ov}}{2} = 0.625\text{mA}$$

$$i_{D,\,\text{peak}} = \frac{500\text{mVpK}}{2\text{k}\Omega} = 250\mu\text{Apk}$$

$i_{D,\,max} = 0.625\ \text{mA} + 250\ \mu\text{A} = 0.875\ \text{mA}$

$i_{D,\,min} = 0.625\ \text{mA} - 250\ \mu\text{A} = 0.375\ \text{mA}$

$v_{sig} = v_{gs} + v_o = 550\text{mVpk}$

5.68

$I_D = 2\,\text{mA} = \frac{1}{2} \times 80 \times 10^{-3} \times \frac{240}{6} \times (V_{GS} - 1.2)^2 \Rightarrow$

$V_{GS} = 2.32\,V$

$R_D I_D = \frac{15}{3} = 5^V \Rightarrow R_D = \frac{5}{2^{mA}} = 2.5^{k\Omega}$

$R_S I_D = 5^V \Rightarrow R_S = \frac{5}{2} = 2.5\,k\Omega$

$V_G = 5 + V_{GS} = 7.32\,V$

$\frac{15}{R_{G1} + R_{G2}} \times R_{G2} = 7.32 \qquad R_{G1} = 22\,M\Omega \Rightarrow R_{G2} = 20.97\,^{M\Omega}$

$V_{DS} = 5^V$

at the edge of saturation $V_{DS} = V_{GS} - V_t$ or

$V_{DS} = 2.32 - 1.2 = 1.12^V$. So V_{DS} is $5 - 1.12 = 3.88^V$

away from the edge of saturation.

5.69

$I_D = 2\,mA = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2 \Rightarrow 2 = \frac{1}{2} \times 50 \times 10^{-3} \times \frac{200}{4} V_{OV}^2$

$V_{OV} = 1.26\,V$

$V_{DS} = V_{OV}$ edge of triode

Midway of cutoff ($V_{DS} = V_{DD}$) and beginning of triode operation ($V_{DS} = V_{OV}$) is when $V_{DS} = \frac{30 + 1.26}{2}$

$V_{DS} = 15.63\,V$

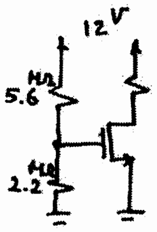
$V_{GS} = 2.32\,V \Rightarrow V_S = -2.32\,V \Rightarrow R_S = \frac{-2.32 + 15}{2}$

$\underline{R_S = 6.34\,k\Omega}$

$V_D = V_S + V_{DS} = -2.32 + 15.63 = 13.31^V \Rightarrow R_D = \frac{15 - 13.31}{2}$

$\underline{R_D = 0.85\,k\Omega}$

5.70



$V_G = 12 \times \frac{2.2}{2.2 + 5.6} = 3.4^V$

$k_n' \frac{W}{L} = 220$ to $380\,\mu A/V^2$

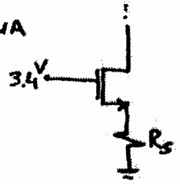
$V_t = 1.3$ to $2.4\,V$

$I_D = \frac{1}{2} \times k_n' \frac{W}{L} (3.4 - V_t)^2$

$I_{Dmin} = \frac{1}{2} \times 220 (3.4 - 2.4)^2 = 110\,\mu A$

$I_{Dmax} = \frac{1}{2} \times 380 (3.4 - 1.3)^2 = 838\,\mu A$

to limit I_{Dmax} to $150\,\mu A$:



$150 = \frac{1}{2} \times 380 (3.4 - 0.15 R_S - 1.3)^2$

$R_S = 8.1\,k\Omega$

Select $\underline{R_S = 8.2\,K\Omega}$

$I_{Dmax} = \frac{1}{2} \times 380 \times (3.4 - I_{Dmax} \times 8.2 - 1.3)^2$

$I_{Dmax} = 0.15\,mA$ or $0.\cancel{44}\,mA$

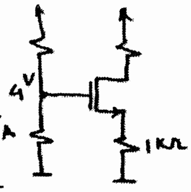
The second answer results in negative VGS

and therefore it is not acceptable.

$I_{Dmin} = \frac{1}{2} \times 0.22 \times (3.4 - 8.2 I_{Dmin} - 2.4)^2$

$\underline{I_{Dmin} = 0.04\,mA}$

5.71



$V_t = 2^V$, $k_n' \frac{W}{L} = 2\,mA/V^2$

$I_D = \frac{1}{2} \times 2 \times (4 - I_D \times 1 - 2)^2$

$I_D = 4 + I_D^2 - 4I_D \Rightarrow \underline{I_D = 1^{mA}}, \cancel{4^{mA}}$

$I_D = 4^{mA}$ results in $V_{GS} = 0$ which is not acceptable, therefore $I_D = 1\,mA$.

For $k' \frac{W}{L}$ 50% larger, i.e. $k' \frac{W}{L} = 3\,mA/V^2$

$I_D = \frac{1}{2} \times 3 (4 - I_D - 2)^2 \Rightarrow \underline{I_D = 1.13\,mA}$

I_D increases by 13%.

5.72

$V_{GS} = 5-2 = 3V$, $I_D = \frac{V_S}{R_S} = \frac{2}{1} = 2mA$

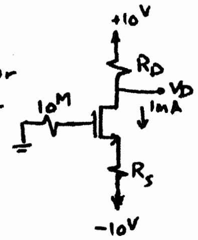
$I_D = 2 = \frac{1}{2} \times 2 \times (3-V_t)^2 \Rightarrow \pm 1.41 = 3 - V_t \Rightarrow \underline{V_t = 1.59^V}$

For a device with $V_t = 1.59 - 0.5 = 1.09^V$:

$I_D = \frac{1}{2} \times 2 \times (5 - I_D \times 1 - 1.09)^2 \Rightarrow \underline{I_D = 2.37^{mA}}$

$\underline{V_S = 2.37V}$

5.73



To maximize gain, we design for the lowest possible V_D consistant with allowing a 2V p-p signal swing. $V_{D\min} = V_D - 1$

$V_{D\min} = V_G - V_t = 0 - 2$

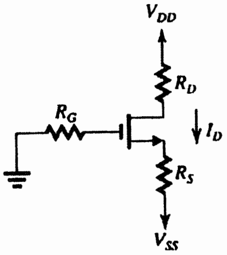
$V_D - 1 = -2 \Rightarrow V_D = -1V \Rightarrow R_D = \frac{10-(-1)}{1mA} = 11K\Omega$

$I_D = \frac{1}{2} \times 2\left[0-(-10+1 \times R_S) - 2\right]^2 = 1 \Rightarrow 1 = (8-R_S)^2$

$\underline{R_S = 7K\Omega}$

5.74

$$k = \frac{1}{2}k'\frac{W}{L}$$



a) $I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2$

$$I_D = K(0 + V_{SS} - R_S I_D - V_t)^2$$

$$\frac{\partial I_D}{\partial K} = (V_{SS} - R_S I_D - V_t)^2 +$$

$$+ 2K(V_{SS} - R_S I_D - V_t)(-R_S)\frac{\partial I_D}{\partial k}$$

$$\frac{\partial I_D}{\partial K} = \frac{I_D}{K} - 2R_S\sqrt{\frac{I_D}{K}}K\frac{\partial I_D}{\partial k}$$

$$\frac{\partial I_D}{\partial K}(1 + 2\sqrt{KI_D}\,R_S) = \frac{I_D}{K} \Rightarrow$$

$$S_K^{I_D} = \frac{\partial I_D}{\partial K}\frac{K}{I_D} = \frac{1}{1 + 2\sqrt{KI_D}\,R_S}$$

b) $K = 100\mu A / V^2, \frac{\Delta K}{K} = \pm 10\%,$

$V_t = 1\ V, I_D = 100\ \mu A$

$$\frac{\Delta I_D}{I_D} = \pm 1\ \%$$

$$S_K^{I_D} \simeq \frac{\partial I_D / I_D}{\frac{\partial k}{k}} = \frac{1}{10} = 0.1$$

$$= \frac{1}{1 + 2\sqrt{100 \times 10^{-3} \times 100 \times 10^{-3}}R_S}$$

$\Rightarrow R_S = 45\ k\Omega$

Now find V_{GS} and V_{SS} when $I_D = 100\ \mu A$ and $K = 100\ \mu A/V^2$ $100 = 100\,(V_{GS} - 1)^2$

$\Rightarrow V_{GS} = 2\ V$

Also $V_{GS} = V_{SS} - I_D R_S$

$2 = V_{SS} - 100 \times 10^{-6} \times 45 \times 10^3$

$\Rightarrow V_{SS} = 6.5\ V$

C. For $V_{SS} = 5\ V$

$$R_S = \frac{-V_{GS} + V_{SS}}{I_D}$$

$$= \frac{-2+5}{100 \times 10^{-6}} = 30\,k\Omega$$

$$S_k^{I_D} = \frac{1}{1 + 2\sqrt{100 \times 10^{-6} \times 100 \times 10^{-6} \times R_S}}$$

$= 0.14$

$\therefore$ For $\frac{\Delta K}{K} = \pm 10\%, \frac{\Delta I_D}{I_D} = \pm 1.4\%$

5.75

Both cases are in saturation region, because

$V_{DG} \geqslant V_t$.

$V_D = 10 - 5 \times 1 = 5^V$

a) $1 = \frac{1}{2} \times 0.5 \times (V_{GS}-1)^2 \Rightarrow V_{GS} = 3V$, $V_S = -3V$

$V_{DS} = 8V$

b) $1 = \frac{1}{2} \times 1.25 \times (V_{GS}-2)^2 \Rightarrow V_{GS} = 3.3V$, $V_S = -3.3^V$

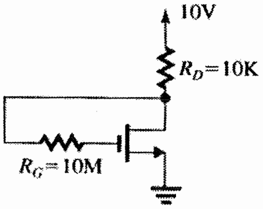
$V_{DS} = 8.3V$

5.76

$V_D = V_G = V_{GS}$

$V_{DS} \geq V_{GS} - V_t \Rightarrow V_{DG} \geq -V_t$

$V_{DG} = 0$



a) $\frac{10 - V_D}{10} = \frac{1}{2} \times 0.5 \times (V_D - 1)^2$

$\Rightarrow V_D = 2.7$ V

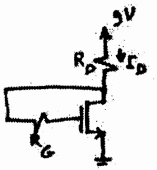
$V_G = 2.7$ V

b) $\frac{10 - V_D}{10} = \frac{1}{2} \times 1.25 \times (V_D - 2)^2$

$\Rightarrow V_D = 3.05$ V

$V_G = 3.05$ V

5.77



For $I_D = 0.2$ mA:

$0.2 = \frac{1}{2} \times 0.4 \times (V_{GS}-1)^2$

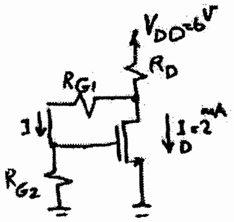
$V_{GS} = 2V$, $V_D = V_G = V_{GS} = 2V$

$R_D = \frac{9-2}{0.2} = 35 K\Omega$

Select $R_D = 36 K\Omega \Rightarrow \frac{9-V_D}{R_D} = \frac{1}{2} \times 0.4 (V_D - 1)^2$

$\frac{9-V_D}{36} = 0.2 (V_D - 1)^2 \Rightarrow V_D = 2V$, $I_D = 0.21$ mA.

5.78



$I_D = 2 = \frac{1}{2} \times 3.2 \times (V_{GS} - 1.2)^2$

$V_{GS} - 1.2 = 1.12 \Rightarrow V_{GS} = 2.32^V$

$V_G = 2.32^V$

$V_{DSmin} = V_{GS} - V_t = 1.12^V$

$V_{DS} = V_{DSmin} + 2 = 3.12^V$

$\underline{R_{G_2} = 22M\Omega} \Rightarrow I = \frac{2.32}{22} = 0.11 \mu A$

$R_{G_1} = \frac{3.12 - 2.32}{0.11} = \underline{7.58 M\Omega}$

$R_D = \frac{6 - 3.12}{2 + 0.11 \times 10^{-3}} = \underline{1.44 K\Omega}$

5.79

a)

$$A_{vo} = -2\frac{(V_{DD} - V_D)}{V_{OV}} = -\frac{2(10 - 2.5)}{1}$$

$= -15$ V / V

b) if V_{OV} is halved ($V_{OV} = 0.5$) then I_D is divided by 4, i.e. $I_D = \frac{0.5}{4} = 0.125$ mA

Since V_D is kept unchanged at 2.5 V then:

$$R_D = \frac{10 - 2.5}{0.25} = 60 \text{ k}\Omega,$$

$$g_m = \frac{2I_D}{V_{OV}} = 0.5 \text{ m}\frac{\text{A}}{\text{v}}$$

$$r_o = \frac{V_A}{I_D} \Rightarrow r_o = 4 \times r_{o1} = 4 \times \frac{75}{0.5} = 600 \text{ k}\Omega$$

$A_{vo} = -15 \times 2 = -30$V / V (without r_o)

c) If we take r_o into account :

$$A_{vo} = -g_m(r_o \| R_D) = -0.5\left(600^k \| 60^k\right)$$
$$= -27.3V/V$$

$$R_{\text{out}} = R_D \| r_o = 600^k \| 60^k = 54.5 \text{ k}\Omega$$

d) $R_{in} = R_G = 4.7 \text{ M}\Omega$

$R_o = 54.5 \text{ k}\Omega$

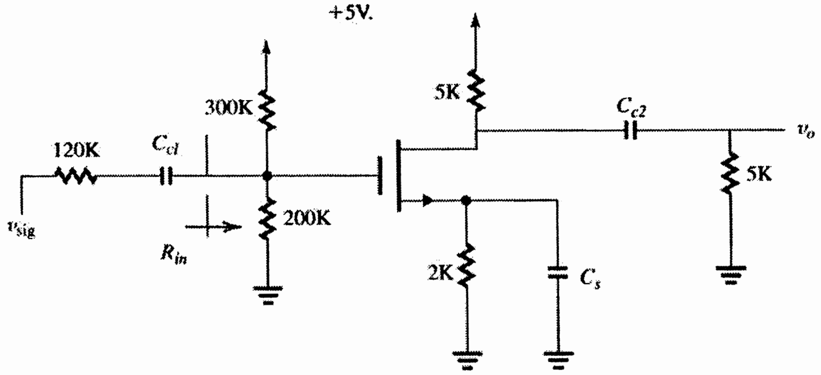
$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} A_{vo} \frac{R_L}{R_L + R_o}$$
$$= \frac{4.7}{4.7 + 0.1} \times 27.3 \times \frac{15}{15 + 54.5}$$
$$G_v = 5.77 \text{ V/V}$$

e) As we can see by reducing V_{OV} to half of its value or equivalently multiplying drain current by 4, A_{vo} is almost doubled, while R_{out} is multiplied by 4.

As a result G_v which is proportional to both A_{vo} and $\frac{1}{R_{\text{out}}}$ is only slightly reduced.

(G_v was -7 V/V before and it is 5.8 V/V now)

5.80



$V_t = 0.7$ V.
$V_A = 50$ V.
a) with $I_D = 0.5$ mA

$V_G = +2\text{V} \quad V_S + 1\text{V}. \quad V_{GS} = +1\text{V}.$

$V_{OV} = 0.3\text{V}$

$$0.5 \text{ mA} = \frac{1}{2} k_n V_{oV}^2 \Rightarrow k_n = 11.1 \frac{\text{mA}}{\text{V}^2}$$

$V_D = 5 - (5\text{ K})(0.5\text{ mA}) = +2.5\text{V}.$

$V_{GD} = -0.5\text{V} < V_t \quad \therefore$ Saturation

b) $R_{in} = 200\text{ K} \| 300\text{ K} = 120\text{ k}\Omega$

$$G_V = \frac{v_o}{v_{sig}} = -\frac{R_{in}}{120\text{ K} - R_{in}} g_m$$

$(5\text{ K} \| r_O \| 5\text{ K})$

$$g_m = \frac{2I_D}{V_{oV}} = 3.33\text{mS}$$

$$r_O = \frac{V_A}{I_D} = 100\text{ k}\Omega$$

$G_V = -4.1$

c) $v_{sig} = \hat{v}_{sig} \sin\omega t$

$g_m(5\text{K} \| 5\text{K} \| 100\text{K}) = 8.12$

$$V_{gd} + V_{GD} = \hat{v}_o + \frac{\hat{v}0}{8.12} - 0.5$$

$\leq V_t = 0.7\text{V}.$

$\hat{v}_{o'} \text{ max} = 1.07 \text{ V}_{\text{pk}}$

$$\therefore \hat{v}_{g'} \text{ max} = \frac{\hat{v}_{o'} \text{ max}}{8.12} = 132 \text{ mv}_{\text{pk}}$$

$$\hat{v}_{sig'} \text{ max} = \frac{\hat{v}_{o'} \text{ max}}{4.1} = 261 \text{ mv}_{\text{pk}}$$

d) Add $R_S = \dfrac{1}{g_m} = 300\ \Omega,$

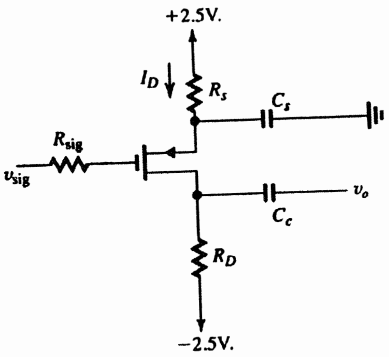
then $v_{gs} = \dfrac{v_g}{1 + g_m R_s} = \dfrac{v_g}{2}$

$$\frac{g_m R_L'}{1 + g_m R_S} = \left| \frac{v_O}{v_g} \right| = 4.06$$

$$\hat{v}_O + \frac{\hat{v}}{4.06} - 0.5 \leq 0.7 \text{ V}.$$

$\Rightarrow \hat{v}_{o'} \text{ max} = 0.96 \text{ V}.$

5.81



$V_{tp} = -0.7\text{V}, \quad V_A \to \infty$

a) for $I_D = 0.3$ mA, $|V_{OV}| = 0.3$ V.

$V_{SG} = 1.0$ V., $V_G = 0$

$V_S = 2.5 - I_D R_S = 1.0$ V.

$\therefore R_S = 5.0\ \text{k}\Omega$

b) $g_m = \dfrac{2I_D}{V_{OV}} = 2$ mS

$G_V = \dfrac{v_0}{v_{sig}} = -g_m R_D = -10$

$\therefore R_D = 5.0\ \text{k}\Omega$

c) $v_{gd} + V_{GD} \geq V_{tp} = -0.7$

$-\left|\hat{v}_o + \dfrac{\hat{v}_o}{10}\right| + 1\text{V} \geq -0.7$

$\hat{v}_O \leq 1.55\text{V}_{\text{pk}}$

$\hat{v}_{sig} \leq \dfrac{\hat{v}_{o},\max}{10} = 0.155\text{V}_{\text{pk}}$

d) for $\hat{v}_{sig} = 50$ mV, changed R_D

$-\left|\hat{v}_O + \dfrac{\hat{v}_O}{g_m R_D}\right| + (2.5 - I_D R_D) \geq -0.7$

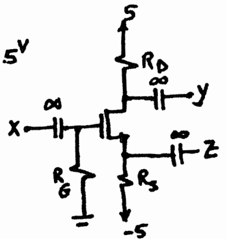
for $g_m = 2$ mS, $I_D = 0.3$ mA

$-\left|\dfrac{1 + g_m R_D}{g_m R_D}\right| g_m R_D \hat{v}_{sig} + 2.5 - I_D R_D \geq -0.7$

$R_D \leq 7.88\ \text{k}\Omega \qquad (\hat{v}_{sig} = 50\ \text{mV})$

$G_V = -g_m R_D = -15.8$

5.82



a) $I_D = 0.1 = \frac{1}{2} \times 0.8 \times V_{ov}^2 \Rightarrow V_{ov} = 0.5^V$

$\Rightarrow V_{GS} = 0.5 + 1 = 1.5^V$

$V_G = 0 \Rightarrow V_S = -1.5\text{V}$

$R_S = \dfrac{-1.5-(-5)}{0.1} = \underline{35\text{K}\Omega}$

$V_{DS} = 5 - R_D \times 0.1$

Largest possible R_D is achieved for $V_{DS\min}$

$V_{DS} \geqslant V_{GS} - V_t \Rightarrow V_{DS\,min} = V_{ov} \Rightarrow V_{DS} - 1 = V_{ov}$

$\Rightarrow V_{DS} = 1 + 0.5 = 1.5\text{V} \Rightarrow R_D = \dfrac{5-1.5}{0.1} = \underline{35\text{K}\Omega}$

$\underline{R_G = 10\text{M}\Omega}$.

b) $g_m = \dfrac{2 I_D}{V_{ov}} = \dfrac{2 \times 0.1}{0.5} = 0.4\ \text{mA/V}$

$r_o = \dfrac{V_A}{I_D} = \dfrac{40}{0.1} = 400\text{K}\Omega$

c) IF z is grounded then the circuit becomes a common-source configuration. The voltage gain according to Eq. 4.82:

$G_v = -\dfrac{R_G}{R_G + R_{sig}}\, g_m\, (r_o \| R_D \| R_L)$

$G_v = \dfrac{10\text{M}}{10^\text{M} + 1^\text{m}} \times 0.4 \times (400^\text{K} \| 35^\text{K} \| 40^\text{K}) = 6.5\ \text{V/V}$

$\underline{G_v = 6.5\ \text{V/V}}$

d) IF y is grounded, then the circuit becomes a source follower configuration.

Eq. 4.103 : $A_{v_o} = \dfrac{r_o}{r_o + \frac{1}{g_m}} = \dfrac{400}{400 + \frac{1}{0.4}} = 0.99\text{V/V}$

$R_{out} = \frac{1}{g_m} \| r_o = \frac{1}{0.4} \| 400$

$\underline{R_{out} = 2.48\text{K}\Omega}$

e) If x is grounded, the circuit becomes a common-gate configuration.

$R_{in} = \frac{1}{g_m} \| R_S = 35\text{K} \| \frac{1}{0.4} = 2.33\text{K}\Omega$

Eq. 4.98: $i_i = i_{sig} \dfrac{R_{sig}}{R_{sig} + R_{in}} \Rightarrow$

$i_i = 10\mu\text{A}\ \dfrac{100^\text{K}}{100^\text{K} + 2.33^\text{K}} = 9.77\mu\text{A}$

$v_y = R_D \times i_i = 35 \times 9.77\mu\text{A} = \underline{0.34\text{V}}$

5.83

a) is a source Follower:

$$A_{vo} = \frac{r_o}{r_o + \frac{1}{g_m}},$$

$$r_o \gg \frac{1}{g_m} \Rightarrow A_{vo} \simeq 1\text{V / V}$$

$$R_{\text{out}} = \frac{1}{g_m} = \frac{1}{5} = 0.2\ \text{k}\Omega$$

b) is a common - gate configuration:

$$R_{\text{in}} = \frac{1}{g_m} = \frac{1}{5} = 0.2\ \text{k}\Omega$$

$$A_v = g_m(R_D \| R_L) = 5(5\text{K} \| 2\text{K}) = 7.1\ \text{V/V}$$

c) If we connect both stages together, then: for the first stage: $A_v = A_{vo}\frac{R_L}{R_L + R_{\text{out}}}$

where R_L is fact R_{in} of the second stage.

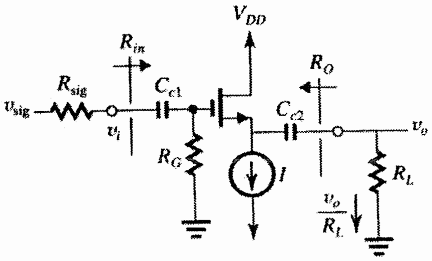
Therefore: $A_{v_1} = 1 \times \frac{0.2\text{K}}{0.2 + 0.2} = 0.5\text{V / V}$

For the second stage : $A_{v_2} = 7.1\ \text{V / V}$

overall gain

$$A_v = A_{v_1}A_{v_2} = 7.1 \times 0.5 = 3.55\text{V / V}$$

5.84



5.85

$V_{to} = 1.0$ V.

$r = 0.5$ V. $^{1/2}$

$2\phi_f = 0.6$ V.

$0 < V_{SB} < 4$V.

$$V_t = V_{to} + r[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}]$$

for $0 < V_{SB} < 4$V., $V_{to} < V_t < V_{to} + 0.685$V.

so 1V. $< V_t <$ 1.68 V.

Since $r = \sqrt{\frac{2qN_A\epsilon_S}{C_{ox}}}$, an increase of 4x in t_{ox} makes C_{ox} 4x lower. and V_t becomes

1V $< V_t <$ 3.74 V.

5.86

The test for region of operation for a depletion mode MOSFET is the same as for a enhancement mode MOSFET. The threshold voltage is negative; however.

$v_t = -3$ Volts, $v_G = 0$, $v_S = 0$ P$v_{GS} = 0$

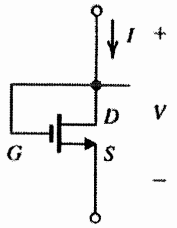
a) $v_D = 0.1$ Volts P$v_{DS} = 0.1$ and $v_{GS} - v_t = 3$ Pv_{DS} – v_t, so transistor is in the triode region

b) $v_D = 1$ Volts P$v_{DS} = 1$ and $v_{GS} - v_t = 3$ P$v_{DS} < v_{GS} - v_t$, so transistor is in the triode region.

c) $v_D = 3$ Volts P$v_{DS} = 3$ and $v_{GS} - v_t = 3$ P$v_{DS} = v_{GS} - v_t$, so transistor is at triode-saturation boundary.

d) $v_D = 5$ Volts P$v_{DS} = 51$ and $v_{GS} - v_t = 3$ P$v_{DS} > v_{GS} - v_t$, so transistor is in the saturation region.

5.87



$V_{GS} = V_{DS} = V$ V_t is negative so

$V_{DS} < V_{GS} - V_t$ (always)

First, when $V = V_{GS} > V_t$

- From TABLE 5.1, this is triode region

$$i_D = k_n'\left(\frac{W}{L}\right)\left[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

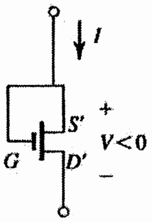
$$= k_n'\left(\frac{W}{L}\right)\left[(V - V_t)V - \frac{1}{2}V^2\right]$$

$$= k_n'\left(\frac{W}{L}\right)\left[\frac{1}{2}V^2 - V_tV\right]$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)[V^2 - 2V_tV]$$

Note that when $V < 0$, $I = i_D$ is negative.

- When $V = V_{GS} < V_t$, AND assuming the device can operate symmetrically with D acting as the source and S acting as the drain, the circuit can be modeled as below. In this configuration



$V_{GS'} = 0 > V_t \;\; V_{D'S'} = -V$

($V_{D'S'}$ is therefore positive)

Since $V_{GD'} = V < V_t$ this is saturation region (see Table 5.1)

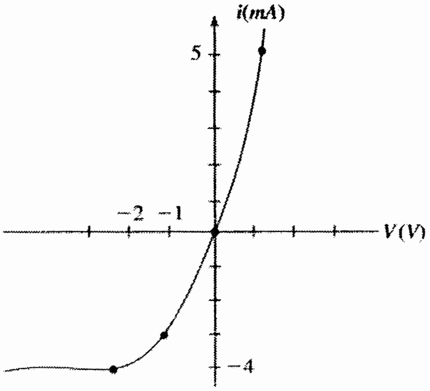
so

$$I = -i_D = -\frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{G'S'} - V_t)^2$$

$$= -\frac{1}{2}k_n'\left(\frac{W}{L}\right)(O - V_t)^2$$

$$= -\frac{1}{2}k_n'\left(\frac{W}{L}\right)V_t^2$$

$$V_t = -2 \text{ V},\; k_n'\left(\frac{W}{L}\right) = 2 \text{ m}\frac{\text{A}}{\text{V}^2}$$



$$i = -\frac{1}{2}k_n'\left(\frac{W}{L}\right)(V^2 - 2V_t v) \qquad V \geq V_t$$

$$= 1(\text{mA}/\text{V}^2)(V^2 + 4\text{V}/V)$$

$$i = -\frac{1}{2}k_n'\left(\frac{W}{L}\right)V_t^2 \quad V \leq V_t$$

$$= -\frac{1}{2}(2 \text{ mA}/\text{V}^2)(-2 \text{ V})^2$$

$$= -4 \text{ mA}$$

6.1

For I = 10 μA:

$$g_m = \frac{I}{V_T} = \frac{10\ \mu\text{A}}{25\ \text{mV}} = 0.4\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.4\ \text{mA/V}} = 250\ \text{k}\Omega$$

$$r_o = \frac{V_A}{I} = \frac{10\ \text{V}}{10\ \mu\text{A}} = 1\ \text{M}\Omega$$

$$A_o = g_m r_o = \frac{V_A}{V_T} = \frac{10\ \text{V}}{0.025\ \text{V}} = 400\ \text{V/V}$$

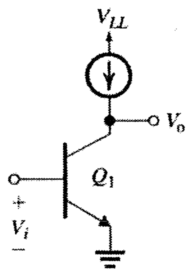
For I = 100 μA:

$$g_m = \frac{100\ \mu\text{A}}{25\ \text{mV}} = 4\ \text{mA/V}$$

$$r_\pi = \frac{100}{4\ \text{mA/V}} = 25\ \text{k}\Omega$$

$$r_o = \frac{10\ \text{V}}{100\ \mu\text{A}} = 100\ \text{k}\Omega$$

$$A_o = 4\text{mA/V}(100\ \text{k}\Omega) = 400$$



For I = 1 mA:

$$g_m = \frac{1\ \text{mA}}{25\ \text{mV}} = 40\ \text{mA/V}$$

$$r_\pi = \frac{100}{40\ \text{mA/V}} = 2.5\ \text{k}\Omega$$

$$r_o = \frac{10\ \text{V}}{1\ \text{mA}} = 10\ \text{k}\Omega$$

$$A_o = 40\ \text{mA/V}\ (10\ \text{K}) = 400$$

I	g_m	r_π	r_o	A_o
10 μA	0.4 mA/V	250 kΩ	1 MΩ	400
100 μA	4.0 mA/V	25 kΩ	100 kΩ	400
1 mA	40 mA/V	2.5 kΩ	10 kΩ	400

6.2

$$g_m = \frac{I_D}{\frac{V_{OV}}{2}}, \text{so}$$

$$I_D = \frac{g_m V_{OV}}{2} = \frac{2\ \text{mA/V}(0.25\ \text{V})}{2} = 0.25\ \text{mA}$$

From chapt. 5, $k_n' = \mu_n C_{OX}$

since $g_m = \sqrt{2\mu_n C_{ox}(W/L)}\sqrt{I_D}$,

$$2\ \text{mA/V} = \sqrt{2(200\ \mu\text{A/V}^2)(W/L)(250\ \mu\text{A})}$$

yielding

$W/L = 40$

so that

$W = 40(0.5\ \mu\text{m}) = 20\ \mu\text{m}$

6.3

Assuming that the MOSFET is operating above V_t,

$$A_o = \frac{V_A'\sqrt{2(\mu_n C_{ox})(WL)}}{\sqrt{I_D}}$$

If I_D is decreased to 25 μA,

A_o is increased by $\frac{1}{\sqrt{1/4}} = 2$

$$g_m = \sqrt{2(\mu_n C_{ox})(W/L)} \cdot \sqrt{I_D}$$

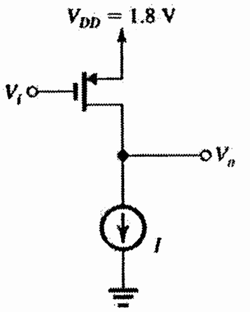
so, g_m is decreased by

$\sqrt{1/4} = 1/2$

If I_D is increased to 400 μA,

A_o is decreased by $\frac{1}{\sqrt{4}} = \frac{1}{2}$

g_m increases by $\sqrt{4} = 2$



The edge of the Saturation region is defined as when $|V_{DS}| = |V_{GS}| - |V_t| = |V_{OV}|$

$\therefore$ The highest instantaneous output voltage is

$V_{DD} - |V_{OV}| = 1.8 - 0.3 = 1.5$ V

6.4

$$L = 2\ (0.18\ \mu m) = 0.36\ \mu m$$

a) $g_m = \dfrac{I_D}{\frac{V_{OV}}{2}} = \dfrac{10\ \mu A}{\frac{0.25}{2}} = 80\ \dfrac{\mu A}{V}$

$$V_A' = 5\ V/\mu m$$

so,

$$r_O = \frac{V_A' L}{I_D} = \frac{5\ V/\mu m\ (2)\ (0.18\ \mu m)}{10\ \mu A} = 180\ k\Omega$$

$$A_O = \frac{2V_A' L}{V_{OV}} = \frac{2\ (5\ V/\mu m)\ (0.36\ \mu m)}{0.25\ V}$$

$= 14.4$ V/V

b) with $I_D = 10\ \mu A$

$$k_n = \frac{2I_D}{V_{OV}^2} = \frac{2(10\ \mu A)}{(0.25\ V)^2} = 320\ \mu V/A^2$$

Solving for V_{OV} with $I_D = 100\ \mu A$:

$$V_{OV} = \frac{2I_D}{k_n} \rightarrow$$

$$V_{OV} = \sqrt{\frac{2(100\ \mu A)}{320\ \mu A/V^2}} = 0.79\ \text{Volts}$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{100\ \mu A}{0.79\ V/2} = 253\ \mu A/V$$

$$r_O = \frac{V_A' L}{I_D} = \frac{5V/\mu m\ (0.36\mu m)}{100\ \mu A} = 18\ k\Omega$$

$$A_O = g_m r_O = 253\ \mu A/V(18\ k\Omega) = 4.56\ V/V$$

c) Now, with a new W and $V_{OV} = 0.25$ V,

$I_D = 100\ \mu A$,

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{100\ \mu A}{0.25\ V/2} = 800\ \mu A/V$$

$$r_O = \frac{V_A' L}{I_D} = \frac{5\ V/\mu m(0.36\ \mu m)}{100\ \mu A} = 18\ k\Omega$$

$$A_O = \frac{2V_A' L}{V_{OV}} = \frac{(2)(5V/\mu m)(0.36\ \mu m)}{0.25\ V}$$

$= 14.4$ V/V

d) I_D is now $10\ \mu A$, first, find k_n:

$$k_n = \frac{2I_D}{V_{OV}^2} = \frac{2(100\ \mu A)}{(0.25\ V)^2} = 3\ mA/V^2$$

so, now with $I_D = 10\ \mu A$,

$$V_{OV} = \sqrt{\frac{2I_D}{k_n}} = \sqrt{\frac{2(10\ \mu A)}{3.2\ mA/V}} = 0.079\ V$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{10\ \mu A}{0.079/2V} = 253\ \mu A/V$$

$$r_O = \frac{V_A' L}{I_D} = \frac{(5V/\mu m)(0.36\ \mu m)}{10\ \mu A} = 180\ k\Omega$$

$$A_O = \frac{2V_A' L}{V_{OV}} = \frac{2(5V/\mu m)(0.36\ \mu m)}{0.079\ V}$$

$= 45.6$ V/V

e) The lowest A_O is 4.56 V/V

when $V_{OV} = 0.79$ V, $I_D = 100\ \mu A$,

$L = 0.36\ \mu m$

The highest A_O is 45.6 V/V

with $I_D = 10\ \mu A$, $V_{OV} = 0.079$ V

If W/L is held constant, and L is increased 10 times,

since $A_O = \dfrac{2V_A' L}{V_{OV}}$ (or since g_m remains constant, and r_O is increased by L)

Each gain is increased by a factor of 10:

Low $A_O = 45.6$ V/V

High $A_O = 456$ V/V

6.5

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$$

$$\frac{W}{L} = \frac{2I_D}{k_n' V_{OV}^2} = \frac{2(100\ \mu A)}{200\ \mu A/V^2(0.25\ V)^2} = 16$$

so, $W = 16(0.4\ \mu m) = 6.4\ \mu m$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{100\ \mu A}{\left(\frac{0.25\ V}{2}\right)} = 800\ \mu A/V$$

$$r_o = \frac{V_A' L}{I_D} = \frac{20\ V/\mu m(0.4\ \mu m)}{100\ \mu A} = 80\ k\Omega$$

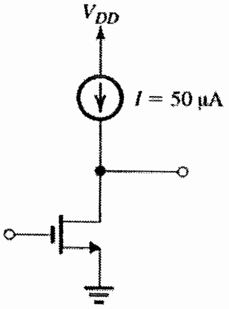
If $L = 0.8\ \mu m$,

$W = 0.8\ \mu m(16) = 12.8\ \mu m$

$$g_m = \frac{100\ \mu A}{\left(\frac{0.25\ V}{2}\right)} = 800\ \mu A/V$$

$$r_o = \frac{V_A' L}{I_D} = \frac{20\ V/\mu m(0.8\ \mu m)}{100\ \mu A} = 160\ k\Omega$$

6.6



Since $A_0 = \frac{2V_A' L}{V_{OV}}$, and the current source is ideal,

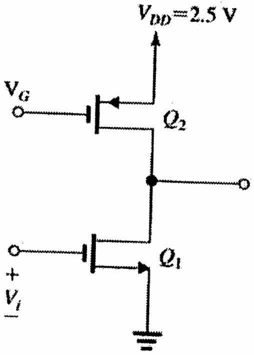
$$L = \frac{A_0 V_{OV}}{2V_A'} = \frac{100(0.2\ V)}{2(20\ V/\mu m)} = 0.5\ \mu m$$

Since $I_D = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)V_{OV}^2$,

$$\frac{W}{L} = \frac{2I_D}{(\mu_n C_{ox})V_{OV}^2}$$

$$= \frac{2\ (50\ \mu A)}{(200\ \mu A/V^2)(0.2\ V)^2} = 12.5$$

6.7



$$V_G = V_{DD} - V_{SD2}$$
$$= V_{DD} - |V_{tp}| - |V_{ov}|$$
$$= 2.5 - 0.5 - 0.3 = 1.7\ V$$

Since $I_{D_1} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)_1 V_{OV}^2$

$$\left(\frac{W}{L}\right)_1 = \frac{2I_{D1}}{(\mu_n C_{ox})V_{OV}^2}$$

$$= \frac{2(100\ \mu A)}{(200\ \mu A/V^2)(0.3V)^2} = 11.1$$

for Q$_2$, $\left(\frac{W}{L}\right)_2 = \frac{2I_{D2}}{(\mu_p C_{ox})|V_{ov}|^2}$

$$= \frac{2\ (100\ \mu A)}{(100\ \mu A/V^2)(0.3)^2} = 22.2$$

Since $V_{An}' = |V_{Ap}'| = 20\ V/\mu m$

$$r_{O1} = r_{O2} = r_O = \frac{V_A' L}{I} = \frac{20\ V/\mu m\ (0.5\ \mu m)}{100\ \mu A}$$

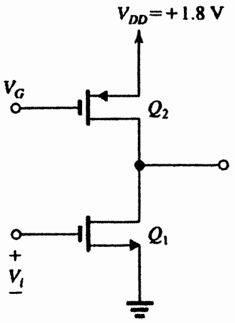
$$g_m = \frac{I_{D1}}{\frac{V_{OV}}{2}} = \frac{100\ \mu A}{0.3/2\ V} = 667\ \mu A/V$$

$$r_o = 100\ k\Omega$$

so,

$$A_V = \frac{1}{2}g_m r_o = -\frac{1}{2}(667\ \mu A/v)(100\ k\Omega)$$
$$= -33.3\ V/V$$

6.8



$V_G = V_{DD} - |V_{tp}| - |V_{OV}|$

$= 1.8 - 0.5 - 0.2 = 1.1$ V

$g_{m1} = \frac{I_D}{V_{OV}/2} = \frac{100\ \mu\text{A}}{0.2\text{V}/2} = 1$ mA/V

$A_v = -g_{m1}(r_{O1} \| r_{O2})$ so we must find r_{O1} and r_{O2}

$r_{O1} \| r_{O2} = \frac{A_v}{-g_{m1}} = \frac{-40}{-1\ \text{mA/V}} = 40\ \text{k}\Omega$

since $r_{O1} = \frac{V'_{An}}{I_D}$ and $r_{O2} = \frac{|V'_{Ap}|\text{L}}{I_D}$

$r_{O1} = \frac{5\ \text{V}/\mu\text{m}}{100\ \mu\text{A}} \cdot L = \frac{50\ \text{K}}{\mu\text{m}} \cdot \text{L}$

$r_{O2} = \frac{6\ \text{V}/\mu\text{m}}{100\ \mu\text{A}} \cdot L = \frac{60\ \text{K}}{\mu\text{m}} \cdot \text{L}$

so,

$40\ \text{k}\Omega = \frac{50\ \text{k}\Omega/\mu\text{m}(60\ \text{k}\Omega/\mu\text{m}).\text{L}^2}{50\ \text{k}\Omega/\mu\text{m} \cdot \text{L} + 60\ \text{k}\Omega/\mu\text{m} \cdot \text{L}}$

or $L = 1.47\ \mu$m

since $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \text{V}_{OV}^2$,

$\left(\frac{W}{L}\right)_1 = \frac{2I_{D1}}{\mu_n C_{ox} V_{OV}^2}$

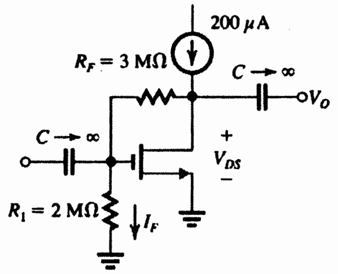
$= \frac{2(100\ \mu\text{A})}{387\ \mu\text{A}/\text{V}^2(0.2\ \text{V})^2} = 12.9$

similarly,

$\left(\frac{W}{L}\right)_2 = \frac{2I_{D2}}{\mu_p C_{ox}|V_{OV}|^2}$

$= \frac{2(100\ \mu\text{A})}{86\ \mu\text{A}/\text{V}^2\ (0.2\ \text{V})^2} = 58.1$

6.9



(a) If we neglect the current through R_F,

$I_D = 200\ \mu\text{A} = \frac{1}{2}k'_n(W/L)V_{OV}^2$

$V_{OV} = \sqrt{\frac{2I_D}{k'_n(W/L)}} = \sqrt{\frac{2(200\ \mu\text{A})}{2\ \text{mA/V}^2}} = 0.45$ V

$V_{GS} = V_t + V_{OV} = 0.5 + 0.45 = 0.95$ V

The current through the feedback network is

$I_F = \frac{V_G}{R_1} = \frac{0.95\ \text{V}}{2\ \text{M}\Omega} = 0.475\ \mu\text{A}$

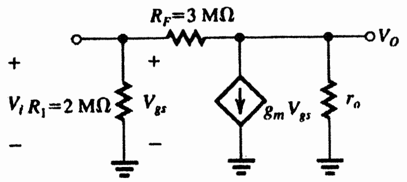
This is << 200 μA, so this assumption is justified.

$V_{DS} \simeq$

$I_F(R_F + R_1) = 0.475\ \mu\text{A}(3\text{M}\Omega + 2\text{M}\Omega)$

$= 2.38\ \text{V} \simeq 2.4\ \text{V}$

(b) small-signal model:



KCL at the output node yields

$\frac{V_O}{r_O} + g_m V_{gs} + \frac{V_O - V_i}{R_F} = 0$

since $V_{gs} = V_i$

$\frac{V_O}{r_O} + g_m V_i + \frac{v_O}{R_F} - \frac{V_i}{R_F} = 0$ or

$\frac{V_O}{V_i} = \frac{\left(\frac{1}{R_F} - g_m\right)}{\left(\frac{1}{r_O} + \frac{1}{R_F}\right)}$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(200\ \mu\text{A})}{0.45\ \text{V}} = 0.89\ \text{mA / V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20\ \text{V}}{200\ \mu\text{A}} = 100\ \text{k}\Omega$$

so,

$$\frac{V_D}{V_i} = \frac{\dfrac{1}{3000\ \text{K}} - 0.89\ \text{mA / V}}{\dfrac{1}{100\ \text{K}} + \dfrac{1}{3000\ \text{K}}} = -86.1\ \text{V / V}$$

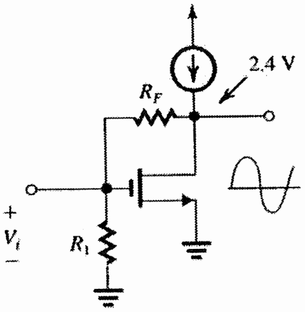
To find the peak of the maximum sinewave output possible, we note that the current source is assumed to be ideal. Therefore, sinewave amplitude will be limited by the negative excursion. Since this happens when

$V_{DS} = V_{OV} = 0.45\ \text{V},$

the maximum peak amplitude will be

$2.4 - 0.45 = 1.95\ \text{V}$

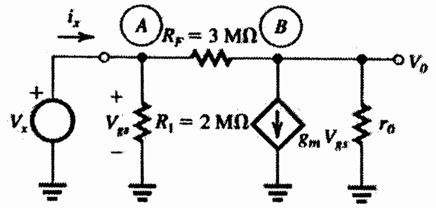
(That is, the output will vary between 0.45V and $2.4 + 1.95 = 4.35\ \text{V}$.)



The corresponding input voltage is

$$V_{i_{\text{peak}}} = \frac{V_{o_{\text{peak}}}}{|A_V|} = \frac{1.95\ \text{V}}{86.1\ \text{V / V}} = 23\ \text{mV}_{\text{peak}}$$

(c) To find R_{in}, we apply a test voltage V_X to the input



KCL at node A:

$$i_X = \frac{V_X}{R_1} + \frac{V_X - V_O}{R_F}$$

KCL at node B:

$$\frac{V_X - V_O}{R_F} = \frac{V_O}{r_O} + g_m V_X$$

$$\Rightarrow V_O = \frac{V_X\left(\dfrac{1}{R_F} - g_m\right)}{\dfrac{1}{r_O} + \dfrac{1}{R_F}}$$

Substituting into the first equation, we get

$$i_X = \frac{V_X}{R_1} + \frac{V_X}{R_F} - \frac{V_X}{R_F}\,\frac{\left(\dfrac{1}{R_F} - g_m\right)}{\left(\dfrac{1}{r_O} + \dfrac{1}{R_F}\right)}$$

so that

$$R_{\text{in}} = \frac{V_X}{i_X}$$

$$= \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_F} - \dfrac{\dfrac{1}{R_F{}^2}}{1/r_O + 1/R_F} + \dfrac{g_m/R_F}{\dfrac{1}{r_O} + \dfrac{1}{R_F}}}$$

$$R_{\text{in}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_F} - \dfrac{1}{\dfrac{(R_F)^2}{r_O} + R_F} + \dfrac{g_m}{\dfrac{R_F}{r_O} + 1}}$$

$$R_{\text{in}} = \frac{1}{\dfrac{1}{2\ \text{m}\Omega} + \dfrac{1}{3\ \text{m}\Omega} - \dfrac{1}{\dfrac{(3\ \text{m}\Omega)^2}{0.1\ \text{m}\Omega} + 3\ \text{m}\Omega} + \dfrac{0.89\ \text{mA / V}}{\dfrac{3\ \text{m}\Omega}{0.1\ \text{m}\Omega} + 1}}$$

$$R_{\text{in}} = 33.9\ \text{k}\Omega$$

6.10

the transfer characteristic of the amplifier over the region labeled as segment III, is quite linear.

$V_{OA} = V_{DD} - V_{OV3} = 5 - 0.53 = 4.47\text{ V}$

Now to Find the linear equation for segment III, we can write $i_{D1} = i_{D2}$:

$$\frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 (v_I - v_{tn})^2\left(1 + \frac{v_O}{V_{An}}\right)$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2 (v - |v_{tp}|)^2\left(1 + \frac{V_{DD} - v_O}{V_{AP}}\right)$$

$$\Rightarrow 200(V_I - 0.6)^2\left(1 + \frac{v_O}{20}\right)$$

$$= 65 \times 0.53^2 \times \left(1 + \frac{V_{DD} - V_O}{10}\right)$$

$$(V_{56} - |V_{tp}|)^2\left(1 + \frac{V_{DD} - v_O}{V_{A_P}}\right)$$

$$\frac{200}{65 \times 0.53^2}(V_I - 0.6)^2 = \frac{1.5 - v_O/10}{1 + \frac{v_O}{20}}$$

$$7.3(v_I - 0.6)^2 = \frac{1 - v_O/15}{1 + \frac{v_O}{20}}$$

$$= \frac{1 - 0.067\ v_O}{1 + 0.05\ v_O} \simeq 1 - 0.117\ v_O$$

$$\Rightarrow v_O = 8.57 - 62.57(v_I - 0.6)^2$$

If we substitute for $v_{OA} = 4.47\text{ V}$, then

$V_{IA} = 0.86\text{ V}$

To determine coordinates of B, note that

$V_{IB} - V_{tn} = V_{OB}$ or $V_{IB} - 0.6 = V_{OB}$

Substitute in 1:

$V_{OB} = 8.57 - 62.57 v_{OB}^2 \Rightarrow \quad V_{OB} = 0.36\text{ V}$

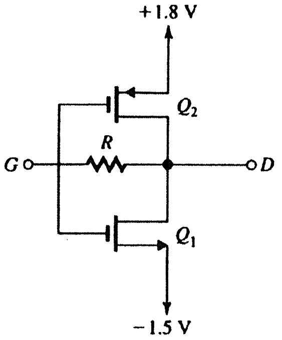
$V_{IB} = 0.6 + 0.36 = 0.96\text{ V}$

Therefore the linear region is :

$0.86\text{ V} \le V_I \le 0.96\text{ V}$ or

$0.36\text{ V} \le V_O \le 4.47\text{ V}$

6.11



(a) If G and D are open, and no current flows to either gate,

$V_D = V_G$ and $I_{D1} = I_{D2}$

$$I_{D1} = \frac{1}{2}k_n'(W/L)_1(V_G - V_S - V_t)^2$$

$$= I_{D2} = \frac{1}{2}k_p'(W/L)_2(V_{DD} - V_G - |V_t|)^2$$

or, $(V_G - (-1.5V) - 0.5V)^2 =$

$(1.5V - V_G - 0.2\text{ V})^2$

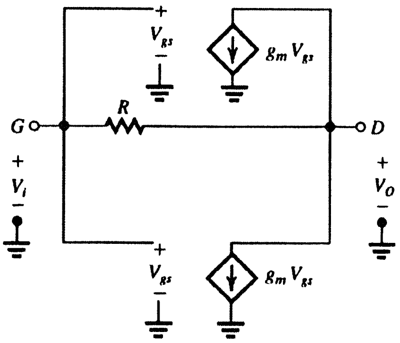
$(V_G + 1)^2 = (1 - V_G)^2 \Rightarrow V_G = 0$

so,

$$I_{D2} = I_{D1} = \frac{1}{2}(1\text{ mA/V}^2)(0 + 1)^2$$

$$= 0.5\text{ mA}$$

(b) For $r_O = \infty$, the small-signal model becomes:



$V_O = V_i - 2(g_m V_{gs})R$

$V_{gs} = V_i$, so

$V_O = V_i - 2\ g_m R v_i$

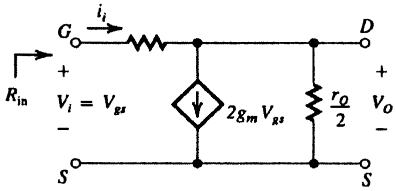
$$Av = \frac{v_O}{V_i} = 1 - 2\ g_m R$$

substituting values,

$$Av = 1 - 2(1\ \text{mA/V})(1\ \text{M}\Omega) = -1999\ \text{V/V}$$

(c) $r_O = \dfrac{|V_A|}{|I_D|} = \dfrac{20\ \text{V}}{0.5\ \text{mA}} = 40\ \text{k}\Omega$

Adding r_{O_1} and r_{O_2} to the model, we get



KCL at D yields,

$$\frac{V_i - V_O}{R} = 2\,g_m V_{gs} + \frac{V_O}{r_{O/2}} \quad \text{and since}$$

$V_{gs} = V_i$,

$$\frac{V_i}{R} - 2\,g_m V_i = \frac{V_O}{R} + \frac{2V_O}{r_O} \quad \text{so that}$$

$$Av = \frac{v_O}{v_i} = \frac{\frac{1}{R} - 2g_m}{\frac{1}{R} + \frac{2}{r_O}} = \frac{1 - 2g_m R}{1 + \frac{2R}{r_O}}$$

Substituting numbers, we get:

$$Av = \frac{1 - 2(1\ \text{mA/V})(1000\ \text{k}\Omega)}{1 + \frac{2000\ \text{k}\Omega}{40\ \text{k}\Omega}}$$

$$= -39.2\ \text{V/V}$$

To find R_{in}, note that

$$R_{\text{in}} = \frac{V_i}{i_i}$$

$$i_i = \frac{V_i - V_O}{R} \quad \text{since} \quad V_O = V_i\left(\frac{1 - 2\,g_m R}{1 + \frac{2R}{r_O}}\right),$$

$$i_O = \frac{V_i\left[1 - \left(\frac{1 - 2\,g_m R}{1 + \frac{2R}{r_O}}\right)\right]}{R}$$

so that,

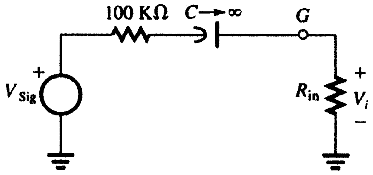
$$R_{\text{in}} = \frac{V_i}{i_i} = \frac{R}{1 - \left(\frac{1 - 2\,g_m R}{1 + \frac{2R}{r_O}}\right)}$$

Substituting in numerical values,

$$R_{\text{in}} = \frac{1\ \text{M}\Omega}{1 - \left[\frac{1 - 2(1\ \text{mA/V})(1\ \text{M}\Omega)}{1 + (2)(1\ \text{mA/V})(40\ \text{k}\Omega)}\right]} = 24.9\ \text{K}$$

$\simeq 25\ \text{k}\Omega$

(d) If the gate is driven as shown:



$$\frac{V_D}{V_{\text{sig}}} = \frac{R_{\text{in}}}{100\ \text{k}\Omega + R_{\text{in}}} \cdot A_v$$

$$= \frac{25\ \text{k}\Omega}{100\ \text{k}\Omega + 25\ \text{k}\Omega} \cdot (-39.2\ \text{V/V}) = -7.84\ \text{V/V}$$

(e) $|v_{DS}|$ must be $\geq |V_{OV}|$

with $V_G = 0$, $V_{GS1} = 1.5\ \text{V}$, $V_{SG2} = 1.5\ \text{V}$

$\therefore |V_{OV}| = 1.5 - 0.5 = 1.0\ \text{V}$

Given the ± 1.5 V supplies,

$-0.5\ \text{V} \leq v_O \leq 0.5\ \text{V}$

6.12

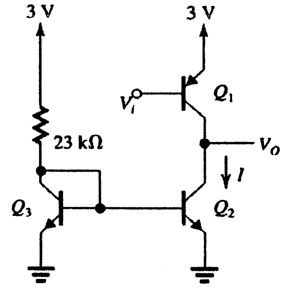
a) $I_{\text{REF}} = I_{C3} = \dfrac{3 - V_{BE3}}{23\ \text{k}\Omega}$

$I_{\text{REF}} = \dfrac{3 - 0.7}{23}$

$I_{\text{REF}} = 0.1\ \text{mA}$

$\Rightarrow I_{C2} = 5I_{C3}$

$I_{C2} = I = 0.5\ \text{mA} \Rightarrow I = 0.5\ \text{mA}$



b)

$|V_A| = 50V \Rightarrow r_{O1} = \dfrac{|V_A|}{I} = \dfrac{50}{0.5} = 100\ \text{k}\Omega$

$r_{O2} = \dfrac{50}{0.5} = 100\ \text{k}\Omega$

Total resistance at the collector of Q_1 is equal to $r_{O1} \parallel r_{O2}$, thus:

$r_{tot} = 100\text{ k}\Omega \parallel 100\text{ k}\Omega = 50\text{ k}\Omega$

$r_{tot} = 50\text{ k}\Omega$

c) $g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.5}{0.025} = 20\text{ mA/V}$

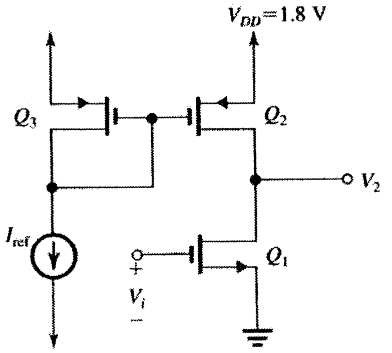
$r_{\pi 1} = \frac{\beta}{g_m} = \frac{50}{20} = 2.5\text{ k}\Omega$

d) $R_{in} = r_{\pi 1} = 2.5\text{ k}\Omega$

$R_O = r_{O1} \parallel r_{O2} = 100\text{ k}\Omega \parallel 100\text{ k}\Omega = 50\text{ k}\Omega$

$A_V = -g_{m1} R_O = -20 \times 50 = -1000\text{ V/V}$

6.13



For an output of 1.6 V,

$V_{SD_{2\min}} = |V_{OV}| = 1.8 - 1.6 = 0.2\text{ V},$

$V_{SD_{1\min}} = 0.2\text{ V}$

Since $I_{D2} = I_{D3} = I_{D1} = 50\ \mu\text{A},$

and $I_D = \frac{1}{2}(\mu_p C_{ox})(W/L)V_{OV}^2$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \frac{2I_{D2}}{(\mu_p C_{OX})(V_{OV})^2}$$

$$= \frac{2(50\ \mu\text{A})}{(86\ \mu\text{A/V}^2)(0.2\text{ V})^2} = 29.1$$

For Q_1,

$$\left(\frac{W}{L}\right)_1 = \frac{2(50\ \mu\text{A})}{(387\ \mu\text{A/V}^2)(0.2\text{ V})^2} = 6.46$$

A_V must be at least -10 V/V,

and $A_V = -g_{m1}(r_{O1} \parallel r_{O2})$

If we want to make r_{O1} and r_{O2} equal,

$A_V = -\frac{1}{2} g_{m1} r_O$

so, $r_O = \frac{A_V}{-1/2\ g_{m1}}$

$g_m \cdot \frac{I_{D1}}{V_{OV/2}} = \frac{50\ \mu\text{A}(2)}{0.2\text{ V}} = 0.5\text{ mA/V}$

$r_O = \frac{-10\text{ V/V}}{-(1/2)\ (0.5\text{ mA/V})} = 40\text{ k}\Omega$

$r_O = \frac{|V'_A| L}{|I_D|}$ so,

for Q_1, $L_1 = \frac{40\text{ k}\Omega(0.05\text{ mA})}{5\text{ V}/\mu\text{m}} = 0.4\ \mu\text{m}$

for Q_2 and Q_3,

$L_2 = L_3 = \frac{40\text{ k}\Omega(0.05\ \mu\text{A})}{6\text{V}/\mu\text{m}} = 0.33\ \mu\text{m}$

Since we want $L_1 = L_2 = L_3$ and L be an integer multiple of $0.18\ \mu\text{m}$, we choose

$L = 3(0.18\ \mu\text{m}) = 0.54\ \mu\text{m}$

(Note: Choosing $0.36\ \mu\text{m}$ results in slightly less than -10 V/V.)

checking,

$r_{O1} = \frac{V'_{An} L}{I_D} = \frac{5\text{V}/\mu\text{m}\ (0.54\ \mu\text{m})}{0.05\text{ mA}} = 54\text{ k}\Omega$

$r_{O2} = r_{O3} = \frac{6\text{V}/\mu\text{m}\ (0.54\ \mu\text{m})}{0.05\text{ mA}}$

$= 64.8\text{ k}\Omega$

$A_V = -g_{m1}(r_{O1} \parallel r_{O2})$

$= -0.5\text{ mA/V}(54\text{ K}1164.8\text{ K})$

$= -14.7\text{ V/V}$

If the gain is to be doubled, and the $\frac{W}{L}$ ratios kept the same, $r_{O1} \parallel r_{O2}$ must double.

If r_{O1} and r_{O2} had been equal, this would have meant doubling L and W, making the area 4 times greater.

For a gain of -20 V/V,

$L_1 = 0.8\ \mu\text{m}$

$L_2 = 0.67\ \mu\text{m}$

The closest integer multiple that satisfies our requirement is $(0.18\ \mu\text{m})(5) = 0.9\ \mu\text{m}$.

so, with $L_1 = L_2 = L_3$,

$r_{O1} = \frac{5\text{ V}/\mu\text{m}\ (0.9\ \mu\text{m})}{0.05\text{ mA}} = 90\text{ k}\Omega$

$r_{O2} = \frac{6\text{ V}/\mu\text{m}\ (0.9\ \mu\text{m})}{0.05\text{ mA}} = 133\text{ k}\Omega$

This results in a gain of

$A_V = -(0.5\text{ mA/V})(90\text{ k}\Omega \parallel 133\text{ k}\Omega)$

$A_V = -26.8\text{ V/V}$

This represents an increase in area of

$\left(\frac{0.9}{0.54}\right)^2 = 2.78$ (instead of 4)

6.14

$$K = 40 = g_{m2}r_{O2} = \frac{V_A}{V_{OV/2}}$$

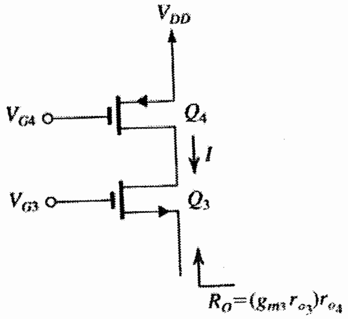
so that

$$V_A = \frac{KV_{OV}}{2} = \frac{40(0.2V)}{2} = 4\text{ V}$$

If $V_A' = 5\text{ V}/\mu\text{m}$,

$$L = \frac{V_A}{V_A'} = \frac{4\text{ V}}{5\text{ V}/\mu\text{m}} = 0.8\ \mu\text{m}$$

6.15



$V_{OV_3} = V_{OV_4} = V_{OV}$

$L_4 = L_3 = L$

$V_{A3}' = V_{A4}' = V_A'$

$$g_{m3}r_{O3} = A_{O3} = \frac{|V_A'|L}{|V_{OV/2}|}$$

$$V_{O4} = \frac{|V_A'|L}{I}$$

so that

$$R_O \simeq \frac{2|V_A'|L}{|V_{OV}|} \cdot \frac{|V_A'|L}{I}$$

Finally, $IR_O = \dfrac{2|V_A'|^2L^2}{|V_{OV}|}$

Now, $|V_A'| = 5\text{ V}/\mu\text{m}$, $|V_{OV}| = 0.2\text{ V}$

For $L = 0.18\ \mu\text{m}$:

$$IR_O = \frac{2(5\text{ V}/\mu\text{m})^2(0.18\ \mu\text{m})^2}{(0.2\text{ V})} = 8.1\text{ V}$$

$$R_O = \frac{IR_O}{I} = \frac{8.1\ V}{0.01\text{ mA}} = 810\text{ k}\Omega$$

$$g_m = \frac{|I|}{|V_{OV}|/2} = \frac{0.01\text{ mA}}{(0.2\text{ V}/2)} = 0.1\text{ mA/V}$$

$2WL = 2(0.18\ \mu\text{m})(0.18\ \mu\text{m})n = 0.065\ n$

Assuming that the driving NMOS transistors have similar g_m and R_O,

$$A_V = -\frac{1}{2}g_m R_O$$

$$A_V = -\frac{1}{2}(0.1\text{ mA/V})(810\text{ K}) = -40.5\text{ V/V}$$

For $L = 0.36\ \mu\text{m}$:

$$IR_O = \frac{2(5\text{ V}/\mu\text{m})^2(0.36\ \mu\text{m})^2}{(0.2\text{ V})} = 32.4\text{ V}$$

$$R_O = \frac{32.4\text{ V}}{0.01\text{ mA}} = 3.240\text{ k}\Omega$$

g_m remains unchanged

$$A_V = -\frac{1}{2}(0.1\text{ mA/V})(3.240\text{ K}) = -162\text{ V/V}$$

Area $= 2\text{ LW} = (0.36\ \mu\text{m})^2n(2) = 0.26\ n\mu\text{m}^2$

For $L = 0.54\ \mu\text{m}$:

$$IR_O = \frac{2(5\text{ V}/\mu\text{m})^2(0.54\ \mu\text{m})^2}{(0.2\text{ V})} = 72.9\text{ V}$$

$$R_O = \frac{72.9\text{ V}}{0.01\text{ mA}} = 7.290\text{ k}\Omega$$

$$A_v = -\frac{1}{2}(0.1\text{ mA/V})(7.290\text{ k}\Omega)$$

$$= -364.5\text{ V/V}$$

Area $= 2(0.54\ n)(0.54) = 0.58n\ \mu\text{m}^2$

Now, use $I = 0.1$ mA:

$L = 0.18\ \mu\text{m}$

Since $I_D = \frac{1}{2}k_p'(W/L)V_{Ov}^2$,

W/L will be ten times larger (10n)

$$g_m = \frac{(0.1\text{ mA})(2)}{(0.2\text{ v})} = 1\text{ mA/V}$$

$$R_O = \frac{IR_O}{I} = \frac{8.1\text{ V}}{0.1\text{ mA}} = 81\text{ k}\Omega$$

$$A_v = -\frac{1}{2}(1\text{ mA/v})(81\text{ K}) = -40.5\text{ V/V}$$

Area $= 2\ WL = 2(10\ n)(0.18\ \mu\text{m})^2$

$= 0.65\ n\mu\text{m}^2$

For $L = 0.36\ \mu\text{m}$:

$$R_O = \frac{32.4\text{ V}}{0.1\text{ mA}} = 324\text{ k}\Omega$$

$$A_v = \frac{1}{2}(1\text{ mA/V})(324\text{ K}) = -162\text{ V/V}$$

Area $= 2\ WL = 2(10\ n)(0.36\ \mu\text{m})^2$

$= 2.59\ n\mu\text{m}^2$

For $L = 0.54\ \mu\text{m}$:

$$R_O = \frac{72.9\text{ V}}{0.1\text{ mA}} = 729\text{ k}\Omega$$

	$L = L_{\min} = 0.18\ \mu\text{m}$ $IR_O = 8.1\ \text{V}$				$L = 2L_{\min} = 0.36\ \mu\text{m}$ $IR_O = 32.4\ \text{V}$				$L = 3L_{\min} = 0.54\ \mu\text{m}$ $IR_O = 72.9\ \text{V}$			
	g_m	R_O	A_{vo}	2WL	g_m	R_O	A_{vo}	2WL	g_m	R_O	A_{vo}	2WL
	mA / V	kΩ	V / V	μm^2	mA/V	kΩ	V / V	μm^2	mA/V	kΩ	V / V	μm^2
$I = 0.01$ mA $W/L = \text{n}$	0.1	810	−40.5	0.065 n	0.1	3,240	−162	0.26 n	0.1	7,290	−364, 5	0.58 n
$I = 0.01$ mA $W/L = 10\ \text{n}$	1.0	81	−40.5	0.65 n	1.0	324	−162	2.6 n	1.0	729	−364, 5	5.8 n
$I = 0.01$ mA $W/L = 100\ \text{n}$	10.0	8.1	−40.5	6.5 n	10.0	32.4	−162	26 n	10.0	72.9	−364, 5	58 n

$$A_v = -\frac{1}{2}(1\ \text{mA/V})(729\ \text{K}) = -364.5\ \text{V/V}$$

$$\text{Area} = 2\,WL = (2)(10\ \text{n})(0.54\ \mu\text{m})^2$$
$$= 5.8\ \text{n}\ \mu\text{m}^2$$

Now, for $I = 1.0$ mA,

For $L = 0.18\ \mu$m:

$$\frac{W}{L} = 100\ \text{n}$$

$$g_m = \frac{1\ \text{mA}(2)}{(0.2\ \text{V})} = 10\ \text{mA/V}$$

$$R_O = \frac{8.1\ \text{V}}{1\ \text{mA}} = 8.1\ \text{k}\Omega$$

$$A_V = -\frac{1}{2}(10\ \text{mA/V})(8.1\ \text{k})$$
$$= -40.5\ \text{V/V}$$

$$\text{Area} = 2\ \text{WL} = 2(100\ \text{n})(0.18\ \mu\text{m})^2$$
$$= 6.5\ \text{n}\ \mu\text{m}^2$$

For $L = 0.36\ \mu$m:

$$R_O = \frac{32.4\ \text{V}}{1\ \text{mA}} = 32.4\ \text{k}\Omega$$

$$A_V = -\frac{1}{2}(10\ \text{mA/V})(32.4\ \text{K}) = -162\ \text{V/V}$$

$$\text{Area} = 2\ \text{WL} = 2(100\ \text{n})(0.36\ \mu\text{m})^2$$
$$= 26\ \text{n}\ \mu\text{m}^2$$

For $L = 0.54\ \mu$m:

$$R_O = \frac{72.9\ \text{V}}{1\ \text{mA}} = 72.9\ \text{k}\Omega$$

$$A_V = -\frac{1}{2}(10\ \text{mA/V})(72.9\ \text{K})$$
$$= -364.5\ \text{V/V}$$

$$\text{Area} = 2\ \text{WL} = 2(100\ \text{n})(0.5\ \mu\text{m})^2$$
$$= 58\ \text{n}\ \mu\text{m}^2$$

The table summarizes the calculations.

Comments:

(a) R_O and A_V are increased but at the cost of larger device area. As L increases by a factor of X, A_V and R_O increase by a factor of X^2. The device area increases at this same rate.

(b) g_m increases with $|I|$, but R_O decreases with $\frac{1}{|I|}$.

The device area increases with $|I|$.

(c) Smallest area = $0.065\ \text{n}\ \mu\text{m}^2$

Largest area = $58\ \text{n}\ \mu\text{m}^2$ Gain and g_m have been increased, but at the expense of increased device area.

6.16

$$g_{m1} = \frac{2I_D}{V_{OV}}, \text{ so,}$$

$$I_D = \frac{g_{m1}V_{OV}}{2} = \frac{1\text{ mA/V}(0.2\text{ V})}{2} = 100\ \mu\text{A}$$

$$R_O = (g_{m2}r_{O2})r_{O1}$$

However, if we make $g_{m1} = g_{m2} = g_m$

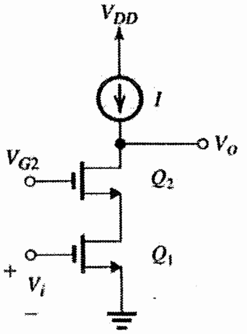
and $r_{O1} = r_{O2} = r_O$, we can say

that $400\text{ k}\Omega = 1\text{ mA/V}\cdot r_O^2$

$$r_O^2 = \frac{400\text{ k}\Omega}{1\text{ mA/V}} \Rightarrow r_O = 20\text{ k}\Omega$$

since $r_O = \dfrac{V_A'L}{I_D}$,

$$L = \frac{I_D r_O}{V_A'} = \frac{100\ \mu\text{A}(20\text{ k}\Omega)}{5\text{ V}/\mu\text{m}} = 0.4\ \mu\text{m}$$



$$g_m = \sqrt{2\,\mu_n C_{ox}(W/L)}\cdot\sqrt{I_D} \quad \text{so that}$$

$$\frac{W}{L} = \frac{g_m^2}{2\,\mu_n C_{ox} I_D}$$

$$= \frac{(1000\ \mu\text{A/V})^2}{2(400\ \mu\text{A/V}^2)(100\ \mu\text{A})} = 12.5$$

For maximum negative excursion at the output, we want the MOSFETs to be biased so that each transistor can reach $V_{DS} = V_{OV} = 0.2$ V.

$\therefore$ Set $V_{G2} = V_{tn} + V_{OV} + V_{OV}$

$= 0.5 + 0.2 + 0.2 = 0.9$ V

minimum output voltage will be

$2\,V_{OV} = 0.4$ V

6.17

$$g_{m1} = \frac{I_{D_1}}{\frac{V_{OV}}{2}} = \frac{100\ \mu\text{A}}{(0.25\text{ V})/2} = 800\ \mu\text{A/V}$$

Since all devices have the same V_A and I_D,

$$r_{O1} = r_{O2} = r_{O3} = r_{O4}$$

$$= \frac{|V_A|}{I_D} = \frac{4\text{ V}}{0.1\text{ mA}} = 40\text{ k}\Omega$$

$$R_{on} = g_m r_O^2 = (0.8\text{ mA/V})(40\text{ k}\Omega)^2$$

$$= 1.28\text{ M}\Omega$$

$$R_{op} = g_m r_O^2 = 1.28\text{ M}\Omega$$

$$R_O = R_{on} \parallel R_{op} = 640\text{ k}\Omega$$

$$A_V = -g_{m1}R_O$$

$$= -800\ \mu\text{A/V}\ (640\text{ k}\Omega)$$

$$= -512\text{ V/V}$$

6.18

Since $A_V = -g_{m1}R_O$

$$R_O = \frac{A_V}{-g_{m1}} = \frac{-200}{-2\text{ mA/V}} = 100\text{ k}\Omega$$

If all have the same I_D and V_A', and

since $R_O = R_{on} \parallel R_{op}$, and

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = g_m,$$

$$R_O = (g_m r_O^2) \parallel (g_m r_O^2) = \frac{1}{2}g_m r_O^2$$

solving for r_O, we get

$$r_O = \sqrt{\frac{2R_O}{g_m}} = \sqrt{\frac{2(100\text{ k}\Omega)}{2\text{ mA/V}}} = 10\text{ k}\Omega$$

$$I = \frac{g_m|V_{OV}|}{2} = \frac{2\text{ mA/V}(0.2\text{ V})}{2}$$

$$= 0.2\text{ mA} = 200\ \mu\text{A}$$

Since $r_O = \dfrac{|V_A'|L}{I_D}$,

$$L = \frac{r_O I}{|V_A'|} = \frac{10\text{ k}\Omega(0.2\text{ mA})}{5\text{ V}/\mu\text{m}} = 0.4\ \mu\text{m}$$

Since $g_m = \sqrt{(2\,\mu_n C_{ox})(W/L)} \cdot \sqrt{I_D}$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{g_m^2}{2\,\mu_n C_{ox} I_D}$$

$$= \frac{(2\text{ mA/V})^2}{2\,(400\ \mu\text{A/V}^2)(200\ \mu\text{A})} = 25$$

Similarly,

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4$$

$$= \frac{(2\text{ mA/V})^2}{2\,(100\ \mu\text{A/V}^2)(200\ \mu\text{A})} = 100$$

6.19

a) $I = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$

$$\Rightarrow \quad \text{For same } I\text{: } \frac{V_{OVb}^2}{V_{OVa}^2} = \frac{\left(\frac{W}{L}\right)_a}{\left(\frac{W}{L}\right)_b}$$

For same I, if $\frac{W}{L}$ is divided by 4,

then V_{OV}^2 is multiplied by 4, or equivalently

V_{OV} is doubled $g_m = \mu_n C_{ox} \frac{W}{L} V_{OV}$ Thus g_m for circuit (b) is half of the one for circuit(a).

$A_O = g_m r_O = \frac{2I_D}{V_{OV}} \times \frac{V_A}{I_D} = \frac{2V_A' L}{V_{OV}}$. Thus, if L is multiplied by 4, and V_{OV} is halved, then A_O is doubled for circuit(b).

In summary, for circuit (b), V_{OV} is doubled, gm is halved, A_O is doubled.

(b) Each transistor is circuit (c) has the same V_{OV} as the one in circuit (a).

$A_{VO} = -A_O^2 = -(g_m r_O)^2$

$G_m \approx g_{m1} = g_m$ (same as circuit (a))

Note that for the transistor in (c), the g_m and r_O are the same as those in circuit (a). In summary, for circuit(b), V_{ov} is doubled, g_m is halved A_o is doubled.

(b) Each transistor in circuit (c) has the same V_{ov} as the one in circuit (a).

$A_{VO} = -A_o^2 = -(g_m r_o)^2$

$G_m = g_{m1} = g_m$ (same as circuit (a))

Note that for the transistor in (c), the g_m and r_o are the same as those is circuit (a).

Thus, the intrinsic gain for circuit (c), $A_{vo} = -A_0^2$ where A_o is the intrinsic gain for circuit (a).

In general, circuit (c) has a higher output resistance, and for the same V_{OV} of transistors it has lower output swing. The output swing is limited to 2 V_{OV} on the low side for circuits (b) and (c), but limited to only V_{OV} in circuit (a)

6.20

For Q_1,

$V_{OV} = V_i - V_{tn} = 0.8 - 0.5 = 0.3$ V

Since all transistors are identical, and

$k_{n1} = k_{n2} = k_{p3} = k_{p4}$

with $I_{D1} = I_{D2} = I_{D3} = I_{D4}$,

$|V_{OV}| = 0.3$ V (since $I_D = \frac{1}{2} k |V_{OV}|^2$.)

with V_{G2} and V_{G3} fixed,

$V_{S2} = V_{G2} - V_{GS2}$

$= 1.2 - 0.5 - 0.3 = 0.4$ V

$V_{S3} = V_{G3} + V_{GS3}$

$= 1.3 + 0.5 + 0.3 = 2.1$ V

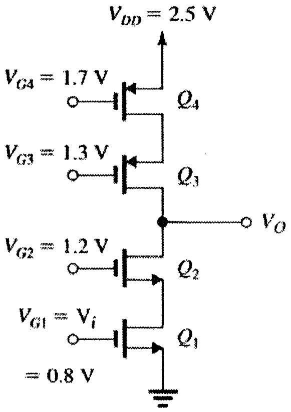
The lowest V_O is

$V_{S2} + V_{OV2} = 0.4 + 0.3 = 0.7$ V

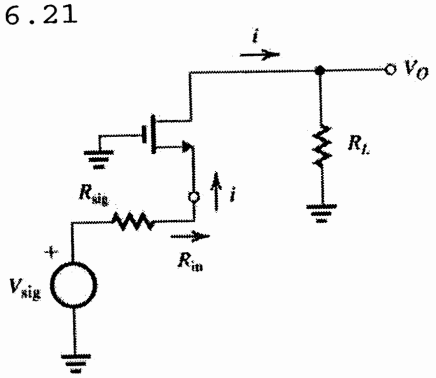
The highest V_O is

$V_{S3} - V_{OV3} = 2.1 - 0.3 = 1.8$ V

so the output range is 0.7 V to 1.8 V



6.21



a)

$$R_{in} = \frac{R_L + r_O}{1 + g_m r_O} = \frac{R_L}{g_m r_O} + \frac{1}{g_m}$$

(b) $V_O = i R_L$ and

$$i = \frac{V_{sig}}{R_{sig} + R_{in}} = \frac{V_{sig}}{R_{sig} + \dfrac{R_L + r_O}{1 + g_m r_O}}$$

multiplying and dividing by V_{sig}, we get

$$\frac{V_O}{V_{sig}} = \frac{R_L}{R_{sig} + \dfrac{R_L + r_O}{1 + g_m r_O}} \simeq \frac{R_L}{R_{sig} + \dfrac{R_L}{g_m r_O} + \dfrac{1}{g_m}}$$

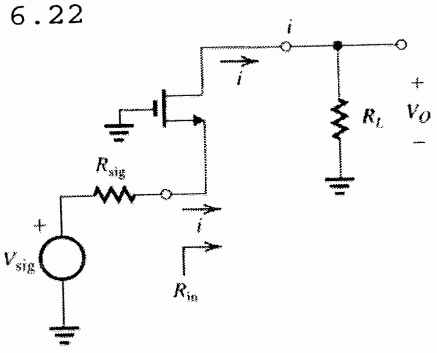
c) If $R_L = r_O = 10\ \text{k}\Omega$, $A_O = 20$,

$$R_{sig} = 1\text{K},\ g_m = \frac{A_O}{r_O} = \frac{20}{10\text{k}\Omega} = 2\text{mA/V}$$

$$R_{in} \simeq \frac{10\ \text{k}\Omega}{20} + \frac{1}{2\ \text{mA/V}} = 1\ \text{k}\Omega$$

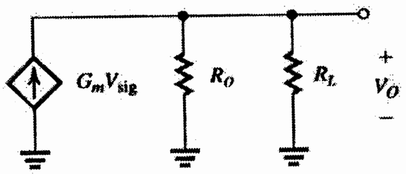
$$\frac{V_O}{V_{sig}} = \frac{R_L}{R_{sig} + R_{in}} = \frac{10\ \text{k}\Omega}{1\ \text{k}\Omega + 1\ \text{k}\Omega} = 5\ \text{V/V}$$

6.22



a) If d is shorted to ground, the current flowing through the short is

$$i = \frac{V_{sig}}{R_{sig} + R_{in}} = \frac{V_{sig}}{R_{sig} + \dfrac{1}{g_m}}$$



$$G_m = \frac{i}{V_{sig}} = \frac{1}{R_{sig} + \dfrac{1}{g_m}}$$

From Fig. 713,

$R_O = r_O + R_{sig} + (g_m r_O)R_{sig}$

b) If $r_O + 10\ \text{k}\Omega$, and

$$g_m = \frac{A_O}{r_O} = \frac{20}{10\ \text{k}\Omega} = 2\ \text{mA/V},$$

$$G_m = \frac{1}{R_{sig} + \dfrac{1}{g_m}} = \frac{1}{1\ \text{k}\Omega + \dfrac{1\ \text{V}}{2\ \text{mA}}}$$

$$= 0.67\ \text{mA/V}$$

$R_O + 10\ \text{k}\Omega + 1\ \text{k}\Omega + (20)(1\ \text{k}\Omega) = 31\ \text{k}\Omega$

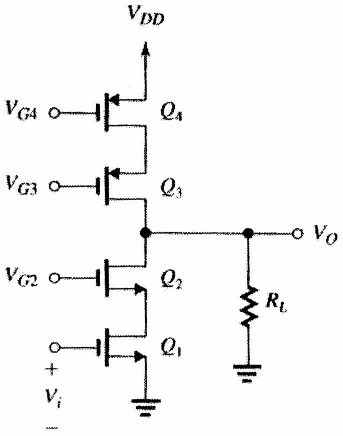
Using the new model,

$V_O = G_m V_{sig}(R_O \parallel R_L)$

$$\frac{V_O}{V_{sig}} = G_m(R_O \parallel R_L)$$

$= 0.67\ \text{mA/V}\ (31\ \text{k}\Omega \parallel 10\ \text{k}\Omega) = 5.04\ \text{V/V}$

6.23



If $\left|V'_{Ap}\right| = V'_{An}$,

$$r_{O1} = r_{O2} = r_{O3} = r_O = \frac{V'_A L}{I_D}$$

$$= \frac{(5\ \text{V}/\mu\text{m})(0.36\ \mu\text{m})}{200\ \mu\text{A}}$$

$r_O = 9\ \text{k}\Omega$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV1}^2$$

$$V_{OV1} = \sqrt{\frac{2I_D}{k_n'(W/L)}}$$

$$= \sqrt{\frac{2(200\ \mu\text{A})}{400\ \mu\text{A}/\text{V}^2\left(\frac{5.4}{0.36}\right)}} = 0.26\ \text{V}$$

$$g_{m1} = \frac{I_{D1}}{\frac{V_{OV}}{2}} = \frac{200\ \mu\text{A}}{0.26/2} = 1.54\ \text{mA/V}$$

$$V_{OV3} = V_{OV4} = \sqrt{\frac{2(200\ \mu\text{A})}{100\ \mu\text{A}/\text{V}^2\left(\frac{5.4}{0.36}\right)}}$$

$$= 0.52\ \text{V}$$

$$g_{m3} = g_{m4} = \frac{200\ \mu\text{A}}{(0.52\ \text{V})/2} = 769\ \mu\text{A/V}$$

$$R_{on} \simeq (g_{m3}r_{O3})r_{O1}$$

$$= (1.54\ \text{mA/V})(9\ \text{k}\Omega)^2 = 125\ \text{k}\Omega$$

$$R_{OP} = (g_{m3}r_{O3})r_{O4}$$

$$= (0.769\ \text{mA/V})(9\ \text{k}\Omega)^2 = 62.3\ \text{k}\Omega$$

$$R_o = R_{op} \parallel R_{on}$$

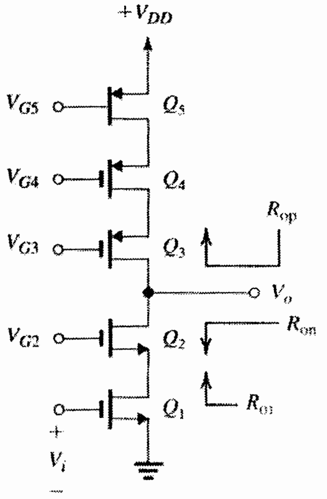
$$= 62.3\ \text{k}\Omega \parallel 125\ \text{k}\Omega = 41.6\ \text{k}\Omega$$

The value of $R_o \parallel R_L$ needed is

$$R_O \parallel R_L = \frac{A_V}{-g_{m1}} = \frac{-100}{1.54\ \text{mA/V}} = 64.9\ \text{k}\Omega$$

This is greater than R_O!

This can't be done with the present design. One thing we could do is double cascode the current source to raise R_{op}:



This raises R_{op} to

$$R_{op} = (g_{m3}r_{O3})(g_{m4}r_{O4})r_{O5}$$

$$= (0.769\ \text{mA/V})^2(9\ \text{k}\Omega)^3 = 431\ \text{k}\Omega$$

We can now find an R_L that will allow a gain of -100 V/V: Since

$$431\ \text{k}\Omega \parallel 125\ \text{k}\Omega = 96.9\ \text{k}\Omega$$

Setting $\frac{R_L(96.9)}{R_L + (96.9)} = 65\ \text{k}\Omega$, we get

$$R_L = 197\ \text{k}\Omega$$

To find the gain of the CS amplifier, we calculate R_{o1}:

$$R_{O1} = (R_{op} \parallel R_L)(g_{m3}r_{O3}) \parallel r_{O1}$$

$$R_{O1} = [(431\ \text{k}\Omega \parallel 197\ \text{k}\Omega)(1.5\ \text{mA/V})(9\ \text{k}\Omega)] \parallel (9\ \text{k}\Omega)$$

$$R_{O1} = 9\ \text{k}\Omega$$

so, $A_{V1} = -g_{m1}R_{O1} = (1.54\ \text{mA/V})(9\ \text{k}\Omega)$

$$= -13.9\ \text{V/V}$$

6.24

$$R_1 = r_O$$

$$R_2 = g_m r_O^2$$

$$R_3 \simeq \frac{1}{g_m} + \frac{g_m r_O^2}{g_m r_O}$$

$$R_3 \simeq \frac{1}{g_m} + r_O$$

b) $i_1 = V_i g_m$

By current division,

$$i_2 = \frac{i_1 R_3}{r_O + R_3} = \frac{V_i g_m\left(\frac{1}{g_m} + r_O\right)}{r_O + \frac{1}{g_m} + r_O}$$

$$= \frac{V_i(1 + g_m r_O)}{\frac{1}{g_m} + 2r_O}$$

also,

$$i_3 = \frac{i_1 r_O}{r_O + R_3} = \frac{V_i g_m r_O}{r_O + \frac{1}{g_m} + r_O} = \frac{V_i g_m r_O}{\frac{1}{g_m} + 2r_O}$$

$$i_4 = i_5 = i_7 = i_3 = \frac{V_i g_m r_O}{\frac{1}{g_m} + 2r_O}$$

c)

$$V_1 = -V_i g_m(r_O \| R_3)$$

$$= \frac{-V_i g_m(r_O)\left(\frac{1}{g_m} + r_O\right)}{r_O + \frac{1}{g_m} + r_O}$$

$$V_1 = \frac{-V_i g_m r_O}{1 + \frac{r_O}{\frac{1}{g_m} + r_O}}$$

$$= \frac{-V_i g_m r_O}{1 + \frac{1}{1 + \frac{1}{g_m r_O}}} \simeq \frac{-1}{2} V_i g_m r_O$$

$$V_2 = V_i g_m[(g_m r_O^2) \| (g_m \| r_O^2)]$$

$$= \frac{1}{2}(g_m r_O)^2$$

$$V_3 = \frac{-V_i g_m r_O}{\frac{1}{g_m} + 2r_O} r_O = \frac{-V_i g_m r_O}{\frac{1}{g_m r_O} + 2}$$

$$\simeq -\frac{1}{2} V_i g_m r_O$$

d) $V_1(t) \simeq \frac{-1}{2} V_i g_m r_O$

with $V_{i\text{peak}} = 5\ \text{mV}$,

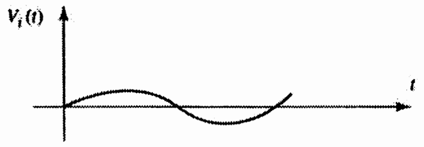
$$V_{1\text{peak}} = -\frac{1}{2}(5\ \text{mV})(20) = -50\ \text{mV}_{\text{peak}}$$

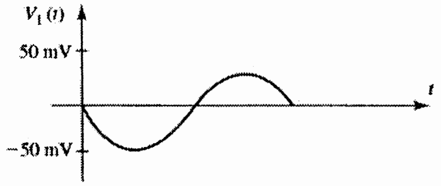
$$V_2(t) = -\frac{1}{2} V_i (g_m r_O)^2$$

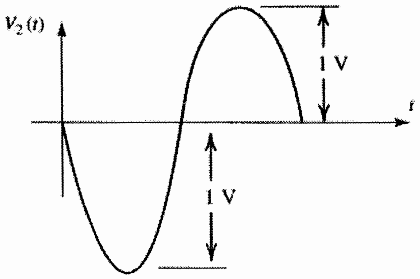
$$V_{2\text{peak}} = -\frac{1}{2}(5\ \text{mV})(20)^2 = -1\ \text{V}_{\text{peak}}$$

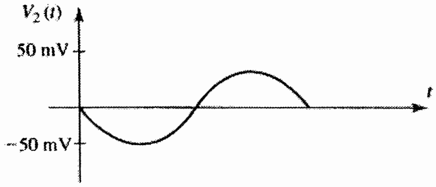
$$V_3(t) \simeq -\frac{1}{2} V_i (g_m r_O)$$

$$V_{3\text{peak}} = -\frac{1}{2}(5\ \text{mV})(20) = -50\ \text{mV}_{\text{peak}}$$









6.25

Since $I_D = \frac{1}{2}(\mu_p C_{ox})\left(\frac{W}{L}\right)|V_{OV}|^2$,

$$\frac{W}{L} = \frac{2I_D}{(\mu_p C_{ox})|V_{OV}|^2}$$

$$= \frac{2(100\ \mu\text{A})}{(100\ \mu\text{A}/\text{V}^2)(0.2\ \text{V})^2}$$

$$\frac{W}{L} = 50$$

(for all transistors)

$$r_O = \frac{|V_A'|L}{I_D} = \frac{(6\ \text{V}/\mu\text{m})(0.18\ \mu\text{m})}{100\ \mu\text{A}}$$

$$= 10.8\ \text{k}\Omega$$

To permit the maximum swing, each $V_{DS_{\min}}$ should equal $|V_{OV}|$. So,

$V_{G1} = V_{DD} - |V_{tp}| - |V_{OV}|$

$= 1.8 - 0.5 - 0.2 = 1.1\ \text{V}$

$V_{G2} = V_{D1_{\max}} - |V_{tp}| - |V_{OV}|$

$= (1.8 - 0.2) - 0.5 - 0.2 = 0.9\ \text{V}$

$V_{G3} = V_{D2_{\max}} - |V_{tp}| - |V_{OV}|$

$= (1.8 - 0.2 - 0.2) - 0.5 - 0.2 = 0.7\ \text{V}$

$R_O \simeq r_{O1}(g_{m2}r_{O2})(g_{m3}r_{O3})$

$\simeq$

$g_m^2 r_O^3 = (1\ \text{mA/V})^2 (10.8\ \text{k}\Omega)^3 = 1.26\ \text{M}\Omega$

6.26

a) Assuming that all transistors have the same g_m and r_o,

$R_{o1} = r_o$

$R_{o1} = r_o$

$R_{o3} = (g_{m3}\ r_{o3})$.

$R_{O1} \| R_{O2} = g_m r_O\left(\frac{1}{2} r_O\right) = \frac{1}{2} g_m r_O^2$

$R_{o4} \simeq (g_{m4}\ r_{o4})\ \ r_{o5} = g_m r_o^2$

$$R_{in3} = \frac{1}{g_m} + \frac{R_{O4}}{g_m r_o} = \frac{1}{g_m} + \frac{g_m r_o^2}{g_m r_o} = \frac{1}{g_m} + r_o$$

b) $R_o = R_{o3} \| R_{o4} = \left(\frac{1}{2} g_m r_o^2\right) \| \ (g_m r_o^2)$

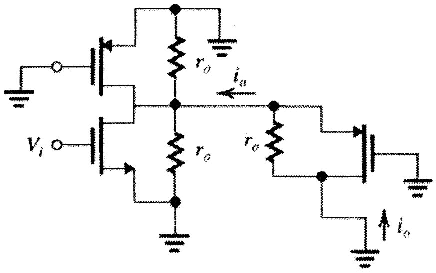
$$R_O = \frac{\frac{1}{2} g_m^2\ r_O^4}{\frac{1}{2} g_m r_O^2 + g_m r_O^2}$$

$= \frac{1}{3} g_m r_O^2$

c) If V_o is shorted to ground,

$$R_{in3} = \frac{1}{g_m} + \frac{0}{g_m r_o} = \frac{1}{g_m}$$

Using current division,



$$i_o = g_{m1} V_i \frac{\frac{1}{2} r_o}{\frac{1}{2} r_o + \frac{1}{g_m}} = \frac{g_{m1} V_i}{1 + \frac{2}{g_m r_o}}$$

$$G_m = \frac{i_o}{V_i} = \frac{g_{m1}}{1 + \frac{2}{g_m r_o}} = g_{m1}$$

d) If $R_L = R_{O4}$,

$$R_{in3} = \frac{1}{g_m} + r_o$$

$$i_o = \frac{V_i g_m\left(\frac{r_o}{2}\right)}{r_o/2 + \frac{1}{g_m} + r_o} = \frac{V_i g_m r_o}{3r_o + \frac{2}{g_m}}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{g_m r_o (g_m r_o^2)}{3r_o + \frac{2}{g_m}}$$

Calculating: $r_o = \frac{A_o}{g_m} = \frac{20}{2\ \text{mA/V}} = 10\ \text{k}\Omega$

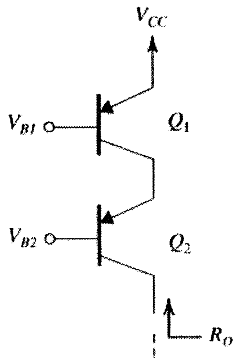
$$\frac{V_o}{V_i} = \frac{-(g_m r_o)^2 r_o}{3r_o + 2/g_m} = -\frac{(20)^2 10\ \text{k}\Omega}{3(10\ \text{k}\Omega) + \frac{2}{2\ \text{mA/V}}}$$

$= -129\ \text{V/V}$

6.27

$\beta = 50$, $V_A = 5\ \text{V}$, $I = 0.5\ \text{mA}$

If the base currents are ignored, we can use the same r_o and g_m for each transistor.



$$g_m = \frac{I_c}{V_t} = \frac{0.5\ \text{mA}}{25\ \text{mV}} = 20\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{20\ \text{mA/V}} = 2.5\ \text{k}\Omega$$

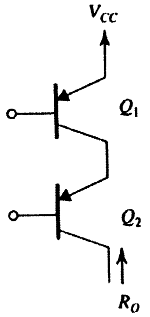
$$r_o = \frac{|V_A|}{I_c} = \frac{5\ \text{V}}{0.5\ \text{mA}} = 10\ \text{k}\Omega$$

$R_o = (g_{m2} r_{o2})(r_{o4} \| r_{\pi 3})$

$R_o = \left(\frac{20\ \text{mA}}{\text{V}}\right)(10\ \text{k}\Omega)(10\ \text{k}\Omega \| 2.5\ \text{k}\Omega)$

$R_o = 400\ \text{k}\Omega$

6.28



If the transistors are identical,

$$r_{O1} = r_{O2} = r_O = \frac{|V_A|}{I_C}$$

$$g_{m1} = g_{m2} = g_m = \frac{|I_C|}{V_T}$$

$$r_{\pi 1} = r_{\pi 2} = r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{|I_C|}$$

$$R_o \simeq (g_{m2} r_{O2})(r_{O1} \parallel r_{\pi 2})$$

$$R_o = \left(\frac{|I_C|}{V_T} \cdot \frac{|V_A|}{|I_C|}\right)\left(\frac{|V_A|}{|I_C|} \parallel \frac{\beta V_T}{|I_C|}\right)$$

$$R_o = \frac{|V_A|}{V_T} \cdot \left[\frac{\frac{|V_A|}{|I_C|} \cdot \frac{\beta V_T}{|I_C|}}{\frac{|V_A|}{|I_C|} + \frac{\beta V_T}{|I_C|}}\right] \text{ with } I_C = I,$$

$$IR_o = \frac{|V_A|}{V_T}\left[\frac{|V_A| \cdot \beta V_T}{|V_A| + \beta V_T}\right]$$

$$IR_o = \frac{|V_A|}{V_T} \cdot \frac{\beta V_T}{1 + \frac{\beta V_T}{|V_A|}} = \frac{|V_A|}{V_T} \cdot \frac{1}{\frac{1}{\beta V_T} + \frac{1}{|V_A|}}$$

$$IR_o = \frac{|V_A|}{(V_T / |V_A|) + (1/\beta)}$$

For $|V_A| = 5\text{ V}, \beta = 50$

If $I = 0.1$ mA,

$$R_o = \frac{5\text{ V}}{\frac{0.025\text{ V}}{5\text{ V}} + \frac{1}{50}} \cdot \frac{1}{0.1\text{ mA}} = 2\text{ M}\Omega$$

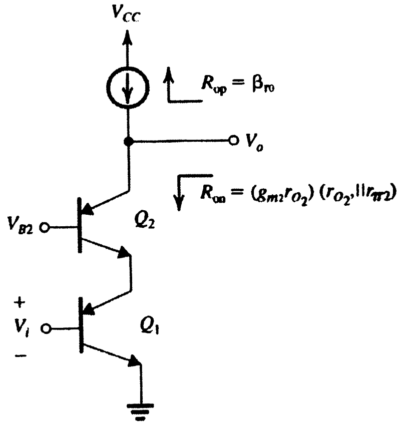
If $I = 0.5$ mA,

$$R_o = 2\text{ M}\Omega\left(\frac{.1\text{ mA}}{.5\text{ mA}}\right) = 400\text{ k}\Omega$$

If $I = 1.0$ mA,

$$R_o = 2\text{ M}\Omega\left(\frac{.1\text{ mA}}{1\text{ mA}}\right) = 200\text{ k}\Omega$$

6.29



$$r_{o1} = r_{o2} = r_o = \frac{|V_A|}{I} = \frac{100\text{ V}}{0.1\text{ mA}} = 1\text{ M}\Omega$$

$$g_{m1} = g_{m2} = g_m = \frac{|I_C|}{V_T} = \frac{0.1\text{ mA}}{25\text{ mV}}$$

$$= 4\text{ mA/V}$$

$$r_{\pi 1} = r_{\pi 2} = r_\pi = \frac{\beta}{g_m} = \frac{100}{4\text{ mA / V}} = 25\text{ k}\Omega$$

$$A_V = -g_{m'}(R_{on} \parallel R_{op})$$

$$R_{op} = \beta r_o = 100(1\text{ M}\Omega) = 100\text{ M}\Omega$$

$$R_{on} = (g_{m2} r_{o2})(r_{o1} \parallel r_{\pi 2})$$

$$= (4\text{ mA/V} \cdot 1\text{ M}\Omega)(1\text{ M}\Omega \parallel 25\text{ k}\Omega)$$

$$R_{on} = 100\text{ M}\Omega$$

so, $A_V = -4\text{ mA/V }(100\text{ M} \parallel 100\text{M})$

$$= 200,000\text{ V/V}$$

6.30

$$R_o \approx r_o[1 + g_m(R_e \parallel r_\pi)]$$

$$g_m = \frac{|I_C|}{V_T} = \frac{0.5\text{ mA}}{25\text{ mV}} = 0.02\text{ A / V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.02\text{ A / V}} = 5\text{ k}\Omega$$

when $R_e = 0$, $R_o = r_o$

a) For $R_o = 5 \cdot r_o$,

$$5 = [1 + g_m(R_e \parallel r_\pi)]$$

$$5 = 1 + 0.02\text{ A/V }(R_e \parallel 5\text{ k}\Omega)$$

$R_e \| 5\ \text{k}\Omega = \dfrac{4}{0.02\ \text{A/V}} = 0.2\ \text{k}\Omega$

Solving, $\dfrac{R_e \cdot 5\ \text{k}\Omega}{R_e + 5\ \text{k}\Omega} = 0.2\ \text{k}\Omega$

$R_e = \dfrac{5\ \text{k}\Omega(0.2\ \text{k}\Omega)}{4.8\ \text{k}\Omega} = 208\ \Omega$

b) For $R_o = 10 \cdot r_o$,

$10 = 1 + (0.02\ \text{A/V}) \cdot (R_e \| 5\text{k})$

So that $\dfrac{R_e \cdot 5\ \text{k}\Omega}{R_e + 5\ \text{k}\Omega} = \dfrac{9}{0.02\ \text{A/V}} = 450\ \Omega$

Solving, $R_e = 495\ \Omega$

c) For $R_o = 50 \cdot r_o$,

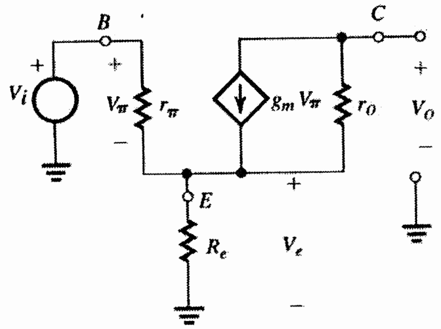
$50 = 1 + 0.02\ \text{A/V}(R_e \| 5\ \text{k}\Omega)$

$\dfrac{R_e \cdot 5\ \text{k}\Omega}{R_e + 5\ \text{k}\Omega} = \dfrac{49}{0.02\ \text{A/V}} = 2.45\ \text{k}\Omega$

$R_e = 4.8\ \text{k}\Omega$

6.31

With the output unloaded, the small-signal model can be drawn as follows:



Since no current flows out the collector,

$V_O = -g_m V_\pi r_O + V_e$ By voltage division,

$V_e = \dfrac{V_i R_e}{r_\pi + R_e}$ and $V_\pi = \dfrac{V_i r_\pi}{r_\pi + R_e}$

substituting, we get

$$A_{VO} = \frac{V_O}{V_i} = \frac{-g_m r_O r_\pi + R_e}{r_\pi + R_e}$$

$$A_{VO} = -g_m r_O = \frac{r_\pi - \dfrac{R_e}{g_m r_O}}{r_\pi + R_e}$$

dividing by r_π,

$$A_{VO} = -g_m r_O \cdot \frac{1 - \dfrac{R_e}{g_m r_\pi r_O}}{1 + \dfrac{R_e}{r_\pi}}$$

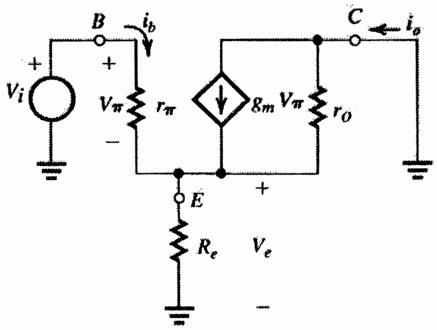
since $g_m r_\pi = \beta$,

$$A_{VO} = -g_m r_O \cdot \frac{1 - \dfrac{R_e}{\beta r_O}}{1 + \dfrac{R_e}{r_\pi}}$$

There are several ways to derive the equation for G_m.

Method 1:

Take the basic small-signal model:



Note that $V_\pi = V_i - V_e$

$$i_O = \frac{0 - V_e}{r_O} + g_m V_\pi$$

$$i_O = -\frac{V_e}{r_O} + g_m(V_i - V_e)$$

Assuming that $i_O >> i_b$

$V_e \simeq i_O R_e$ Then,

$$i_O = \frac{-i_O R_e}{r_O} + g_m V_i + i_O R_e g_m$$

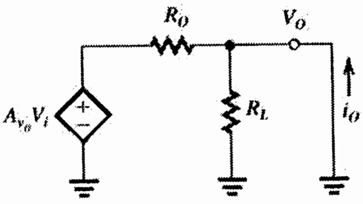
$i_O\left(1 + \dfrac{R_e}{r_O} + R_e g_m\right) = g_m V_i$ so that,

$$G_m = \frac{i_O}{V_i} \simeq \frac{g_m}{1 + \dfrac{R_e}{r_O} + g_m R_e}$$

since $\dfrac{R_e}{r_O} << 1$ usually,

$$G_m = \frac{i_O}{V_i} \simeq \frac{g_m}{1 + g_m R_e}$$

Method 2:
Consider the model



$$R_O = r_O + (Re \| r_\pi) + (g_m r_O)(R_e \| r_\pi)$$

or

$$R_O \approx r_O[1 + g_m(Re \| r_\pi)]$$

Shorting the output removes R_L from the CKT.

$$-A_{VO} = \frac{g_m r_O r_\pi - R_e}{r_\pi + R_e} \quad \text{(from part 1 above)}$$

$$G_m = \frac{i_O}{V_i} = \frac{-A_{VO}}{R_O} = \frac{\dfrac{g_m r_O r_\pi - R_e}{r_\pi + R_e}}{r_O + g_m r_O \dfrac{R_e r_\pi}{r_\pi + R_e}}$$

$$G_m = \frac{g_m r_O r_\pi - R_e}{r_O(r_\pi + R_e) + g_m r_O R_e r_\pi}$$

Dividing by $r_O r_\pi$, we get

$$G_m = \frac{g_m - \dfrac{R_e}{r_O r_\pi}}{\dfrac{r_O(r_\pi + R_e)}{r_O r_\pi} + g_m R_e}$$

since $\dfrac{r_\pi + R_e}{r_\pi} \simeq 1$ and $\dfrac{R_e}{r_O r_\pi} << g_m$,

$$G_m \simeq \frac{g_m}{1 + g_m R_e}$$

with $\beta = 100$, $r_O = 100\ \text{k}\Omega$,

$I_C = 0.2\ \text{mA}$, and $R_e = 250\ \Omega$,

$$g_m = \frac{|I_C|}{V_T} = \frac{0.2\ \text{mA}}{25\ \text{mV}} = 8\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{8\ \dfrac{\text{mA}}{\text{V}}} = 12.5\ \text{k}\Omega$$

$$R_O \simeq r_O + (Re \| r_\pi)(1 + g_m)$$

$$\simeq r_O + r_O g_m(Re \| r_\pi)$$

$$R_O \simeq 100\ \text{k}\Omega + (0.25\ \text{k}\Omega \| 12.5\ \text{k}\Omega)$$

$$(100\ \text{k}\Omega)\left(8\ \frac{\text{mA}}{\text{V}}\right)$$

$$= 296\ \text{k}\Omega$$

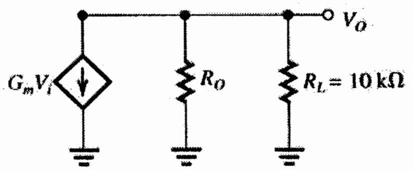
$$A_{VO} = -g_m r_O \frac{1 - \dfrac{R_e}{\beta r_O}}{1 + \dfrac{R_e}{r_\pi}}$$

$$= -(8\ \text{mA/V})(100\ \text{k}\Omega).\ \frac{1 - \dfrac{0.25\ \text{k}\Omega}{100(100\ \text{k}\Omega)}}{1 + \dfrac{0.25\ \text{k}\Omega}{12.5\ \text{k}\Omega}}$$

$$A_{VO} = -784\ \text{V/V}$$

$$G_m \simeq \frac{g_m}{1 + g_m R_e} = \frac{8\ \text{mA/V}}{1 + 8\ \text{mA/V}(12.5\ \text{k}\Omega)}$$

$$= 2.67\ \text{mA/V}$$



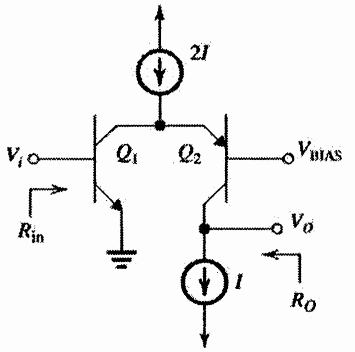
$$A_V = \frac{V_O}{V_i} = -G_m(R_O \| R_L)$$

$$= -2.67\ \text{mA/V}\ (296\ \text{k}\Omega \| 10\ \text{k}\Omega)$$

$$A_V = -25.9\ \text{V/V}$$

Note: Depending upon the approximations taken, the values of A_V may very slightly.

6.32



$$g_{m1} = g_{m2} = \frac{|I_C|}{V_T} = \frac{0.1\text{ mA}}{25\text{ mV}} = 4\text{ mA/V}$$

$$r_{\pi 1} = r_{\pi 1} = \frac{\beta}{g_m} = \frac{100}{4\text{ mA/V}} = 25\text{ k}\Omega$$

$$r_{O1} = r_{O2} = \frac{|V_A|}{I} = \frac{5\text{ V}}{0.1\text{ mA}} = 50\text{ k}\Omega$$

$R_{in} = r_{\pi 1} = 25\text{ k}\Omega$

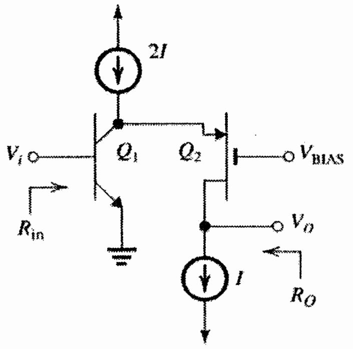
$R_O \simeq (g_{m2} r_{O2})(r_{O1} \| r_{\pi 2})$

$R_O \simeq (4\text{ mA/V})(50\text{ k}\Omega)(50\text{ k}\Omega \| 25\text{ k}\Omega)$

$= 3.33\text{ M}\Omega$

$A_{vo} = -G_m R_O \approx -g_{m1} R_O$

$= -(4\text{ mA/V})(3.33\text{ M}\Omega) = -13.3 \times 10^3\text{ V/V}$



$g_{m1} = 4\text{ mA/V}$

$$g_{m2} = \frac{|I_D|}{\frac{|V_{OV}|}{2}} = \frac{0.1\text{ mA}}{0.2\text{ V/2}} = 1\text{ mA/V}$$

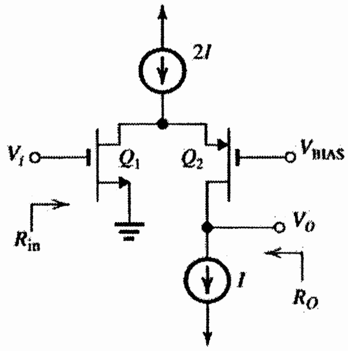
Again, $R_{in} = r_{\pi 1} = 25\text{ k}\Omega$

$R_O = (g_{m2} r_{O2}) r_{O1} = (1\text{ mA/V})(50\text{ K})^2$

$= 2.5\text{ M}\Omega$

$A_{vo} = -G_m R_O \approx -g_{m1} R_O$

$= -(4\text{ mA/V})(2.5\text{ M}\Omega) = -10 \times 10^3\text{ V/V}$



From part (b),

$g_{m1} = g_{m2} = 1\text{ mA/V}$

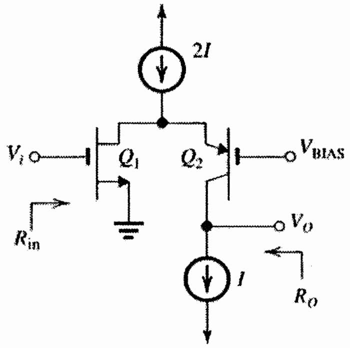
$R_{in} = \infty$

$R_O = (g_{m2} r_{O2}) r_{O1}$

$R_O = (1\text{ mA/V})(50\text{ k}\Omega)^2 = 2.5\text{ M}\Omega$

$A_{vo} = -G_m R_O = -g_{m1} R_O$

$A_{vo} = -(1\text{ mA/V})(2.5\text{ M}\Omega) = -2,500\text{ V/V}$



From above,

$g_{m1} = 1\text{ mA/V}$

$g_{m2} = 4\text{ mA/V},\ r_{\pi 2} = 25\text{ k}\Omega$

$R_{in} = \infty$

$R_O = (g_{m2} r_{O2})(r_{O1} \| r_{\pi 2})$

$R_O = (4\text{ mA/V})(50\text{ k}\Omega)(50\text{ k}\Omega \| 25\text{ k}\Omega)$

$= 3.33\text{ M}\Omega$

$A_{vo} = -G_m R_O$

$A_{vo} \approx -g_{m1} R_O = -1\text{ mA/V}(3.33\text{ M}\Omega)$

$-3.33 \times 10^3\text{ V/V}$

Comments:

(1) A MOSFET for Q_1 makes $R_{in} \rightarrow \infty$.

(2) The output resistance when Q_2 is a BJT is limited by $r_{\pi 2}$. In cases (a) and (d), R_o was higher due to the value of r_O and g_{m2}.

(3) In these four cases, A_{VO} was highest with two BJTs A_{vo} was lowest with two MOSFETs. These results could be changed with different biasing.

6.33

$I_o = I_{REF} = 50\mu A$, $L = 0.5\mu m$, $W = 5\mu m$, $V_t = 0.5^V$

$I_o = I_D = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2$ $\qquad k_n' = 250\mu A/_{V^2}$

$50 = \frac{1}{2} \times 250 \times \frac{5}{0.5} (V_{GS} - 0.5)^2 \Rightarrow V_{GS} = 0.7V$, 0.3^V

$V_{GS} = 0.3V < V_t$ is not acceptable, therefore

$\underline{V_{GS} = 0.7V}$

$I_D = I_{RE} = \frac{V_{DD} - V_{GS}}{R} \Rightarrow \frac{1.8 - 0.7}{R} = 0.050 \Rightarrow \underline{R = 22k\Omega}$

Q_1 and Q_2 have the same V_{GS}. The lowest value of V_o or V_{DS2} is when $V_{DS} = V_{GS} - V_t = 0.7 - 0.5 = 0.2^V$

hence $V_{omin} = 0.2^V$

$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{0.05} = 200k\Omega$

$\Delta I_o \cong \frac{\Delta V_o}{r_o} = \frac{1}{200^k} = 5\mu A \Rightarrow \underline{\Delta I_o = 5\mu A}$

6.34

$\mu_n C_{ox} = 250\mu A/_{V^2}$, $V_A' = 20V/_{\mu m}$, $V_t = 0.6V$

$\frac{\Delta I_o}{I_o} = 5\%$ $\Rightarrow \Delta I_o = 5\mu A$ for $\Delta V_o = 1.8 - 0.25 = 1.55^V$

$r_o = \frac{\Delta V_o}{\Delta I_o} = \frac{1.55}{5\mu} = 310k\Omega$

$r_o = \frac{V_A' L}{I_O} \Rightarrow L = I_D \times \frac{r_o}{V_A'} = 0.1 \times \frac{310}{20} = \underline{\underline{1.55\mu m}}$

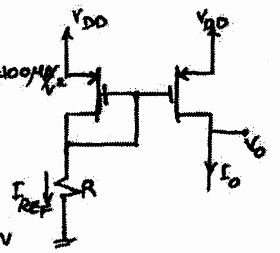
$V_{omin} = V_{GS} - V_t = 0.25 \Rightarrow V_{GS} = 0.25 + 0.6 = 0.85^V$

$R = \frac{V_{DD} - V_{GS}}{I_D} = \frac{1.8 - 0.85}{0.1} = \underline{\underline{9.5k\Omega}}$

$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow W = \frac{2 L I_D}{\mu_n C_{ox} (V_{GS} - V_t)^2}$

$\Rightarrow W = \frac{2 \times 1.55 \times 100}{250 (0.85 - 0.6)^2} = \underline{\underline{19.84\mu m}}$

6.35



$V_{DD} = 1.8V$, $|V_t| = 0.6V$, $\mu_p C_{ox} = 100\mu A/_{V^2}$

$I_{REF} = 80\mu A$ $\quad V_{omax} = 1.6^V$

$V_{DS} \leq V_{GS} - V_t$

$V_{Omax} = V_{DSmax} = V_{GS} - V_t \Rightarrow$

$1.6 - 1.8 = V_{GS} + 0.6 \Rightarrow V_{GS} = -0.8^V$

$\Rightarrow V_G = 1.8 - 0.8 = 1^V$

$R = \frac{V_G}{I_D} = \frac{1}{0.080} = \underline{12.5k\Omega}$

$I_D = \frac{1}{2} \mu_p C_{ox} (V_{GS} - V_t)^2 \frac{W}{L} \Rightarrow W = \frac{2 L I_D}{\mu_p C_{ox} (V_{GS} - V_t)^2}$

$\frac{W}{L} = \frac{2 \times 80}{100(-0.8 + 0.6)^2} = \underline{40}$

6.36

$W_2 = 4W_1$ $\quad L_1 = L_2$ $\quad V_{ov} = 0.3^V$, $I_{REF} = 20\mu A$

$I_o = I_{REF} \frac{(W/L)_2}{(W/L)_1} = 20 \times 4 = \underline{80\mu A}$

$V_{omin} = V_{ov} = \underline{0.3V}$

$V_t = 0.5V$. According to Eq. 6.11 $I = \frac{(W/L)_2}{(W/L)_1} I_{REF} \left(1 + \frac{V_o - V_{GS}}{V_{A2}}\right)$

$V_{ov} = V_{GS} - V_t \Rightarrow V_{GS} = 0.3 + 0.5 = 0.8V$

$1 + \frac{V_o - V_{GS}}{25} = 1 \Rightarrow \underline{V_o = 0.8V}$

Or we could simply say $V_{DS1} = V_{DS2} = V_o$ and

Since $V_{DS1} = V_{GS1} = 0.8^V \Rightarrow \underline{V_o = 0.8^V}$

$r_{o2} = \frac{V_A}{I_{D2}} = \frac{25}{0.08} = 312.5k\Omega$

$r_{o2} = \frac{\Delta V_o}{\Delta I_o} = \frac{1}{\Delta I_o} \Rightarrow \Delta I_o = \frac{1}{312.5^k} = \underline{3.2\mu A}$

6.37

$V_{GS1} = V_{GS2}$ so that $\frac{I_{D2}}{I_{D1}} = \frac{(W/L)_2}{(W/L)_1}$ and

$$I_{D2} = I_{REF}\frac{(W/L)_2}{(W/L)_1}$$

$$I_{D2} = I_{D3}$$

$V_{GS3} = V_{GS4}$ so that $\frac{(W/L)_3}{(W/L)_4} = \frac{I_{D3}}{I_{D4}} = \frac{I_{D2}}{I_{D4}}$

$$I_O = I_{D4} = I_{REF}\frac{(W/L)_2}{(W/L)_1}\cdot\frac{(W/L)_4}{(W/L)_3}$$

6.38

IF the transistor with $w=10$ is diode-connected, then: $I_2 = 100\times\frac{20}{10} = 200\mu A$

$I_3 = 100\times\frac{40}{10} = 400\mu A$

IF the transistor with $w=20$ is diode-connected then: $I_2 = 100\times\frac{10}{20} = 50\mu A$

$I_3 = 100\times\frac{40}{20} = 200\mu A$

IF the transistor with $w=40$ is diode-connected, then: $I_2 = 100\times\frac{10}{40} = 25\mu A$

$I_3 = 100\times\frac{20}{40} = 50\mu A$

So for cases that only one transistor is diode connected, 4 different output currents are possible. (depending on the configuration we choose).

IF 2 transistors are diode-connected; then they act as an equivalent transistor whose width is the sum of the widths of each transistor:

IF $W_{eff} = 10+20$ then $I_o = 100\times\frac{40}{30} = 133\mu A$

IF $W_{eff} = 20+40$ then $I_o = 100\times\frac{10}{60} = 16.7\mu A$

IF $W_{eff} = 40+10$ then $I_o = 100\times\frac{20}{50} = 40\mu A$

So 3 different output currents are possible depending on which two transistors are diode-connected. Now we calculate V_{SG}:

$100 = \frac{1}{2}\times 80\times\frac{30}{1}(V_{SG}-0.7)^2 \Rightarrow V_{SG} = 1^V$ for $W_{eff} = 30\,\mu m$

all have the same V_{SG} for any given configuration.

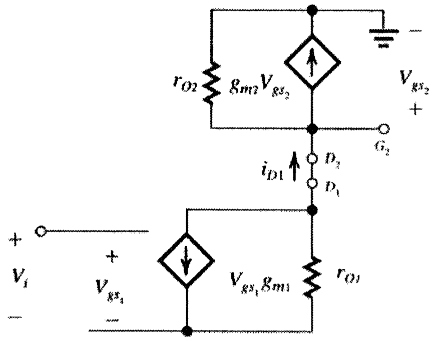
For $W_{eff} = 60 \Rightarrow 100 = \frac{1}{2}\times 80\times\frac{60}{1}(V_{SG}-0.7)^2$

$\Rightarrow V_{SG} = 0.9^V$

For $W_{eff} = 50 \Rightarrow 100 = \frac{1}{2}\times 80\times\frac{50}{1}(V_{SG}-0.7)^2$

$\Rightarrow VSG = 0.93V$

6.39 , the small-signal model can be drawn as follows:



(1) $V_O = -gm_3(r_{O3}\,\|\,R_L)V_{gs2}$

$V_{gs2} = (i_{D1} - g_{m2}\,V_{gs2})\,r_{O2}$

(2) $V_{gs2} = i_{D1}\,r_{O2} - g_{m3}\,V_{gs2}\,r_{O2}$

(3) $i_{D1} = -V_{gs1}\,g_{m1} - \frac{V_{gs2}}{r_{O1}}$

substituting (3) into (2), we get

$$V_{gs2} = -V_{gs1}\,g_{m1}\,r_{O2} - \frac{V_{gs2}\,r_{O2}}{r_{O1}} - g_{m2}\,V_{gs2}\,r_{O2}$$

(4) $V_{gs2} = \dfrac{-V_{gs1}\,g_{m1}\,r_{O2}}{\left(1 + \frac{r_{O2}}{r_{O1}} + g_{m2}\,r_{O2}\right)}$

substituting (4) into (1), we get

$$V_O = -g_{m3}(r_{O3}\parallel R_L)\left[\frac{-V_{gs1}\,g_{m1}\,r_{O2}}{\left(1+\frac{r_{O2}}{r_{O1}}+g_{m2}\,r_{O2}\right)}\right]$$

since $V_{gs1} = V_i$,

$$\frac{V_O}{V_i} = g_{m3}(r_{O3}\parallel R_L)\left[\frac{g_{m1}\,r_{O2}}{\left(1+\frac{r_{O2}}{r_{O1}}+g_{m2}\,r_{O2}\right)}\right]$$

divide out r_O:

$$\frac{V_O}{V_i} = \frac{g_{m1}\,g_{m3}(r_{O3}\parallel R_L)}{\left(\frac{1}{r_{O2}}+\frac{1}{r_{O1}}+g_{m2}\right)}$$

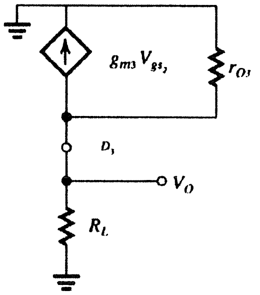
Assuming all r_O values are >> 1,

$$\frac{V_O}{V_i} = \frac{g_{m1}\,g_{m3}\,R_L}{g_{m2}}$$

Since $I_D = \frac{1}{2}k'_p\left(\frac{W}{L}\right)V_{OV}^2$ and $V_{GS2} = V_{GS3}$,

$V_{OV2} = V_{OV3}$. Also, $g_m = \dfrac{I_D}{V_{OV}/2}$

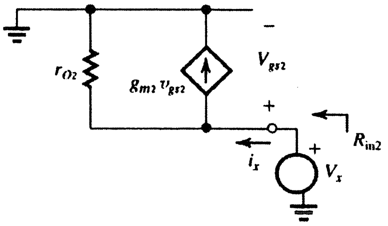
making $g_m \alpha I_D$ which is $\alpha \dfrac{W}{L}$



Here, $L_2 = L_3$, so we could also express the gain as

$$\frac{V_O}{V_i} = g_{m1}R_L\left(\frac{W_3}{W_2}\right)$$

Now, to find the resistance looking into the diode-connected drain of Q_2, we apply a test voltage V_X:



$$i_X = \frac{V_X}{V_{O2}} + g_{m2}V_{gs2}$$

since $V_{gs2} = V_X$, $i_X = \dfrac{V_X}{r_{O2}} + g_{m2}V_X$

$$\frac{i_X}{V_X} = \frac{1}{r_{O2}} + g_{m2}$$

$$R_{in2} = \frac{V_X}{i_X} = r_{O2}\parallel\frac{1}{g_{m2}}$$

The CS gain is

$$\frac{V_{d1}}{V_i} = -g_{m1}\left(\frac{1}{g_{m2}}\parallel r_{O2}\parallel r_{O1}\right)$$

6.40

$I_S = 10^{-15}A$

a) $I_{REF} = I_S\, e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln\dfrac{I_{REF}}{I_S}$

$I_{REF} = 10\mu A \rightarrow V_{BE} = 0.025 \ln\dfrac{10\times10^{-6}}{10^{-15}} = 0.576^V$

$I_{REF} = 10mA \Rightarrow V_{BE} = 0.025 \ln\dfrac{10\times10^{-3}}{10^{-15}} = 0.748^V$

Therefore:

$10\mu A \leqslant I_{REF} \leqslant 10mA \Rightarrow 0.576^V \leqslant V_{BE} \leqslant 0.748^V$

Since β is very high, I_B is negligible and hence

$I_o \alpha I_{REF}$: $\underline{10\mu A \leqslant I_o \leqslant 10\ mA}$

b) $I_o = I_{REF}\dfrac{1}{1+2/\beta}$

For $0.1^{mA} \leqslant I_C \leqslant 5^{mA}$, β remains constant at 100.

$I_{REF} = 10mA \Rightarrow I_o = \dfrac{10}{1+\frac{2}{70}} = 9.72mA$

$I_{REF} = 0.1mA \Rightarrow I_o = \dfrac{0.1}{1+2/100} = 0.098mA$

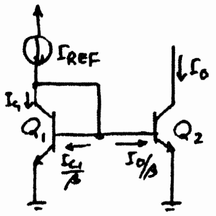
$I_{REF} = 1mA \Rightarrow I_o = \dfrac{1}{1+2/100} = 0.98mA$

$I_{REF} = 10\mu A \Rightarrow$

6.41

$I_{S2} = I_{S1} \times m$, $I_{C1} = I_C$

$I_{REF} = I_C + \frac{I_C}{\beta} + \frac{I_O}{\beta}$ ①



I_{REF}
I_O
I_{C1}
Q_1
Q_2
$\frac{I_{C1}}{\beta}$
I_O/β

$V_{BE1} = V_{BE2} \Rightarrow$

$V_T \ln \frac{I_C}{I_{S1}} = V_T \ln \frac{I_O}{I_{S2}}$

$\Rightarrow \frac{I_O}{I_C} = \frac{I_{S2}}{I_{S1}} = m \Rightarrow I_C = I_O/m$

by substituting for I_C in ①:

$I_{REF} = \frac{I_O}{m} + \frac{I_O}{m\beta} + \frac{I_O}{\beta} \Rightarrow \frac{I_O}{I_{REF}} = \frac{m}{1 + \frac{1}{\beta} + \frac{m}{\beta}}$

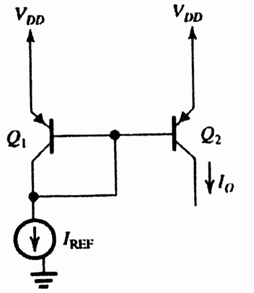
$\frac{I_O}{I_{REF}} = \frac{m}{1 + \frac{1+m}{\beta}}$

This result is the same as Eq. 6.22.

For large β, $I_O/I_{REF} = m$, with finite β this ratio drops to $I_O/I_{REF} = \frac{m}{1+\frac{1+m}{\beta}}$. To keep the introduced error within 5% : $0.95m = \frac{m}{1+\frac{1+m}{\beta}}$

$\beta_{min} = 80 \Rightarrow 0.95 = \frac{1}{1 + \frac{1+m}{80}} \Rightarrow \underline{m = 3.21}$

6.42



V_{DD}
V_{DD}
Q_1
Q_2
I_O
I_{REF}

For identical transistors, the transfer ratio is the same as eq. (7.69):

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + 2/\beta} = \frac{1}{1 + \frac{2}{20}} = 0.91$$

6.43

$I_{C1} = I_{C2} = I_{R1}$

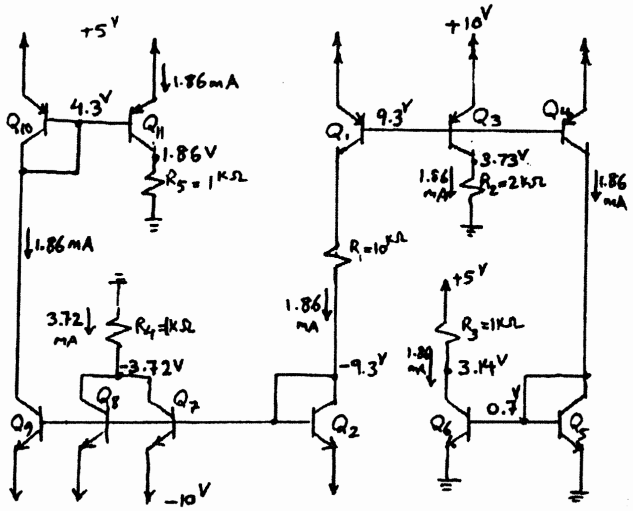
$V_{B1} = 10 - 0.7 = 9.3V$, $V_{B2} = -10 + 0.7 = -9.3^V$, $I_{R1} = \frac{9.3+9.3}{10}$

$\Rightarrow I_{R1} = 1.86\,mA = I_{C1} = I_{C2} = I_{C3} = I_{C4} = I_{C5} = I_{C6}$

$V_{C3} = 1.86^m \times 2^K = 3.72V$, $V_{C5} = 0.7^V$

$V_{C6} = 5 - 1.86 \times 1 = 3.14V$, $I_{C9} = I_{C8} = I_{C7} = I_{C2} = 1.86^{mA}$

$I_{R4} = 2 \times 1.86 = 3.72mA \Rightarrow V_{C7} = -3.72 \times 1 = -3.72V$



$+5^V$
1.86mA
4.3^V
Q_{10}
Q_{11}
1.86V
$R_5 = 1K\Omega$
1.86mA
3.72 mA
$R_4 = 1K\Omega$
-3.72V
Q_9
Q_8
Q_7
-10^V
$+10^V$
Q_1
9.3^V
Q_3
Q_4
3.73^V
1.86 mA
$R_2 = 2K\Omega$
1.86 mA
$R_1 = 10^{K\Omega}$
1.86 mA
-9.3^V
Q_2
$+5^V$
$R_3 = 1K\Omega$
1.86 mA
3.14^V
0.7^V
Q_6
Q_5

$I_{C10} = I_{C9} = 1.86mA$

$V_{C9} = V_{C10} = V_{B10} = 5 - 0.7 = 4.3^V$

$I_{C11} = I_{C10} = 1.86mA$

$V_{C11} = 1.86 \times 1 = 1.86V$

6.44

a)

$R = 10K\Omega$

$V_1 = -0.7^V \Rightarrow I_{C_1} = \frac{-0.7-(-10.7)}{10^K} = 1mA$

$\underline{I_{C_1} = 1mA}$

$V_2 = 5.7 - 0.7 = 5V$

$I = I_{C3} + I_{C4}$, $I_{C3} = I_{C4} = I_{C_1} \Rightarrow I = 2\times1 = \underline{2mA}$

$V_3 = 0 + 0.7 = 0.7^V$

$V_4 = -10.7 + 1\times10^K = -0.7V$

$V_5 = -10.7 + 1\times\frac{10^K}{2} = -5.7^V$

b) $R = 100K\Omega$

$V_1 = -0.7V \Rightarrow I_{C_1} = \frac{-0.7+10.7}{100^K} = \underline{0.1mA}$

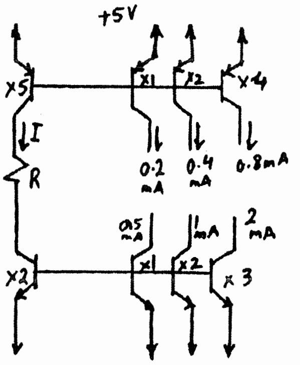
$I = 2I_{C_1} = \underline{0.2mA}$

$V_3 = 0.7V$, $V_2 = 5.7 - 0.7 = 5^V$

$V_4 = -10.7 + \frac{1}{10}\times100 = -0.7^V$

$V_5 = -10.7 + 0.1\times\frac{100}{2} = -5.7^V$

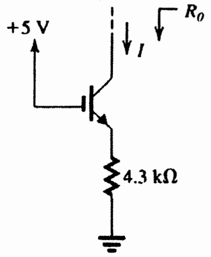
6.45



$I = \frac{10-1.4}{R} = 1^{mA}$

$\Rightarrow \underline{R = 8.6K\Omega}$

6.46



$$I_E = \frac{5 - V_{BE}}{4.3\ \text{k}\Omega} = \frac{5 - 0.7}{4.3\ \text{k}\Omega} = 1\ \text{mA}$$

since $\beta >> 1$, $I = I_C \approx I_E \approx 1$ mA

To find the output resistance, we can use eq. (7.50) or since $g_m r_O >> 1$,

$$R_O \approx r_O + g_m r_O (R_e \| r_\pi)$$

In this case,

$$g_m = \frac{I_C}{V_T} = \frac{1\ \text{mA}}{25\ \text{mV}} = 0.04\ \text{A/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{.04} = 2.5\ \text{k}\Omega$$

$$r_O = \frac{V_A}{I_C} = \frac{100\ \text{V}}{1\ \text{mA}} = 100\ \text{k}\Omega$$

R_O = 100 K + (.04 A / V)(100 kΩ) (4.3 kΩ ∥ 2.5 kΩ)

R_O = 6.42 MΩ

If the collector voltage changes by 10 V,

$$\Delta I = \frac{\Delta V}{R_O} = \frac{10\ \text{V}}{6.42\ \text{M}\Omega} = 1.56\ \mu\text{A}$$

6.47

All the transistors in this problem are operating at a bias current of 0.5mA and thus have:

$r_e = 50\Omega$, $g_m = 20\, mA/V$, $r_\pi = 5K\Omega$

$C_\pi + C_\mu = \frac{20^m}{2\pi \times 400^M} = 8\,pF$

Since $C_\mu = 2pF \Rightarrow C_\pi = 6pF$, $r_o = \infty$, $r_x = 0$

a) Common-Emitter amplifier:

$R_{sig} = 10k\Omega$, $R_C = 10k\Omega$

$A_M = -\frac{r_\pi}{R_{sig}+r_\pi} g_m R_C = -\frac{5}{10+5} 20\times 10 = \underline{-66.7 V/V}$

$f_H = \frac{1}{2\pi (R_{sig} \| r_\pi)\left[C_\pi + (1+g_m R_C) C_\mu\right]} \Rightarrow$

$f_H = \frac{1}{2\pi (10^k \| 5^k)\left[6^p + (1+20\times 10)2\right]} = \underline{117 kHz}$

b) Cascode:

$A_M = -\frac{\beta_1 \alpha_2 R_C}{R_{sig}+r_\pi} = -\frac{100\times 0.99\times 10}{10+5} = -66 V/V$

Input pole: $f_{P1} = \frac{1}{2\pi (R_{sig} \| r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$

$f_{P1} = \frac{1}{2\pi (10^k \| 5^k)(6+4)^p} = 4.77 MHz$

output pole: $f_{P3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2^p \times 10^k} = 7.96\, MHz$

pole at midband node:

$f_{P2} = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi \times 6^p \times 50} = 530.5\, MHz$

Very high

$f_H = \sqrt{\frac{1}{\left(\frac{1}{f_{P1}}\right)^2 + \left(\frac{1}{f_{P2}}\right)^2}} = 4.1 MHz$

c) CC-CB Cascade (Modified diff. amplifier)

$A_M = \frac{\beta R_C}{R_{sig}+2r_\pi} = \frac{100\times 10}{10+10} = 50 V/V$

Input pole: $f_{P1} = \frac{1}{2\pi (R_{sig} \| 2r_\pi)(C_\pi/2 + C_\mu)}$

$f_{P1} = \frac{1}{2\pi (10^k \| 10^k)(3+2)^p} = 6.4 MHz$

Output pole: $f_{P2} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2^p \times 10^k} = 7.96\, MHz$

Thus: $f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{P1}}\right)^2 + \left(\frac{1}{f_{P2}}\right)^2}} = 5 MHz$

d) CC-CE Cascade:

$A_M = -\frac{(\beta_1+1)\beta_2 R_C}{R_{sig} + r_{\pi 1} + (\beta_1+1) r_{\pi 2}} = -\frac{101\times 100\times 10}{10+5+101\times 5} = -194 V/V$

Refer to Example 6.13 in:

$R_{\mu 1} = (R_{sig} \| R_{in}) = 10^k \| (\beta+1)\left[r_{e1} + r_{\pi 2}\right]$

$R_{\mu 1} = 10^k \| 101 \times [0.05+5] = 9.81 K\Omega$

$R_{\pi 1} = r_{\pi 1} \| \frac{R_{s} + r_{\pi 2}}{1 + g_{m1} r_{\pi 2}} = 5 \| \frac{10+5}{1+20\times 5} = 144\Omega$

$R_T = r_{\pi 2} \| \frac{r_{\pi 1} + R_{sig}}{\beta+1} = 5^k \| \frac{5+10}{101} = 144\Omega$

where $C_T = C_{\pi 2} + C_{\mu 2}(1 + g_{m2} R_C) = 6 + 2(1+200)$

$C_T = 408pF$

$R_{\mu 2} = R_C = 10K\Omega$

$\tau_H = C_{\mu 1} R_{\mu 1} + C_{\pi 1} R_{\pi 1} + C_T R_T + C_{\mu 2} R_{\mu 2}$

$\tau_H = 2\times 9.81 + 6\times 0.144 + 408\times 0.144 + 2\times 10$

$\tau_H = 19.62 + 0.86 + 58.75 + 20 = 99.2 ns$

$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 99.2^n} = \underline{1.6 MHz}$

e) Folded Cascode:

$A_M = -\frac{\beta_1 \alpha_2 R_C}{R_{sig} + r_{\pi 1}} = -\frac{100\times 0.99\times 10}{10+5} = -66 V/V$

Input pole:

$f_{P1} = \frac{1}{2\pi (R_{sig} \| r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})} = \frac{1}{2\pi (10 \| 5)(6+4)}$

$f_{P1} = 4.77 MHz$

At middle: $f_{P2} = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi \times 6^p \times 0.05} = 530 MHz$ very high!

At output: $f_{P3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2\times 10} \Rightarrow$

$f_{P3} = 7.96 MHz$

Thus: $f_H \simeq \frac{1}{\sqrt{\frac{1}{4.77^2} + \frac{1}{7.96^2}}} = \underline{4.1 MHz}$

f) CC_CB Cascade:

$$A_M = \frac{(\beta_1+1)\alpha_2 R_C}{R_{sig}+(\beta_1+1)2r_e} = \frac{101\times 0.99\times 10}{10+101\times 0.1} \simeq 50\,V/V$$

Input pole: $f_{P1} = \dfrac{1}{2\pi(R_{sig}\parallel 2r_\pi)(C_\pi/2 + C_\mu)}$

$$f_{P1} = \frac{1}{2\pi(10^k \parallel 10^k)(3^P + 2^P)} = 6.4\,\text{MHz}$$

Output pole: $f_{P2} = \dfrac{1}{2\pi R_C C_\mu} = \dfrac{1}{2\pi\times 10^k\times 2^P} = 7.96\,\text{MHz}$

$$f_H \cong \frac{1}{\sqrt{\frac{1}{6.4^2}+\frac{1}{7.96^2}}} = \underline{5\,\text{MHz}}$$

Summary of results:

Configuration	A_M (V/V)	f_H (MHz)	G.B. (MHz)
a) CE	−66.7	0.117	7.8
b) Cascode	−66	4.1	271
c) CC_CB cascade	+50	5.0	250
d) CC_CE cascade	−194	1.6	310
e) Folded cascode	−66	4.1	271
f) CC_CB cascade	+50	5.0	250

6.48

$I_{REF} = 80\,\mu A = I_4 = I_1 = I_2 = I_3$

All transistors have the same g_m, r_o, V_{OV} values.

$I = \frac{1}{2}k_n' \frac{W}{L} V_{OV}^2 \Rightarrow 0.08 = \frac{1}{2}\times 4\times V_{OV}^2 \Rightarrow V_{OV} = 0.2V$

$V_{GS} = V_{OV} + V_t = 0.2 + 0.5 = 0.7V$

$V_{G1} = V_{GS} = 0.7V = V_{S4} \Rightarrow V_{G4} = 0.7 + V_{GS4} = 1.4^V$

$\Rightarrow V_{G3} = 1.4^V \Rightarrow V_{S3} = 1.4^V - V_{GS} = 0.7^V$

$\Rightarrow V_{O_{min}} = V_{O3} = V_{S3} + V_{OV} = \underline{0.9V}$

As explained, the voltage at the gate of Q_3 is $2V_{GS}$ which implies voltage of $V_{GS} = V_{OV} + V_t$ at the source of Q_3. For minimum allowable voltage at the output, $V_{DS} = V_{OV}$ or equivalently $V_{O_{min}} = V_{OV} + V_{GS}$

$V_{Omin} = V_{OV} + V_{OV} + V_t = 2V_{OV} + V_t$.

$g_m = \frac{2I_D}{V_{OV}} = \frac{2\times 0.08}{0.2} = 0.8\,mA/V$ $\quad r_o = \frac{V_A}{I_D} = \frac{8}{0.08} = 100\,k\Omega$

Using Eq. 6.189: $R_o = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}]r_{o2}$

$R_o = 100K + [1 + 0.8\times 100]\times 100 = \underline{8.2\,M\Omega}$

6.49

$I_{REF} = 25\ \mu A,$

$I_4 = 25\ \mu A = I_1 \quad W_1 = W_4 = 2\ \mu m$

$W_2 = W_3 = 40\ \mu m$

$I_1 = \frac{1}{2}k_n' \frac{W_1}{L_1} V_{OV1}^2 \Rightarrow 25$

$= \frac{1}{2}\times 200\times\frac{2}{1}V_{OV_1}^2 \Rightarrow V_{OV_1} = 0.354\ V$

$V_{OV1} = V_{OV2} \Rightarrow \frac{I_2}{I_1} = \frac{(W/L)_2}{\left(\frac{W}{L}\right)_1}$

$\Rightarrow I_2 = 25\times\frac{40}{2} = 500\ \mu A$

$I_2 = 0.5\ mA = I_3$

$I_O = 0.5\ mA$

$V_{GS1} = V_{OV1} + V_t = 0.354 + 0.6 = 0.954\ V$

$V_{G1} = 0.954\ V$

$V_{G4} = V_{GS1} + V_{GS4},$

Since $I_1 = I_4$ and $W_1 = W_4$ then

$V_{GS1} = V_{GS4} \Rightarrow V_{G4} = 2V_{GS1}$

$= 1.191\ V = V_{G3}$

The lowest possible voltage for the output is when Q_3 has $V_{DS3} = V_{OV3}$ or

$V_{O\,min} = V_{G3} - V_{GS3} + V_{OV3}$

since $V_{GS1} = V_{GS2}$ and $I_2 = I_3$ then

$V_{GS3} = V_{GS1}$

$\Rightarrow V_{O\,min} = 1.91 - 0.954 + 0.354 = 1.31\ V$

$g_{m2} = g_{m3} = \frac{2I_D}{V_{OV}} = \frac{2\times 0.5}{0.354}$

$= 2.82\ mA/V$

6.50

$$r_{o2} = r_{o3} = \frac{V_A}{I_D} = \frac{20}{0.5} = 40\ \text{k}\Omega$$

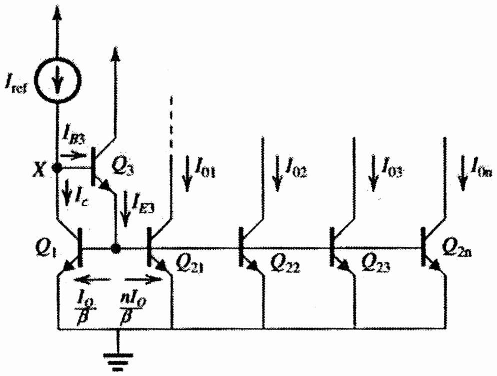
Eq. 6.189:

$$R_O = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}]r_{o2}$$

$$R_O = 40\ \text{k}\Omega + [1 + 2.82\ \text{k}\Omega \times 40\ \text{k}\Omega \times 40\ \ \text{k}\Omega]$$

$$\times 40\ \ \text{k}\Omega\ \ = 4.6\ \text{M}\Omega$$

6.51



I_{ref}
I_{B3}
X
Q_3
I_C
I_{E3}
I_{01}
I_{02}
I_{03}
I_{0n}
Q_1
Q_{21}
Q_{22}
Q_{23}
Q_{2n}
$\frac{I_0}{\beta}$
$\frac{nI_0}{\beta}$

$I_{01} = I_{02} = I_{03} \dots = I_{0n} = I_0$

The emitter of Q_3 supplies the base currents for all transistor so

$$I_{E3} = \frac{(n+1)I_0}{\beta}$$

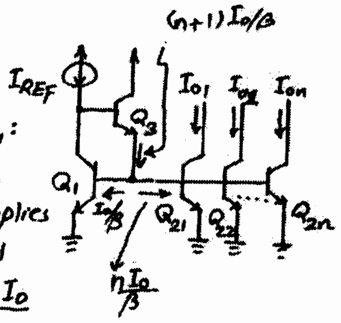
$$I_{REF} = I_{B3}\ rI_0 = \frac{(n+1)I_0}{\beta(\beta+1)} + I_0$$

$$\frac{I_0}{I_{REF}} = \frac{1}{1 + \frac{(n+1)}{\beta(\beta+1)}}$$

for deviation of .1% from unity:

$$\frac{99.9}{100} = \frac{1}{1 + \frac{(n+1)}{100(101)}} \Rightarrow n \cong 9$$

6.52



$(n+1)I_o/\beta$
I_{REF}
Q_3
I_{o1}
I_{o2}
I_{on}
Q_1
I_o/β
Q_{21}
Q_{22}
Q_{2n}
$\frac{nI_o}{\beta}$

Since $Q_{21}, Q_{22}, \dots Q_{2n}$ are all matched to Q_1:

$I_{o1} = I_{o2} = \dots = I_{on} = I_0$

The emitter of Q_3 supplies the basecurrent for all trasistors, so $I_{c3} = \frac{(n+1)I_o}{\beta}$

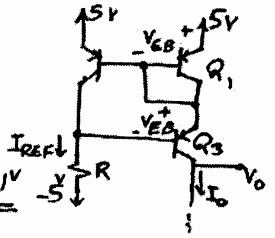
A node equation at the base of Q_3 yields:

$I_{REF} = I_o + \frac{(n+1)I_o}{\beta(\beta+1)}$, Thus : $\frac{I_o}{I_{REF}} = \frac{1}{1 + \frac{n+1}{\beta^2}}$

For a deviation from unity of less than 0.1% : $\frac{99.9}{100} = \frac{1}{1 + \frac{n+1}{\beta^2}} \Rightarrow \frac{n+1}{\beta^2} = \frac{1}{999}$

$\Rightarrow n = \frac{\beta^2}{999} - 1 \Rightarrow \underline{n \cong 9}$

6.53



5V
+5V
V_{EB}
Q_1
V_{EB}
Q_3
I_{REF}
R
−5V
V_o
I_o

$I_{REF} = 0.1\text{mA} = \frac{5 - 0.7 - 0.7 - (-5)}{R}$

$\Rightarrow \underline{R = 86\text{K}\Omega}$

V_{omax} is obtained when Q_3 is saturated: $V_{omax} = 5 - 0.7 - 0.2 = \underline{\underline{4.1^V}}$

6.54

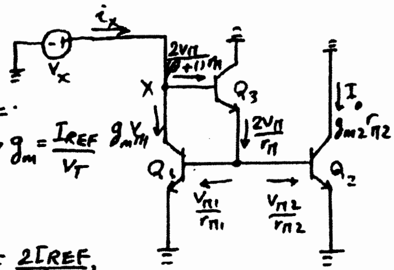
Q_1 and Q_2 are biased at I_{REF}.

$r_{e1} = r_{e2} = \frac{V_T}{I_{REF}} \Rightarrow g_m = \frac{I_{REF}}{V_T}$

$r_{\pi 1} = \frac{\beta V_T}{I_{REF}}$

Q_3 is biased at $\frac{2I_{REF}}{\beta}$,

Thus $r_{e3} = \frac{\beta V_T}{2I_{REF}}$



i_x
V_x
X
$\frac{2V_\pi}{(\beta+1)r_\pi}$
Q_3
$g_m V_{\pi 1}$
$\frac{2V_\pi}{r_\pi}$
I_o
$g_{m2} r_{\pi 2}$
Q_1
Q_2
$\frac{V_{\pi 1}}{r_{\pi 1}}$
$\frac{V_{\pi 2}}{r_{\pi 2}}$

$v_{\pi 1} = v_{\pi 2}$

small-signal model

Refer to the small_circuit analysis performed directly on the circuit. Since the current in the emitter of Q_3 is $\frac{2V_{\pi 1}}{r_{\pi 1}}$, the voltage $V_{\pi 3}$ will be: $V_{\pi 3} = \frac{2V_{\pi 1}}{r_{\pi 1}} \times r_{e3}$.

$V_x = V_{\pi 2} + V_{\pi 1} = \frac{2V_{\pi 1} \cdot r_{e3}}{r_{\pi 1}} + V_{\pi 1} = V_{\pi 1}\left(1 + 2\frac{r_{e3}}{r_{\pi 1}}\right)$

$V_x = V_{\pi 1}\left(1 + 2\frac{\beta V_T}{2I_{REF}} \times \frac{I_{REF}}{\beta V_T}\right) = 2V_{\pi 1}$

and $i_x \simeq g_{m1} V_{\pi 1}$. Thus: $R_{in} = \frac{V_x}{I_x} = \frac{2}{g_{m1}} = \frac{2V_T}{I_{REF}}$

For $I_{REF} = 100\,\mu A \Rightarrow R_{in} = \frac{2 \times 0.025}{0.1} = \underline{0.5\,K\Omega}$

6.55

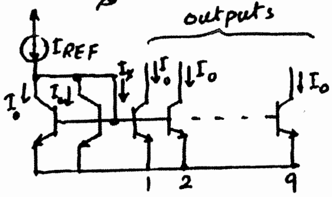
All the output currents are equal to I_0, then we have: $I_{REF} = 2I_0 + 11\frac{I_0}{\beta} \Rightarrow \frac{I_0}{I_{REF}} = \frac{1}{2 + 11/\beta}$

I_0 is idealy $I_{REF}/2$, For 5% lower I_0:

$$0.95 \times \frac{I_{REF}/2}{I_{REF}} = \frac{1}{2 + \frac{11}{\beta}} \Rightarrow \beta = 104.5 \simeq 105$$

$\underline{\underline{\beta = 105}}$

$I_x = 11 \times \frac{I_0}{\beta}$



I_{REF}
outputs
I_0
I_0
I_x
I_0
I_0
I_0
1
2
9

6.56

a) See the analysis on the circuit.

$$I_{REF} = I + \frac{\beta + 2}{\beta(\beta + 1)}I = I\frac{\beta^2 + 2\beta + 2}{\beta(\beta + 1)}$$

$$I_{01} = I_{02} = \frac{1}{2}\frac{\beta + 2}{\beta + 1}I$$

$$\frac{I_{01}}{I_{REF}} = \frac{I_{02}}{I_{REF}} = \frac{1}{2}\frac{\beta(\beta + 2)}{\beta^2 + 2\beta + 2}$$

$$= \frac{1}{2} \times \frac{1}{1 + 2/(\beta^2/2\beta)}$$

$$\frac{I_{01}}{I_{REF}} = \frac{1}{2}\frac{1}{1 + 2/\beta^2}$$

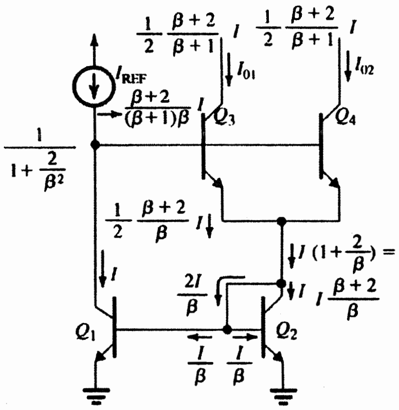
Observe that the deviation factor $\frac{1}{1 + 2/\beta^2}$ is independent of the number of outputs or the value of each output, i.e.:

The current I_{REF} can be split into any number of outputs through an appropriate combinations of parallel-connected transistors. (Q_3 and Q_4 in this case) The reason the error factor remains unchanged at $\frac{1}{1 + 2/\beta^2}$ is that the base current that need to be supplied by I_{REF} (subtract from I_{REF}) remains unchanged.

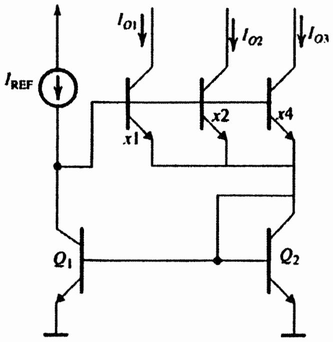
b) The I mA reference current can be used to generate three output currents of 1, 2, 4 mA by using 3 transistors in parallel having relative area ratios of 1, 2, 4 as shown:

$$\frac{I_{01}}{I_{REF}} = \frac{1}{7}\frac{1}{1 + 2/\beta^2} \Rightarrow I_{01}$$

$= 0.998$ mA (1 mA ideally)



$\frac{1}{2}\frac{\beta+2}{\beta+1}I$
$\frac{1}{2}\frac{\beta+2}{\beta+1}I$
I_{01}
I_{02}
I_{REF}
$\frac{\beta+2}{(\beta+1)\beta}I$
Q_3
Q_4
$\frac{1}{1+\frac{2}{\beta^2}}$
$\frac{1}{2}\frac{\beta+2}{\beta}I$
$I\left(1+\frac{2}{\beta}\right) =$
$I\frac{\beta+2}{\beta}$
I
$\frac{2I}{\beta}$
I
Q_1
Q_2
$\frac{I}{\beta}$
$\frac{I}{\beta}$



$$\frac{I_{O2}}{I_{REF}} = \frac{2}{7}\frac{1}{1+2/\beta^2} \Rightarrow I_{O2}$$

$= (1.996)$ mA (2 mA ideally)

$$\frac{I_{O3}}{I_{REF}} = \frac{4}{7}\frac{1}{1+2/\beta^2} \Rightarrow I_{O3}$$

$= 3.992$ mA (4 mA ideally)

6-57

(a) First, we need an estimate for V_{ov} and V_{GS}. Since the currents are all approximately the same, and $I_D = \frac{1}{2}(\mu_n C_{ox})(W/L)V_{OV}^2$,

6.57

$$V_{OV} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}$$

$$V_{OV} = \sqrt{\frac{2(100\ \mu A)}{(400\ \mu A/V)(12.5)}} = 0.2\ V$$

since no value is given for V_{tn}, we have to estimate this with $\mu_n C_{ox} = 400\ \mu A/V^2$,

this fabrication process is similar to the 0.18 µm technology. We will therefore approximate V_{tn} as approximately 0.5 V.

$V_{GS} = V_{tn} + V_{ov} = 0.5\ V + 0.2\ V = 0.7\ V$

(b) $V_{DS2} = V_{GS1} + V_{GS3} = 1.4$ V, which is $\simeq (2 \cdot V_{os1})$

$$r_o = \frac{V_A}{I} = \frac{20\ V}{0.1\ mA} = 200\ k\Omega$$

$$\Delta I = \frac{\Delta V_{os}}{r_o} = \frac{1.4 - 0.7}{200\ k\Omega} = 0.35\ \mu A$$

$I_o \approx I_{REF} - \Delta I$

so that,

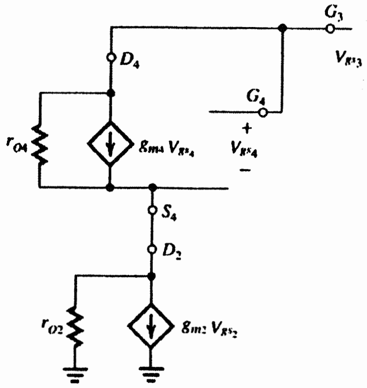
$I_o \approx 100 - 3.5 = 96.5\ \mu A$

(c) $I_o \approx I_{REF}$

(d) $V_{o_{min}} = V_{tn} + 2V_{ov}$

$= 0.5 + 2(0.2\ V) = 0.9\ V$

(e) If a small-signal model is added to account for Q_4, the circuit is changed to



Since $V_{Ds4} = V_{GS4} = -g_{m4}V_{gs4}r_{o4}$ (no current into gate 3)

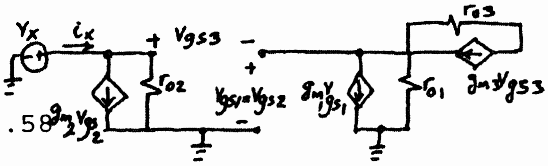
$V_{GS4} = V_{DS4} = 0$ so that $V_{D2} = V_{G3}$ and there is no effect.

$R_o \approx (g_{m3}r_{o3})r_{o2}$

$$g_m = \frac{I_D}{V_{ov}/2} = \frac{0.1\ mA}{0.2\ v/2} = 1\ mA/V$$

$R_o \approx (1\ mA/V)(200\ k\Omega)^2 = 40\ M\Omega$

6.58



$i_x = g_{m2}V_{gs2}$ ①

$V_{gs2} + V_{gs3} = V_x$.

Since Q_2 and Q_3 have the same parameters and same current, therefore $V_{gs2} = V_{gs3}$

$V_x = 2V_{gs2} \Rightarrow V_{gs2} = \frac{V_x}{2}$

Substitute for V_{gs2} in ①:

$i_x = g_{m2} \times \frac{V_x}{2}$

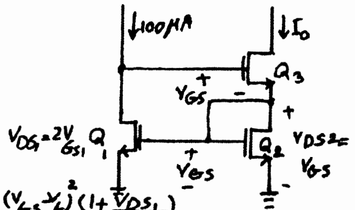
$R_{in} = \frac{V_x}{i_x} = \frac{2}{g_{m2}}$

6.59

$I_{REF} = 100\mu A$

$V_{DS1} = 2V_{GS}$

$V_{DS2} = V_{GS}$



$I_{D1} = I_{REF} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_t)^2 (1 + \frac{V_{DS1}}{V_A})$

$100 = \frac{1}{2} \times 2000 (V_{GS} - 0.6)^2 (1 + \frac{2V_{GS}}{20})$

$1 = (V_{GS} - 0.6)^2 (10 + V_{GS})$

$V_{GS} \simeq 0.91V$ (by iteration)

$I_o = I_{D2} = \frac{1}{2} \times 2000 (V_{GS} - 0.6)^2 (1 + \frac{V_{GS}}{20})$

$I_o = 100.47\mu A$

Thus there is $\frac{0.47}{100}$ or 0.5% error. Modifying the circuit as Fig. 6.61C ensures that Q_1 and Q_2 have the same V_{DS} and thus eliminate the above error.

6.60 $I_{REF} = 100\mu A$ $I_o = 10\mu A$

a) $V_{BE1} = 0.7 + V_t \ln \frac{100}{1000} = 0.642V$

$V_{BE2} = 0.7 + V_t \ln \frac{10}{1000} = 0.585V$

$I_o = \frac{V_{BE1} - V_{BE2}}{R_E} = 10\mu A \Rightarrow R_E = 5.7\ k\Omega$

b) $r_{\pi 2} = (\beta + 1) \frac{V_T}{10\mu A} = 503k\Omega >> R_E$

$r_{o2} = \frac{V_A}{I_o} = 10M\Omega \Rightarrow R_o = (1 + g_{m2} R_E) r_{o2} = 33M\Omega$

$\underline{R_o = 33M\Omega}$

$\Delta I_o = \frac{\Delta V_o}{R_o} = \frac{5}{R_o} = 0.15\mu A$

6.61

a) $\frac{I_o}{I_{REF}} = 0.9 \Rightarrow I_o = 90\mu A$

$V_{RE} = V_T \ln \frac{1}{0.9} = 2.63mV$

$R_E = \frac{2.63mV}{90\mu A} = 29.3\Omega$

$r_o = \frac{V_A}{I_o} = 1.11M\Omega$

$g_m = 3.6\ mA/V$

$R_o = (1 + g_m R_E) r_o = \underline{1.23M\Omega}$ Compare to $r_o = 1.11\ M\Omega$

b) $\frac{I_o}{I_{REF}} = 0.1 \Rightarrow I_o = 10\ \mu A$

$V_{R_E} = V_T \ln 10 = 57.56\ mV$

$R_E = \frac{57.56m}{10\mu} = 5.76k\Omega$

$r_o = \frac{100}{10\mu} = 10M\Omega$

$g_m = 0.4m\ A/V$

$R_o = (1 + g_m R_E) r_o = 33M\Omega$ Compare to $r_o = 10M\Omega$

c) $\frac{I_o}{I_{REF}} = 0.01 \Rightarrow I_o = 1\mu A$

$V_{RE} = V_T \ln 100 = 115mV$

$R_E = \frac{115}{1} = 115k\Omega$

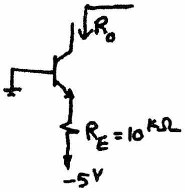
$r_o = \frac{100}{1} = 100\ M\Omega$

$g_m = 0.04\ mA/V$

$R_o = (1 + g_m R_E) r_o = 560M\Omega$ Compare to $r_o = 100\ M\Omega$

6.62

$R_o = [1 + g_m (R_E \| r_\pi)] r_o$



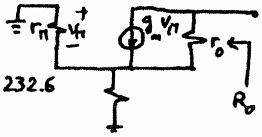
$I_E = \frac{-0.7 - (-5)}{R_E} = 0.43mA$

$g_m = \frac{I_C}{V_T} = \frac{0.43}{0.025} = 17.2mA/V$

$r_o = \frac{VA}{I_C} = \frac{100}{0.43} = 232.6k\Omega$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{17.2} = 5.8k\Omega$

$R_E = 10^{k\Omega}$



$R_o = [1 + (10^k \| 5.8^k) \times 17.2] \times 232.6$

$\underline{R_o = 14.92M\Omega}$

6.63

$I = 2mA \Rightarrow g_m = \frac{2}{0.025} = 80mA/V \quad , \quad r_\pi = \frac{\beta}{g_m} = 1.25^{K\Omega}$

$r_e = \frac{r_\pi}{\beta+1} = 12.4\Omega$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_\pi + C_\mu = \frac{80^m}{2\pi \times 400\times10^6} = 31.85^{pF}$

$\Rightarrow C_\pi = 31.85 - 2 = 29.85pF$

$A_M = \frac{R_L}{\frac{R_{sig}+r_\pi}{\beta+1} + R_L} = \frac{1}{\frac{R_{sig}}{101} + 0.0124 + 1} = \underline{\underline{\frac{1}{1.0124 + \frac{R_{sig}}{101}}}}$

$R'_L = R_L = 1K\Omega \quad , \quad R'_{sig} = R_{sig} + r_x = R_{sig}$

$R_\mu = R'_{sig} \| [r_\pi + (\beta+1)R'_L] \quad$ (Eq. 6.179)

$R_\mu = R_{sig} \| (1.25 + 101\times1) = \underline{\underline{R_{sig} \| 102.25^K}}$

$R_\pi = \frac{R'_{sig} + R'_L}{1 + \frac{R'_{sig}}{r_\pi} + \frac{R'_L}{r_e}} \quad$ (Eq. 6.180)

$R_\pi = \frac{R_{sig} + 1K}{1 + 0.8R_{sig} + 80} = \underline{\underline{\frac{R_{sig}+1}{0.8R_{sig} + 81}}}$

$f_H = \frac{1}{2\pi(R_\pi C_\pi + R_\mu C_\mu)} = \frac{1}{2\pi(29.85R_\pi + 2R_\mu)}$

a) $R_{sig} = 1^{K\Omega}$: $\quad A_M = 0.978 V/V$

$R_\mu = 0.99^{K\Omega}, R_\pi = 24.4\Omega \Rightarrow f_H = 58.8MHz$

b) $R_{sig} = 10K\Omega$: $\quad A_M = 0.9 V/V$

$R_\mu = 9.11K\Omega, R_\pi = 124\Omega \Rightarrow f_H = 7.27MHz$

c) $R_{sig} = 100K\Omega$: $\quad A_M = 0.499 V/V$

$R_\mu = 50.6K\Omega, R_\pi = 627\Omega \Rightarrow f_H = 1.34MHz$

6.64

Each of the transistors is operating at a bias current of approximately $100\mu A$. Thus:

$g_m = \frac{0.1}{0.025} = 4mA/V$, $r_\pi = \frac{100}{4} = 25K\Omega$

$r_e \simeq 250\Omega$, $r_o = \frac{100}{0.1} = 1M\Omega$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{4^m}{2\pi \times 400^M} = 1.59pF \Rightarrow C_\pi = 1.39pF$

a) $R_{in} = (\beta+1)\,[r_{e1} + (r_{\pi 2} \| r_{o1})]$

$R_{in} = 101\,[250\times10^{-3} + 25^K \| 1M\Omega] \simeq 2.5^{M}\Omega$

$A_M = -\frac{R_{in}}{R_{in}+R_{sig}} \times \frac{r_{\pi2}\|r_{o1}}{r_{e1}+(r_{\pi2}\|r_{o1})} \times g_{m2} r_{o2}$

$A_M = -\frac{2.5^M}{2.5^M+0.01} \times \frac{25^K \| 1^M}{0.25+(25^K\|1^M)} \times 4\times 1^M$

$A_M = -3943.6\ V/V$

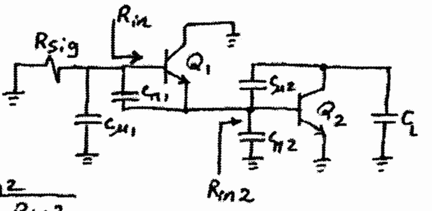
b) To calculate f_H, refer to

$R_{\mu1} = R_{sig} \| R_{in} = 10^K \| 2.5^M \simeq \underline{10K\Omega}$

$R_{in2} = r_{\pi2} \| r_{o1}$

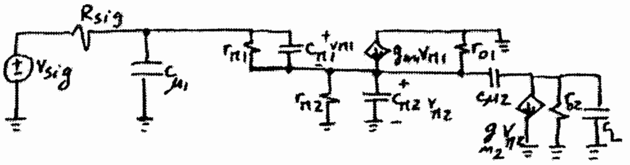
$R_{in2} = 25^K \| 1^M$

$R_{in2} = 24.4K\Omega$

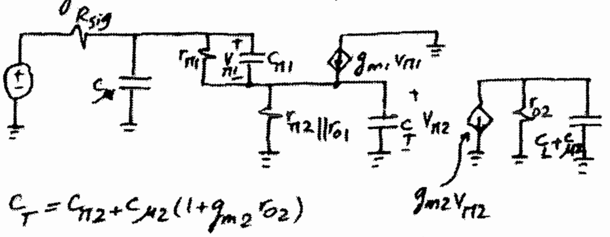


$R_{\pi1} = \frac{R_{sig} + R_{in2}}{1 + \frac{R_{sig}}{r_{\pi1}} + \frac{R_{in2}}{r_{e1}}}$

$R_{\pi1} = \frac{10 + 24.4}{1 + \frac{10}{25} + \frac{24.4}{0.25}} = 0.35K\Omega$



Using Miller's Theorem for $C_{\mu2}$:



$C_T = C_{\pi2} + C_{\mu2}(1 + g_{m2} r_{o2})$

$C_T = 1.39 + 0.2(1 + 4\times1000) = 801.6pF$

$R_T = r_{\pi2} \| r_{o1} \| \frac{r_{\pi1}+R_{sig}}{\beta+1} = 25^K \| 1000^K \| \frac{25+10}{101}$

$R_T = 342\Omega$

$R_{\mu2} = r_{o2} = 1000K$

$\tau_H = C_{\mu1}R_{\mu1} + C_{\pi1}R_{\pi1} + C_T R_T + (C_{\mu2} + C_L)R_{\mu2}$

$\tau_H = 0.2\times10 + 1.39\times0.35 + 801.6\times0.342 + (0.2+1)\times1000$

$\tau_H = 2 + 0.49 + 274.15 + 1200\ ns$

Thus $(C_L + C_{\mu2})R_{\mu2}$ is the dominating term, The second most significant term is $C_T R_T$. So $(C_L + C_{\mu2})$ dominates and then C_T or equivalently $C_{\mu2}$.

$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi\times1476.6^n} = 107.8MHZ$

c) Increasing the bias currents by a factor of 10:

$g_m = 40mA/V$, $r_\pi = 2.5K\Omega$

$r_e \simeq 25\Omega$, $r_o = 100K\Omega$

$C_\pi = C_{je} + C_{de}\times10 = 0.8 + 0.59\times10 = 6.7pF$

$C_\mu = 0.2pF$

$R_{in} = 101\left[0.025 + (2.5^K \| 100^K)\right] = 249K\Omega$

R_{in} is almost decreased by a factor of 10.

$A_M = -\frac{249}{249+10} \times \frac{2.5^K \| 100^K}{0.025 + (2.5^K\|100^K)} \times 4000$

$A_M = -3807\ V/V$

A_M remains almost constant.

$C_T = 6.7 + 0.2(1 + 40\times100) = 806.9$ (almost constant)

$R_{\mu1} = R_{sig} \| R_{in} = 10^K \| 249^K = 9.61K\Omega$

$R_{\mu1}$ stays almost the same.

$R_T = 2.5^K \| 10^K \| \frac{2.5+10}{101} = 117.8\Omega$

R_T is almost reduced by a factor of 3.

$R_{in2} = r_{\pi2} \| r_{o1} = 2.44K\Omega$

$R_{\pi1} = \frac{10^K + 2.44}{1 + \frac{10}{2.5} + \frac{2.44}{0.025}} = 120\Omega$

$R_{\pi1}$ is almost decreased by a factor of 3.

$R_{\mu2} = r_{o2} = 100K\Omega$ (decreased by a factor of 10)

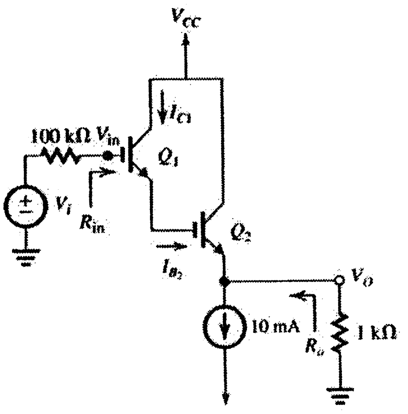
$\tau_H = 0.2\times9.61 + 6.7\times0.120 + 806.9\times0.118 + 1.2\times100$

$\tau_H = 1.92 + 0.8 + 95.2 + 120 = 217.92ns$

Thus the dominant effect, that of the output pole, is reduced by a factor of 10. This occurs because $(C_L + C_{\mu2})$ remains constant while r_{o2} decreases by a factor of 10. The second most significant factor (that due to C_T or $C_{\mu2}$ with Miller Effect) also decreases, but only by a factor of 3. The overall result is an increase in f_H.

Cont.

6.65



$$I_{B2} = \frac{I_{E2}}{\beta + 1} = \frac{10\text{ m}}{101} = 99\ \mu\text{A}$$

$$I_{E1} = I_{B2} = 99\ \mu\text{A}$$

$$I_{C1} = \alpha I_{E1} = \frac{100}{101}(99\ \mu) = 98\ \mu\text{A}$$

$$I_{C2} = \alpha I_{E2} = \frac{100}{101}(10\text{ m}) = 9.9\text{ mA}$$

Neglecting Early effect
using resistance reflection rule:

$$R_{in} = r_{\pi 1} + r_{\pi 2}(\beta_1 + 1) + 1\text{ K}(\beta_2 + 1)(\beta + 1)$$

$$R_{in} = \frac{\beta V_T}{I_{C1}} + \frac{\beta V_T}{I_{C2}}(101) + 1\text{k}(101)^2$$

$$= \frac{100(25\text{ m})}{98\ \mu} + \frac{100(25\text{ m})}{9.9\text{ m}}(101) + 10.2\text{ M}$$

$$R_{in} = 25.5\text{ K} + 25.5\text{ K} + 10.2\text{ M} = 10.252\text{ M}$$

$$R_O = \frac{r_{\pi 2}}{\beta_2 + 1} + \left[\frac{r_{\pi 1}}{\beta_1 + 1} + \frac{100\text{ K}}{\beta_1 + 1}\right]\left(\frac{1}{\beta_2 + 1}\right)$$

$$R_O = \frac{100(25\text{ m})}{(101)(9.9\text{ m})} + \left[\frac{25.5\text{ K}}{101} + \frac{100\text{ K}}{101}\right]\left(\frac{1}{101}\right)$$

$$R_O = 2.5 + (253 + 990)\left(\frac{1}{101}\right) = 14.8\ \Omega$$

$$A_{VO} = 1,000\text{ V/V}$$

$$A_V = \frac{1 \times 1,000}{14.8 + 1,000} = 0.985\text{ V/V}$$

6.66

$I_{E2} = 10\text{mA} \Rightarrow r_{e2} = 2.5\Omega$, $r_{\pi 2} = 253\Omega$

$I_{E1} = \frac{10}{101} \cong 0.1\text{mA} \Rightarrow r_{e1} = 250\Omega$, $r_{\pi 1} = 25.3\text{K}\Omega$

$R_{in} = 101 \times \left[0.25 + 101(0.0025 + 1)\right] = 10.3\text{M}\Omega$

$\underline{R_{in} = 10.3\text{M}\Omega}$

$$R_{out} = r_{e2} + \frac{1}{\beta_2 + 1}\left[r_{e1} + \frac{R_{sig}}{\beta_1 + 1}\right]$$

$$R_{out} = 2.5 + \frac{1}{101}\left[250 + \frac{100000}{101}\right] = \underline{14.8\Omega}$$

Neglecting r_o:

$A_{v_o} = 1000\text{ V/V}$, $A_v = \frac{1 \times 1000}{14.8 + 1000} = 0.985\text{ V/V}$

6.67

$I_1 = I_2 = I = 1\text{mA} \Rightarrow g_m = 40\text{ mA/V}$, $r_\pi = \frac{120}{40} = 3\text{K}\Omega$

$r_e = \frac{3}{121} \cong 25\Omega$, $C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40^m}{2\pi \times 700^M} = 9.1\text{pF} \Downarrow \underline{C_\pi = 8.6\text{pF}}$

Using Eq. 6.185:

$$A_M = \frac{v_o}{v_{sig}} = \frac{1}{2}\left(\frac{R_{in}}{R_{in} + R_{sig}}\right) g_m R_L$$

$$R_{in} = 2r_\pi = 2 \times 3\text{K}\Omega = 6\text{K}\Omega$$

$$A_M = \frac{1}{2} \times \frac{6}{6 + 20} \times 40 \times 10^K = 46.15\text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi\left(\frac{C_\pi}{2} + C_\mu\right)(R_{sig} \| 2r_\pi)} = \frac{1}{2\pi\left(\frac{8.6}{2} + 0.5\right)(20^K \| 6^K)}$$

$$f_{P1} = 7.19\text{MHz}$$

$$f_{P2} = \frac{1}{2\pi C_\mu R_L} = \frac{1}{2\pi \times 0.5^p \times 10^K} = 31.8\text{ MHz}$$

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{P1}}\right)^2 + \left(\frac{1}{f_{P2}}\right)^2}} = 7.01\text{MHz}$$

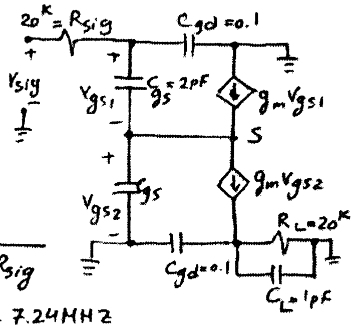
6.68

$v_i = 2v_{gs}$

$v_o = R_L \times g_m v_{gs}$

$\frac{v_o}{v_i} = \frac{v_o}{v_{sig}} = g_m R_L/2$

$A_M = \frac{5 \times 20}{2} = \underline{50\text{ V/V}}$



$$f_{P1} = \frac{1}{2\pi\left(\frac{C_{gs}}{2} + C_{gd}\right) R_{sig}}$$

$$f_{P1} = \frac{1}{2\pi(1^p + 0.1^p)20^K} = 7.24\text{MHz}$$

$$f_{P2} = \frac{1}{2\pi(C_{gd} + C_L) R_L} = \frac{1}{2\pi(0.1 + 1) \times 20^K} = 7.24\text{MHz}$$

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{P1}}\right)^2 + \left(\frac{1}{f_{P2}}\right)^2}} = \underline{5.12\text{MHz}}$$

	0.5 μm		0.25 μm		0.18 μm		0.13 μm	
	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS
V_{ov}(V)	0.32	−0.54	0.27	−0.46	0.23	−0.48	0.2	−0.42
V_{GS}(V)	1.02	−1.34	0.7	−1.08	0.71	−0.93	0.6	−0.82

	0.5 μm		0.25 μm		0.18 μm		0.13 μm	
	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS
g_m (mA / V)	0.62	0.37	0.73	0.43	0.88	0.41	1.02	0.48

6.69 If the area of the emitter-base junction is changed by a factor of 10, then I_S is changed by the save factor. If V_{BE} is kept constant, then I_C is also changed by the same factor:

$$I_C = I_S e^{V_{BE}/V_T}$$

$$I_S \alpha A, I_C \alpha I_S \Rightarrow I_C \alpha A$$

$$A_2 = 10 A_1 \Rightarrow I_{C2} = 10 I_{C1}$$

If I_C is kept constant, then V_{BE} changes:

$$I_{S2} = 10 I_{S1} \Rightarrow I_S e^{V_{BE}/V_T}$$

$$e^{\frac{V_{BE1} - V_{BE2}}{V_T}} = 10 \Rightarrow V_{BE1} - V_{BE2}$$

$$= V_T \ln 10 = 0.058 \text{ V or } 58 \text{ mV}$$

6.70 $\frac{W}{L} = 10, I_D = 100 \text{ μA},$

$$I_D = \frac{1}{2} k' \frac{W}{L} V_{OV}^2$$

$$V_{OV} = \sqrt{\frac{2I_D}{k'_n \frac{W}{L}}} = \sqrt{\frac{2 \times 100}{k'_n \times 10}} = \sqrt{\frac{20}{k'_n}}$$

$$V_{GS} = V_t + V_{OV}$$

6.71

$$|V_{OV}| = 0.25 \text{ V}, I_D = 100 \text{ μA}$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{k'_n V_{OV}^2}$$

For NMOS:

$$k'_n = 267 \frac{\text{μA}}{\text{V}^2} \Rightarrow \left(\frac{W}{L}\right)_n$$

$$= \frac{2 \times 100}{267 \times 0.25^2} = 11.98 \simeq 12$$

For PMOS:

$$k'_n = 93 \frac{\text{μA}}{\text{V}^2} \Rightarrow \left(\frac{W}{L}\right)_p$$

$$= \frac{2 \times 100}{93 \times 0.25^2} = 34.4 \simeq 34$$

6.72

$$i_{Dn} = i_{Dp} \Rightarrow \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n V_{OVn}^2$$

$$= \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p V_{OVp}^2 \quad (1)$$

we also have $g_{mn} = g_{mp}$

$$g_m = \frac{2I_D}{V_{OV}} \Rightarrow V_{OVn} = V_{OVp} \quad (2)$$

$$①, ② \Rightarrow \frac{\left(\frac{W}{L}\right)_p}{\left(\frac{W}{L}\right)_n} = \frac{\mu_n}{\mu_p} = \frac{460}{160} = 2.88$$

6.73

$V_{OV} = 0.25$ V

for an npn transistor:

$g_m = \frac{I_C}{V_T} = \frac{0.1}{0.025} = 4$ mA/V

For an NMOS with the same g_m, i.e.

$g_m = 4$ mA/V

we will have :

$g_m = \frac{2I_D}{V_{OV}} \Rightarrow I_D = g_m \times \frac{V_{OV}}{2} = 0.5$ mA

$I_D = 0.5$ mA

6.74

Assuming large r_o For both transistors, for case (a) we have $r = \frac{1}{g_m} = \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}}$

$r = \frac{10^3}{\sqrt{2 \times 200 \times 10 \times 0.1 \times 10^3}} = 1.58$ kΩ

For case (b) we have

$r = r_\pi \| \frac{1}{g_m} = \frac{\beta}{(\beta + 1)g_m}$

$r = \frac{\beta V_T}{(\beta + 1)I_C} \approx \frac{V_T}{I_c} = \frac{0.025}{0.1} = 0.25$ kΩ

$r = 250\ \Omega$

6.75

$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 100 \times 10^{-3}}{0.5} = 0.4$ mA/V

$r_o = \frac{V_A}{I_D} = \frac{V'_A L}{I_D} = \frac{25 \times 1}{0.1} = 250$ kΩ

$A_o = g_m r_o = 0.4 \times 250 = 100$ V/V

$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV} \Rightarrow$

$W = \frac{g_m \times L}{\mu_n C_{ox} V_{OV}} = \frac{0.4 \times 1}{127 \times 10^{-3} \times 0.5}$

$W = 6.3\ \mu$m

6.76

$L = 0.3\ \mu$m, $I_D = 100\ \mu$A,

$V_{OV} = 0.2$ V

$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 100 \times 10^{-3}}{0.2} = 1$ mA/V

$r_o = \frac{V_A}{I_D} = \frac{V_A \times L}{I_D} = \frac{5 \times 0.3}{0.1} = 15$ kΩ

$A_o = g_m r_o = 1 \times 15 = 15$V/V

$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV} \Rightarrow$

$W = \frac{g_m \times L}{\mu_n C_{ox} V_{OV}} = \frac{1 \times 0.3}{387 \times 10^{-3} \times 0.2}$

$W = 3.88\ \mu$m

6.77

$L = 0.3\ \mu$m, $W = 6\ \mu$m

$V_{OV} = 0.2$ V

$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{OV}^2$

$= \frac{1}{2} \times 387 \times \frac{6}{0.3} \times 0.2^2 = 155\ \mu$A

$I_D = 0.155$ mA

$g_m = \frac{2I_D}{V_{OV}} = 1.55$ mA/V

$C_{gs} = \frac{2}{3} \frac{W}{L} C_{ox} + C_{OV} = \frac{2}{3} WLC_{ox}$

$+ WL_{OV} C_{ox}$

$C_{gs} = \frac{2}{3} \times 6 \times 0.3 \times 8.6 + 6 \times 0.37$

$= 12.54$ fF

$C_{gd} = C_{OV} W = 0.37 \times 6 = 2.22$ fF

$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$

$= \frac{1.55 \times 10^{-3}}{2\pi(12.54 + 2.22) \times 10^{-15}} = 16.7$ GHz

If we use the approximation formula:

$f_T \approx \frac{1.5\ \mu_n V_{OV}}{2\pi L^2}$ when

$C_{gs} >> C_{gd}$, $C_{gs} \simeq \frac{2}{3} \frac{W}{L} C_{ox}$

$f_T \simeq \frac{1.5 \times 450 \times 10^{-4} \times 0.2}{2 \times \pi \times 0.3^2 \times 10^{-12}} = 23.9$ GHz

The approximation formula over estimates f_T because it ignores $WL_{OV} C_{ox}$ or C_{OV} in C_{gs} and C_{gd} calculation.

6.78

$I_C = 10\ \mu\text{A}$, High-voltage process:

$$g_m = \frac{I_C}{V_T} = \frac{10 \times 10^{-3}}{0.025} = 0.4\ \text{mA/V}$$

$$C_{de} = \tau_F g_m = 0.35 \times 10^{-9} \times 0.4 \times 10^{-3}$$

$$= 140 \times 10^{-15}\ \text{F} = 140\ \text{fF}$$

$$C_{je} = 2C_{jeo} = 2 \times 1 = 2\text{pF} = 2000\ \text{fF}$$

$$C_\pi = C_{je} + C_{je} = 2140\ \text{fF}$$

$$C\mu = C\mu_O = 0.3\ \text{pF} = 300\ \text{fF}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C\mu)}$$

$$= \frac{0.4 \times 10^{-3}}{2\pi(2140 + 300 \times 10^{-15})} = 26.1\ \text{MHz}$$

$I_C = 100\ \mu\text{A}$, High-voltage process:

$$g_m = 10 \times 0.4 = 4\ \text{mA/V},$$

$$C_{de} = 10 \times 140 = 1400\ \text{fF}$$

$$C_\pi = 3400\ \text{fF} \Rightarrow$$

$$f_T = \frac{4 \times 10^{-3}}{2\pi(3400 + 300) \times 10^{-15}} = 172.1\ \text{MHz}$$

$I_C = 10\ \mu\text{A}$, Low-voltage process

$$g_m = \frac{10 \times 10^{-3}}{0.025} = 0.4\ \text{mA/V}$$

$$C_{de} = 10 \times 10^{-12} \times 0.4 \times 10^{-3} = 4\ \text{fF}$$

$$C_{je} = 2 \times 5\ \text{fF} = 10\ \text{fF}$$

$$C_\pi = C_{de} + C_{je} = 14\ \text{fF}$$

$$C\mu \simeq C\mu_O = 5\ \text{fF}$$

$$f_T = \frac{g_m}{2\pi(C_\mu + C_\pi)} = \frac{0.4 \times 10^{-3}}{2\pi(5 + 14) \times 10^{-15}}$$

$$= 3.35\ \text{GHz}$$

$I_C = 100\ \mu\text{A}$, Low-voltage process

$$g_m = \frac{100 \times 10^{-3}}{0.025} = 4\ \text{mA/V}$$

$$C_{de} = 10 \times 4 = 40\ \text{fF}$$

$$C_\pi = 40 + 10 = 50\ \text{fF},\ C_\mu = 5\ \text{fF}$$

$$f_T = \frac{4 \times 10^{-3}}{2\pi(50 + 5) \times 10^{-15}} = 11.6\ \text{GHz}$$

In Summary:

	Standard High-Voltage npn		Standard low-Voltage npn	
	$I_C =$ 10 μA	$I_C =$ 100 μA	$I_C =$ 10 μA	$I_C =$ 100 μA
f_T	26.1 MHz	172.1 MHz	3.35 GHz	11.6 MHz

6.79

$$I_C = 1\ \text{mA} \Rightarrow g_m = \frac{I_C}{V_T} = 40\ \text{mA} / V$$

For pnp:

$$C_{de} = \tau_F g_m = 30 \times 10^{-9} \times 40\ \text{mA/V} = 1200\ \text{pF}$$

$$C_{je} = 2C_{jeo} = 2 \times 0.3 = 0.6\ \text{pF}$$

$$C_\pi = 1200.6\ \text{pF}$$

$$C_\mu \simeq 1\ \text{pF}$$

$$\Rightarrow f_T = \frac{g_m}{2\pi(c_\pi + c_\mu)} = \frac{40\ \text{mA/V}}{2\pi(1200.6 + 1)\ \text{pF}}$$

$$f_T = 5.3\ \text{MHz}$$

For npn:

$$C_{de} = \tau_F g_m = 0.35\ \text{ns} \times 40\ \text{mA/V} = 14\ \text{pF}$$

$$C_{je} = 2 \times 1 = 2\ \text{pF}$$

$$C_\mu = \simeq 0.3\ \text{pF}$$

$$C_\pi = 14 + 2 = 16\ \text{pF}$$

$$\Rightarrow f_T = \frac{40\ \text{mA/V}}{2\pi(16 + 0.3)\ \text{pF}} = 391\ \text{MHz}$$

6.80

$$A_O = g_m r_O = \frac{2I_D}{V_{OV}} \times \frac{V_A}{I_D} = \frac{2V_A}{V_{OV}} = \frac{2V'_A L}{V_{OV}}$$

Therefore A_O is only determined by setting values for L and V_{OV}.

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$= \frac{2I_D/V_{OV}}{2\pi\left(\frac{2}{3}WLC_{ox} + C_{OV} + C_{OV}\right)}$$

If we assume that C_{OV} is very small or equivalently $C_{gs} >> C_{gs}$ and $C_{gs} = \frac{2}{3}WLC_{ox}$:

(replace I_D with $\frac{1}{2}k'_n\frac{W}{L}V_{OV}^2$)

$$f_T \simeq \frac{k'_n W/L\ V_{OV}}{2\pi \times \frac{2}{3}WLC_{ox}} = \frac{3\mu_n}{4\pi}V_{OV}/L^2$$

$$= \frac{3}{4\pi}\mu_n\frac{V_{OV}}{L^2}$$

As we can see f_T can be determined after knowing V_{OV} and L, it is not dependent on either I_D or W.

6.81

$V_{OV} = 0.2\text{V}, L = 0.2\ \mu\text{m}, 0.3\ \mu\text{m}, 0.4\ \mu\text{m}$

$$A_O = g_m r_O = \frac{2V_A}{V_{OV}} = \frac{2V_A' \cdot L}{V_{OV}} = \frac{2 \times 5 \times L}{0.2}$$

$$= 50\ \text{L V/V}$$

$$f_T \simeq \frac{1.5\mu_n V_{OV}}{2\pi L^2} = \frac{1.5 \times 450 \times 10^{-4} \times 0.2}{2 \times 3.14 \times L^2 \times 10^{-2}}$$

$$= \frac{2.15}{L^2}\ \text{GHz}$$

$L(\mu\text{m})$	0.2	0.3	0.4
A_O (V / V)	10	15	20
f_t (GHz)	53.75	23.9	13.4

6.82

$L = 0.5\ \mu\text{m}, V_{OV} = 0.3\ \text{V}, C_L = 1\ \text{pF},$
$f_T = 100\ \text{MHz}$

$$f_T = \frac{g_m}{2\pi C_L} \Rightarrow g_m = 2\pi C_L f_T$$

$$= 2\pi \times 1\ \text{pF} \times 100\ \text{MHz} = 628\ \mu\text{A/V}$$

$$g_m = \frac{2I_D}{V_{OV}} \Rightarrow I_D = g_m \times V_{OV}/2 = 6.28 \times \frac{0.3}{2}$$

$$I_D = 94.2\ \mu\text{A}$$

$$I_D = \frac{1}{2}k_n' \frac{W}{L} V_{OV}^2 \Rightarrow W = \frac{2LI_D}{k_n' V_{OV}^2}$$

$$= \frac{2 \times 0.5 \times 94.2}{190 \times 0.3^2} = 5.51\ \mu\text{m}$$

$$W = 5.51\ \mu\text{m}$$

$$r_O = \frac{V_A}{I_D} = \frac{V_A' \cdot L}{I_D} = \frac{20 \times 0.5}{94.2 \times 10^{-3}} = 106.2\ \text{k}\Omega$$

$$A_O = g_m r_O = \frac{628}{1000} \times 106.2 = 66.7\ \text{V/V}$$

$$f_{3db} = \frac{1}{2\pi C_L r_O} = \frac{1}{2\pi \times 1\text{pF} \times 106.2\ \text{k}\Omega} = 1.5\ \text{MHz}$$

7.1

$V_{DD} = V_{SS} = 2.5\,V$

$K_n' \frac{W}{L} = 3\,\frac{mA}{V^2}$; $V_{tn} = 0.7\,V$

$I = 0.2\,mA$; $R_D = 5\,K\Omega$

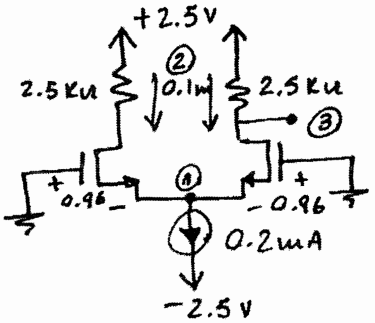
(a) $V_{ov} = \sqrt{I / K_n' W/L}$

$= \sqrt{0.2/3} = \underline{\underline{0.26}}\,V$

$V_{GS} = V_{ov} + V_t = 0.26 + 0.7$

$= \underline{\underline{0.96\,V}}$

(b)



① $V_{S1} = V_{S2} = V_{CM} - V_{GS}$

$= 0 - 0.96 = -\underline{\underline{0.96}}\,V$

② $I_{D1} = I_{D2} = \frac{I}{2} = 0.1\,mA$

③ $V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} \times R_D$

$= +2.5 - 0.1 \times 2.5 = \underline{\underline{2.25}}\,V$

(c) If $V_{CM} = +1\,V$

$V_{S1} = V_{S2} = +1 - 0.96 = \underline{\underline{0.04}}\,V$

$I_{D1} = I_{D2} = \underline{\underline{0.1\,mA}}$

$V_{D1} = V_{D2} = \underline{\underline{2.25\,V}}$

(d) If $V_{CM} = -1\,V$

$V_{S1} = V_{S2} = -1 - 0.96 = -\underline{\underline{1.96}}\,V$

$I_{D1} = I_{D2} = \underline{\underline{0.1\,mA}}$

$V_{D1} = V_{D2} = \underline{\underline{2.25}}\,V$

(e) $V_{CMAX} = V_t + V_{DD} - \frac{I}{2} R_D$

$= 0.7 + 2.5 - 0.1 \times 2.5 = +\underline{\underline{2.95}}\,V$

(f) $V_{CMIN} = -V_{SS} + V_{CS} + V_t + V_{ov}$

$= -2.5 + 0.3 + 0.7 + 0.26$

$= -\underline{\underline{1.24}}\,V$

$V_{S\,min} = V_{CM\,min} - V_{GS}$

$= -1.24 - 0.96 = -\underline{\underline{2.2\,V}}$

7.2

(a) $V_{ov} = -\sqrt{I / K_p' (W/L)}$

$= -\sqrt{0.7/3.5} = -\underline{\underline{0.45}}\,V$

$V_{GS} = V_{ov} + V_t = -0.45 - 0.8$

$= -\underline{\underline{1.25}}\,V$

$V_{S1} = V_{S2} = V_G - V_{GS}$

$= 0 + 1.25 = +\underline{\underline{1.25}}\,V$

$V_{D1} = V_{D2} = \frac{I}{2} \times R_D - V_{DD}$

$= \frac{0.7}{2} \times 2 - 2.5 = -\underline{\underline{1.8}}\,V$

(b) For Q_1 and Q_2 to remain in saturation:

$V_{DS} \le V_{GS} - V_t$

$\rightarrow V_{CM} \geq \left(\frac{I}{2} R_D - V_{DD}\right) + V_t$

$V_{CM\,min} = \frac{0.7}{2} \times 2 - 2.5 - 0.8$

$= -\underline{\underline{2.6}}\,V$

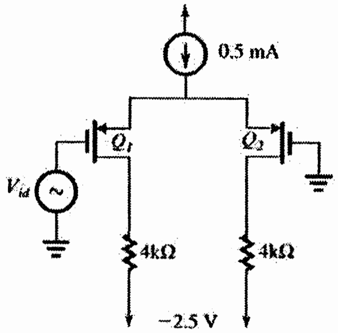
To allow sufficient voltage for the current source to operate properly:

$V_{CM} \le V_{SS} - V_{CS} + (V_t + V_{ov})$

$\rightarrow V_{CM\,max} = 2.5 - 0.5 - 1.25$

$= 0.75\,V$ //

7.3



$V_{G2} = 0$

$V_{G1} = v_{id}$

When all the current is on Q_1:

$$I = \frac{1}{2}\left(k_p'\frac{w}{L}\right)(V_{GS1} - V_t)^2$$

$$\Rightarrow V_{GS1} = V_t + \sqrt{\frac{2I}{k_p' w / L}}$$

$= V_t + \sqrt{2} V_{OV}$

and V_{GS2} is reduced to V_t thus $V_S = -V_t$.

Then $v_{id} = v_{GS1} + v_S$

$= V_t + \sqrt{2}\, V_{OV} - V_t = \sqrt{2}\, V_{ov}$

In a similar manner as for the NMOS Differential Amplifier, as v_i reaches $-\sqrt{2}\, V_{OV}$ Q_1 turns off and Q_2 on. Thus the steerine range is

$\sqrt{2}\, V_{OV} \le V_i \le -\sqrt{2}\, V_{OV}$

For this particular case

$$V_{ov} = \sqrt{\frac{0.25\text{ mA}}{4\text{ mA}/\text{V}^2}} = 0.25\text{ V}$$

$\sqrt{2} \times -0.25 \le v_{id} \le \sqrt{2} \times 0.25$

$-0.35 \le v_{id} \le 0.35$

when $V_{id} = -0.35$ V,

$i_{D1} = 0.5\text{ mA},\ i_{D2} = 0$

$V_S = -V_{t2} = +0.8\text{ V}$

$V_{D1} = 4\text{ k}\Omega \times 0.5\text{ mA} - 2.5 = -0.5\text{ V}$

$V_{D2} = 0 - 2.5\text{ V} = -2.5\text{ V}$

when $v_{id} = +0.35$ V,

$i_{D1} = 0\ ;\ i_{D2} = 0.5\text{ mA}$

$V_S = v_{id} - v_{GS1} = v_{id} - V_{t1}$

$= 0.35\text{ V} + 0.8\text{ V} = 1.15\text{ V}$

$V_{D1} = -2.5\text{ V}$

$V_{D2} = -0.5\text{ V}$

7.4

$V_{G1} = v_{id}\ i_{D1} = 0.11\text{ mA}$

$V_{G2} = 0 \quad i_{D2} = 0.09\text{ mA}$

$$I_D = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2$$

For Q_1:

$$0.11\text{ m} = \frac{1}{2}\, 5\text{ m}(V_{GS1} - 0.5)^2$$

$\rightarrow V_{GS1} = 0.71\text{ V}$

For Q_2:

$$0.09\text{ m} = \frac{1}{2}\, 5\text{ m}(V_{GS2} - 0.5)^2$$

$\rightarrow V_{GS2} = 0.69\text{ V}$

$V_S = -V_{GS2} = -0.69\text{ V}$

$v_{id} = V_S + V_{GS1} = -0.69 + 0.71$

$= 0.02\text{ V}$

$V_{D2} - V_{D1} = 10\text{ k}\Omega\ (i_{D1} - i_{D2})$

$= 10\text{ kV } (0.11\ 2\ 0.09)\text{ m}$

$= 0.2\text{ V}$

thus

$$\frac{V_{D2} - V_{D1}}{v_{id}} = \frac{0.2}{0.02} = 10$$

when $i_{D1} = 0.09$ mA and

$i_{D2} = 0.11$ mA

is the reverse condition from the case we just studied, thus $v_{id} = -0.02$ V

7.5

$V_{GS} = V_{tn} + V_{OV} = 0.5\text{ V} + 0.2\text{ V} = 0.7\text{ V}$

$V_{D4} = V_{G4} = -V_{SS} + V_{GS} = -1.2\text{ V} + 0.7\text{ V}$

$= -0.5\text{ V}$

$$R = \frac{V_{DD} - V_{D4}}{0.1\text{ mA}} = \frac{1.2\text{V} - (-0.5\text{ V})}{0.1\text{ mA}} = 17\text{ k}\Omega$$

$$R_D = \frac{V_{DD} - V_{D1}}{0.4\text{ mA}/2} = \frac{1.2\text{V} - 0.2\text{ V}}{0.2\text{ mA}} = 5\text{ k}\Omega$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{0.4\text{ mA}}{2}\left[k_n' V_{ov}^2\right]$$

$= 0.2\text{ mA}[(0.25\text{ mA}/\text{V}^2)(0.2\text{V})^2]^{-1} = 20$

$$\left(\frac{W}{L}\right)_3 = 0.4\text{ mA}[0.01\text{ mA}]^{-1} = 40$$

$$\left(\frac{W}{L}\right)_4 = 0.1\text{ mA}[0.01\text{ mA}]^{-1} = 10$$

$V_{Cm(max)} = V_{tn} + V_{DD} - (I/2)R_D$

$= 0.5\text{ V} + 1.2\text{ V} - (0.2\text{ mA})(5\text{ k}\Omega) = 0.7\text{ V}$

$V_{Cm(min)} = -V_{SS} + V_{OV3} + V_{tn} + V_{OV1}$

$= -1.2\text{ V} + 0.2\text{ V} + 0.5\text{ V} + 0.2\text{ V} = -0.3\text{ V}$

7.6

We Know that there is a linear relationship between Vov & Vid since:

$$V_{ov} = \frac{V_{id}/2}{\sqrt{0.1}}$$

Then from the data in table 7.3 we can tell that for $V_{i\,max} = 150\,mV$

$$V_{ov} = 0.2 \times \frac{150}{126} = \underline{\underline{0.238}}\,V$$

For W/L: $\frac{W}{L} = \frac{1}{(V_{ov})^2} \cdot \frac{I}{K}$

where I and K are constant thus, for W/L:

$$\left(\frac{W}{L}\right)_2 = \frac{50}{\left(\frac{150}{126}\right)^2} = 35.3$$

For gm: $g_m = \frac{I}{V_{ov}}$ where I is constant

$$\longrightarrow g_{m2} = \frac{g_{m1}}{\left(\frac{150}{126}\right)} = \frac{2}{\frac{150}{126}} = \underline{\underline{1.68\,\frac{mA}{V}}}$$

7.7

$$\left(\frac{v_{idmax}/2}{V_{OV}}\right)^2 = \mathrm{K}$$

$$\Rightarrow 2V_{OV}\sqrt{\mathrm{K}} = v_{idmax}$$

Q.E.D.

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2}\sqrt{1-K}$$

$$i_{D1} = \frac{I}{2} \pm \frac{I}{V_{OV}} \cdot \frac{2}{2}V_{OV}\sqrt{K} \cdot \sqrt{1-K}$$

$$\rightarrow i_{D1} = \frac{I}{2} \pm I\sqrt{K(1-K)}$$

thus $\Delta I = 2I\sqrt{K(1-K)}$

Q.E.D.

For K = 0.01

$$\Delta I = 2I\sqrt{0.01(1-0.01)}$$
$$= 0.198 \times I$$

$$V_{idmax} = 2V_{OV}\sqrt{0.01} = 0.2V_{OV}$$

For K = 0.1

$$\Delta I = 2I\sqrt{0.1(1-0.1)} = 0.8I$$

$$V_{idmax} = 2V_{OV}\sqrt{0.2}$$
$$= 0.894.\,V_{OV}$$

7.8

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_t)^2$$

$$\frac{200}{2} = \frac{1}{2} \times 90 \times \frac{100}{1.6}(V_{GS} - 0.8)^2$$

$$\Longrightarrow V_{GS} = \underline{\underline{1.19\,V}}$$

$$g_m = \frac{2I_D}{V_{GS} - V_t} = \frac{2 \times 100}{(1.19 - 1)} = \underline{\underline{1.06\,\frac{mA}{V}}}$$

$$V_{id}\Big|_{\substack{\text{full current}\\ \text{switching}}} = \sqrt{2}\,(V_{GS} - V_t)$$
$$= \underline{\underline{0.27\,V}}$$

To double this value, VGS − Vt must be doubled which means that ID should be quadrupled. i.e. I changed to: $\underline{\underline{800\,\mu A}}$

7.9

$$g_m = \frac{2I_D}{V_{OV}} \rightarrow 1m = \frac{I}{0.2}$$

$$\rightarrow I = \underline{\underline{0.2mA}}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2$$

$$100 = \frac{1}{2} \times 90 \times \frac{W}{L} \times (0.2)^2$$

$$\Rightarrow \frac{W}{L} = \underline{\underline{55.6}}$$

7.10

$$i_D = \frac{1}{2}k_n\frac{W}{L}(V_{GS} - V_t)^2$$

$$50 = \frac{1}{2} \times 400(V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 1.5\text{ V}$$

For $v_{G1} = v_{G2} = 0$, $v_S = -1.5$ V

For $v_{G1} = v_{G2} = 2$ V, $v_S = +0.5$ V

The drain currents are equal in both cases.

For $V_{G2} = 0$:

To reduce i_{D2} by 10%,

$i_{D2} = 0.9 \times 50 = 45$ μA

$i_{D1} = 55$ μA

$$v_{GS2} = \sqrt{\frac{2i_{D2}}{400}} + 1 = 1.47\text{ V}$$

$$v_{GS1} = \sqrt{\frac{2 \times 55}{400}} + 1 = 1.52\text{ V}$$

Thus, $V_{G1} \equiv v_{GS1} - v_{GS2} = 0.05$ V

To increase i_{D2} by 10%

$i_{D2} = 55$ μA

$i_{D1} = 45$ μA

$v_{GS2} = 1.52$ V

$v_{GS1} = 1.47$ V

$\Rightarrow V_{G1} = -0.05$ V

i_{D2}/i_{D1}	i_{D2} (μA)	i_{D1} (μA)	V_{GS2} (V)	V_{GS1} (V)	$V_G - V_{G1}$ (V)
1	50	50	1.5	1.5	0
0.5	33.3	66.7	1.408	1.577	−0.17
0.8	47.4	52.6	1.487	1.513	−0.026
0.99	47.75	50.25	1.4886	1.5012	−0.013

For $i_{D1}/i_{D2} = 20 \Rightarrow i_{D2} = 4.76$ μA

$i_{D1} = 95.24$ μA

$V_{GS2} = 1.154$ V, $V_{GS1} = 1.690$

Thus $V_{G1} - V_{G2} = 0.536$ V

7.11

(a) $V_{od} = V_{D2} - V_{D1} =$

$(V_{DD} - i_{D2}R_D) - (V_{DD} - i_{D1}R_D) = (i_{D1} - i_{D2})R_D$

$$V_{od} = \left[\left(\frac{I}{V_{OV}}\right)\left(\frac{V_{id}}{2}\right)\sqrt{1 - \left(\frac{V_{id}/2}{V_{OV}}\right)^2}\right.$$

$$\left. + \left(\frac{I}{V_{OV}}\right)\left(\frac{V_{id}}{2}\right)\sqrt{1 - \left(\frac{V_{id}/2}{V_{OV}}\right)^2}\right]R_D$$

$$= IR_D\frac{V_{id}}{V_{OV}}\sqrt{1 - \left(\frac{V_{id}/2}{V_{OV}}\right)^2}$$

(b) see plot

slope of linear portion

$$= \frac{d}{dV_{id}}\left(\frac{IR_D}{V_{OV}} \cdot V_{id}\right) = IR_D/V_{OV}$$

(c) see plot

when the bias current is doubled, V_{OV} so

$$V_{od}/V_{id} = \frac{2IR_D}{\sqrt{2}\,V_{OV}}\sqrt{1 - \left(\frac{V_{id}/2}{\sqrt{2}\,V_{OV}}\right)^2}$$

increases by a factor of $\sqrt{2}$ the slope of the linear part has increased by a factor of $\sqrt{2}$

(d) see plot

If W/L is doubled, V_{OV} reduces by a factor at $\sqrt{2}$

$$\text{so } V_{od}/V_{id} = \frac{2IR_D}{\sqrt{2}\,V_{OV}}\sqrt{1 - \left(\frac{V_{id}/\sqrt{2}}{V_{OV}}\right)^2}$$

The slope of the linear part has increased by factor of $\sqrt{2}$ compared to (b)

7.12

$$V_{ov} = \sqrt{I / K_n'\frac{W}{L}} = \sqrt{\frac{0.5}{0.25\times 50}} = \underline{\underline{0.2V}}$$

$$g_m = \frac{I}{V_{ov}} = \frac{0.5mA}{0.2V} = \underline{\underline{2.5\frac{mA}{V}}}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{(0.5m/2)} = \underline{\underline{40K\Omega}}$$

$$A_d = g_m \times (R_D \,//\, r_o)$$
$$= 2.5\frac{mA}{V}(4K\Omega \,//\, 40K\Omega)$$
$$= \underline{\underline{9.09}}\ V/V$$

7.13

$$\left(\frac{V_{id}/2}{V_{ov}}\right)^2 = 0.1 \rightarrow \left(\frac{0.2/2}{V_{ov}}\right)^2 = 0.1$$
$$\rightarrow V_{ov} = \sqrt{0.1} = \underline{\underline{0.316}}\ V$$

$$g_m = \frac{I}{V_{ov}} \rightarrow 3\frac{mA}{V} = \frac{I}{0.316}$$
$$\rightarrow I = \underline{\underline{0.95mA}}$$

$$\text{also: } V_{ov} = \sqrt{\frac{I}{K_n' W/L}}$$
$$\Rightarrow (0.316)^2 = \frac{0.95\,mA}{0.1\frac{mA}{V^2}\times\left(\frac{W}{L}\right)}$$
$$\rightarrow \frac{W}{L} = \underline{\underline{95}}$$

$$\text{if } R_D = 5K\Omega \Rightarrow$$
$$A_d = g_m R_D = 3\frac{mA}{V}\times 5K\Omega = \underline{\underline{15\frac{V}{V}}}$$

$$\text{if } V_{id} = 0.2 \Rightarrow V_{od} = V_{id}\times A_d$$
$$= 0.2\times 15 = \underline{\underline{3V}}$$

7.14

(a) $g_m = \frac{A_d}{R_D} = \frac{20}{47\ k\Omega} = 0.426\ mA/V$

(b) $I = g_m V_{OV} = (0.426\ mA/V)(0.2V)$

$= 85\ \mu A$

(c) $V_{RD} = \frac{I}{2}R_D = (85\ \mu A/2)(47\ k\Omega)$

$= 2\ V$

(d) $V_{id(MAX)} = V_{CM} + 10\ mV = 0.51\ V$

$V_{DD} \geq V_{id(MAX)} - V_t + I_D R_D$

$= 0.51\ V - V_t + (85\ \mu A/2)(47\ k\Omega)$

$= 2.51\ V - V_t$

7.15

For a CS amplifier $A_v = -g_m R_D$

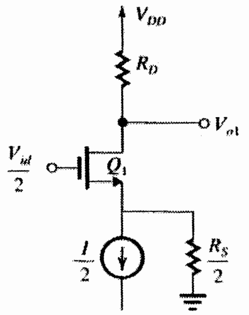
For a differential amplifier $A_d = g_m R_D$ with $I = 2I_D$

So the differential pair requires twice the bias current as the CS amplifier.

The power dissipation at the diff amp is also twice as high.

7.16

HALF-CIRCUIT



small-signal analysis

$$V_{gs} = \frac{V_{id}}{2} - g_m V_{gs}\frac{R_s}{2}$$

$$V_{gs} = \frac{V_{id}/2}{1 + g_m\frac{R_s}{2}}$$

$$V_{O1} = -g_m V_{gs} R_D = -g_m\left[\frac{V_{id}/2}{1 + g_m R_s/2}\right]R_D$$

$$A_d = \frac{V_{Od}}{V_{id}} = \frac{g_m R_D}{1 + g_m R_s/2}$$

when $R_S = 0$ $A_d = g_m R_D$ (agrees with Eqtn. 8.35)

when $R_S = \frac{2}{g_m}$ the differential gain is reduced by half

7.17

(a) $V_{G1} = V_{G2} = OV$

$V_{S1} = V_{S2}$ assuming matching components

$$V_{S1} = V_{G1} - V_{GS1} = OV - (V_t + V_{OV})$$
$$= -(V_t + V_{OV})$$

(b) zero current flows through Q_3

$$V_{OV3} = V_C - V_{S1} - V_t = V_C - (-(V_t + V_{OV})) - V_t$$
$$= V_C + V_t$$
$$= V_C + V_{OV}$$

(c) $V_{G1} = -V_{G2} = V_{id}/2$

V_{S1} is now more negative than in (a) and V_{S2} is now less negative than in (a) so there is a voltage across Q_3. If this voltage is small and if V_C is such that $V_{GS3} > V_t$ then Q_3 will operate in triode.

$$r_{Ds3} = \left[k_n' \frac{W}{L} V_{ov3}\right]^{-1}$$

$$g_{m1} = g_{m2} = \frac{1/2\, k_n' \frac{W}{L} V_{ov}^2}{V_{ov}} = 1/2\, k_n' \frac{W}{L} V_{ov}$$

$$\text{so } r_{Ds3} = \left[g_{m1} \frac{V_{ov3}}{V_{ov}}\right]^{-1} = \frac{V_{ov}}{V_{ov3}\, g_{m1}}$$

(d) $r_{DS3} = \frac{V_{OV}}{V_{OV3}} \cdot \frac{1}{g_{m1}}$

(i) $R_S = \frac{1}{g_{m1}}$ $\therefore V_{OV3} = V_{OV}$

From (b) $V_{OV3} = V_C + V_{OV}$ so $V_C = 0\text{V}$

(ii) $R_s = \frac{1}{2\, g_{m1}}$ $\therefore V_{OV3} = 2\, V_{OV}$

so $V_C = V_{OV}$

7.18

(a) $V_{G1} = V_{G2} = 0\text{V}$

$V_{S1} = V_{S2} = -(V_t + V_{OV})$

Zero current flows through Q_3 and Q_4

Q_3 and Q_4 have the same overdrive voltage as Q_1 and Q_2

$$r_{DS3} = r_{DS4} = \left[k_n'\left(\frac{W}{L}\right)_{3,4} V_{OV3,4}\right]^{-1}$$

$$= \left[k_n'\left(\frac{W}{L}\right)_{3,4} V_{OV1,2}\right]^{-1}$$

$$g_{m1,2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{1,2} V_{OV1,2}$$

$$V_{OV1,2} = g_{m1,2}\left[\frac{1}{2}k_n'\left(\frac{W}{L}\right)_{1,2}\right]^{-1}$$

$$r_{DS3} = r_{DS4} = r_{DS3,4}$$

$$= \left[k_n'\left(\frac{W}{L}\right)_{3,4} g_{m1,2}\left[\frac{1}{2}k_n'\left(\frac{W}{L}\right)_{1,2}\right]^{-1}\right]^{-1}$$

$$= \left[2g_{m1,2}\left(\frac{W}{L}\right)_{3,4} / \left(\frac{W}{L}\right)_{1,2}\right]^{-1}$$

$$= \frac{\left(\frac{W}{L}\right)_{1,2}}{\left(\frac{W}{L}\right)_{3,4} \cdot 2 \cdot g_{m1,2}}$$

$$R_S = 2r_{DS3,4} = \frac{\left(\frac{W}{L}\right)_{1,2}}{g_{m1,2}\left(\frac{W}{L}\right)_{3,4}}$$

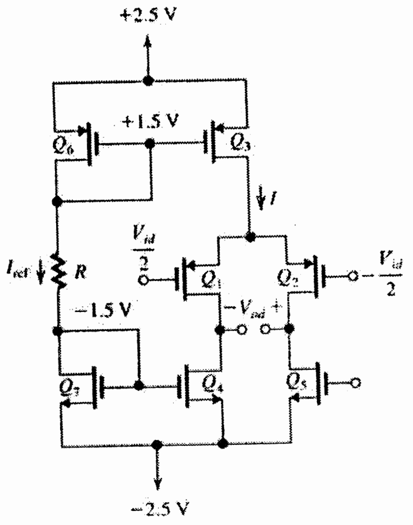
(b) $A_d = V_{Od}/V_{id} = \frac{g_m R_D}{1 + g_m R_S/2}$

(See solution to 8.21)

$$= \frac{g_{m1,2} R_D}{1 + g_{m1,2}\left[\frac{\left(\frac{W}{L}\right)_{1,2}}{\left(\frac{W}{L}\right)_{3,4} \cdot 2 \cdot g_{m1,2}}\right]}$$

$$= \frac{g_{m1,2} R_D}{1 + \frac{\left(\frac{W}{L}\right)_{1,2}}{2\left(\frac{W}{L}\right)_{3,4}}}$$

7.19



For $I_{REF} = 100\ \mu A$,

$$R = \frac{V_{D6} - V_{D7}}{I_{REF}} = \frac{1.5 - (-1.5)}{0.1\ mA} = 30\ k\Omega$$

$V_{GS7} = V_{GS4} = V_{GS5} = -1.5 - (-2.5) = 1\ V$

$V_{OV7} = V_{OV4} = V_{OV5} = V_{GS} - V_{tn} = 1 - 0.7 = 0.3\ V$

$V_{GS6} = V_{GS3} = 1.5 - 2.5 = -1\ V$

$V_{OV6} = V_{OV3} = V_{GS} - V_{tp} = -1 - (-0.7) = -0.3\ V$

From section 8.23, we know that

$A_d = g_{m1}(r_{o1} \| r_{o4})$

Since Q_1 and Q_2 circuits are symmetical

With $I = I_{REF} = 100\ \mu A$

$$I_D = \frac{1}{2} = 50\ \mu A$$

$$r_{o1} = r_{o2} = r_{o4} = r_{o5} = \frac{|V_A|}{I_D} = \frac{20\ V}{50\ \mu A} = 400\ k\Omega$$

So,

$80\ V/V = g_{m1}(400\ K \| 400\ K)$

and

$g_{m1} = 400\ \mu A/V$

$$\text{Since } g_m = \frac{|I_D|}{|V_{ov}|/2},$$

$$|V_{OV1}| = |V_{OV2}| = |V_{OV4}| = |V_{OV5}| = \frac{2I_D}{g_m}$$

$$= \frac{2(50\ \mu A)}{400\ \mu A/V} = 0.25\ V$$

so,

$V_{GS1} = V_{GS2} = V_{OV} + V_{tp} = -0.25 - 0.7$

$= -0.95\ V$

For $\left(\frac{W}{L}\right)$ ratios

$$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{OV})^2$$

So that

$$\frac{W}{L} = \frac{2I_D}{\mu C_{ox} V_{OV}^2}$$

For Q_7,

$$\left(\frac{W}{L}\right)_7 = \frac{2(100\ \mu A)}{90\ \mu A/V^2(0.3\ V)^2} = 24.7$$

For Q_4 and Q_5,

$$\left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = \frac{2(50\ \mu A)}{90\ \mu A/V^2(0.3)^2} = 12.3$$

For Q_1 and Q_2,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2(50\ \mu A)}{30\ \mu A/V^2(0.25)^2} = 53.3$$

For Q_6 and Q_3,

$$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_3 = \frac{2(100\ \mu A)}{30\ \mu A/V^2(0.3\ V)^2} = 74.1$$

In summary, the results are as follows:

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	
μC_{ox}	30	30	30	90	90	30	90	μA/V2
I_D	50	50	100	50	50	100	100	μA
V_{OV}	−0.25	−0.25	−0.3	0.3	0.3	−0.3	0.3	V
$\frac{W}{L}$	53.3	53.3	74.1	12.3	12.3	74.1	24.7	
V_{GS}	−0.95	−0.95	−1	1	1	−1	1	V

7.20

$$(a)\ I_{D1} = \frac{1}{2}k_n'\frac{W}{L}(V_{GS1} - V_t)^2$$

$$I_{D2} = \frac{1}{2}k_n'\left(2 \times \frac{W}{L}\right)(V_{GS2} - V_t)^2$$

Since $V_{GS} - V_t$ is equal for both transistors :

$$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{1}{2}; I_{D2} = 2I_{D1}$$

but $I = I_{D1} + I_{D2} = 3I_{D1}$

$$I_{D1} = I/3$$

$$I_{D2} = 2I/3$$

(b) $V_{OV} = V_{GS} - V_t$

$V_{OV1} = V_{OV2} = V_{OV}$

For $Q1$: $\frac{I}{3} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$

$$\Rightarrow V_{OV} = \sqrt{\frac{2}{3}\frac{I}{k_n' W/L}}$$

(c) $g_m = \frac{2I_D}{V_{OV}} \rightarrow g_{m1} = \frac{2I}{3V_{OV}}$

$g_{m2} = \frac{4}{3}\frac{I}{V_{OV}}$

$$v_{O1} = -g_{m1} \times \frac{v_{id}}{2} \cdot R_D$$

$$= -\frac{2}{3}\frac{I}{V_{OV}} \cdot R_D \cdot v_{id}$$

$$v_{O2} = +g_{m2} \times \frac{v_{id}}{2} \cdot R_D$$

$$= \frac{4}{3}\frac{I}{V_{OV}} \cdot R_D \cdot v_{id}$$

$$\Rightarrow \frac{v_{O2} - v_{O1}}{v_{id}} = \left(\frac{4}{3} + \frac{2}{3}\right)\frac{I}{V_{OV}} \cdot R_D$$

$$= 2 \times \frac{I}{V_{OV}} \cdot R_D$$

7.21

$$A_d = g_{m1}(R_{on} \| R_{op})$$

$$= g_{m1}[(g_{m3}r_{O3})r_{O1} \| (g_{m5}r_{O5})r_{O7}]$$

If all transistors have the same channel length and the same $|V_{OV}|$ and $|V_A|$ Since $g_m = \frac{2I_D}{V_{OV}}$ and $r_O = \frac{V_A}{I_D}$ and with g_m and r_o the same for all devices.

$$A_d = \frac{2I_D}{V_{OV}}\left(\left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{I_D}\right)\frac{V_A}{I_D}\right) \| \left(\left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{I_D}\right)\frac{V_A}{I_D}\right)$$

$$= \frac{2I_D}{V_{OV}}\left[\frac{2\,V_A^2}{V_{OV}\,I_D} \| \frac{2\,V_A^2}{V_{OV}\,I_D}\right]$$

$$= \left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A^2}{V_{OV}\,I_D}\right)$$

$$= \frac{2\,V_A^2}{V_{OV}^2} = 2\left(\frac{|V_A|}{|V_{OV}|}\right)^2$$

For $A_d = 1000$ V/V and $|V_{OV}| = 0.2$ V

$$1000 = 2\frac{|V_A|^2}{|V_{OV}|^2}$$

$$V_A = \sqrt{500} \cdot 0{,}2\text{ V} = 4.47\text{ V}$$

If $|V_A'| = 10$ V/µA

$$L = \frac{4.47\text{ V}}{10\text{ V}/\mu\text{M}} = 0.447\ \mu\text{m}$$

For high g_m the bias current should be high, but with ±0.9 V Supplies the bias current must not exceed $\frac{1\text{ mW}}{1.8\text{ V}} = 0.556$ mA to keep power dissipation at 1 mW

7.22

$$V_{OV} = \sqrt{\frac{I}{K_n' W/L}} = \sqrt{\frac{0.2}{3}} = 0.26\text{V}$$

$$g_m = \frac{I}{V_{OV}} = \frac{0.2\text{mA}}{0.26\text{V}} = 0.77\frac{\text{mA}}{\text{V}}$$

(a) Single-ended output:

$$|A_d| = \frac{1}{2} g_m \times R_D = \frac{0.77 \times 10}{2}$$

$$= \underline{\underline{3.85}}\ \text{V/V}$$

$$|A_{cm}| = \frac{R_D}{2R_{SS}} = \frac{10}{2 \times 100} = \underline{\underline{0.05}}\ \text{V/V}$$

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right| = \frac{3.85}{0.05} = 77$$

i.e $\underline{\underline{37.7\text{ dB}}}$

(b) Differential output, and 1% mismatch in R_D's:

$$|A_d| = g_m R_D$$

$$= 0.77 \times 10 = \underline{\underline{7.7}}\ V/V$$

$$|A_{cm}| = \frac{R_D}{2R_{SS}} \times \left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{10}{2\times 100} \times 0.01 = \underline{\underline{0.5\, m\, V/V}}$$

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{7.7}{0.5\times 10^{-3}} = 15,400$$

i.e $\underline{\underline{83.7}}$ dB

7.23

$$V_{OV} = -\sqrt{\frac{I}{K_p' W/L}} = -\sqrt{\frac{0.7mA}{3.5\frac{mA}{V^2}}}$$

$$= \underline{\underline{-0.45V}}$$

$$g_m = \frac{I}{|V_{OV}|} = \frac{0.7mA}{0.45V} = 1.56\frac{mA}{V}$$

$$|A_d| = g_m R_D = 1.56 \times 2 = \underline{\underline{3.12}}\ V/V$$

$$|A_{cm}| = \frac{R_D}{2R_{SS}} \cdot \left(\frac{\Delta R_D}{R_D}\right) = \frac{2}{2\times 30} \times 0.02$$

$$= \underline{\underline{6.7\times 10^{-4}}}$$

$$CMRR = \frac{3.12}{6.7\times 10^{-4}} = 4680 \rightarrow \underline{\underline{73.4}}\ dB$$

7.24

(a) $R_{D1} = R_D + \frac{\Delta R_D}{2}$ $R_{D2} = R_D - \frac{\Delta R_D}{2}$

$$g_{m1} = g_m + \frac{\Delta g_m}{2} \quad g_{m2} = g_m - \frac{\Delta g_m}{2}$$

$$i_{d1} = \frac{g_{m1}V_{icm}}{g_m R_{SS}} \quad i_{d2} = \frac{g_{m2}V_{icm}}{2g_m R_{SS}}$$

$$i_{d1} - i_{d2} = (g_{m1} - g_{m2})\frac{V_{icm}}{2g_m R_{SS}}$$

$$= \Delta g_m \frac{V_{icm}}{2g_m R_{SS}} \quad (1)$$

$$i_{d1} + i_{d2} = (g_{m1} + g_{m2})\frac{V_{icm}}{2g_m R_{SS}}$$

$$= (2\,g_m)\frac{V_{icm}}{2g_m R_{SS}} = \frac{V_{icm}}{R_{SS}} \quad (2)$$

$$V_{od} = V_{O2} - V_{O1} = -i_{d2}R_{D2} + i_{d1}R_{D1}$$

$$= -i_{d2}\left(R_D - \frac{\Delta R_D}{2}\right) + i_{d1}\left(R_D + \frac{\Delta R_D}{2}\right)$$

$$V_{Od} = R_D(i_{d1} - i_{d2}) + \frac{\Delta R_D}{2}(i_{d2} + i_{d1})$$

Now substitute (1) and (2)

$$V_{Od} = R_D\left(\Delta g_m \frac{V_{icm}}{2g_m R_{SS}}\right) + \frac{\Delta R_D}{2}\left(\frac{V_{icm}}{R_{SS}}\right)$$

$$A_{CM} = \frac{V_{Od}}{V_{icm}} = \frac{R_D}{R_{SS}} \cdot \frac{\Delta g_m}{2g_m} + \frac{\Delta R_D}{2R_{SS}}$$

$$= \frac{R_D}{2R_{SS}}\left[\frac{\Delta g_m}{g_m} + \frac{\Delta R_D}{R_D}\right]$$

(b) $R_D = 5$ KΩ $R_{SS} = 25$ KΩ

If $A_{cm} = 0.002$ V/V, use the result of (a)

$$A_{cm} = 0.002 = \frac{R_D}{2R_{SS}}\left[\frac{\Delta g_m}{g_m} + \frac{\Delta R_D}{R_D}\right]$$

So, ΔR_D can compensate for Δg_m

$$0.002 = \frac{5\text{ k}\Omega}{2.25\text{ k}\Omega} \cdot \frac{\Delta R_D}{5\text{ k}\Omega}$$

$$\Delta R_D = 0.002(50\text{ k}\Omega) = 100\ \Omega$$

so a 100 ohm compersation in R_D(a 2% adjustment) is sufficient.

7.25

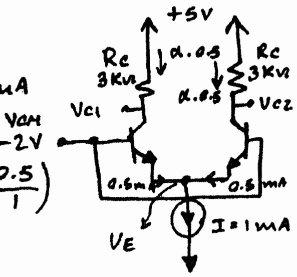
If $A_o = 100$ (40dB), R_{SS} and therefore $CMRR$ will increase by 40 dB.

$$A_o = \frac{V_A}{V_{OV}/2}$$

$$V_A = 100 \cdot \frac{V_{OV}}{2} = 100\left(\frac{0.2\text{ V}}{2}\right) = 10\text{ V}$$

for $V_A = \frac{10\text{ V}}{\mu\text{m}}$, $L = 1\ \mu$m

7.26


+5V
Rc
3KΩ
α.0.5
α.0.5
Rc
3KΩ
Vc1
Vc2
VCM
-2V
0.5m
0.5 mA
I=1mA
VE

$V_{BE} = 0.7$ @ $i_C = 1mA$

→ at $i_C = 0.5mA$

$V_{BE} = 0.7 + 25m \ln\left(\frac{0.5}{1}\right)$

$= 0.683V$

Thus,

$V_E = V_{CM} - V_{BE}$

$= -2 - 0.683 = \underline{-2.683V}$

$i_{C1} = i_{C2} = \alpha \times 0.5 = \frac{100}{101} \times 0.5$

$= 0.495mA$

$V_{C1} = V_{C2} = V_{CC} - i_C R_C$

$= 5 - 0.495 \times 3$

$= +\underline{3.515V}$

7.27

$I = 0.5$ mA So $I_{C1} = I_{C2} = 0.25$ mA

$V_E = V_B - V_{BE}$ $V_{BE} = 0.7 + 0.025 \cdot \ln\left(\frac{i_E}{1}\right)$

for $i_E = 0.5$ mA , $V_{BE} = 0.683$V

if $V_{B1} = 0.5$ V and $V_{B2} = 0$V , $V_{id} = 0.5$ V

$$i_{E1} = \frac{I}{1 + e^{-V_{id}/V_T}} = \frac{0.5 \text{ mA}}{1 + e^{-0.5 \text{ V}/.025 \text{ V}}}$$

$$= \frac{0.5 \text{ mA}}{1 + e^{-20}} = 0.5 \text{ mA}$$

$$i_{E2} = \frac{I}{1 + e^{V_{id}/V_T}} = \frac{0.5 \text{ mA}}{1 + e^{0.5/.025}}$$

$$= \frac{0.5 \text{ mA}}{4.85 \times 10^8} \approx 1 \times 10^{-12} \text{A}$$

$$i_{C1} = \frac{100}{101} 0.5 \text{ mA} = 0.495 \text{ mA}$$

$V_{C1} = 2.5$ V $- (0.495$ mA$)(8$ kΩ$)$

$= -1.46$ V

$i_{C2} \approx 0$ $V_{C2} = 2.5$ V

$V_E = 0.5$ V $- 0.683$ V $= -0.183$ V

if $V_{B1} = -0.5$ V and $V_{B2} = 0$ V

$V_{id} = -0.5$ V $i_{E1} \approx 0$ $i_{E2} \approx 0.5$ mA

(Same equations as above)

$V_{C1} = 2.5$ V $V_{C2} = -1.46$ V

$V_E = 0 - 0.683$ V $= -0.683$ V

7.28

$$V_{cm\,max} = V_{CC} - \alpha\frac{I}{2}R_C + 0.4 \text{ V}$$

$$= 2.5 \text{ V} - \frac{100}{101}\left(\frac{0.5 \text{ mA}}{2}\right)8 \text{ k}\Omega + 0.4\text{V} = 0.92 \text{ V}$$

$V_{CM\,min} = -V_{EE} + V_{CS} + V_{BE}$

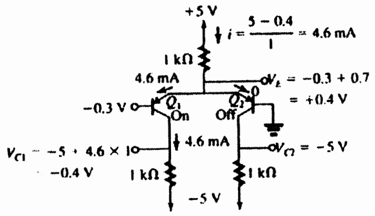
$= -2.5$ V $+ 0.3$ V $+ V_{BE}$

$$V_{BE} = 0.7 \text{ V} + 0.025\ln\left(\frac{0.25 \text{ mA}}{1 \text{ mA}}\right) = 0.665 \text{ V}$$

$V_{CM\,min} = -2.2$ V $+ 0.665$ V $= -1.53$ V

So -1.53 V $< V_{CM} <$ 0.92 V

7.29


+5 V
i = (5 − 0.4)/1 = 4.6 mA
1 kΩ
4.6 mA
V_E = −0.3 + 0.7 = +0.4 V
−0.3 V
Q1
On
Q2
Off
0
4.6 mA
V_C1 = −5 + 4.6 × 1 = −0.4 V
V_C2 = −5 V
1 kΩ
1 kΩ
−5 V

7.30

$V_{BE} = 690$ mV at $i_C = 1$ mA $\beta = 50$

$V_{CE}(SAT) = 0.3$V

$R_C = 82$ kΩ $V_{CC} = -V_{EE} = 1.2$ V

$I = 20$ μA

(a)

$$V_{BE} = 690 \text{ mV} + 25 \text{ mV} \ln\left(\frac{10\mu\text{A}}{1000\mu\text{A}}\right) = 575 \text{ mV}$$

$V_E = V_B - V_{BE} = -575$ mV

$V_{C1} = V_{C2} = 1.2$ V $- (10$ μA$)(82$ kΩ$) = 0.38$ V (b).

$V_{CM\,MAX} = V_{CC} - \alpha\frac{I}{2}R_C + 0.4\text{ V}$

$= 1.2\text{V} - \frac{50}{51}10\ \mu\text{A} \cdot 82\text{ k}\Omega + 0.4\text{ V}$

$= 0.8\text{V}$

$V_{CM\ MIN} = V_{EE} + V_{CS} + V_{BE}$

$= -1.2\text{ V} + 0.3\text{ V} + 0.575\text{ V} = -0.325\text{ V}$

So $-0.325\text{ V} < V_{CM} < 0.80\text{ V}$

(c)

$i_{E1} = 1.1\left(\frac{I}{2}\right) = \frac{1}{1 + e^{-V_{id}/V_T}} \frac{1}{0.55} = 1 + e^{-V_{id}/V_T}$

$0.82 = e^{-V_{id}/V_T}$

$-V_T\ln(0.82) = V_{id} = 5\text{ mV}$

if $V_{B2} = 0, V_{B1} = +5\text{ mV}$

7.31

With only common-mode at the inputs

$V_{C1} = V_{C2} = V_{CC} - \alpha\frac{I}{2}R_C + V_r$

therefore the ripple voltage directly appears at the single-ended output V_{C1} and V_{C2}

However, because the differential output $V_{od} = V_{C2} - V_{C1}$ does not include the common-mode output, the ripple voltage does not appear on the differential output.

This is an advantage of using the differential output compared to using the single ended output.

7.32

(a) $v_{CM\,max} = v_{C1,2} = V_{CC} - \frac{I}{2} \cdot R_C$

(b) If the current is steered to Q_1, then

$v_{C1} = V_{CC} - IR_C$, a change of: $-\frac{I}{2}R_C$

$v_{C2} = V_{CC}$, a change of $+\frac{I}{2}R_C$

(c) $v_{CM\,max} = 3 = 5 - \frac{I}{2}R_C$

$\Rightarrow IR_C = 4V$

(d) $\frac{I/2}{\beta+1} \leq 2\mu A$

$\Rightarrow I \leq 4(\beta+1)\mu A$

Thus, $I = 4 \times 101\mu A = 0.404\mu A$

Select $I = 0.4mA$

$R_C = \frac{4V}{I} = \frac{4V}{0.4mA} = 10k\Omega$

7.33

$i_{E1} - \frac{I}{1 + e^{-\frac{v_d}{V_T}}}, v_d = v_{B1} - v_{B2}$

$\frac{\Delta i_{E1}}{I} = \frac{i_{E1} - I/2}{I} = \frac{I_{E1}}{I} - 0.5$

Define normalized Gain

$G_n = \frac{\Delta i_{E1} I}{v_d}$

vd (mV)	5	10	20	30	40
G_n	9.97	9.87	9.50	8.95	8.30

Observe that the gain stays relatively constant upto V_d nearly 20 mV. Then it decreases significantly with the increase in signal level. Whenever gain depends on signal level, nonlinear, distortion occurs.

7.34

With:

$V_{B1} - V_{B2} = 10\,mV$

$$i_{E1} = \frac{I}{1+e^{-10/25}} = 0.598\,I$$

Since $i_{E1} + i_{E2} = I$

$i_{E2} = 0.402\,I$

For a collector resistance R_C

$$\begin{aligned} v_O = v_{C1} - v_{C2} &= (V_{CC} - i_{C1} R_C) \\ &\quad - (V_{CC} - i_{C2} R_C) \\ &= -(i_{C2} - i_{C1}) R_C \\ &= -\alpha\,(i_{E2} - i_{E1}) R_C \\ &\simeq -0.196\,I \cdot R_C \end{aligned}$$

Thus, for

$v_O = 1V$; $0.196\,I R_C = 1$

$I R_C = 5.102$

Now $I = 2mA$, thus

$R_C = \underline{\underline{2.5\,K\Omega}}$

DC (bias) voltage at each collector

$$= V_{CC} - \frac{I}{2} R_C = 10 - 1 \times 2.5 = 7.5V$$

For a $-1V$ output swing, the minimum voltage at each collector is:

$7.5 - 0.5 = 7.0V$

Thus, $V_{ICM}|_{max} = \underline{\underline{7V}}$

7.35

$$i_{E1} = \frac{I}{1+e^{-V_{id}/V_T}} \text{ and } i_{E2} = \frac{I}{1+e^{V_{id}/V_T}}$$

with $V_{id} = v_{B1} - v_{B2} = 5\text{ mV}$, and $\alpha = 1$,

$$i_{C1} \simeq i_{E1} = \frac{I}{1+e^{-5mV/25mV}} = 0.55\,I$$

$$i_{C2} \simeq i_{E2} = \frac{I}{1+e^{5mV/25mV}} = 0.45\,I$$

$$V_{C2} - V_{C1} = (V_{CC} - i_{C2} R_C) - (V_{CC} - i_{C1} R_C)$$

$$= -0.45\,IR_C + 0.55\,IR_C = 0.1\,IR_C$$

$$A_v = \frac{v_O}{V_{id}} = \frac{(0.1)\,IR_C}{0.005\text{ V}} = (20\,IR_C)\text{V/V}$$

(b) Each collector is biased at $V_{CC} - \frac{I}{2} R_C$

If we want to maintain the same differential input, each collector should be allowed to fall by $\frac{0.1\,IR_C}{2}$ below its bias value.

so,

$$V_{C(\min)} = V_{CC} - 0.5\,IR_C - 0.05\,IR_C$$

$$= V_{CC} - 0.55\,IR_C$$

If this is permitted until $v_{CB} = 0$.

$$V_{ICM(\max)} = V_{C(\min)} = V_{CC} - 0.55\,IR_C$$

If the gain is $20\,IR_C$

$IR_C = \frac{A_v}{20}$ so that

$$V_{ICM(\max)} = V_{CC} - \frac{0.55\,A_v}{20} = V_{CC} - 0.0275\,A_v$$

so, for a given V_{CC}, A_v reduces the maximum allowed V_{ICM}.

A_v (V/V)	100	200	300	400
$V_{ICM(\max)}$ (V)	V_{CC} − 2.75	V_{CC} − 5.5	V_{CC} − 8.25	V_{CC} − 11
IR_C (V)	5	10	15	20
R_C (kΩ)	5	10	15	20

For example, if $V_{CC} = 10$ V, a gain of 200 can be achived by increasing R_C to 10 k Ω, the maximum common-mode input voltage would be $V_{CC} - 5.5 = 4.5$ V. If a gain of 300 is required, if can be achieved by changing RC to 15 kΩ. Howerver this means that $V_{ICM(\max)} = V_{CC} - 8.25 = 1.75$ V.

7.36

$I = 6\,mA$

The current will divide between the two transistors in proportion to their emitter areas. Thus with no input,

$I_{E1} = 1.5\, I_{E2}$

$I_{E1} + I_{E2} = 2.5\, I_{E2} = 6\,mA$

$I_{E2} = 2.4\,mA$

$I_{E1} = 3.6\,mA$

For $\alpha \simeq 1$

$I_{C1} = \underline{\underline{3.6\,mA}}$

$I_{C2} = \underline{\underline{2.4\,mA}}$

To equalize the collector currents we apply a difference signal $v_d = v_{B2} - v_{B1}$ whose value can be determined as follows:

$i_{E1} = I_{SE1}\, e^{((v_{B1} - v_E)/V_T)}$

$i_{E2} = I_{SE2}\, e^{((v_{B2} - v_E)/V_T)}$

where $I_{SE1}/I_{SE2} = 1.5$

Now, $i_{E1} = i_{E2}$ when

$1 = 1.5\, e^{(v_{B1} - v_{B2})/V_T}$

$v_d = v_{B2} - v_{B1} = V_T \ln 1.5 = \underline{\underline{10.1\,mV}}$

7.37

(a)

$$V_{BE} = 690\text{ mV} + 25\text{ mV}\ln\left(\frac{0.2/2}{1}\right) = 632\text{ mV}$$

$R_e = 0,\ V_{id} = 0$

(b) Eqn 8.73

$$i_{C1} = \alpha\, i_{E1} \approx i_{E1} = \frac{200\ \mu\text{A}}{1 + e^{-20/25}} = 138\ \mu\text{A}$$

$$i_{C2} = \alpha\, i_{E2} \approx i_{E2} = \frac{200\ \mu\text{A}}{1 + e^{20/25}} = 62\ \mu\text{A}$$

$R_e = 0,\ V_{id} = 20\text{ mV}$

(c)

$$V_{BE1} = 690\text{ mV} + 25\text{ mV}\ln\left(\frac{.138}{1}\right) = 640\text{ mV}$$

$$V_{BE2} = 690\text{ mV} + 25\text{ mV}\ln\left(\frac{.062}{1}\right) = 620\text{ mV}$$

$$V_{BE1} - V_{BE2} = 20\text{ mV}$$

$$200\text{ mV} = V_{id} = V_{B1} - V_{B2}$$

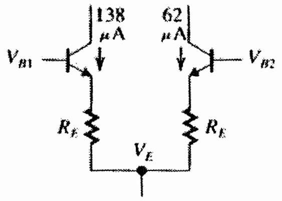
$$= (V_{BE1} + 138\ \mu\text{A}\ R_E + V_E)$$

$$-(V_{BE2} + 62\ \mu\text{A}\ R_E + V_E)$$

$$200\text{mV} = V_{B1} - V_{B2} + (138\ \mu\text{A} - 62\ \mu\text{A})R_E$$

$$180\text{ mA} = 76\ \mu\text{A} \cdot R_E$$

$$R_E = 2.37\text{ k}\Omega$$



(d) Without R_E, a V_{id} of 20 mV causes a differential current of 76 μA

$$G_m = \frac{76\ \mu\text{A}}{20\text{ mV}} = 3.8\text{ mA/V} = (263\ \Omega)^{-1}$$

with $R_E = 2.37\text{ k}\Omega$, a v_{id} of 200 mV causes a differential current of 76 μA

$$G_m = \frac{76\ \mu\text{A}}{200\text{ mV}} = 0.38\text{ mA/V} = (2.63\text{ k}\Omega)^{-1}$$

So G_m has been reduced by a factor of 10. This is the same factor by which V_{id} increased. So we have traded differential gain for a wider usable input range.

7.38

Each device is operating at a current of $150\,\mu A = 0.15\,mA$. Thus,

$$g_m = \frac{0.15\,mA}{25\,mV} = \underline{\underline{\frac{6\,mA}{V}}}$$

$$R_{id} = 2(\beta+1)r_e = 2r_\pi$$

$$= 2 \times \frac{150}{4} = \underline{\underline{75\,K\Omega}}$$

7.39

$R_{id} \geq 10K\Omega$; $A_d = 200\ V/V$;

$\beta \geq 100$; $V_{cc} = 10V$

$R_{id} = 10^4 = 2r_\pi = 2 \times \frac{100}{g_m}$

$\Rightarrow g_m = 20\ mA/V$

Thus each device is operating at 0.5mA and $I = \underline{\underline{1mA}}$

Voltage gain $= g_m . R_c$

$200 = 20 R_c$

$\Rightarrow R_c = \underline{\underline{10K\Omega}}$

7.40

$\underline{\underline{5mV}}$

$r_e = \frac{V_T}{I/2} = \frac{25mV}{50\mu A} = \underline{\underline{500\Omega}}$

Half-circuit gain $= \frac{\alpha R_c}{r_e} \simeq \frac{R_c}{r_e}$

$= \frac{10K}{500} = \underline{\underline{20}}\ V/V$

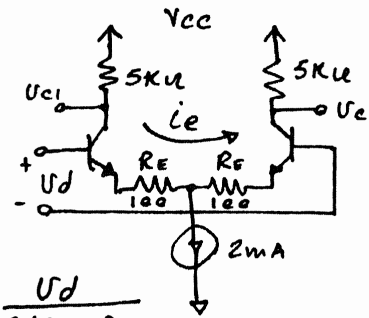
At one collector we expect a signal of (+100mV) and at the other a signal of (-100mV)

7.41

$r_e = \frac{V_T}{I_E}$

$= \frac{25m}{1m}$

$= 25\Omega$



(a) $i_e = \frac{v_d}{2(r_e + R_E)}$

$= \frac{0.1\ V}{2(25 + 100)\Omega} = \underline{\underline{0.4}}mA$

(b) $i_{E1} = 1 + 0.4 = \underline{\underline{1.4}}\ mA$

$i_{E2} = 1 - 0.4 = \underline{\underline{0.6mA}}$

(c) $v_{c1} = -i_e R_c \simeq -0.4 \times 5 = \underline{\underline{-2V}}$

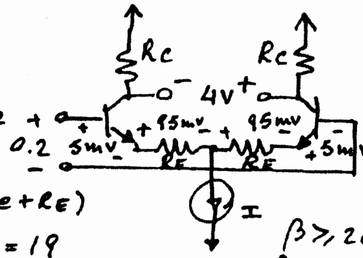
$v_{c2} = \underline{\underline{+2V}}$

(d) $v_{od} = 4V$

$A_d = v_{od} / v_{id} = \frac{4}{0.1} = \underline{\underline{40}}\ V/V$

7.42

$\frac{2R_c}{2(r_e + R_E)} \simeq \frac{4}{0.2}$



$R_c = 20(r_e + R_E)$

$\frac{R_E}{r_e} = \frac{95}{5} = 19$

$\beta \geq 200$

$R_{in} \geq 80K$

$R_{in} = 2(\beta+1)(r_e+R_E)$

$= 2 \times 201 \times 20 r_e = 80K\Omega$

$\Rightarrow r_e \simeq \frac{80000}{8000} = 10\Omega$

Thus each device is operating at a current of $\frac{25mV}{10\Omega} = 2.5mA$

$\Rightarrow I = \underline{\underline{5mA}}$

$R_E = 19 \times 10 = \underline{\underline{190\Omega}}$

$R_C = 20 \times 200 = \underline{\underline{4K\Omega}}$

7.43

(a) $V_{BC} \le 0.4$ V

$V_B - V_C \le 0.4$ V

$(V_{CM} + V_{id}/2) - (V_{CC} - i_{C1}R_C) \le 0.4$ V

So $V_{CM\,max} = V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2} - i_{C1}R_C$

$= V_{CC} + 0.4\text{ V} - \frac{\hat{V}_{id}}{2} - \left(I_C + g_m\frac{\hat{V}_{id}}{2}\right)R_C$

$A_d = g_mR_C$ and $g_m = \frac{I_C}{V_T}$

$I_C = g_mV_T$

$V_{CM\,max} = V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2}$

$-\left[(g_m V_T R_C) + \left(g_m \frac{\hat{V}_{id}}{2} R_C\right)\right]$

$= V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2} - \left[A_d V_T + A_d \frac{\hat{V}_{id}}{2}\right]$

$= V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2} - A_d\left[V_T + \frac{\hat{V}_{id}}{2}\right]$

(b)

$V_{CM\,max} = V_{CC} + 0.4\text{V} - \frac{\hat{V}_{id}}{2} - A_d\left(V_T + \frac{\hat{V}_{id}}{2}\right)$

$= 5\text{ V} + 0.4\text{ V} - \frac{10\text{ mV}}{2} - 100\left(25\text{ mV} + \frac{10\text{ mV}}{2}\right)$

$= 5\text{ V} + 0.4\text{ V} - 5\text{ mV} - 100\ (30\text{ mV})$

$= 2.395$ V

$\hat{V}_{od} = A_d \cdot \hat{V}_{id} = 100 \cdot 10\text{ mV} = 1$ V

$IR_C = 2I_CR_C$ Eqn 8.80 $g_m = \frac{I_C}{V_T}$

$I_C = g_mV_T$ $IR_C = 2(g_m V_T) R_C$ Eqn 8.93

$A_d = g_m R_C$

$IR_C = 2V_TA_d = (2)(25\text{ mV})(100) = 5$ V

$I = \frac{\text{quiescent power}}{V_{CC} - (-V_{EE})} = \frac{5\text{ mW}}{10\text{ V}} = 0.5$ mA

$R_C = \frac{5\text{ V}}{I} = 10\text{ k}\Omega$

(c) For $V_{CM\,max} = 0V$

$= 5\text{ V} + 0.4\text{ V} - \frac{\hat{V}_{id}}{2} - A_d\left(25\text{ mV} + \frac{\hat{V}_{id}}{2}\right)$

$0V = 5.4\text{ V} - 5\text{ mV} - A_d\ (30\text{ mV})$

$A_d = \frac{5.395\text{ V}}{30\text{ mV}} = 180\text{V}/\text{V}$

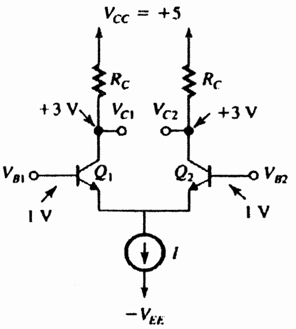
(for $\hat{V}_{id} = 10$ mV)

7.44

with $IR_C = 4\text{V}$, and assuming that $\alpha = 1$,

$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2} \cdot R_C$

$= 5 - 2 = 3$ V



(a) $v_{B1} = 1 + 0.005 \sin(\omega t)$

$v_{B2} = 1 - 0.005 \sin(\omega t)$

we see

that since $\frac{V_{id}}{V_T} = \frac{10.0 \text{ mV}}{25 \text{ mV}} = 0.4$,

the output will be fairly linear. With the information given,

since $i_C = i_E$

$i_{C1} \simeq \frac{I}{1 + e^{(-V_{id}/V_T)}}$ and $i_{C2} \simeq \frac{I}{1 + e^{(V_{id}/V_T)}}$

$v_{od} = v_{C2} - v_{C1}$

$= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C)$

or

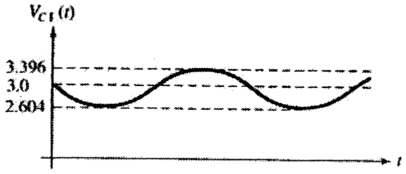
$$V_{od} = \frac{I\,R_C}{1 + e^{-V_{id}/V_T}} - \frac{I\,R_C}{1 + e^{V_{id}/V_T}}$$

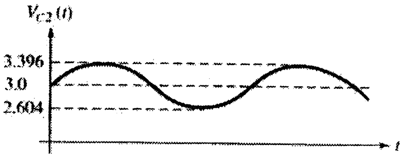
with $I_{RC} = 4$ V and $|V_{id}| = 10$ mV,

$$v_{od\,max} = 5 \text{ V}\left(\frac{1}{1 + e^{-10/25}} - \frac{1}{1 + e^{10/25}}\right)$$

$= 989$ mV

so, $A_d = \frac{V_{od\,max}}{V_{id\,max}} = \frac{792 \text{ mV}}{10 \text{ mV}} = 79.2$





(b) $v_{B1} = 1 + 0.1 \sin(\omega t)$

$v_{B2} = 1 - 0.1 \sin(\omega t)$

Here, $\frac{V_{id}}{V_T} = \frac{200 \text{ mV}}{25 \text{ mV}} = 8$

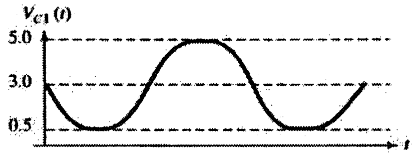
see that this will clearly represent large-signal operation with significant distortion.
Using the same equation,

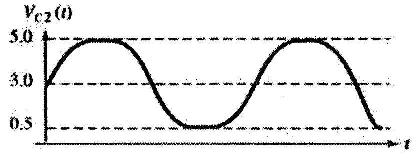
$$v_{od\,max} = 4 \text{ V}\left(\frac{1}{1 + e^{-200/25}} - \frac{1}{1 + e^{200/25}}\right)$$

$\simeq 4.0$ V

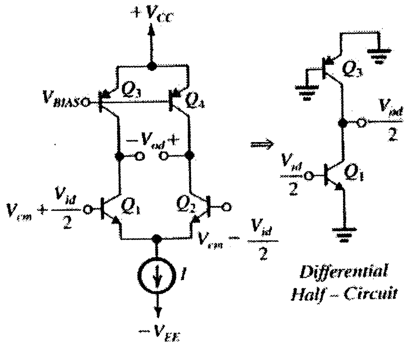
$A_d = \frac{V_{od}}{V_{id}} = \frac{5 \text{ V}}{0.2 \text{ V}} = 25$

waveform is distorted; upper excursions are limited to 5 V.





7.45



$|A_d| = \frac{V_{od}}{V_{id}} = g_{m1}(r_{O1} \| r_{O3})$ Assuming that

$I_C = I_E = \frac{I}{2}$,

$r_{O1} = r_{O3} = \frac{|V_A|}{I_C} = \frac{10 \text{ V}}{I_C}$ and

$g_m = \frac{|I_C|}{V_T} = \frac{I_C}{25 \text{ mV}}$

$A_d = \frac{I_C}{25 \text{ mV}}\left(\frac{1}{2}\right)\left(\frac{10 \text{ V}}{I_C}\right) = \frac{5 \text{ V}}{25 \text{ mV}} = 200$

7.46

$-\frac{v_{od}}{2} = i_b\beta R_C$

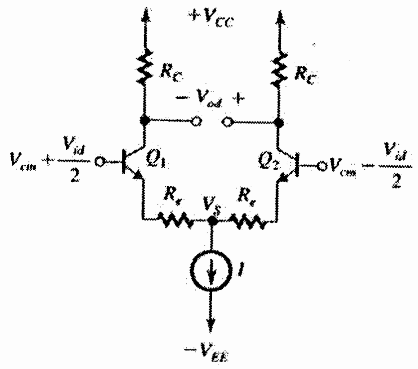
$$i_b = \frac{\frac{v_{id}}{2}}{r\pi + (\beta + 1)R_e}$$

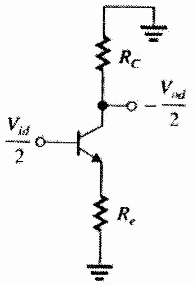
So,

$$-\frac{v_{od}}{2} = \frac{\frac{v_{id}}{2}\beta R_C}{r\pi + (\beta + 1)R_e}$$

$\frac{V_{od}}{V_{id}} = \frac{-R_C}{\frac{r\pi}{\beta} + \frac{\beta + 1}{\beta}R_e}$ If $\alpha \simeq 1$ and

(a)


+V_CC
R_C
R_C
− V_od +
V_cm + V_id/2
Q_1
Q_2
V_cm − V_id/2
R_e
V_S
R_e
I
−V_EE


R_C
V_od/2
V_id/2
R_e

Differential Half – Circuit

noting

$r_e = \frac{V_T}{I_E}$,

$|A_d| = \left|\frac{V_{od}}{V_{id}}\right| \simeq \frac{R_C}{r_e + R_e}$ which is identical to

The half circuit has

$R_i = r\pi + (\beta + 1)R_e$

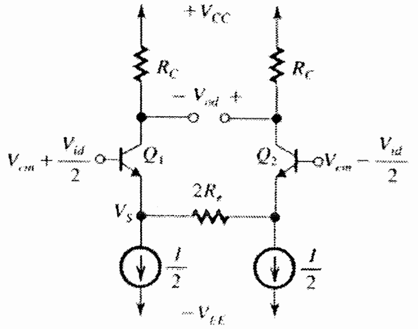
$R_{id} = 2r\pi + (\beta + 1)(2R_e)$

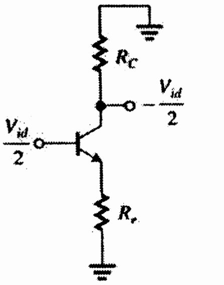
This is equivalent to

$R_{id} = (\beta + 1)(2r_e + 2R_e)$

$V_{Cm} = V_{BE} + \frac{I}{2}R_e + V_S$

(b)


+V_CC
R_C
R_C
− V_od +
V_cm + V_id/2
Q_1
Q_2
V_cm − V_id/2
2R_e
V_S
I/2
I/2
−V_EE


R_C
V_id/2
V_id/2
R_e

Differential Half – Circuit is the same as the circuit of Part (a)

So, using the same derivation,

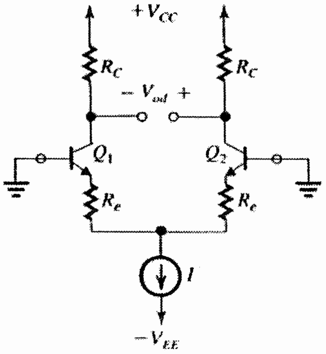
$|A_d| = \left|\frac{V_{od}}{V_{id}}\right| \simeq \frac{R_C}{r_e + R_e}$

$R_{id} = 2r\pi + 2(\beta + 1)R_e = (\beta + 1)(2r_e + 2R_e)$

$V_{Cm} = V_{BE} + V_S$

Since the quiescent emitter currents do not pass through the $2R_e$ resistance, there is no drop so that V_{Cm} can be lower in case (b) than case (a)

7.47


+V_CC
R_C
R_C
− V_od +
Q_1
Q_2
R_e
R_e
I
−V_EE

$V_{Re} = 4V_T$ $\quad V_{RC} = 40V_T$

From Eq. (8.94).

$$A_d \simeq \frac{R_C}{r_e + R_e} = \frac{\frac{40V_T}{I_C}}{\frac{V_T}{I_E} + \frac{4V_T}{I_E}}$$

If $\alpha \simeq 1$, $I_C \simeq I_E$, and

$$A_d = \frac{40V_T}{5V_T} = 8$$

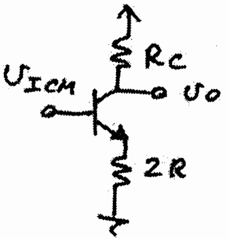
7.48

2MΩ

$$\frac{v_o}{v_{Icm}} \simeq \frac{R_C}{2R} = \frac{20\text{K}\Omega}{2\text{M}\Omega}$$

$$= 0.01 \text{ V/V}$$

or −40dB



7.49

$$\frac{v_o}{v_i} = \frac{\alpha \times 20\text{K}\Omega}{(2r_e + 2\times 200)\,\Omega}$$

Where $r_e = \frac{V_T}{0.5/2} = \frac{0.05\text{V}}{0.5\text{mA}} = 100\Omega$

$$\frac{v_o}{v_i} \simeq \frac{20\,000}{600} = 33.3\text{V/V}$$

$$R_i = (\beta+1)(2r_e + 2\times 200)$$

$$= 101 \times 2 \times 300 \simeq 60\text{K}\Omega$$

7.50

Each transistor is operating at $I_E = 1\text{mA}$, thus

$r_e = 25\Omega$ and $r_\pi = 101 \times 25$

$= 2525\Omega$

$$\frac{v_o}{v_i} = \frac{\alpha \times 7.5\text{K}\Omega}{(2r_e + 200)\,\Omega} \simeq \frac{7500}{250} = 30\frac{\text{V}}{\text{V}}$$

$$R_i = (\beta+1)(r_e + 200 + r_e) \simeq 25\text{K}\Omega$$

7.51

with $V_{icm} = 0$, $V_E = -0.7$ V

(a) $I = \dfrac{V_E - V_{EE}}{R_E} = \dfrac{-0.7-(-5)}{4.3\text{ K}} = 1$ mA

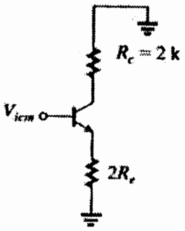
$$r_e = \frac{V_T}{I_E} = \frac{25\text{ mV}}{1\text{ mA}/2} = 50\ \Omega$$

Since $R_e >> r_e$,

$\left|\dfrac{V_o}{V_{id}}\right| \simeq \dfrac{1}{2}, \dfrac{\alpha R_C}{r_e}$ If $\alpha \simeq 1$,

$$\left|\frac{V_o}{V_{id}}\right| \simeq \frac{R_C}{2r_e} = \frac{2\text{ K}}{2(50)} = 20\text{ V/V}$$

(b) $A_{cm} \simeq \dfrac{\alpha R_C}{2R_e + r_e}$



common-mode half circuit

If $\alpha = 1$,

$$A_{cm} \simeq \frac{2\text{ K}}{2(4.3\text{ K}) + 50} = 0.23$$

(c) CMRR (dB) = 20 $\log_{10}$

$$\left|\frac{V_o/V_{id}}{A_{cm}}\right| = 20\log_{10}\left|\frac{20}{0.23}\right| = 38.8\text{ dB}$$

(d)

$V_{B1} = 0.1 \sin 2\pi \times 60t + 0.005 \sin 2\pi \times 1000\,t$

$V_{B2} = 0.1 \sin 2\pi \times 60t - 0.005 \sin 2\pi \times 1000t$

$V_{id} = 0.01 \sin 2\pi \times 1000\,t$

$V_{icm} = 0.1 \sin 2\pi \times 60\,t$

so that

$$V_o = \left|\frac{V_o}{V_{id}}\right| \cdot V_{id} + A_{cm} \cdot V_{icm}$$

$V_o(t) = 20\,[0.01 \sin 2\pi \times 1000\,t] + 0.23$

$[0.1 \sin 2\pi \times 60\,t]$

$V_o(t) = 0.2 \sin 2\pi \times 1000\,t + 0.023 \sin 2\pi \times 60\,t$

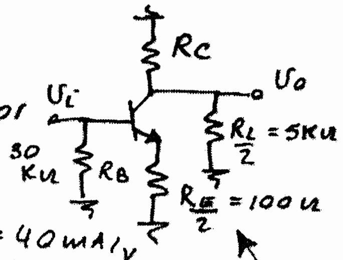
7.52

Each transistor is biased at 1mA. Thus,

$r_e = 25\Omega$, $g_m = 40\text{mA/V}$

$r_o = 100/1 = 100\text{K}\Omega$

The differential half-circuit is



R_C
v_o
v_i
30 KΩ
R_B
$\frac{R_L}{2} = 5\text{K}\Omega$
$\frac{R_E}{2} = 100\Omega$

$$A_d = \frac{v_o}{v_i} = \frac{\alpha\,[R_C \,//\, (R_L/2)]}{r_e + R_E/2}$$

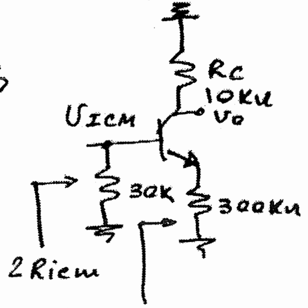
$$\simeq \frac{10//5}{0.025 + 0.100} = \underline{\underline{26.7}}\ \text{V/V}$$

$$R_{id} = 2\,[R_B // (\beta+1)(r_e + R_E/2)]$$

$$= 2\,[30 // 101\,(0.025 + 0.100)]$$

$$= \underline{\underline{17.8}}\ \text{K}\Omega$$

The common-mode halfcircuit



$$A_{cm} = \frac{v_o}{v_{Icm}} \simeq \frac{10}{300}$$

$$= \frac{1}{30} = \underline{\underline{0.033}}\ \text{V/V}$$

$$2R_{icm} = 30\text{K} // 7.5\text{M}\,\Omega$$

$$= 30\text{K}\Omega$$

$$R_{icm} = \underline{\underline{15\text{K}\Omega}}$$

$$(\beta+1)(300) // (\beta+1)\,r_o$$

$$\simeq 30 // 10 = 7.5\text{M}\Omega$$

Without the R_B resistors $R_{icm} = \underline{\underline{3.75}}\ \text{M}\Omega$

7.53

(a) $A_d\big|_{\substack{\text{single-ended}\\ \text{output}}} = \dfrac{\alpha\,(R_C // r_o)}{2r_e}$

where $r_e = \dfrac{0.025\text{V}}{0.25\text{mA}} = 100\Omega$

$r_o = \dfrac{200\text{V}}{0.25\text{mA}} = 800\text{K}\Omega$

$A_d\big|_{\substack{\text{single}\\ \text{ended}}} \simeq \dfrac{20}{2 \times 0.1} = \underline{\underline{100}}\ \text{V/V}$

(b) $A_d\big|_{\substack{\text{diff}\\ \text{output}}} = 2 \times A_d\big|_{\substack{\text{single}\\ \text{ended}}}$

$= \underline{\underline{200}}\ \text{V/V}$

(c) $R_{id} = 2r_\pi = 2 \times 201 \times 100$

$= \underline{\underline{40.2}}\ \text{K}\Omega$

(d) $A_{cm}\big|_{\substack{\text{single-ended}\\ \text{output}}} = \dfrac{R_C}{2R}$

$= \dfrac{20}{2000} = \underline{\underline{0.1}}\ \text{V/V}$

(e) $A_{cm}\big|_{\text{diff out}} = 0$

7.54

$I = 100\ \mu\text{A}$, $\beta = 50$, $V_A = 20$ V

For Q_3,

$$R_{EE} = r_{o3} = \frac{V_A}{I} = \frac{20\text{ V}}{0.1\text{ mA}} = 200\text{ k}\Omega$$

$$r_o = r_{o1} = r_{o2} = \frac{V_A}{I/2} = \frac{20\text{ V}}{0.05\text{ mA}} = 400\text{ k}\Omega$$

Using Eq. (8.103),

$$R_{icm} \simeq \beta R_{EE}\frac{1 + \dfrac{R_C}{\beta r_o}}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

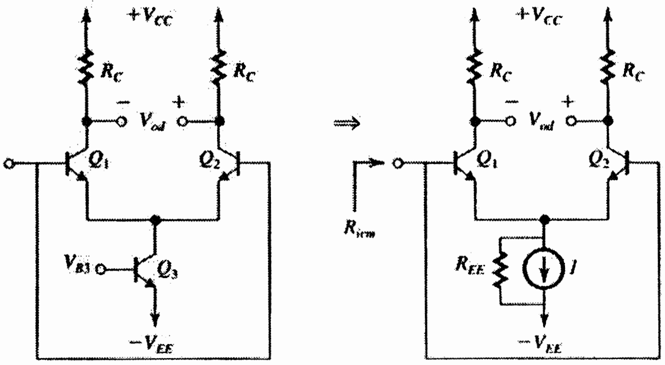
$$R_{icm} \simeq 50(200\text{ k})\cdot\frac{1 + \dfrac{R_C}{(50)(400\text{ k})}}{1 + \dfrac{R_C + 2(200\text{ k})}{400\text{ k}}}$$

If $R_C << R_{EE}$

and $R_C << r_o$,

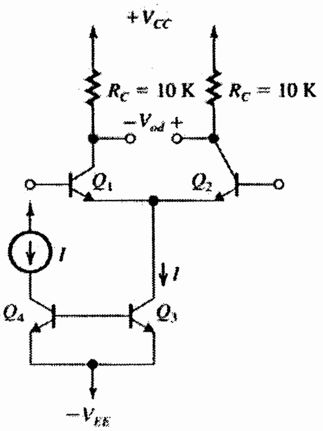
$R_{icm} \simeq 50\ (200\text{ K})\ (.5) = 5\text{ M}\Omega$

This figure is for 7.54


$+V_{CC}$
R_C
R_C
$-$ V_{od} $+$
Q_1
Q_2
V_{B3}
Q_3
$-V_{EE}$
⇒
R_{icm}
R_{EE}
I

(c)

7.55


$+V_{CC}$
$R_C = 10\text{ K}$
$R_C = 10\text{ K}$
$-V_{od}+$
Q_1
Q_2
I
I
Q_4
Q_3
$-V_{EE}$

Equivalent

$$R_{EE} = r_{o3} = \frac{V_A}{I} = \frac{10\text{ V}}{0.5\text{ mA}} = 20\text{ k}\Omega$$

$$r_{e2} = r_{e1} = r_e = \frac{V_T}{I/2} = \frac{25\text{ mV}}{0.5\text{ mA}/2} = 100\ \Omega$$

$$\text{Since } \alpha = \frac{\beta}{\beta+1} = \frac{100}{101} = 1,$$

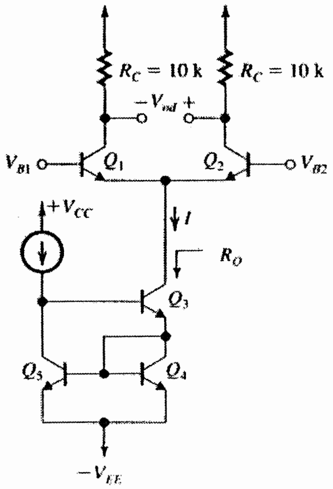
$$A_d \simeq \frac{R_C}{r_e} = \frac{10\text{ k}}{0.1\text{ k}} = 100\text{ V/V}$$

(b) $A_{cm} \equiv \dfrac{\alpha \Delta R_C}{2R_{EE} + r_e} = \dfrac{(0.02)(10\text{ k})}{2(20\text{ k}) + 0.1\text{ k}}$

$= 0.00499\text{ V/V}$

$$\text{CMRR(dB)} = 20\log_{10}\left|\frac{A_d}{A_{cm}}\right| = 20\log_{10}\left|\frac{100}{0.00499}\right|$$

$= 86\text{ dB}$


$R_C = 10\text{ k}$
$R_C = 10\text{ k}$
$-V_{od}+$
V_{B1}
Q_1
Q_2
V_{B2}
$+V_{CC}$
I
R_O
Q_3
Q_5
Q_4
$-V_{EE}$

From Eq. (7.88)

$$R_o \simeq \frac{1}{2}\beta_3 r_{o3}$$

$$R_o \simeq \frac{1}{2}(100)(20\text{ k}) = 1\text{ M}\Omega$$

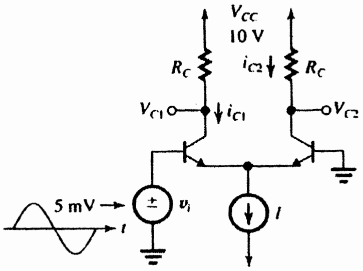
$$A_{cm} \simeq \frac{\Delta R_C}{2R_o + r_o} \simeq$$

$$\frac{(0.02)(10\text{ k})}{2(1\text{ m}) + 0.1\text{ k}} = 0.0001\text{ V/V}$$

$$\text{CMRR(dB)} = 20\log_{10}\left|\frac{A_d}{A_{cm}}\right| = 20\log_{10}\left|\frac{100}{0.0001}\right|$$

$= 120\text{ dB}$

7.56



$$i_{c1} = \frac{I}{2} + \left(\frac{I/2}{V_T}\right)\left(\frac{5}{2}\right)\sin\omega t$$

$$i_{c2} = \frac{I}{2} - \left(\frac{I/2}{V_T}\right)\left(\frac{5}{2}\right)\sin\omega t$$

$$v_{C1} = V_{CC} - \frac{I}{2}R_C - \frac{I/2}{V_T}R_C\frac{5}{2}\sin\omega t$$

$$v_{C2} = V_{CC} - \frac{I}{2}R_C - \frac{I/2}{V_T}R_C\frac{5}{2}\sin\omega t$$

$V_{C1}, V_{C2} \geqq 0$

$\Rightarrow 10 - 5I - 0.5I = 0$

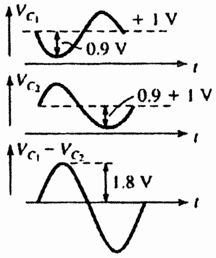
$I = 1.8$ mA

$V_{C1} = V_{C2} = 1$ V

$A_d = \dfrac{20\text{ k}\Omega}{2r_e}$, where $r_e = \dfrac{25}{0.9} = 27.8\ \Omega$

Thus, $A_d = 360$ V/V

$v_{C2} - v_{C1} = 1.8 \sin \omega t$, V



7.57

Taken single-endedly $Acm_s = \dfrac{\alpha R_C}{2R_o}$

Let collector resistors be R_c & $R_c + \Delta R_c$, then

$$Acm = \frac{\alpha}{2R_o}(R_C + \Delta R_C - R_C)$$

$$= \alpha\frac{\Delta R_C}{2R_o}$$

Which can be written as

$$Acm_d = \frac{\alpha R_C}{2R_o}\cdot\frac{\Delta R_C}{R_C} = Acm_s\frac{\Delta R_C}{R_C}$$

$$\text{CMRR} = \frac{A_d}{A_{cmd}} = \frac{2.A_s}{Acm_s\dfrac{\Delta R_C}{R_C}}$$

$$= \frac{A_s}{Acm_s}\cdot\frac{2}{\dfrac{\Delta R_C}{R_C}}$$

Thus, $20 \log \dfrac{2}{\dfrac{\Delta R_C}{R_C}} = 40$ dB

$\rightarrow \Delta R_C / R_C = 2\%$

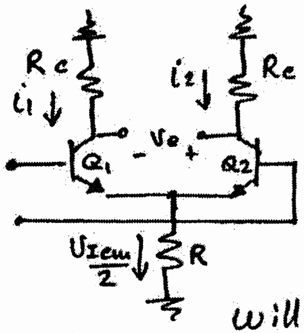
7.58

The bias current will split between the two transistors according to their area ratio. Thus the large-area device will carry twice the current of the other device.

That is, the bias currents will be $2I/3$ and $I/3$.

Now with v_{Icm} applied, the CM signal current will ↳ v_{Icm}/R

split between Q_1 and Q_2 in the same ratio. This is



because their r_e values will be related in the same way. Thus, if Q_1 is the large device r_{e1} will be half the value of r_{e2}.

The result will be that

$$i_1 = \frac{2}{3}\frac{v_{Icm}}{R} \quad \text{and} \quad i_2 = \frac{1}{3}\frac{v_{Icm}}{R}$$

Thus the differential output voltage V_o will be

$$V_o = 0 - i_2 R_c - (-i_1 R_c) = (i_1 - i_2) R_c$$

$$= \frac{1}{3}\frac{v_{Icm}}{R}\cdot R_c$$

$$A_{cm} = \frac{1}{3}\frac{R_c}{R} = \frac{1}{3}\times\frac{12}{1000} = \underline{\underline{0.004}}\ \frac{V}{V}$$

7.59

For $I = 200\ \mu A$:

$$g_m = \sqrt{2 k_n' W/L\, I_D} = \sqrt{2\times 4\times 0.1}$$

$$= 0.89\ mA/V$$

$R_D = 10\ k\Omega$

Thus, $A_d = g_m R_D = 10\times 0.89 = \underline{\underline{8.9}}\ V/V$

$$V_{OS} = \frac{(V_{GS} - V_t)}{2}\cdot\frac{\Delta R_D}{R_D}$$

where $\frac{\Delta R_D}{R_D} = 0.02$ (worst case)

and $V_{GS} - V_t = \sqrt{\frac{2 I_D}{k_n' W/L}} = \sqrt{\frac{2\times 0.1}{4}}$

$$= \underline{\underline{0.223V}}$$

Thus, $V_{OS} = \frac{1}{2}\times 0.223\times 0.02$

$$= \underline{\underline{2.23 mV}}$$

For $I = 400\ \mu A$:

$g_m = \sqrt{2\times 4\times 0.2} = 1.265\ mA/V$

$A_d = \underline{\underline{12.65}}\ V/V$

$V_{ov} = V_{GS} - V_t = 0.316\ V$

$V_{OS} = \frac{1}{2}\times 0.316\times 0.02 = \underline{\underline{3.16}}\ mV$

Thus both A_d and V_{OS} increase by the same ratio since both are proportional to $\sqrt{I}$

7.60

Worst cases: $\Delta V_t = 10 mV$

$\frac{\Delta R_D}{R_D} = 0.04$; $\frac{\Delta(W/L)}{(W/L)} = 0.04$

$$V_{OS1}\ (\text{due to } \Delta R_D) = \frac{V_{ov}}{2}\frac{\Delta R_D}{R_D} = \frac{0.3\times 0.04}{2}$$

$$= 6 mV$$

V_{OS2} (due to $\Delta W/L$) $= \frac{V_{ov}}{2}\frac{\Delta W/L}{W/L} = \frac{0.3 \times 0.04}{2}$

$= 6\,mV$//

V_{OS3} (due to ΔV_t) $= \Delta V_t = \underline{\underline{10\,mV}}$

Since these offsets are not correlated

$$V_{OS} = \sqrt{V_{OS1}^2 + V_{OS2}^2 + V_{OS3}^2}$$

$$V_{OS} = \sqrt{6^2 + 6^2 + 10^2} = \underline{\underline{13.11}}\,mV$$

The major contribution is due to the the threshold mismatch ΔV_t.

To find the required mismatch ΔR_D that can correct for V_{OS}

$$13.11\,mV = \frac{V_{ov}}{2} \cdot \frac{\Delta R_D}{R_D}$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = \frac{2 \times 13.11\,mV}{0.3\,V}$$

$= 0.087$ or $\underline{\underline{8.7\%}}$

If ΔV_t is reduced by a factor of 10 to 1 mV, V_{OS} reduces to:

$$\sqrt{6^2 + 6^2 + 1^2} = 8.54\,mV$$

and $\frac{\Delta R_D}{R_D} = \frac{2 \times 8.54\,mV}{0.3\,V} = \underline{\underline{5.69\%}}$

7.61

$$V_{ov} = \sqrt{\frac{I}{k_n' W/L}} = \sqrt{\frac{100}{100 \times 20}} = 0.223\,V$$

we obtain V_{OS} due to $\Delta R_D/R_D$ as:

$$V_{OS} = \frac{V_{ov}}{2}\frac{\Delta R_D}{R_D} = \frac{0.223}{2} \times 0.05$$

$= \underline{5.57\,mV}$

From Eqn. (7.117), V_{OS} due to $\Delta(W/L)/(W/L)$ is:

$$V_{OS} = \left(\frac{V_{ov}}{2}\right)\frac{\Delta W/L}{W/L} = \frac{0.223}{2} \times 0.05$$

$= \underline{5.57}\,mV$

The offset arising from ΔV_t is

$$V_{OS} = \Delta V_t = \underline{\underline{5\,mV}}$$

Worst case offset is:

$5.57 + 5.57 + 5 = 16.15\,mV$

Applying the root-sum-of-squares

$$V_{OS} = \sqrt{2(5.57m)^2 + 5m^2} = \underline{\underline{9.33\,mV}}$$

7.62

$$\Delta V_C = \Delta R_C \cdot \frac{I}{2}$$

$$A_d = \frac{R_C}{r_e} = \frac{R_C}{V_T / \frac{I}{2}} = \frac{I R_C}{2 V_T}$$

$$\Rightarrow V_{OS} = \frac{\Delta V_C}{A_d} = \frac{\Delta R_C}{R_C} \cdot V_T$$

$= 0.1 \times 25 = \underline{\underline{2.5\,mV}}$

7.63

$$V_{OS} = V_T \cdot \frac{\Delta I_S}{I_S}$$

$$= 25 \times 0.1 = \underline{\underline{2.5\,\text{mV}}}$$

7.64

$$\Delta v_{EE} = \Delta R_C \frac{I}{2}$$

$$A_d = \frac{R_C}{r_e + R_e} = \frac{R_C}{\frac{2V_T}{I} + R_E} = \frac{I R_C}{2V_T + I R_E}$$

$$V_{OS} = \frac{\Delta v_C}{A_d} = \frac{\Delta R_C}{R_C}\left(V_T + \frac{I R_E}{2}\right)$$

7.65

CASE 1: BJT Diff. Amp.
From Eq. (8.121)

$$|V_{OS}| = V_T\left(\frac{\Delta R_C}{R_C}\right) = 25\text{ mV }(0.04) = 1\text{ mV}$$

CASE 2: MOSFET Diff. Amp.

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\Delta R_D}{R_D}\right) = \frac{300\text{ mV}}{2}(0.04) = 6\text{ mV}$$

If the MOSFET widths, are increased by a factor of 4, and since I_D must remain constant, we see that since

$$I_D = \frac{1}{2}K_n'\left(\frac{W}{L}\right)V_{OV}^2,$$

The new $V_{OV} = \sqrt{\dfrac{2I_D}{(4)K_n'\left(\frac{W}{L}\right)}}$ which is $\sqrt{\frac{1}{4}}$ or $\frac{1}{2}$ of its orginal value.

So, the new offset voltage is

$$V_{OS} = \left(\frac{150\text{ mV}}{2}\right)(0.04) = 3\text{ mV}$$

7.66

Since the two transistors are matched except for their V_A value, we can express the collector currents when the input terminals are grounded as,

$$I_{C1} = I_C\left(1 + \frac{V_{CE}}{V_{A1}}\right)$$

$$I_{C2} = I_C\left(1 + \frac{V_{CE}}{V_{A2}}\right)$$

Where I_C can be determined from

$$I_{C1} + I_{C2} = I$$

$$\Rightarrow I_C = \frac{I}{2 + \frac{V_{CE}}{V_{A1}} + \frac{V_{CE}}{V_{A2}}}$$

Note that for $V_{CE} \ll V_{A1}, V_{A2}, I_C \approx \frac{I}{2}$ Thus, the differential gain A_d can still be written as

$$A_d \approx \frac{R_C}{r_e} = \frac{I R_C}{2V_T}$$

The offset voltage at the output can be found from

$$\Delta V_C = v_{C2} - V_{C1} = (I_{C1} - I_{C2})R_c$$

$$= I_C R_C\left(\frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}}\right)$$

$$\approx \frac{I}{2}R_C\left(\frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}}\right)$$

Thus, $V_{OS} = \dfrac{\Delta V_C}{A_d}$

$$V_{OS} = V_T\left(\frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}}\right)$$

For

$V_{CE} = 10$ V, $V_{A1} = 100$ V and

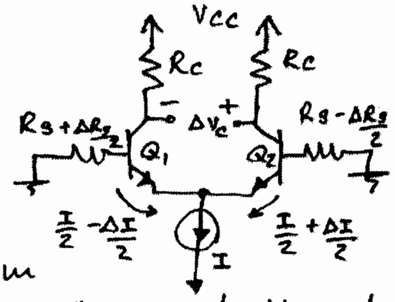
$V_{A2} = 300$ V

$$V_{OS} = 25\left(\frac{10}{100} - \frac{10}{300}\right)$$

$= 1.7$ mV

7.67

Equating the incremental changes in voltage from ground to emitter on both sides of the pair (and neglecting second-order terms i.e. Δ×Δ terms):



$$\frac{I}{2(\beta+1)} \cdot \frac{\Delta R_s}{2} - \frac{\Delta I}{2(\beta+1)} \cdot R_s - \frac{\Delta I}{2} \cdot r_e$$

$$\simeq \frac{-I}{2(\beta+1)} \frac{\Delta R_s}{2} + \frac{\Delta I}{2(\beta+1)} R_s + \frac{\Delta I}{2} r_e$$

$$\Delta I \left[r_e + \frac{R_s}{\beta+1} \right] = \frac{I}{2(\beta+1)} \cdot \Delta R_s$$

$$\Delta I = \frac{I\,\Delta R_s}{2(\beta+1)} \cdot \frac{1}{r_e + \frac{R_s}{\beta+1}}$$

$$\Delta V_C = -\Delta I \,.\, R_C$$

$$= \frac{-I R_C \,\Delta R_s}{2(\beta+1)} \cdot \frac{1}{r_e + R_s/(\beta+1)}$$

$A_d = R_C/r_e$

Thus, $V_{OS} = \Delta V_C / A_d$

$$= \frac{-I\,\Delta R_s}{2(\beta+1)} \cdot \frac{r_e}{r_e + \frac{R_s}{\beta+1}}$$

For $\frac{R_s}{\beta+1} \ll r_e$ and $\beta \gg 1$,

$$|V_{OS}| \simeq \frac{I}{2\beta}(\Delta R_s) \qquad Q.E.D.$$

7.68

(a) $R_{C1} = 5 \times 1.05 = 5.25\,\text{K}\Omega$

$R_{C2} = 5 \times 0.95 = 4.75\,\text{K}\Omega$

Perfect offset nulling will be achieved when x is such that

$R_{C1} + (x \times 1\,\text{K}\Omega) = R_{C2} + (1-x) \times 1\,\text{K}\Omega$

$\Longrightarrow 5.25 + x = 4.75 + 1 - x$

$\Longrightarrow x = \underline{\underline{0.25}}$

(b) $I_{C1} = 1.05\,\text{mA}$

$I_{C2} = 0.95\,\text{mA}$

Offset nulling is achieved when x is such that

$1.05\,(x + 5) = 0.95\,((1-x) + 5)$

$x = \underline{\underline{0.225}}$

7.69

$$I_{B\max} = \frac{I/2}{\beta_{\min}+1} = \frac{300}{80+1} = \underline{\underline{3.7}}\,\mu\text{A}$$

$$I_{B\min} = \frac{I/2}{\beta_{\max}+1} = \frac{300}{200+1} = \underline{\underline{1.5}}\,\mu\text{A}$$

$$I_{OS} = I_{B\max} - I_{B\min} = \underline{\underline{2.2}}\,\mu\text{A}$$

7.70

$I_{E1} = \frac{2}{3} I$ and $I_{E2} = \frac{1}{3} I$

(Q_1 twice the area of Q_2)

$$\Delta V_C = V_{C2} - V_{C1} \simeq \underline{\underline{\frac{1}{3} I R_C}}$$

Nominally,

$$A_d = \frac{R_C}{r_e} = \frac{I R_C}{2V_T}$$

$$V_{OS} = \frac{\Delta V_C}{A_d} = \frac{2}{3} V_T = \underline{\underline{16.7}}\,\text{mV}$$

Thus, small-signal analysis predicts that a 16.7 mV DC voltage applied as $v_{B2} - v_{B1} =$ 16.7 would restore the current balance in the pair and reduce ΔV_C to zero.

Using large-signal analysis:

$$i_{E1} = I_{S1} \cdot e^{\frac{v_{B1}-v_E}{V_T}}$$

$$i_{E2} = I_{S2} \cdot e^{\frac{v_{B2}-v_E}{V_T}}$$

Thus,

$$\frac{i_{E1}}{i_{E2}} = \frac{I_{S1}}{I_{S2}} \cdot e^{\frac{v_{B1}-v_{B2}}{V_T}}$$

To restore balance, $i_{E1} = i_{E2}$, thus

$$1 = 2e^{\frac{V_{B1}-V_{B2}}{V_T}}$$

$$\Rightarrow V_{B1} - V_{B2} = -V_T \ln 2$$

$$V_{B2} - V_{B1} = \underline{\underline{17.3\,mV}}$$

Nominally

$$I_B = \frac{I/2}{\beta+1} \simeq \frac{100}{2\times100} = \underline{\underline{0.5\,\mu A}}$$

But with the imbalance,

$$I_{B1} \simeq \frac{2I/3}{\beta} = \frac{2\times 100}{300} = 0.67\,\mu A$$

$$I_{B2} = \frac{I/3}{\beta} = \frac{100}{300} = 0.33\,\mu A$$

$$I_B = \frac{I_{B1}+I_{B2}}{2} = \underline{\underline{0.5\,\mu A}}$$

$$I_{OS} = |I_{B1} - I_{B2}| = \underline{\underline{0.34\,\mu A}}$$

7.71

$R_C = 20K\Omega$; $A_d = 90\,V/V$

$V_{OS} = \pm 3mV$

Worst case $|V_{OS}|$ is 3mV

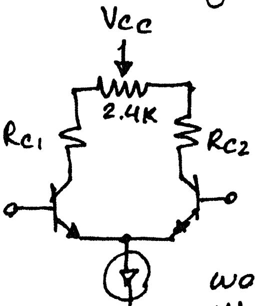
$|V_{OS}| = V_T\left(\frac{\Delta R_C}{R_C}\right)$ $|V_{OS}| = 3mV$

$$\Rightarrow \frac{3m \times 20}{25m}K = 2.4K, \ \Delta R_C = 2.4K\Omega$$

This is the maximum mismatch that occurs in R_C.

Thus, if the lowest collector resistor is adjusted from $R_{C\,min} + \Delta R$ with ΔR varying between zero and 2.4KΩ, then the offset would be eliminated!

This can be achieved with the following circuit:



When R_{C1} & R_{C2} are equal the potentiometer is tuned to the middle point. In the worst case, when either R_C is higher by 2.4KΩ, the potentiometer is adjusted to one extreme such as to increase the lowest R_C by 2.4KΩ. In all other cases when ΔR_C is distributed between R_{C1} and R_{C2} the potentiometer is adjusted

7.72

For each transistor $I_D = I/2$.

$r_{o2} = r_{o4} = r_o$

7 $A_d = \frac{1}{2} g_m r_o$

but $g_m = \frac{2I_D}{V_{ov}}$ and $r_o = \frac{V_A}{I_D}$

$\Longrightarrow A_d = \frac{1}{2}\left(\frac{2I_D}{V_{ov}}\right)\frac{V_A}{I_D} = \frac{V_A}{V_{ov}}$

$\longrightarrow 80\,V/V = 20V/V_{ov}$

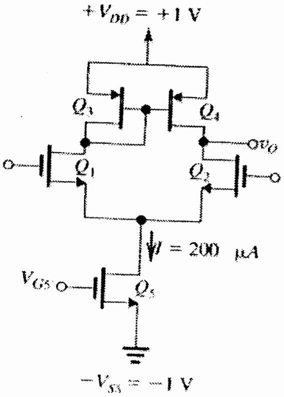
$\longrightarrow V_{ov} = 20/80 = 0.25V$

Finally,

$I = 2I_D = K\frac{W}{L}V_{ov}^2$

$= 3.2\,\frac{mA}{V^2}\,(0.25V)^2 = \underline{\underline{0.2\,mA}}$

7.73



(a) $I_{D5} = I = 200\ \mu A$

$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2}$

$= \frac{200\ \mu A}{2} = 100\ \mu A$

$$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

so,

$$\left(\frac{W}{L}\right)_{1-2} = \frac{2I_D}{\mu_n C_{ox} V_{OV}^2} = \frac{2(100\ \mu A)}{400\ \mu A/V\ (0.2\ V)^2}$$

$= 12.5$

For Q_3 and Q_4,

$$\left(\frac{W}{L}\right)_{3-4} = \frac{2I_D}{\mu_p C_{ox} V_{OV}^2} = \frac{2(100\ \mu A)}{100\ \mu A/V^2\ (0.2\ V)^2}$$

$= 50$

(b)

$$A_d = \frac{V_O}{V_{id}} = g_m(r_{o2} \| r_{o4})$$

Since $r_{o2} = r_{o4} = r_o$, $A_d = \frac{1}{2}g_m r_o$

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = g_m = \frac{I_D}{V_{OV}/2}$$

$= \frac{100\ \mu A}{0.2\ V/2} = 1\ mA/V$

The value of r_o needed is

$$r_O = \frac{2A_d}{g_m} = \frac{(2)(50\ V/V)}{1\ mA/V} = 100\ k\Omega$$

Since $r_O = \frac{|V_A'|}{I_D}\cdot L$,

$$L = \frac{r_O I_D}{|V_A'|} = \frac{100\ K(0.1\ mA)}{20\ V/\mu m} = 0.5\ \mu m$$

(c) If $V_{cm} = 0$, the maximum V_o is $V_{DD} - V_{ov} = 1 - 0.2 = +0.8$ V with a single-NMOS current transistor, the lowest V_o should go is $V_{SS} + 2\ V_{ov} = -1 + 2(0.2\ V) = -0.6$ V so, the range of V_o is -0.6 V to $+0.8$ V

(d) Q_5 delivers $I = 200\ \mu A$, and $L = 0.5\ \mu m$. $V_{ov} = 0.2$ V. So,

$$r_{O5} = \frac{|V_A'|}{I}\cdot L = \frac{(20\ V/\mu m)(0.5\ \mu m)}{0.2\ mA} = 50\ k\Omega$$

$r_{o1} = r_{o4} = r_o = 100\ k\Omega$

$$A_{cm} = \frac{v_O}{v_{icm}} = -\frac{r_{O4}}{2R_{SS}}\cdot\frac{1}{1 + g_{m3}\,r_{O3}}$$

$$A_{cm} = -\frac{100\ k}{2(50\ k)}\cdot\frac{1}{1 + (1\ mA/V)(100\ k)}$$

$= -0.01$

$$CMRR(dB) = 20\log_{10}\frac{|A_d|}{|A_{cm}|}$$

$= 20\log_{10}\left(\frac{50}{0.01}\right) = 74\ dB$

7.74

CMRR = $(g_m r_o)(g_m R_{ss})$

(a) For a simple current mirror

$R_{SS} = r_{OS} \Rightarrow (\text{for } I_D = 1/2)$

CMRR = $(g_m r_o)(g_m r_{os})$

$$= \left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{I_D}\right) \cdot \left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{2I_D}\right)$$

$$= 2 \cdot \frac{V_A}{V_{OV}} \cdot \frac{V_A}{V_{OV}}$$

$$= 2\left(\frac{V_A}{V_{OV}}\right)^2 \quad \text{Q.E.D.}$$

(b) for the modified Wilson current source of

$RSS \simeq g_{m7} \cdot r_{O7} \cdot r_{O5}$

$\Rightarrow$ CMRR $= (g_m r_O)(g_m \cdot g_{m7}\, r_{O7}\, r_{O5})$

For $Q_{5,6,7,8}$:

$$V_{OVS} = \sqrt{\frac{2I}{k' W/L}}$$

while for $Q_{1,2,3,4}$:

$$V_{OV} = \sqrt{\frac{I}{k' W/L}}$$

$\Rightarrow V_{OVS} = \sqrt{2}\, V_{OV}$.

Thus, (for $I = 2I_D$)

$$CMRR = \frac{I}{V_{OV}} \cdot \frac{V_A}{(I/2)} \cdot \frac{I}{V_{OV}} \cdot \frac{2I}{\sqrt{2}V_{OV}} \cdot \frac{V_A}{I} \cdot \frac{V_A}{I}$$

$$= \frac{4}{\sqrt{2}} \frac{VA^3}{V_{OV}^3} = 2 \cdot \sqrt{2}\, \frac{VA^3}{V_{OV}^3}$$

For $k' W/L = 10 \text{ mA/V}^2$

$I = 1$ mA

$|V_A| = 10$ V

$$V_{OV} = \sqrt{\frac{1 \text{ mA}}{10 \text{ mA/V}^2}} = 0.316 \text{ V}$$

$\Rightarrow$ For the simple current mirror case:

$$\text{CMRR} = 2\left(\frac{10}{0.316}\right)^2 = 2000$$

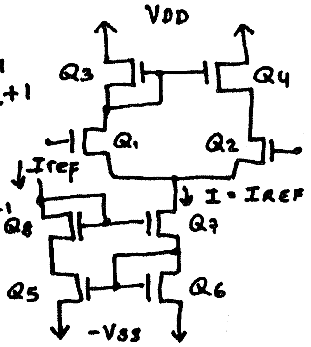
→ 66 dB

For the Wilson source:

$$CMRR = 2\sqrt{2}\frac{(10)^3}{(0.316)^3} = 89442$$

→ 99 dB

7.75



$$V_{GS_{1-4}} = \sqrt{\frac{50\mu \cdot 2}{800\mu}}+1$$

$= 1.35$ V

$$V_{GS\,5-8} = \sqrt{\frac{2\times 100}{800}}+1$$

$= 1.5$ V

For $V_{DS} = V_{GS}$

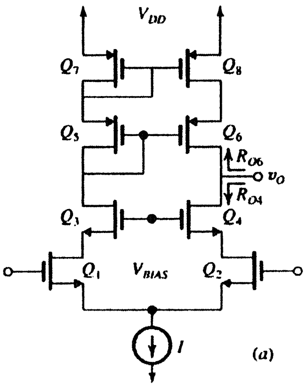
$$-V_{SS} + 2V_{GS_{5-8}} + 2V_{GS_{1-4}} = V_{DD}$$

Thus,

$$V_{DD} + V_{SS} = 2(1.5) + 2(1.35)$$

$$= \underline{\underline{5.7\text{ V}}}$$

7.76



(b) $R_{O4} = (g_{m4} r_{O4}) r_{O2}$

$= g_m r_o^2$

$R_{O6} = (g_{m6} r_{O6})\, r_{O8}$

$= g_m r_o^2$

$A_d = g_m (R_{O4} \| R_{O6})$

$$= g_m \cdot \frac{1}{2} gm^2 r_o^2$$

$$g_m = \frac{2I_D}{V_{OV}} \qquad r_O = \frac{V_A}{I_D}$$

thus, $g_m r_o = 2V_A / V_{ov}$

$$\Rightarrow A_d = 2(V_A / V_{OV})^2$$

Q.E.D.

For $V_{ov} = 0.25$ V & $V_A = 20$ V

$A_d = 2\,(20 / 0.25)^2 = 12800$ V/V

7.77

$$i_1 = \frac{V_O}{r_O} = \frac{\frac{1}{2}(g_m r_O)\, v_{id}}{r_O} = \frac{1}{2} g_m v_{id}$$

$$i_2 = g_{m4} v_{sg4} = \frac{g_m v_{id}}{4}$$

$$i_3 = i_1 - i_2 = \frac{g_m v_{id}}{2} - g_m \frac{v_{id}}{4} = \frac{g_m v_{id}}{4}$$

$$i_4 = -g_{m2} v_{gs2} = -g_m \left[-\frac{v_{id}}{2} - \frac{v_{id}}{4} \right]$$

$$= \frac{3}{4} g_m v_{id}$$

$$i_5 = i_4 = \frac{3}{4} g_m v_{id}$$

$$i_6 = i_4 - i_3 = \frac{3}{4} g_m v_{id} - \frac{1}{4} g_m v_{id} = \frac{1}{2} g_m v_{id}$$

However, if we use KVL,

$$i_6 = \frac{v_o - v_s}{r_o} = \frac{\frac{1}{2} g_m r_o v_{id} - \frac{V_{id}}{4}}{r_o}$$

$$= \frac{1}{2} g_m V_{id} - \frac{V_{id}}{4r_o} \quad \text{inconsistent}$$

$$i_7 = i_5 - i_6 = \frac{3}{4} g_m V_{id} - \frac{g_m V_{id}}{2} = \frac{g_m V_{id}}{4}$$

(which is the same as i_3)

$$i_8 = g_m v_{gs1} = g_m \left(\frac{V_{id}}{2} - \frac{V_{id}}{4} \right) = \frac{1}{4} g_m v_{id}$$

$$i_9 = i_8 = \frac{1}{4} gm\, V_{id}$$

$$i_{10} = i_8 - i_7 = \frac{g_m V_{id}}{4} - \frac{g_m V_{id}}{4} = 0$$

$i_{11} + i_{10} = i_9$ or

$$i_{11} = i_9 - i_{10} = i_9 = \frac{g_m V_{id}}{4}$$

(which is the same as i_7)

$$i_{12} = g_m v_{gs3} = \frac{1}{4} g_m V_{id}$$

$$i_{13} = i_{11} - i_{12} = \frac{1}{4} g_m V_{id} - \frac{1}{4} g_m V_{id} = 0$$

Note, through, that this is inconsistent with KVL.
If $i_{13} = 0$, $V_{D3} = 0$, but $V_{D3} = V_{G3} = -V_{id}/4$.

If $i_{10} = 0$, $V_{D1} = \frac{V_{id}}{4}$, but this conflicts with V_{D3} being $-\frac{V_{id}}{4}$.

It appears that the approximations for V_{gs} and v_s prevent a clean solution. If these were more exact, all current and voltage relutionships should be consistent.

7.78

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2} = \frac{100\ \mu\text{A}}{2} = 50\ \mu\text{A}$$

$$g_{m1} = g_{m2} = \frac{I_D}{V_{ov}/2} = \frac{50\ \mu\text{A}}{0.2\ \text{V}/2} = 0.5\ \text{mA/V}$$

$$G_m = g_{m1} = 0.5\ \text{mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_{An}}{I_D} = \frac{20\ \text{V}}{0.05\ \text{mA}} = 400\ \text{k}\Omega$$

$$r_{o3} = r_{o4} = \frac{|V_{Ap}|}{I_D} = \frac{12\ \text{V}}{0.05\ \text{mA}} = 240\ \text{k}\Omega$$

$$R_o = r_{o2} \| r_{o4} = 400\ \text{k} \| 240\ \text{k} = 150\ \text{k}\Omega$$

$$A_d = G_m R_o = (0.5\ \text{mA/V})(150\ \text{k}) = 75\ \text{V/V}$$

Gain will be reduced by a factor of 2 if
$R_L = R_o = 150$ kΩ

7.79

$$R_{id} = (\beta+1)\, 2r_e \; ; \; r_e = \frac{25\text{mV}}{50\mu A} = 500\Omega$$

$$\rightarrow R_{id} = 101 \times 1000 = \underline{\underline{101 K\Omega}}$$

$$R_o = r_{o4} // r_{o2} = \frac{r_o}{2} \; ; \; r_o = \frac{V_A}{I_C}$$

$$\rightarrow r_o = \frac{160V}{50\mu A} = 3.2 M\Omega$$

Thus, $R_o = \underline{\underline{1.6 M\Omega}}$

$$G_m = g_{m1} = g_{m2} = \frac{50\mu A}{25mV} = \underline{\underline{2mA/V}}$$

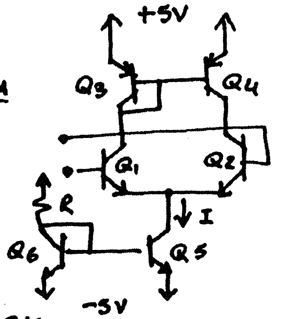
$A_d = G_m R_o = 2 \times 1600 = \underline{\underline{3200\ V/V}}$

With a subsequent stage having a 100KΩ input resistance,

$A_d = G_m\,(R_o \,\|\, 100K\Omega)$

$= \underline{\underline{188.2\ V/V}}$

7.80



$G_m = \frac{I/2}{V_T} = 5\frac{mA}{V}$

$I = \underline{\underline{250\mu A}}$

$R = \frac{5-(-5)-V_{BE}}{I}$

$= \frac{9.3}{0.25} = \underline{\underline{37.2K\Omega}}$

$R_{id} = (\beta+1)\,2r_e$ where,

$r_e = \frac{V_T}{I/2} = \frac{25mV}{0.125mA} = 200\Omega$

$\longrightarrow R_{id} = 151 \times 2 \times 0.2 = \underline{\underline{60.4K\Omega}}$

$r_o = \frac{V_A}{I_C} = \frac{100}{0.125} = 800K\Omega$

$R_o = \frac{r_o}{2} = \underline{\underline{400K\Omega}}$

$A_d = g_m R_o = 5 \times 400 = \underline{\underline{2000}}\ V/V$

$I_B = \frac{I/2}{\beta+1} = \frac{125}{151} = \underline{\underline{0.83\mu A}}$

$v_{ICM}|_{max} \simeq V_{C1} + 0.4V$

$= 5 - 0.7 + 0.4 = \underline{\underline{4.7V}}$

$v_{ICM}|_{min} = V_{BS} - 0.4 + 0.7$

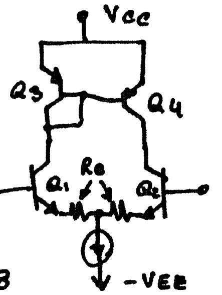
$= -5 - 0.4 + 0.7$

$= -4V$

Thus, the input common-mode range is -4V to +4.7V (where we have assumed that a transistor remains active

7.81

$R_{id} = (\beta+1)\,2\,(r_e + R_E)$



Gm is still equal to: $g_m = 5mA/V$

$\Longrightarrow I = \underline{\underline{250\mu A}}$

(From Problem 7.68 above)

and $r_e = \frac{V_T}{I/2} = 200\Omega$, $r_o = 800K\Omega$

If $R_{id} = 100K\Omega \Longrightarrow$

$100K = 151 \times 2 \times (200 + R_E)$

$\longrightarrow R_E = 131\Omega$

To obtain Ad:

$A_d = G_m \cdot R_o$ (Eqn. 7.165)

As in the derivation of R_{o2} in Eqn. (7.162), R_{o2} can be found using Eqn. (6.159), but this time noting that Re at the emitter of Q2 is:

$r_{e1} + 2R_E$

Thus,

$$R_{o2} = r_{o2}\left[1 + g_m\left((r_{e1} + 2R_E) \parallel r_{\pi 2}\right)\right]$$

$$R_{o2} = 800K\left[1 + 5m\left((200 + 2\times 131) \parallel 30.2K\right)\right.$$

$$\swarrow ((\beta+1)r_e$$

$$R_{o2} = 2620\,K\Omega$$

$$R_o = R_{o2} \parallel r_{o4}$$

$$= (2620 \parallel 800)K = 613K\Omega$$

$$\Rightarrow A_d = 5m \times 613K = \underline{\underline{3065}}\ V/V$$

7.82

$$G_m = g_m = \frac{I/2}{V_T} = \frac{0.5m/2}{25m}$$

$$g_m = \underline{\underline{10mA/V}}$$

$$R_o = r_{o2} \parallel r_{o4} = \frac{V_A}{I_{C2}} \parallel \frac{V_A}{I_{C4}} = \frac{1}{2}\frac{V_A}{I/2}$$

$$= \frac{120}{0.5m} = \underline{\underline{240K\Omega}}$$

$$A_d = G_m R_o = 10 \times 240 = \underline{\underline{2400}}\,\frac{V}{V}$$

$$R_{id} = 2r_\pi \simeq 2\frac{V_T}{I/2}\beta = \frac{25m \times 150}{0.5m}$$

$$R_{id} = \underline{\underline{7.5K\Omega}}$$

For a simple current mirror the output resistance (thus R_{EE}) is r_o

$$\Rightarrow R_{EE} = \frac{V_A}{I} = \frac{120}{0.5m} = \underline{\underline{240K\Omega}}$$

$$A_{cm} = \frac{-r_{o4}}{\beta_3 R_{EE}} = \frac{-(2\times 240K)}{150 \times 240K}$$

$$= -13.3\ mV/V$$

and, $CMRR = \left|\frac{2400}{-13.3m}\right| = 180{,}451$

i.e. $\underline{\underline{105\ dB}}$

$$\frac{v_i}{v_s} = \frac{R_{id}}{R_{id} + R_s} = \frac{7.5K}{7.5K + 10K} = 0.43\frac{V}{V}$$

$\Rightarrow$ Overall gain A:

$$A = \frac{v_i}{v_s}\frac{v_o}{v_i} = 0.43 \times 2400$$

$$= \underline{\underline{1032}}\ V/V$$

7.83

(a) If R_{o1} and R_{o2} can be ignored,

$i_i = G_{mcm} V_{icm}$

$v_o \simeq [A_m i_i - G_{mcm} V_{icm}]R_{om}$

substituting in for i_i,

$v_o = [A_m G_{mcm} V_{icm} - G_{mcm} V_{icm}]\, R_{om}$

$$A_{cm} = \frac{v_o}{V_{icm}} = G_{mcm}\, R_{om}(A_m - 1)$$

(b) , $i_i = i_{ro3} + A_m i_i$

$$A_m = \frac{i_i - i_{ro3}}{i_i} = 1 - \frac{i_{ro3}}{i_i} = 1 - \frac{V_{sg3}}{r_{o3} i_i}$$

$g_m + V_{sg3} = A_m i_i$ so,

$$A_m = 1 - \frac{A_m i_i}{g_{m4} r_{o3}\, i_i} \quad \text{since } g_{m4} = g_{m3},$$

$$A_m = \frac{1}{1 + \frac{1}{g_{m3}\, r_{o3}}}$$

Continuing, we can substitute this into the equation of part (a):

$$A_{cm} = G_{mcm}\, R_{om}\left(\frac{1}{1 + \frac{1}{g_{m3}\, r_{o3}}} - 1\right)$$

Since $V_{icm}G_{mcm} = \frac{V_{icm}}{2R_{ss}}$, $G_{mcm} = \frac{1}{2\,R_{ss}}$

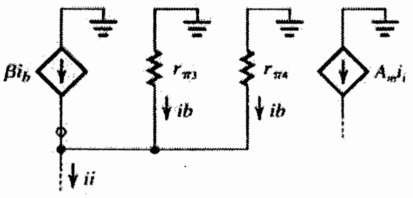
substituting,

$$A_{cm} = \left(\frac{R_{om}}{2R_{ss}}\right)\left[\frac{1-\left(1+\frac{1}{g_{m3}\,r_{o3}}\right)}{1+\frac{1}{g_{m3}\,r_{o3}}}\right]$$

since $R_{om} = r_{o4}$,

$$A_{cm} = \frac{-r_{o4}}{2R_{ss}}\left(\frac{1}{g_{m3}\,r_{o3}+1}\right)$$

(c)



$$i_i = \beta i_b + 2i_b \Rightarrow i_b = \frac{i_i}{\beta+2}$$

$$A_m i_i = i_b\beta$$

$$A_m i_i = \left(\frac{i_i}{\beta+2}\right)\beta$$

$$A_m = \left(\frac{\beta}{\beta+2}\right) = \frac{1}{1+2/\beta}$$

Now, substituting into the resulting equation of part (a),

$$A_{cm} = G_{mcm}R_{om}(A_m - 1)$$

$$A_{cm} = G_{mcm}r_{O4}\left(\frac{\beta}{\beta+2} - 1\right)$$ and since

$$G_{mcm} = \frac{1}{2R_{EE}},$$

$$A_{cm} = \frac{r_{O4}}{2R_{EE}}\cdot\left(\frac{\beta-\beta-2}{\beta+2}\right) = \frac{-r_{O4}}{2R_{EE}}\cdot\left(\frac{2}{\beta+2}\right)$$

since $\beta >> 2$,

$$A_m \simeq \frac{-r_{O4}}{\beta_p R_{EE}}$$

7.84

for a wilson current mirror,

$$\frac{I_O}{I_{REF}} = \frac{1}{1+\frac{2}{\beta(\beta+2)}}$$

As an active load, this means that one collector current will be $\frac{\alpha I}{2}$, while the other is

$$\frac{\alpha I}{2}\left(1+\frac{2}{\beta(\beta+2)}\right)$$

$$|\Delta i| = \frac{\alpha I}{2}\left(1+\frac{2}{\beta(\beta+2)} - 1\right) = \alpha I\left[\frac{1}{\beta(\beta+2)}\right]$$

$$G_m = g_m = \frac{\frac{\alpha I}{2}}{V_T} = \frac{\alpha I}{2V_T}$$

$$|V_{OS}| = \frac{\Delta i}{G_m} = \frac{\frac{\alpha I}{\beta(\beta+2)}}{\frac{\alpha I}{2V_T}} = \frac{2V_T}{\beta(\beta+2)}$$

For $\beta_P = 50$,

$$|V_{OS}| = \frac{2(25\text{ mV})}{50(50+2)} = 19.2\ \mu\text{V}$$

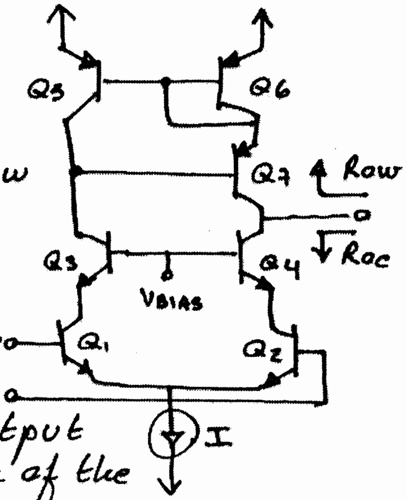
7.85

$A_{vo} = G_m R_o$

$G_m = g_m$.

$R_o = R_{oc} // R_{ow}$

where R_{oc} is the output resistance of the cascode stage & R_{ow} the output resistance of the wilson stage



$R_{oc} = \beta r_o$ & $R_{ow} = \frac{\beta r_o}{2}$

$$\Rightarrow R_o = \beta r_o \,\|\, \frac{\beta r_o}{2} = \frac{\beta r_o \cdot \frac{\beta r_o}{2}}{\beta r_o\left(1+\frac{1}{2}\right)}$$

$$= \frac{\beta r_o}{2\times\frac{3}{2}} = \frac{\beta r_o}{3}$$

$$\Rightarrow A_{vo} = G_m R_o = \underline{g_m \frac{\beta r_o}{3}} \quad \text{Q.E.D}$$

For: $I = 0.4\,mA$, $\beta = 100$, $V_A = 120V$

$$A_{vo} = \frac{I/2}{V_T}\cdot\frac{\beta}{3}\cdot\frac{V_A}{I/2} = \frac{\beta}{3}\frac{V_A}{V_T}$$

$$= \frac{100}{3}\times\frac{120V}{25mV} = \underline{\underline{160000}}$$

i.e. $\underline{\underline{104\,dB}}$

7.86

To obtain maximum positive swing V_{bias} must be as low as possible.

To keep the top current sources out of saturation:

$V_{CC} - 0.2 - 0.7 = V_{bias\ max}$

$V_{bias\ max} = 4.1$ V

And: $V_O - V_{bias\ min} = +0.4$ V

Since $V_O \sim 0 \Rightarrow V_{bias\ min} = -0.4$ V

$\Rightarrow$ Range of V_{bias} is :

$(-0.4 << V_{bias} \leq 4.1)$ V

For: $I = 0.4$ mA, $\beta_P = 50$, $\beta_N = 150$ &

$V_A = 120$

$$G_m = g_{m1} = \frac{0.2\text{ mA}}{25\text{ mV}} = 8\,\frac{\text{mA}}{\text{V}}$$

For the folded cascode: $R_{O4} = \beta_4 r_{O4}$

For the Wilson mirror: $R_{O5} = \beta_5 \frac{r_{O5}}{2}$

$$\Rightarrow R_O = \left[\beta_4 \cdot r_{O4} \,\|\, \beta_5 \cdot \frac{r_{O5}}{2}\right]$$

$r_{O4} = r_{O5} = 120/0.2\text{ mA} = 600\text{ k}\Omega$

$$\rightarrow R_O = \left[50 \times 600\text{ k} \,\|\, 150 \times \frac{600\text{ k}}{2}\right]$$

$= [30\text{ M} \,\|\, 45\text{ M}]$

$= 18\text{ M}\Omega$

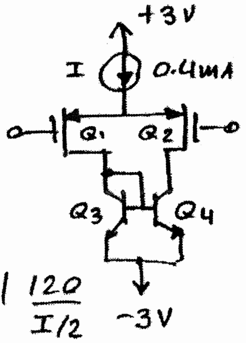
$$A_d = G_m R_O = 8\,\frac{\text{mA}}{\text{V}} \times 18\text{ M}\Omega = 144000$$

7.87

$K_p' W/L = 6.4\,mA/V^2$

$|V_{A_p}| = 10\,V$

$V_{A_{NPN}} = 120.$



$$R_o = r_{o2} \,\|\, r_{o4} = \frac{V_{A_p}}{I/2} \,\|\, \frac{120}{I/2}$$

$$R_o = (10/0.2m) \,\|\, (120/0.2m) = \underline{\underline{46k\Omega}}$$

$$G_m = g_{m1} = \sqrt{I \times K_p' W/L}$$

$$= \sqrt{0.4mA \times 6.4mA/V^2}$$

$$\Rightarrow G_m = \underline{\underline{1.6\,\frac{mA}{V}}}$$

$$A_d = G_m \times R_o = 1.6\,\frac{mA}{V} \times 46\,k\Omega$$

$$\longrightarrow A_d = \underline{\underline{73.6}}\ V/V$$

7.88

$I_{D5} = I_{D8} = I_{D7} = I_{D6} = I = I_{REF} = 225\ \mu A$

$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2} = \frac{225\ \mu A}{2}$

$= 112.5\ \mu A$

From Eq. (8.180), systemic balance will occur in this circuit when

$$\frac{\left(\frac{W}{L}\right)_6}{\left(\frac{W}{L}\right)_4} = 2\frac{\left(\frac{W}{L}\right)_7}{\left(\frac{W}{L}\right)_5}$$

$$\left(\frac{W}{L}\right)_6 = 2\frac{\left(\frac{W}{L}\right)_7}{\left(\frac{W}{L}\right)_5}\cdot\left(\frac{W}{L}\right)_4 = (2)\frac{\left(\frac{60}{0.5}\right)}{\left(\frac{60}{0.5}\right)}\cdot\left(\frac{10}{0.5}\right)$$

$$= \frac{20}{0.5}$$

so, $W_6 = 20$

To find $|V_{OV}|$, we use $I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

$$|V_{OV}|_{1,2} = \sqrt{\frac{2(I_{D1})}{\mu_p C_{Ox}\left(\frac{W}{L}\right)_{1,2}}} = \sqrt{\frac{225\ \mu A}{60\ \mu A V^2\left(\frac{30}{0.5}\right)}}$$

$= 0.25$ V

$$|V_{OV}|_{3,4} = \sqrt{\frac{2I_{D3}}{\mu_n C_{Ox}}} = \sqrt{\frac{2(112.5\ \mu A)}{180\ \mu A/V^2\left(\frac{10}{0.5}\right)}}$$

$= 0.25$ V

$$|V_{OV}|_{5,7,8} = \sqrt{\frac{2I}{\mu_p C_{Ox}\left(\frac{W}{L}\right)_{5,7,8}}} = \sqrt{\frac{2(225\ \mu A)}{60\ \mu A/V\left(\frac{60}{0.5}\right)}}$$

$= 0.25$ V

$|V_{GS}| = |V_t| + |V_{OV}|$, so all are

$|V_{GS}| = 0.75 + 0.25 = 1.0$ V

$$g_{m_{1-4}} = \frac{I/2}{|V_{OV}|/2} = \frac{225\ \mu A}{0.25\ V} = 0.9\ mA/V$$

$$g_{m_{5-8}} = \frac{I}{|V_{OV}|/V} = \frac{2(225\ \mu A)}{0.25\ V} = 1.8\ mA/V$$

$$r_{o_{1-4}} = \frac{|V_A|}{I/2} = \frac{9\ V}{0.225\ mA} = 80\ k\Omega$$

$$r_{o_{5-8}} = \frac{|V_A|}{I/2} = \frac{9\ V}{0.1125\ mA} = 40\ k\Omega$$

$A_1 = -g_{m1}(r_{O2} \| r_{O4})$

$= -(0.9\ mA/V)(80\ k \| 80\ k) = -36$ V/V

$A_2 = -g_{m6}(r_{O6} \| r_{O7})$

$= -(1.8\ mA/V)(40\ k \| 40\ k) = -36$ V/V

$A_O = A_1 \times A_2 = (-36)(-36) = 1296$ V/V

$= 20\log_{10}(1296) = 62.25$ dB

The input common-mode range is determined as follows:

The lower limit is when the input is such that Q_1 and Q_2 leave the saturation region:

$V_{D1} = -V_{SS} + V_{GS3} = -1.5 + 1 = 0.5$ V

with $|V_{DS}| = |V_{OV}|$, this would be when

$V_{S1} = -0.5 + 0.25 = -0.25$ V

$V_{in\,min} = V_{S1} - V_{SG} = -0.25 - 1 = -1.25$ V

The upper limit is when Q_5 leaves saturation:

$V_{D5_{max}} = V_{DD} - |V_{OV}| = 1.5 - 0.25 = 1.25$ V

$V_{in\,max} = V_{S\,max} - V_{SG} = 1.25 - 1.0 = +0.25$ V

so, range is (–1.25 V to +0.25 V)

For the output range, $V_{O\,max}$ is

$V_{Omax} = V_{DD} - |V_{OV}| = 1.5 - 0.25 = 1.25$ V

$V_{Omin} = -V_{SS} + |V_{OV}| = -1.5 + 0.25$

$= -1.25$ V

so the output range is (–1.25 V +1.25 V.)

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
$I_D(\mu A)$	112.5	112.5	112.5	112.5	225	225	225	225
$\|V_{OV}\|$(V)	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$\|V_{GS}\|$(V)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$g_m\left(\frac{mA}{V}\right)$	0.9	0.9	0.9	0.9	1.8	1.8	1.8	1.8
$r_O(k\Omega)$	80	80	80	80	40	40	40	40

7.89

$I_{D8} = I_{D1-4} = I_{REF} = 200\ \mu A$

$I_{D5} = 2I_{D1} = 400\ \mu A$

No requirements are given for Q_6 and Q_7, so choose

* $I_{D6} = I_{D7} = 2I_{REF} = 400\ \mu A$

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
$\left(\frac{W}{L}\right)$	25	25	100	100	50	200	50	25

$I_D = \frac{1}{2}k'(W/L)V_{OV}^2$ so,

$$\left(\frac{W}{L}\right)_{1,2,8} = \frac{2I_{REF}}{k_n'(V_{OV})^2} = \frac{2(200\ \mu A)}{400\ \mu A/V^2(0.2\ V)^2}$$

$= 25$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{2I_{REF}}{k_p'(V_{OV})^2} = \frac{2(200\ \mu A)}{400\ \mu A/V^2(0.2\ V)^2}$$

$= 100$

$$\left(\frac{W}{L}\right)_{5,7} = \frac{2(2I_{REF})}{k_n'(V_{OV})^2} = \frac{2(400\ \mu A)}{400\ \mu A/V^2(0.2\ V)^2}$$

$= 50$

$$\left(\frac{W}{L}\right)_{C} = \frac{4I_{REF}}{k_p'(V_{OV})^2} = \frac{4(200\ \mu A)}{100\ \mu A/V^2(0.2\ V)^2}$$

$= 200$

Ideally, $V_O(dc) = 0$

(b) For the common-mode input range:
The lower limit is when Q_5 is leaving saturation.

$V_{D5} = -V_{SS} + |V_{OS}| = -1\ V + 0.2\ V = -0.8V$

$V_{in(min)} = V_{GS1} + V_{D5} = V_{tn} + V_{OV} + V_{D5}$

$= 0.4 + 0.2 - 0.8 = -0.2\ V$

The upper input limit is when Q_1 and $Q2$ leave the saturation region:

$V_{D1} = V_{DD} - V_{SD3} = 1 - (0.4 + 0.2) = 0.4\ V$

$V_{DS1} = |V_{OV}| = 0.2\ V,$ so

$V_{in(max)} = V_{D1} - V_{OV} + V_{GS1}$

$= V_{D1} + V_{tn} = 0.4\ V = 0.8\ V$

so, the range of input voltage is
(-0.2 V to +0.8 V)

(c) The maximum output voltage is

$V_{O_{(max)}} = V_{DD} - |V_{OV}| = 1 - 0.2 = +0.8\ V$

$V_{O_{(min)}} = -V_{SS} + |V_{OV}| = -1 + 0.2$

$= -0.8\ V$

so range is (−0.8 V to +0.8 V)

(d) $r_{O2} = r_{O4} = \dfrac{|V_A|}{I_{D2}} = \dfrac{5\ V}{0.2\ mA} = 25\ k\Omega$

$r_{O6} = r_{O7} = \dfrac{|V_A|}{I_{D6}} = \dfrac{5\ V}{0.4\ mA} = 12.5\ k\Omega$

$g_{m1} = g_{m2} = \dfrac{|I_D|}{V_{OV}/2} = \dfrac{0.2\ mA}{0.2\ V/2} = 2\ mA/V$

$g_{m6} = \dfrac{|I_{D6}|}{V_{OV}/2} = \dfrac{0.4\ mA}{0.2/2} = 4\ mA/V$

$A_1 = g_{m1}(r_{O2} \| r_{O4}) = (2\ mA/V)(25\ k \| 25\ k)$

$= 25\ V/V$

$A_2 = -g_{m2}(r_{O6} \| r_{O7}) = -4\ mA/V(12.5\ k \| 12.5\ k)$

$= -25\ V/V$

$A_O = A_1 \cdot A_2 = 25(-25) = -625\ V/V$

7.90

$I = \frac{1}{2}k\ V_{OV}^2$

(a) $V_{OV} = \sqrt{\dfrac{2I}{k}}$

If K increases by 4 → V_{OV} decreases by 1/2

$g_m = 2I/V_{OV} = k \cdot V_{OV}$

→ if k increases by 4

gm increases by × 2

(b) $A_1 = gmR_{O1}$

→ A_1 increases × 2 as does A_O

(c) Offsets due to V_t mismatch are unaffected.

Others reduced $\times \frac{1}{2}$ since A_O increases × 2

7.91

$I_{D7} = \dfrac{W_7}{W_8} I_{REF} = \dfrac{50}{40} \times 90\ \mu A$

$= 112.5\ \mu A$

Output offset current $= I_{D7} - I_{D6}$

$= 112.5 - 90 = 22.5\ \mu A$

$\Rightarrow V_o = 22.5\ \mu\ (r_{o6} \| r_{o7})$

$r_{o7} = \dfrac{10}{112.5\ \mu} = 88.9\ k\Omega$

$\Rightarrow V_o = 22.5\ \mu\ (111\ k \| 88.9\ k)$

$= 1.11\ V$

$V_{os} = \dfrac{V_o}{A_o} = \dfrac{1.11V}{1109} = 1\ mV$

7.92

Offset current $= I_{D2} - I_{D4}$

$= I_{D3} - I_{D4}$

$$I_{D3} = \frac{K}{2}(V_{GS} - V_t)^2$$

$$I_{D4} = \frac{K}{2}(V_{GS} - (V_t + \Delta V_t))^2$$

$$I_O = I_{D3} - I_{D4}$$

$$= \frac{K}{2}\Big[(V_{GS} - V_t - V_{GS} + V_t + \Delta V_t) \times (V_{GS} - V_t + V_{GS} - V_t - \Delta V_t)\Big]$$

$$= \Delta V_t . \frac{K}{2}(2V_{GS} - 2V_t - \Delta V_t)$$

$$\simeq K(V_{GS} - V_t) . \Delta V_t$$

$$\underline{\underline{I_O = g_{m3} \Delta V_t}}$$

Recall $I_O = G_{m1} . V_{OS}$

and $G_{m1} = g_{m1}$

$$\Longrightarrow V_{OS} = \frac{g_{m3}}{g_{m1}} . \Delta V_t$$

For $\Delta V_t = 2mV$

$$V_{OS} = \frac{0.3m}{0.3m} \times 2m = \underline{\underline{2mV}}$$

7.93

(a) $\underline{\underline{I_{E1} = I_{E2} = 0.1mA \simeq I_{E3}, I_{E4}}}$

$\underline{\underline{I_{E5} \simeq 1mA}}$ and since the output is held at 0V $\underline{\underline{I_{E6} = 2mA}}$

(b) $r_{e1} = r_{e2} = \dfrac{25mV}{0.1mA} = 250\Omega$

$r_{e5} = \dfrac{25mV}{1mA} = 25\Omega$

$r_{e6} = \dfrac{25mV}{2mA} = 12.5\Omega$

For the active loaded differential pair; Recall from Eqn. (7.161) $G_{m1} = g_{m1}$

$$\simeq \frac{1}{r_{e1}} = \frac{1}{250} = 4\frac{mA}{V}$$

$R_{o1} = (\beta+1) r_{e5}$ Since all r_o's $= \infty$

$R_{o1} = 101 \times 25 = 2525\Omega$

$\Longrightarrow A_1 = G_{m1} R_{o1} = 4\frac{mA}{V} \times 2525\Omega$

$= 10.1 V/V$

For the common-emitter:

$A_5 = -g_{m5} . R_{C5}$

$$\simeq -\frac{\beta R_L}{r_{e5}} = -\frac{100 \times 10K}{25}$$

$= -40,000 V/V$

For the emitter follower:

$A_6 \simeq 1$

$A_{2nd\text{-}stage} = A_5 . A_6 = -40,000 V/V$

$A = A_1 . A_{2nd\text{-}stage} = 10.1 \times -40,000$

$= \underline{\underline{-404,000 V/V}}$

(c) Since the dominant low-frequency pole is set by C_c & $r_{\pi5}$

$$f_P = \frac{1}{2\pi . R_{o1} \underbrace{(A_5 + 1) C_c}_{\text{by Miller effect}}} = 100Hz$$

$$\Longrightarrow C \cong 1/(2\pi \times 2525 \times 40K \times 100)$$

$= \underline{\underline{15.76pF}}$

7.94

$I_B = 225\ \mu A$

$\mu_n C_{OX} = 180\ \mu A / V^2$

$\mu_p C_{OX} = 60\ \mu A / V^2$

For Q_8 & Q_9 : W/L = 60/0.5

$$\Rightarrow |V_{ov}| = \sqrt{\frac{2I_D}{k_p'(W/L)}}$$

$$|V_{ov}|_{8,9} = \sqrt{\frac{2 \times 225\ \mu}{60\ \mu \times 120}} = 0.25\ V$$

$$\text{then } g_{m8,9} = \frac{2I_D}{|V_{ov}|} = \frac{2 \times 225\ \mu}{0.25\ V}$$

$= 1.8$ mA/V

Since g_m of Q_{10}, Q_{11} & Q_{13} are identical to g_m of Q_8 & Q_9 then $V_{ov13} = 0.25$ V

Thus for Q_{13}

$$(0.25)^2 = \frac{2 \times 225\ \mu}{180\ \mu \times (W/L)_{13}}$$

$\rightarrow (W/L)_{13} = 40$ i.e, $(20/0.5)$

Since Q_{12} is 4 times as wide as Q_{13}, then

$$(W/L)_{12} = \frac{4 \times 20}{0.5} = 80/0.5$$

$$R_B = \frac{2}{\sqrt{2\,k_n'\,(W/L)_{12}\,I_B}} \cdot \left(\sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1\right)$$

$$= \frac{2}{\sqrt{2 \times 180\ \mu \times \frac{80}{0.5} \times 225\ \mu}} \cdot \left(\underbrace{\sqrt{\frac{80/0.5}{20/0.5}} - 1}_{\sqrt{4} - 1}\right)$$

$\rightarrow R_B = 555.6\ \Omega$

The voltage drop on R_B is :

$555.6 \times 225\mu = 0.125$ V.

To obtain the gate voltages:(assume $|V_{tn}| = |V_{tp}| = 0.7$ V)

$$V_{ov12} = \sqrt{\frac{2 \times 225\ \mu}{180\ \mu \times \frac{80}{0.5}}} = 0.125\ V$$

$V_{OV12} = V_{GS12} - V_{tn}$

$\rightarrow V_{GS12} = 0.125 + 0.7 = 0.825$ V

thus,

$V_{G12,13} = V_{GS12} + I_B R_B - V_{SS}$

$= 0.825 + 0.125 - 1.5$

$= -0.55$ V

$V_{OV11} = |V_{OV8}| = 0.25$ V

$\Rightarrow V_{GS11} = 0.25 + 0.7 = 0.95$ V

$V_{G11} = -0.55 + 0.95$

$V_{G11} = V_{G10} = 0.4$ V

$V_{G8} = V_{DD} - V_{SG8} = 1.5 + (-0.25 - 0.7)$

$= +0.55$ V

Finally from the results above:

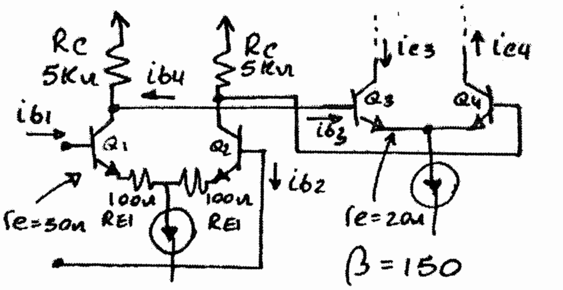
$(W/L)_{10} = 20/0.5$

$(W/L)_{11} = 20/0.5$

$(W/L)_{12} = 80/0.5$

$(W/L)_{13} = 20/0.5$

7.95



$$A_1 = \frac{2R_C \parallel R_{id2}}{2(R_{E1} + r_{e1})}$$

$R_{id2} = ((\beta+1)(2r_{e2}) = 6.04\text{K}\Omega$

$\Rightarrow A_1 = \underline{\underline{12.5}}\ V/V$

$$A_i = \frac{i_{e4}}{i_{b1}} = \beta_1 \cdot \frac{2R_C}{R_{id2} + 2R_C}\,\beta_4$$

$= 1.4 \times 10^4\ \underline{\underline{A/A}}$

7.96

$$R_O \simeq \frac{R_5}{\beta + 1} + r_{e8} = R_C'$$

Thus R_5 affects R_O We want $R_O' \parallel 3\text{ k} = 76$

$\Rightarrow R_O' = 78\ \Omega$

$\Rightarrow R_5 = (78 - r_{e8})(\beta + 1)$

$= 7.34$ kΩ

$$A_3 = \frac{-R_5 \parallel R_{i4}}{r_{e4} + R_4} \quad ; \quad R_{i4} \cong 304\ \text{k}\Omega$$

and $A_3 = -3.09$ V/V

and $A = 8513 \cdot \frac{3.09}{6.42} = 4104$ V / V

The gain has been reduced by a factor of 2.07 and can be restored by reducing R_4 by this same factor to increase A_3. Thus $R_4 = 1.11\ \text{k}\Omega$
(Note that this is a first order approximation).

7.97

(a) $A_3 = \frac{-R_{i4}}{2.325\ \text{k}\Omega} = \frac{-303.5}{2.325}$

$= -130.5$ V/V

i,e A_3 is increased by $\frac{130.5}{6.42}$

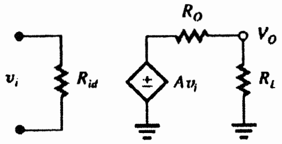
$= 20.33$

$\Rightarrow A = 8513 \times 20.33$

$\doteq 173.1 \times 10^3$ V / V

(b) Let the output resistance of the current source be $R \to \infty$ $R_O = 3\text{ k} \parallel \left(\frac{R}{\beta + 1} + r_e\right) = 3\ \text{k}\Omega$

The amplifier can be modelled as shown:



Thus,

$$A_{LOAD} = \frac{A \cdot R_L}{R_L + R_O}$$

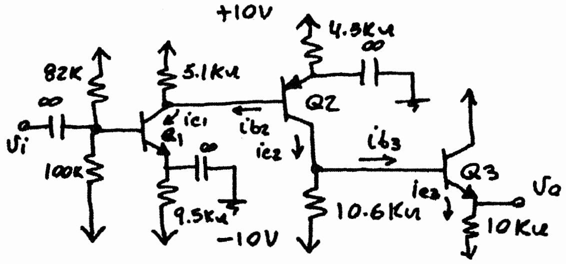
$$= 173.1 \times 10^3 \frac{100}{100 + 3000}$$

$= 5583$ V/V

For the original amplifier:

$$A_{LOAD} = 8513 \times \frac{100}{100 + 152} = 3378\ \frac{\text{V}}{\text{V}}$$

7.98



(a) $I_{E1} = \dfrac{\dfrac{20\text{V} \times 100\text{K}}{82\text{K} + 100\text{K}} - 0.7}{9.5\text{K} + \dfrac{(82\text{K} \parallel 100\text{K})}{\beta + 1}}$

$\beta = 100 \Rightarrow I_{E1} = 1.03\text{mA}$

$\alpha = \frac{100}{101} \Rightarrow I_{C1} = \underline{\underline{1.02\text{mA}}}$

$V_{C1} \simeq 10\text{V} - 1.02\text{mA} \times 5.1\text{K}\Omega = 4.8\text{V}$

$I_{E2} = \frac{(10 - 0.7 - 4.8)\text{V}}{4.5\text{K}\Omega} = 1\text{mA}$

$\rightarrow I_{C2} = \underline{\underline{0.99\text{mA}}}$

$V_{C2} \simeq 0.99\text{m} \times 10.6\text{K} - 10 = 0.5\text{V}$

$\Rightarrow v_{O\,DC} = 0.5 - 0.7 = \underline{\underline{-0.2\text{V}}}$

$I_{E3} = \frac{-0.2 - (-10)}{10\text{K}} = 0.98\text{mA}$

$\rightarrow I_{C3} = \underline{\underline{0.97\text{mA}}}$

Thus all transistors are operating at $I_C \simeq \underline{\underline{1\text{mA}}}$

(b) $R_{in} = 82K \,//\, 100K \,//\, r_{\pi 1}$

where $r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{40m} = 2.5K\Omega$

$\Rightarrow R_{in} = (82 \,//\, 100 \,//\, 2.5)K = \underline{\underline{2.37K\Omega}}$

$R_{out} = 10K \,//\, \left[r_{e3} + \frac{10.6K}{\beta+1}\right]$

$= 10K \,//\, \left[25 + \frac{10.6K}{101}\right]$

$= \underline{\underline{128\,\Omega}}$

(c) $\frac{i_{c1}}{v_i} = g_{m1} = 40mA/V$

$\frac{i_{b2}}{i_{c1}} = \frac{5.1K}{5.1K + r_{\pi 2}} = \frac{5.1}{5.1 + 2.5} = 0.671\ \frac{A}{A}$

$\frac{i_{e2}}{i_{b2}} = \beta_2 = 100\ A/A$

$\frac{i_{b3}}{i_{c2}} = \frac{10.6K}{10.6K + (\beta+1)(r_{e3} + 10K)}$

$= 0.01036\ A/A$

$\frac{i_{e3}}{i_{b3}} = \beta_3 + 1 = 101$

$v_o = i_{e3} \times 10K$

Thus,

$\frac{v_o}{v_i} = 10 \times 101 \times 0.01036 \times 100 \times 0.671 \times 40$

$= \underline{\underline{2.81 \times 10^4\ V/V}}$

(d) $f_{P2} = 1/(2\pi C_2 \cdot R_2)$

where: $R_2 = 5.1K \,//\, r_{\pi 2}$

$= 5.1K // 2.5K = 1.68K\Omega$

$C_2 = C_{\pi 2} + C_{\mu 2}(1 + g_{m2} R_{L2})$

with:

$R_{L2} = 10.6K \,//\, ((\beta+1)(r_{e3} + 10K))$

$= 10.6K \,//\, 101 \times (25 + 10K)$

$= 10.5K\Omega.$

$\Rightarrow C_2 = 10p + 2p(1 + 40m \times 10.5K)$

$= 852pF$

$\Rightarrow f_{P2} = \frac{1}{2\pi \times 852p \times 10.5K}$

$= \underline{\underline{17.8}}\ KHz$

7.99

(a) $I_{D1-5,7} = \frac{I}{2}$

$I_{D6,8} = 2\left(\frac{I}{2}\right) = I$

$g_m = \frac{|I_D|}{\frac{|V_{OV}|}{2}}$ So that

$g_{m1-5,7} = \frac{I/2}{V_{OV}/2} = \frac{I}{|V_{OV}|}$

$g_{m6,8} = \frac{I}{\frac{|V_{OV}|}{2}} = \frac{2I}{|V_{OV}|}$

$r_O = \frac{|V_A|}{|I_D|}$ So that

$r_{O1-5,7} = \frac{2|V_A|}{I}$

$r_{O6,8} = \frac{|V_A|}{I}$

In Summary,

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
I_D	$I/2$	$I/2$	$I/2$	$I/2$	$I/2$	I	$I/2$	I
g_m	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{2I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{2I}{\lvert V_{OV}\rvert}$
r_D	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{\lvert V_A\rvert}{I}$

(b)To find the differential gain, apply $-\frac{V_{id}}{2}$ to Q_1 and $V_{id}/2$ to Q_2

$$V_{g5} = g_{m1}\left(r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{m3}}\right) V_{id}/2$$

since $\frac{1}{g_{m3}} << r_{o1} \parallel r_{o3}$,

$$V_{g5} = g_{m1}\left(\frac{1}{g_{m3}}\right) \cdot \frac{V_{id}}{2}$$

$$i_{d5} = -V_{g5}\, g_{m5} = -g_{m1}\left(\frac{1}{g_{m3}}\right)(g_{m5})\,\frac{V_{id}}{2}$$

$$V_{g8} = V_{g7} = i_{d5}\left(r_{o5} \parallel r_{o7} \parallel \frac{1}{g_{m7}}\right) = i_{d5}\left(\frac{1}{g_{m7}}\right)$$

since $g_{m8} = g_{m7}$,

$$V_{g8} = -g_{m1}\left(\frac{1}{g_{m7}}\right)\frac{V_{id}}{2}$$

since $g_{m8} = 2\, g_{m7}$

$$i_{d8} = +g_{m1}\left(\frac{1}{g_{m7}}\right)(2g_{m7}) \cdot \frac{V_{id}}{2} = +g_{m1}V_{id}$$

with $+\frac{V_{id}}{2}$ applied to Q_2,

$$V_{g4} = -g_{m2}\left(r_{o2} \parallel r_{o4} \parallel \frac{1}{g_{m4}}\right),$$

$$V_{g6} = V_{g4} = -g_{m1}\left(\frac{1}{g_{m4}}\right) \cdot \frac{V_{id}}{2}$$

since $g_{m6} = g_{m4} \times 2$,

$$i_{d6} = -g_{m4}(2)\,(-g_{m1})\left(\frac{1}{g_{m4}}\right)\frac{V_{id}}{2}$$

$$i_{d6} = g_{m1}\ V_{id}$$

$$i_O = g_{m1}\, V_{id} + g_{m1}\ V_{id} = 2\, g_{m1}\, V_{id}$$

$$\frac{A_d}{V_{id}} = \frac{i_O\, R_O}{V_{id}} = 2\, g_{m1}\,(r_{o6} \parallel r_{O8})$$

$$g_{m1} = \frac{I}{\lvert V_{OV}\rvert} \quad r_{o6} = r_{O8} = \frac{\lvert V_A\rvert}{I}$$

$$\frac{A_d}{V_{id}} = 2\frac{I}{\lvert V_{OV}\rvert}\left(\frac{1}{2}\right)\frac{\lvert V_A\rvert}{I} = \frac{V_A}{V_{OV}}$$

(c) If each input transistor (Q_1 and Q_2) is replaced with a current source of $\frac{V_{icm}}{2R_{SS}}$,

From Q_2, with a transfer ratio of $\left(1 - \frac{1}{g_m r_O}\right)$,

$$i_{D6} = -\frac{V_{icm}}{2\,R_{SS}}\left(1 - \frac{1}{g_{m4} r_{O4}}\right)(2)$$

From

$$Q_1, i_{D8} = \frac{V_{iCM}}{2\,R_{SS}}\left(1 - \frac{1}{g_{m3} r_{O3}}\right)\left(1 - \frac{1}{g_{m7} r_{O7}}\right)(2)$$

$$i_O = i_{D8} + i_{D6}$$

$$i_O = \frac{V_{icm}}{R_{SS}}\left[-1 + \frac{1}{g_{m4}\,r_{O4}} + \left(1 - \frac{1}{g_{m3}\,r_{O3}}\right)\left(1 - \frac{1}{g_{m7}\,r_{O7}}\right)\right]$$

Since $g_{m3} = g_{m4}$ and $r_{O3} = r_{O4}$,

$$i_O = \frac{V_{iCM}}{R_{SS}}\left[\left(1 - \frac{1}{g_{m3} r_{O3}}\right)\left(1 - \frac{1}{g_{m7} r_{O7}}\right)\right]$$

$V_O = i_O(r_{o6} \| \ r_{O8})$ and Since $\frac{1}{g_{m3} r_{O3}} >> 1$

$$v_O \simeq \frac{-v_{icm}}{R_{SS}}\left(\frac{1}{g_{m7} r_{O7}}\right)(r_{o6} \| \ r_{O8})$$

$$|A_{CM}| = \left|\frac{V_O}{V_{iCM}}\right| = \frac{(r_{o6} \| \ r_{O8})}{R_{SS}} = \frac{1}{g_{m7} r_{O7}}$$

(d) If the current source is fabricated as a simple current mirror, $R_{SS} = \frac{V_A}{I}$

$r_{O6} = r_{O8} = \frac{V_A}{I}$ so, $r_{o6} \| \ r_{O8} = \frac{V_A}{2I}$

$$g_{m7} = \frac{I}{V_{OV}/2} = \frac{2I}{V_{OV}}$$

$$CMRR = \frac{|A_d|}{|A_{CM}|} = \frac{V_A / V_{OV}}{\frac{r_{o6} \| \ r_{O8}}{R_{SS}} \cdot \frac{1}{g_{m7} r_{O7}}}$$

$$= \frac{V_A / V_{OV}}{\frac{V_A / 2I}{V_A / I} \cdot \frac{1}{\frac{2\,I}{V_{OV}} \cdot \frac{V_A}{I}}}$$

$$CMRR = \frac{2\,V_A / V_{OV}}{\frac{1}{2}\left(\frac{V_{OV}}{V_A}\right)} = 4(V_A / V_{OV})^2$$

(e) To find the input common-mode range, consider both upper and lower limits: Lower limit is when the current source begins to leave the saturation region at $V_{DS} = V_{OV}$

So,

$V_{I(min)} = V_{GS1} + V_{OV} + V_{SS} = V_t + 2V_{OV} - V_{DD}$

The maximum limit occurs when Q_1 or Q_2 begins to leave the saturated region: For example, when

$V_{S1} = V_{DD} - V_{SG3} - V_{OV1}$

$V_{I(max)} = V_t + V_{OV} + V_{S1}$

$V_{I(max)} = V_t + V_{OV} + V_{DD} - V_t + 2V_{OV}$

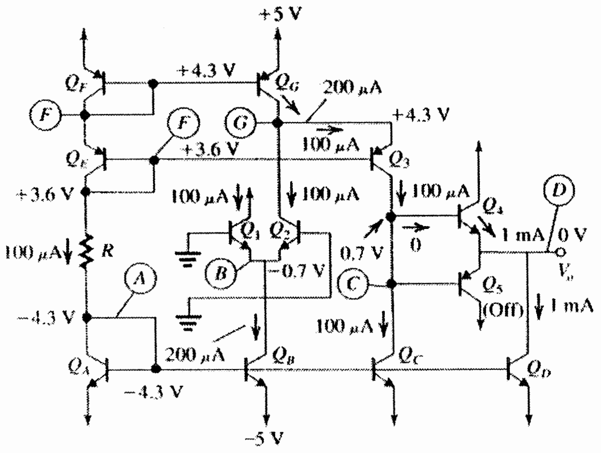
$V_I(\text{max}) = V_{DD} - V_{OV}$

So the range is

$(-V_{DD} + V_t + 2V_{OV} \leq V_{ICM} \leq V_{DD} - V_{OV})$

7.100

(a)



DC Analysis

$$R = \frac{3.6-(-4.3)}{100\ \mu A} = 79\ k\Omega$$

Node voltages:

$V_A = -4.3$ V $\quad V_B = -0.7$ V

$V_C = +0.7$ V $\quad V_D = 0$ V

$V_E = +3.6$ V $\quad V_F = +4.3$ V

$V_G = +4.3$V

(b)

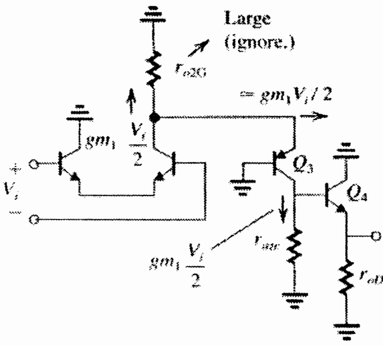
Transistor	I_C(mA)	g_m(mA/V)	r_o(mΩ)
Q_1	0.1	4	2
Q_2	0.1	4	2
Q_3	0.1	4	2
Q_4	1.0	40	0.2
Q_5	0	0	∞
Q_A	0.1		
Q_B	0.2		
Q_C	0.1	....	2
Q_D	1.0	....	0.2
Q_E	0.1		
Q_F	0.1		
Q_G	0.2	....	1

(c) Total resistance at collector Q_3 is

$\approx \beta_3 r_{O3} \| r_{oC} \| \ (\beta_4 + 1)(r_{O4} \| \ r_{OD})$

$= 100 \times 2 \| 2 \| \ 101(0.2 \| \ 0.2)$

$= 1.65$ MΩ


Large
(ignore.)
r_{o2G}
$\approx g m_1 V_i / 2$
gm_1
$\frac{V_i}{2}$
+
V_i
−
Q_3
Q_4
r_{oC}
v_o
$gm_1 \frac{V_i}{2}$
r_{oD}

$$\frac{v_{C3}}{v_i} = + g_{m1} \times \frac{1}{2} \times 1.65 \times 10^3 = 3300 \frac{V}{V}$$

$$\frac{v_O}{v_{C3}} \approx 1$$

Thus, $\frac{v_O}{v_i} \approx 3300$ V/V (Polarity correct)

(d) $R_{in} = 2\, r_{\pi 1}$

$= 2 \times \frac{100}{4} = 50$ kΩ

$$R_{out} = r_{OD} \| r_{O4} \| \ \left[r_{\pi 4} + \frac{r_{O2C} \| \ \beta_3 r_{O3}}{\beta + 1} \right]$$

$$= 0.2 \| 0.2 \| \ \left[25.10^{-6} + \frac{2 \| \ 100 \times 2}{101} \right]$$

≈ 16.4 kΩ

(e) $v_{ICM|min|} = -4.3 - 0.4 + 0.7$

$= -4$ V

$v_{ICM|max|} = V_G + 0.4 = +4.7$ V

(f) The voltage at the base of Q_4 can rise to V_{B3}.

$(V_E) + 0.4 = +4$ V

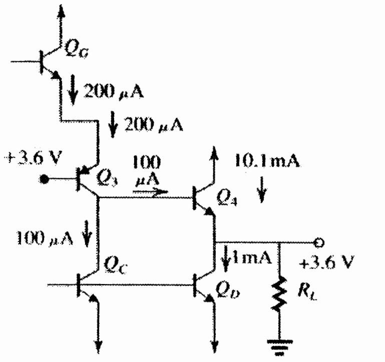
before Q_3 saturating. Thus v_O can go up to + 3.3 V

The voltage at the output can go down to V_{base} of

$Q_D + 0.4 = V_A - 0.4 = -4.3 - 0.4 = -4.7$ V

Thus the linear range at the output is − 4.7 V to +3.3 V

(g) At the positive limit of v_O


Q_G
200 μA
200 μA
+3.6 V
Q_3
100 μA
10.1 mA
Q_4
100 μA
Q_C
1mA
+3.6 V
Q_D
R_L

i.e. $v_O = +3.3$ V and Q_2 just cut off

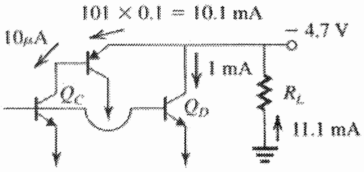
$$R_L = \frac{3.3\ V}{9.1\ mA}$$

$= 363$ Ω

(this is the minimum allowed R_L for + 3.3 V output)

At the negative limit of v_O i.e. $v_O = -3.3$ V and Q_1 has cut-off. Q_3 will also be cut-off, and Q_4 will cut-off.

Thus,


101 × 0.1 = 10.1 mA
10μA
− 4.7 V
1 mA
Q_C
Q_D
R_L
11.1 mA

$R_L = \dfrac{4.7}{11.1 \text{ mA}} = 423\ \Omega$ This is the minimum allowed R_L for a − 4.7 V output.

7.101

DC analysis

(a) $I_{REF} = 10\ \mu\text{A} = \frac{1}{2} \times 40 \times \frac{5}{5}(V_{GS_A} - V_t)^2$

$\Rightarrow V_{GS_A} = 1.71 \text{ V} \approx 1.7 \text{ V}$

$10 = \frac{1}{2} \times 20 \times \frac{5}{5}(V_{GS_{E\,F}} - 1)^2$

$\Rightarrow V_{GS_{E\,F}} = 2 \text{ V}$

$\text{R} = \dfrac{3-(-3.3)}{10\ \mu\text{A}} = 660\ \text{k}\Omega$

(b) See figure above

$V_{GS1} = V_{GS2} = V_{GSA} \approx 1.7 \text{ V}$

$V_{GS3} = \sqrt{\dfrac{2 \times 10}{20 \times \frac{10}{3}}} + 1 = 1.71 \text{ V} \approx 1.7 \text{ V}$

$V_{GS5} = V_{GS3} = 1.7 \text{ V}$

For Q_6: $50 = \frac{1}{2} \times 40 \times \frac{50}{5}(V_{GS6} - V_t)^2$

$\Rightarrow V_{GS6} = 1.50 \text{ V}$

$V_A = -3.3 \text{ V}$ $V_B = -1.7 \text{ V}$

$V_C = +1.5 \text{ V}$ $V_D = 0 \text{ V}$

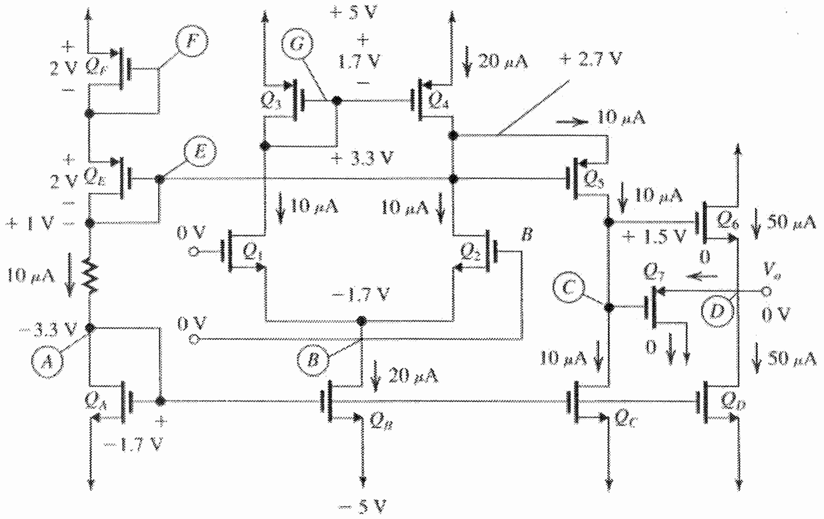
$V_E = +1 \text{ V}$ $V_F = +3 \text{ V}$

$V_G = +3.3 \text{ V}$ $V_H = +2.7 \text{ V}$

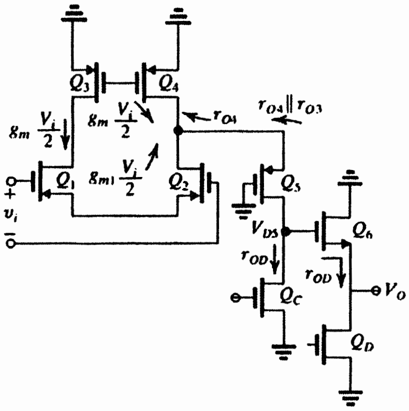
(c)

Transistor	I_D (μA)	V_{GS} (V)	gm (mA/V)	r_O (MΩ)
Q_1	10	1.7	28.3	5
Q_2	10	1.7	28.3	5
Q_3	10	1.7	28.3	5
Q_4	20	1.7	56.6	2.5
Q_5	10	1.7	28.3	5
Q_6	50	1.5	200	1
Q_7	0	−1.5*	0	∞
Q_A	10	1.7	28.3	5
Q_B	20	1.7	56.6	2.5
Q_C	10	1.7	28.3	5
Q_D	50	1.7	141.4	1
Q_E	10	2	20	5
Q_F	10	2	20	5

* Cut-off.


+ 5 V
+
2 V Q_F
−
F
G
+
1.7 V
−
Q_3
Q_4
20 μA
+ 2.7 V
10 μA
+ 3.3 V
+
2 V Q_E
−
E
Q_5
10 μA
10 μA
10 μA
+ 1 V
0 V
Q_1
Q_2
B
Q_6
50 μA
+ 1.5 V
0
Q_7
V_o
C
D
0 V
10 μA
−1.7 V
−3.3 V
0 V
A
B
20 μA
10 μA
0
50 μA
Q_A
+
−1.7 V
Q_B
Q_C
Q_D
− 5 V

(d)



Q_3
Q_4
$g_m \frac{V_i}{2}$
$g_m \frac{V_i}{2}$
r_{O4}
$r_{O4} \| r_{O3}$
$g_{m1} \frac{V_i}{2}$
Q_1
Q_2
Q_5
+
v_i
V_{DS}
Q_6
r_{OD}
r_{OD}
Q_C
V_O
Q_D

Total resistance at the drain of Q_5, R is:

$R = (g_{m5} r_{O5})(r_{O4} \| r_{O2}) \| r_{OC}$

$= [(28.3 \times 5)(2.5 \| 2)] \| 5$

$= 4.9\ \text{M}\Omega$

Thus, $\frac{v_{d5}}{v_i} = g_{m1} R$

$= 28.3 \times 4.9 = 138.7\ \text{V/V}$

and $\frac{v_O}{v_{d5}} = \frac{(r_{OD} \| r_{O6})}{(r_{OD} \| r_{O6}) + \frac{1}{g_{m6}}}$

$= \frac{(1 \| 1)}{(1 \| 1) + \frac{1}{200}} \approx 1$

$\frac{v_O}{v_i} = 138.7\ \text{V/V}$

$R_{in} = \infty$

$R_{out} = r_{OD} \| r_{O6} \| \ 1/g_{m6}$

$= 1 \| 1 \| \ 1/200\ \text{M}\Omega$

$\approx 5\ \text{k}\Omega$

(c) $v_{ICM|max|} = V_G + V_t$

$= +4.3\ \text{V}$

$v_{ICM|min|} = V_{GS1} + V_B\text{min}$

$= V_{GS1} + V_A - V_t$

$= 1.7 - 3.3 - 1 = -2.6\ \text{V}$

(f) $V_{O\,\text{max}} = V_{C\,\text{max}} - V_{GS6}$

$= V_E + |V_t| - V_{GS6}$

$= +1 + 1 - 1.5 = +0.5\ \text{V}$

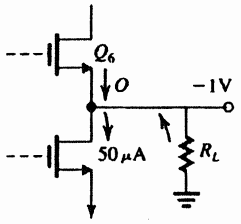
$v_{O\text{min}} = V_A - V_t = -3.3 - 1 = -4.3\ \text{V}$

(g) $Q6$ cuts off

thus,

$\frac{1\,V}{R_L} = 50\ \mu\text{A}$

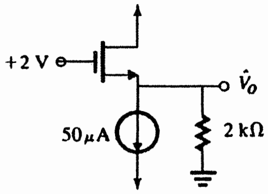
$R_L = \frac{1\,V}{50\ \mu\text{A}} = 20\ \text{k}\Omega$



Q_6
O
-1V
$50\,\mu\text{A}$
R_L

(h)Maximum possible voltage at drain of Q_5 is +2V. At theis value we have:

$I_D = 50\mu\text{A} + \frac{\hat{V}_o}{2}\ \text{mA}$

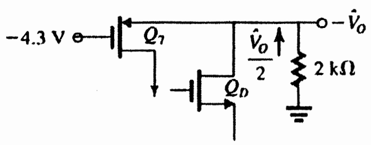


$+2\ \text{V}$
$\hat{V}_o$
$50\,\mu\text{A}$
$2\ \text{k}\Omega$

$I_D = \frac{1}{2}\mu_n C_{OX}\frac{W}{L}(2 - V_o \hat{-} V_t)^2$

$\Rightarrow V_o \approx \hat{0}.17\text{V}$

For the lowest posible output, the circuit becomes



$-4.3\ \text{V}$
Q_7
$-\hat{V}_O$
$\frac{\hat{V}_O}{2}$
$2\ \text{k}\Omega$
Q_D

Where:

Q_6 cuts off and Q_7 conducts

$I_D = \frac{V_O}{2} - 0.05\ \text{mA}$

$= \frac{1}{2}\mu_p C_{OX}\left(\frac{100}{5}\right)(-\hat{V}_O + 4.3 - 1)^2$

$\Rightarrow \hat{V}_O = 1.45\ \text{V}$

That is, the range of v_o is

-1.45 V to $+0.17$ V

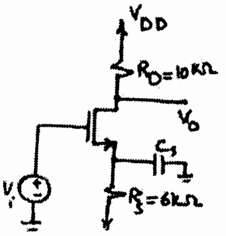
8.1

$I_D = 1\,mA$, $g_m = 1\,mA/V$

using eq. 4.89 we have:

$$A_M = \frac{-g_m R_D}{1+g_m R_S} = -\frac{1\times 10}{1+1\times 6}$$

$A_M = 1.43\,V/V$



$$f_L = \frac{1}{2\pi\left(\frac{1}{g_m} \| R_S\right)C_S} = 10\,Hz$$

$$C_S = \frac{1}{2\pi\times 10\,(1^K \| 6^K)} = \underline{18.57\,\mu F}$$

8.2

$$f_{C_{C_2}} = \frac{1}{2\pi C_{C_2}(R_L + R_D \| r_o)} \leqslant 10^{Hz}$$

$$\Rightarrow C_{C_2} \geqslant \frac{1}{10\times 2\pi\times(10^K + 15^K \| 150^K)} \Rightarrow C_{C_2} \geqslant 0.67\,\mu F$$

$$\Rightarrow C_{C_2} = 0.7\,\mu F \Rightarrow f_{C_{C_2}} = 9.62\,Hz$$

If I_D is doubled with both r_o and R_D halved:

$$f_{C_{C_2}} = \frac{1}{2\pi\times 0.7^{\mu}\left(10^K + \frac{15^K}{2} \| \frac{150^K}{2}\right)} = 13.5^{Hz}$$

8.3

$g_m = 1\,mA/V$

$$A_M = -\frac{R_G}{R_G + R_{sig}}\, g_m\, (R_D \| R_L) \quad \text{where } R_G = 10^M \| 47^M$$

$R_G = 8.25^{M\Omega}$

$$A_M = -\frac{8.25}{8.25+0.1}\times 1\times(4.7^K \| 10^K) = -3.16\,V/V$$

$$f_{P_1} = \frac{1}{2\pi C_{C_1}(R_G + R_{sig})} \quad \text{(Eq. 4.134)}$$

$$f_{P_1} = \frac{1}{2\pi\times 0.01\times 10^{-6}\times(8.25+0.1)\times 10^6} = \underline{1.9^{\,Hz}}$$

$$f_{P2} = \frac{1}{2\pi C_S\left(R_S \| \frac{1}{g_m}\right)} = \frac{1}{2\pi\times 10\times 10^{-6}\times\left(2^K \| \frac{1^K}{1}\right)} = \underline{23.9^{Hz}}$$

$$f_{P3} = \frac{1}{2\pi C_{C_2}(R_D + R_L)} = \frac{1}{2\pi\times 0.1\times 10^{-6}\times(4.7+10)\times 10^3} = \underline{108.3^{Hz}}$$

$\underline{f_L \simeq 108.3\,Hz}$

8.4

$$C_{TOT} = C_S + C_{C1} + C_{C2} = 3\,\mu F$$

$$R_G = 10\,M\Omega,\ R_{sig} = 100\,k\Omega,\ g_m = 2\,\frac{mA}{V},$$

$$R_D = R_L = 10\,k\Omega$$

$$f_{P2} = \frac{g_m}{2\pi\cdot C_S} \Rightarrow C_S = \frac{2\times 10^{-3}}{2\pi}\cdot\frac{1}{f_{P2}}$$

$$= \frac{3.18\times 10^{-4}}{f_{P2}}$$

$$f_{P1} = \frac{1}{2\pi\cdot C_{C1}\cdot(R_G + R_{sig})}$$

$$\Rightarrow C_{C1} = \frac{1.57\times 10^{-8}}{f_{P1}}$$

$$f_{P3} = \frac{1}{2\pi\cdot C_{C2}\cdot(R_D + R_L)}$$

$$\Rightarrow C_{C2} = \frac{7.95\times 10^{-6}}{f_{P3}}$$

If we choose: $f_{P2} = f_L, f_{P1} = f_L/25, f_{P3} = f_L/5$

$$C_{TOT} = 3\,\mu F = C_S + C_{C1} + C_{C2}$$

$$3\,\mu F = \frac{3.18\times 10^{-4}}{f_L} + \frac{1.57\times 10^{-8}}{f_L/25} + \frac{7.95\times 10^{-6}}{f_L/5}$$

$\Rightarrow f_L = 120$ Hz and $C_s = 2.65\,\mu F$,

$C_{C1} = 3.3$ nF, $C_{C2} = 0.33\,\mu F$

If we choose: $f_{P2} = f_L$, $f_{P1} = \frac{f_L}{5}$, $f_{P3} = \frac{f_L}{25}$

$$C_{TOT} = 3\,\mu F = C_S + C_{C1} + C_{C2}$$

$$3\,\mu F = \frac{3.18\times 10^{-4}}{f_L} + \frac{1.57\times 10^{-8}}{f_L/5} + \frac{7.95\times 10^{-6}}{f_L/25}$$

$\Rightarrow f_L = 172.3$ Hz, $C_S = 1.8\,\mu F$,

$C_{C1} = 455$ pF, $C_{C2} = 1.15\,\mu F$

8.5

$R_E = 72\,\Omega$,

$R_{C1} = 7.44\,k\Omega$, $R_{C2} = 13\,k\Omega$

If $C_E = 50\,\mu F$, $C_{C1} = C_{C2} = 2\,\mu F$

$$f_{P1} = \frac{1}{2\pi\cdot C_{C1} R_{C1}}$$

$$= \frac{1}{2\pi\times 2\times 10^{-6}\times 7.44\times 10^3} = 10.7\,Hz$$

$$f_{P2} = \frac{1}{2\pi\cdot C_E\cdot R_E}$$

8.6

$$= \frac{1}{2\pi \cdot 50 \times 10^{-6} \times 72} = 44.2 \text{ Hz}$$

$$f_{P3} = \frac{1}{2\pi C_{C2} R_{C2}}$$

$$= \frac{1}{2\pi \times 2 \times 10^{-6} \times 13 \times 10^{3}} = 6.1 \text{ Hz}$$

From Eq 9.19

$$f_L = f_{P1} + f_{P2} + f_{P3} = 61 \text{ Hz}$$

$g_m = 40 \text{ mA/V}$, $r_\pi = 2.5 \text{ k}\Omega$ and

$r_e = 25 \ \Omega$

If I_C is reduced by half, since

$$g_m = \frac{I_C}{V_T} \rightarrow g_m = 20 \frac{\text{mA}}{\text{V}}$$

$$r_\pi = \frac{\beta}{g_m} \rightarrow r_\pi = 5 \text{ k}\Omega \text{ and}$$

$$r_e = \frac{\alpha}{g_m} \rightarrow r_e = 50 \ \Omega$$

Then:

$$R_{C1} = (R_B \parallel r_\pi) + R_{sig}$$

$$= (100 \text{ K} \parallel 5 \text{ K}) + 5 \text{ K} = 9.76 \text{ k}\Omega$$

$$R_E = r_e + \frac{R_B \parallel R_{sig}}{\beta + 1} = 50 + \frac{100 \text{ K} \parallel 5 \text{ K}}{101}$$

$$= 97 \ \Omega$$

$$R_{C2} = R_C + R_L = 13 \text{ k}\Omega$$

For C_E to contribute 80% of f_L

$$0.8 \times 2\pi \times 100 = \frac{1}{C_E \cdot 97} \rightarrow C_E = 20.51 \ \mu\text{F}$$

For C_{C1} and C_{C2} to contribute 10% of f_L each

$$0.1 \times 2\pi \times 100 = \frac{1}{C_{C1} \cdot 9.76 \times 10^{3}}$$

$$\rightarrow C_{C1} = 1.64 \ \mu\text{F}$$

$$0.1 \times 2\pi \times 100 = \frac{1}{C_{C2} \cdot 13 \times 10^{3}}$$

$$\rightarrow C_{C2} = 1.23 \ \mu\text{F}$$

To verify the value of f_L that results,

$$f_L = \frac{1}{2\pi}\left(\frac{1}{97 \times 20.51 \ \mu} + \frac{1}{9.76 \text{ K} \times 1.64 \ \mu}\right.$$

$$\left. + \frac{1}{13 \text{ K} \times 1.23 \ \mu}\right)$$

$f_L = 99.89$ Hz

8.7

$$R_{sig} = 20 \text{ k}\Omega, \ R_C = 20 \text{ k}\Omega,$$

$$R_B = 200 \text{ k}\Omega, \ R_C = 10 \text{ k}\Omega,$$

$$\beta = 100, \ I_C \approx 100 \ \mu\text{A}$$

$$g_m = \frac{I_C}{V_T} \approx \frac{100 \ \mu\text{A}}{25 \text{ mV}} = 4 \frac{\text{mA}}{\text{V}}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4 \times 10^{-3}} = 25 \text{ k}\Omega$$

$$r_e = \frac{\alpha}{g_m} = \frac{0.99}{4 \times 10^{-3}} = 247.5 \ \Omega$$

Then,

$$R_{C1} = (R_B \parallel r_\pi) + R_{sig}$$

$$= (200 \text{ K} \parallel 25 \text{ K}) + 20 \text{ K}$$

$$R_{C1} = 42.22 \text{ k}\Omega$$

$$R_E = r_e + \frac{R_B \parallel R_{sig}}{\beta + 1}$$

$$= 247.5 + \frac{(200 \text{ K} \parallel 20 \text{ K})}{101} = 427.52 \ \Omega$$

$$R_{C2} = R_C + R_L = 20 \text{ K} + 10 \text{ K} = 30 \text{ K}$$

If we choose $f_L = 100$ Hz and

$$f_{P2} = 0.9 \times f_L$$

$$C_E = \frac{1}{2\pi(0.9 \times f_L) \times R_E}$$

$$= \frac{1}{2\pi \times 90 \times 427.52} = 4.2 \ \mu\text{F}$$

Selecting C_{C1} and C_{C2} such as they contribute 5% of f_L each we have:

$$C_{C1} = \frac{1}{2\pi(0.05 \times 100) \times (42.22 \text{ K})} = 0.8 \ \mu\text{F}$$

$$C_{C2} = \frac{1}{2\pi(0.05 \times 100) \times (30 \text{ K})} = 1 \ \mu\text{F}$$

The resulting f_L is:

$$f_L = \frac{1}{2\pi}\left\{\frac{1}{4.2 \ \mu \times (427.52)}\right.$$

$$\left. + \frac{1}{0.8 \ \mu \times 42.22 \text{ K}} + \frac{1}{1 \ \mu \times 30 \text{ K}}\right\}$$

$f_L = 98.65$ Hz

The total capacitance is:

$$C_T = 0.8 \ \mu + 1 \ \mu + 4.2 \ \mu = 6 \ \mu\text{F}$$

8.8

$R_{in} = R_1 \| R_2 \| (r_x + r_\pi)$

where $R_1 = 33\,K\Omega$, $R_2 = 22\,K\Omega$

$r_x = 50$ and,

$$r_\pi = \frac{\beta_0}{g_m} = \frac{120}{0.3 \times 40} = \frac{120}{12} = 10\,K\Omega$$

$R_{in} = 33 \| 22 \| 10.05 = \underline{\underline{5.7}}\,K\Omega$

$$A_H = -\frac{R_{in}}{R_{in}+R_s} \cdot \frac{r_\pi}{r_\pi + r_x} \cdot g_m \left(R_c \| R_L \| r_o\right)$$

$$= -\frac{5.7}{5.7+5} \cdot \frac{10}{10+0.05} \cdot 12 \left(4.7 \| 5.6 \| 300\right)$$

$$= -\underline{\underline{16.11}}\ V/V$$

$R'_{sig} = r_\pi \| \left[r_x + (R_1 \| R_2 \| R_{sig})\right]$

$= 10K \| \left[50 + (33 \| 22 \| 5)K\right]$

$= \underline{\underline{2.69}}\,K\Omega$

$R'_L = r_o \| R_c \| R_L = 300 \| 4.7 \| 5.6\,(K\Omega)$

$= 2.53\,K\Omega$

$$C_\pi + C_\mu = \frac{g_m}{2\pi \cdot f_T} = \frac{12 \cdot 10^{-3}}{2\pi \times 700 \cdot 10^6} =$$

$= 2.73\,pF$

$C_\pi = (2.73 - 1)\,pF$

$= 1.73\,pF$

$C_{in} = C_\pi + C_\mu (1 + g_m R'_L)$

$= 1.73p + 1p(1 + 12 \times 2.53)$

$= 33\,pF$

$f_H = 1/2\pi C_{in} \cdot R'_{sig}$

$= 1/2\pi \times 33 \cdot 10^{-12} \times 2.69 \cdot 10^3$

$= \underline{\underline{1.79\,MHz}}$

8.9

To select C_E so that it contributes 90% of the value of f_L

$$\frac{1}{2\pi\, C_E \cdot R_E'} = 0.9 \times 100 \quad R_E' = 110.8\ \Omega$$

(From problem 9.11)

$\Rightarrow C_E = 15.9\ \mu F$

To select C_{c1} so that it contributes 5% of f_L:

$R_{C1} = 10.7\ k\Omega$

$$\Rightarrow C_1 = \frac{1}{2\pi \cdot 10.7 \times 10^3 \times 0.05 \times 100}$$

$= 2.97\ \mu F$

To select C_{C2} so that it contributes 5% of f_L: $R_{C2} = 10.3\ k\Omega$

$$\Rightarrow C_{C2} = \frac{1}{2\pi \cdot 10.3 \times 10^3 \times 0.05 \times 100}$$

$= 3.1\ \mu F$

8.10

$R_{C1} = R_S + \left[R_B \| (r_x + r_\pi)\right]$

$= 10 + \left[10 \| (0.1 + 1)\right]$

$= 10.99\,K\Omega$

$$R_E' = R_E \| \frac{r_\pi + r_x + (R_B \| R_S)}{\beta_0 + 1}$$

$$\simeq 1 \| \frac{1 + 0.1 + (10 \| 10)}{100 + 1}$$

$\simeq 57\,\Omega$

For C_E and C_{C1} to contribute equally to the determination of f_L,

$C_E R_E' = C_{C1} \cdot R_{C1}$

$$\Longrightarrow \frac{C_E}{C_{C1}} = \frac{R_{C1}}{R_E'} = \frac{10.99}{0.057} = \underline{\underline{193}}$$

8.11

a) $I_b = \frac{V_S}{R_S + r_\pi}$ $\quad I_C = \beta \cdot I_b = \frac{\beta \cdot V_S}{R_S + r_\pi}$

$$V_O = -I_C\,(R_C \parallel R_L) = -\beta\,\frac{(R_C \parallel R_L)}{R_S + r_\pi} \cdot V_S$$

$$A_M = \frac{V_O}{V_S} = -\beta\frac{(R_C \parallel R_L)}{R_S + r_\pi}$$

b) Pole due to C_E: $\omega_{PE} = \frac{1}{C_E\left(r_e + \frac{R_S}{\beta + 1}\right)}$

Pole due to C_C: $\omega_{PC} = \frac{1}{C_C(R_C + R_L)}$

zeros are both at $s = 0$

c) $A(s) = A_M \cdot \frac{s^2}{(s + \omega_{PE})(s + \omega_{PC})}$

$$A(s) = \frac{-\beta\,(R_C \parallel R_L)}{R_S + r_\pi}$$

$$\frac{s^2}{\left[s + \frac{1}{C_E\left(R_E + \frac{R_S}{\beta + 1}\right)}\right] \times \left[s + \frac{1}{C_C(R_C + R_L)}\right]}$$

d) $A_M = \frac{-100(10 \parallel 10)}{10 + \frac{100}{40}} = -40$ V/V

e) Since the resistance that forms the pole ω_{PE} is very small, we choose to make ω_{PE} the dominant pole, thus :

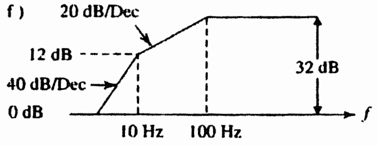
$$f_{PE} = f_L = 100 = \frac{1}{2\pi\, C_E\left(25 + \frac{10\text{ K}}{101}\right)}$$

$$\Rightarrow C_E = \frac{1}{2\pi \cdot 100 \cdot (0.025 + 0.100) \times 10^3}$$
$$= 12.7\ \mu\text{F}$$

$$f_{PC} = 10\text{ Hz} \Rightarrow 10\text{ Hz} = \frac{1}{2\pi\, C_C\,(R_L + R_C)}$$

$$\Rightarrow C_C = \frac{1}{2\pi \times 10(10 + 10) \cdot 10^3} = 0.8\ \mu\text{F}$$

f)



Unity-gain frequency must be an octave lower than 10 Hz i.e. at 5 Hz

g) $A(j\omega) = -A_M \cdot \frac{\omega^2}{(\omega_{PE} + j\omega)(\omega_{PC} + j\omega)}$

$$= +40 \cdot \frac{\omega^2}{(\omega_{PE} + j\omega)(\omega_{PC} + j\omega)}$$

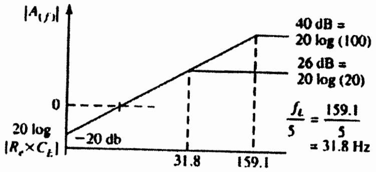
Thus $\phi = \tan^{-1}\left(\frac{\omega}{\omega_{PE}}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{PC}}\right)$

$$= -\left[\tan^{-1}\frac{f}{f_{PE}} + \tan^{-1}\frac{f}{f_{PC}}\right]$$

$$= -\left[\tan^{-1}\frac{f}{100} + \tan^{-1}\frac{f}{10}\right]$$

Thus at $f = 100$ Hz $\quad \phi = -[\tan^{-1}1 + \tan^{-1}10]$

$\simeq 129.3°$



8.12

(a) $I_e = \frac{V_S}{r_e + R_e + \frac{1}{sC_e}}$

$I_c \simeq I_e$

$V_o = -R_c I_c = \frac{-R_c}{r_e + R_e + \frac{1}{sC_E}} \cdot V_s$

$A(s) \equiv \frac{V_o}{V_s} = \frac{-R_c}{r_e + R_e + \frac{1}{sC_E}}$

$= \frac{-R_c}{r_e + R_e} \cdot \frac{s}{s + \frac{1}{C_E(r_e + R_e)}}$

Thus, $A_M = \frac{-R_c}{r_e + R_e}$

$\omega_L = \frac{1}{C_e \cdot (r_e + R_e)}$

(b) A_M is reduced by the factor $\frac{r_e + R_e}{r_e}$

$$= 1 + \frac{R_e}{r_e}$$

(c) ω_L is reduced by the factor $\left(1 + \frac{R_e}{r_e}\right)$ which is the same as the gain reduction factor. Thus, the value of R_e can be used as the parameter for exercising the gain-bandwidth trade off.

(d) $R_e = 0$:

$$|A_M| = \frac{R_C}{r_e} = \frac{10{,}000}{25} = \underline{400} \text{ V/V}$$

$$f_L = \frac{1}{2\pi C_E r_e} = \frac{1}{2\pi \times 100 \times 10^{-6} \times 25}$$

$$= \underline{63.7} \text{ Hz}$$

To lower f_L by a factor of 5 use:

$R_e = 4r_e = \underline{100\,\Omega}$. The gain is also lowered by a factor of 5 to $\underline{80}$ V/V

8.13

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}}{8 \times 10^{-9}}$$

$$= 4.3 \times 10^{-3} \text{ F/m}^2 = 4.3 \text{ fF}/\mu\text{m}^2$$

$$k'_n = \mu_n C_{ox} = 450 \times 10^{-4} \times 4.3 \times 10^{-3}$$

$$= 193.5\ \mu\text{A}/\text{V}^2$$

$$I_D = 100\ \mu\text{A} = \frac{1}{2} \times 193.5 \times \frac{20}{1} V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.23 \text{ V}$$

$$V_{DS} = 1.5 \text{ V} > V_{OV} \Rightarrow \text{Saturation}$$

$$g_m = \frac{2I_D}{V_{OV}} = 880\ \mu\text{A/V},$$

$$r_O = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 0.1} = 200 \text{ k}\Omega$$

$$X = \frac{r}{2\sqrt{2\mu_f + V_{SB}}} = \frac{0.5}{2\sqrt{0.65 + 1}} = 0.19$$

$$g_m b = X g_m = 167.2\ \mu\text{A/V}$$

$$C_{OV} = W L_{OV} C_{ox} = 20 \times 0.05 \times 4.3$$

$$= 4.3 \text{ fF}$$

$$C_{gs} = \frac{2}{3} W L C_{ox} + C_{OV}$$

$$= \frac{2}{3} \times 20 \times 1 \times 4.3 + 4.3 = 61.6 \text{ fF}$$

$$C_{gd} = C_{OV} = 4.3 \text{ fF}$$

$$C_{sb} = \frac{C_{sbo}}{\sqrt{1 + \frac{V_{SB}}{V_O}}} = \frac{15}{\sqrt{1 + \frac{1}{0.7}}} = 9.6 \text{ fF}$$

$$C_{db} = \frac{C_{dbo}}{\sqrt{1 + \frac{V_{db}}{V_O}}} = \frac{15}{\sqrt{1 + \frac{(1 + 1.5)}{0.7}}} = 7 \text{ fF}$$

8.14

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.25} = 0.8 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{0.8 \times 10^{-3}}{2\pi(20 + 5) \times 10^{-15}} = 5.1 \text{ GHz}$$

8.15

$$f_T = \frac{g_m}{2\pi\,(C_{gs} + C_{gd})}$$

$$g_m = \sqrt{2 \cdot \mu_n \cdot C_{ox}\frac{W}{L} \cdot I_D}$$

Also $C_{gs} \simeq \frac{2}{3}WL \cdot C_{ox}$, if $C_{gs} >> C_{gd}$ then we can ignore C_{gd}. If we replace for g_m and C_{gs} then we have :

$$f_T = \frac{\sqrt{2\mu_n \cdot C_{ox}(W/L)I_D}}{2 \cdot \pi \cdot \frac{2}{3}W \cdot L \cdot C_{ox}}$$

$$= \frac{1.5}{\pi \cdot L}\sqrt{\frac{\mu_n \cdot I_D}{2C_{ox} \cdot WL}}$$

Therefore we can see that the higher the current I_D then the higher is f_T. Also the frequency is inversely proportional to the size of the device, i.e. higher frequencies are achievable for smaller devices.

8.16

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \qquad ①$$

For $C_{gs} >> C_{gd}$ and the over lap capacitance of C_{gs} negligibly small : $C_{gs} \simeq \frac{2}{3}WLC_{ox}$

Also $g_m = \frac{2I_D}{V_{OV}} = k_n'\frac{W}{L}V_{OV}$

If we substitude g_m and C_{gs} in ① from the above formulas : $f_T = k_n'\frac{W}{L}V_{OV}\frac{1}{2\pi \times \frac{2}{3}WLC_{ox}}$

$$\Rightarrow f_T = \frac{3\mu_n V_{OV}}{4\pi L^2}$$

Therefore, for a given device f_T is proportional to $V_{OV} \cdot f_T \propto V_{OV}$

For $L = 1\ \mu\text{m}$, $V_{OV} = 0.25$:

$$f_T = \frac{3 \times 450 \times 10^{-4} \times 0.25}{4 \times \pi \times 1 \times 10^{-12}} = 2.7\ \text{GHz}$$

For $V_{OV} = 0.5\ \text{V}$: $\frac{f_{T1}}{f_{T2}} = \frac{V_{OV1}}{V_{OV2}}$

$$\Rightarrow f_{T2} = 2.7 \times \frac{0.5}{0.25}$$

$f_T = 5.4$ GHz

8.17

The intrinsic gain A_O is

$$= g_m \cdot r_O = \left(\frac{2I_D}{V_{OV}}\right) \cdot \left(\frac{V_A}{I_D}\right) = \frac{2V_A}{V_{OV}} \text{ and}$$

$$V_A = V_A' \cdot L = 5\ \text{V}/\mu\text{m} \times L$$

$$A_O = \frac{2 \times 5[\text{V}/\mu\text{m}] \cdot L}{0.2[\text{V}]} = 50 \times L\ \text{V/V} \text{ with}$$

L in mm.

* From problem 9.19

$$f_T = \frac{3\mu_n V_{OV}}{4\pi \cdot L^2} = \frac{3 \times 450\left[\frac{\text{cm}^2}{\text{V} \cdot \text{S}}\right] \cdot 0.2[\text{V}]}{4\pi L^2}$$

$$= \frac{2.15 \times 10^{-3}}{L^2}$$

$$L_{min} = 0.18 \times 10^{-6}\ \text{m}$$

	$1\ L_{min}$	$2\ L_{min}$	$3\ L_{min}$	$4\ L_{min}$	$5\ L_{min}$
A_O [V/V]	9	18	27	36	45
f_T[GHz]	66.35	16.59	7.37	4.14	2.65

8.18

$$f_T = \frac{g_m}{2\pi\,(C_\pi + C_\mu)}$$

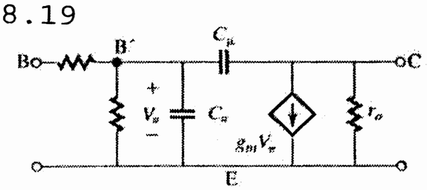
$$= \frac{80 \times 10^{-3}}{2\pi\,(10 + 1) \times 10^{-12}}$$

$$= \underline{\underline{4.24\,\text{GHz}}}$$

$$f_\beta = f_T / \beta_0 = (4.24/150) \times 10^9$$

$$= \underline{\underline{28.26\,\text{MHz}}}$$

8.19



$r_x = 100\ \Omega$

$g_m = \dfrac{I_C}{V_T} = \dfrac{0.5\text{ mA}}{25\text{ mV}} = 20\text{ mA/V}$

$r_\pi = \dfrac{\beta_O}{g_m} = \dfrac{100}{20} = 5\text{ k}\Omega$

$r_O = \dfrac{V_A}{I_C} = \dfrac{50\text{ V}}{0.5\text{ mA}} = 100\text{ k}\Omega$

$C_\mu = \dfrac{C_{\mu o}}{\left(1 + \dfrac{V_{cB}}{V_{OC}}\right)^{0.5}} = \dfrac{30}{\left(1 + \dfrac{2}{0.75}\right)^{0.5}}$

$= 15.7\text{ fF}$

$C_\mu \simeq 2C_{\mu o} = 2 \times 20 = 40\text{ fF}$

$C_{de} = \tau_F\, g_m = 30 \times 10^{-12} \times 20 \times 10^{-3}$

$= 600\text{ fF}$

$C_\pi = C_{je} + C_{de} = 0.640\text{ pF}$

$f_T = \dfrac{g_m}{2\pi(C_\pi + C_\mu)}$

$= \dfrac{20 \times 10^{-3}}{2\pi(0.64 + 0.016) \times 10^{-12}} = 4.85\text{ GHz}$

8.20

$|h_{fe}| \simeq f_T / f$

- At $I_c = 0.2\,mA$, $|h_{fe}| = 2.5$ at $f = 500\,MHz$, thus:

$f_T = 2.5 \times 500 = \underline{\underline{1.25}}\ GHz$

- At $I_c = 1.0\,mA$, $|h_{fe}| = 11.6$ at $f = 500\,MHz$, thus:

$f_T = 11.6 \times 500 = \underline{\underline{5.8}}\ GHz$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} \rightarrow C_\pi = \frac{g_m}{2\pi f_T} - C_\mu$$

$$C_\pi\ (I_c = 0.2\,mA) = \frac{8 \times 10^{-3}}{2\pi \times 1.25 \times 10^9} - 0.05 \cdot 10^{-12}$$

$$= 0.9686\ pF$$

$$C_\pi\ (I_c = 40\,mA) = \frac{40 \times 10^{-3}}{2\pi \times 5.8 \times 10^9} - 0.05 \times 10^{-12}$$

$$= 1.0476\ pF$$

Since $C_\pi = C_{je} + \tau_F g_m$,

$C_{je} + 8 \times 10^{-3} \tau_F = 0.9686 \times 10^{-12}$ (1)

$C_{je} + 40 \times 10^{-3} \tau_F = 1.0476 \times 10^{-12}$ (2)

Solving Eqn. (1) and (2) together yields,

$C_{je} = \underline{\underline{0.95}}\,pF$, $\tau_F = \underline{\underline{247\,ps}}$

8.21

$\omega_T = g_m / (C_\pi + C_\mu)$

$$2\pi \times 5 \times 10^9 = \frac{20 \times 10^{-3}}{(C_\pi + 0.1) \times 10^{-12}}$$

$$C_\pi + 0.1 = \frac{20}{10\pi} = 0.64\,pF$$

$C_\pi = \underline{\underline{0.54}}\ pF$

$g_m = \underline{\underline{20\,mA/V}}$

$r_\pi = \beta / g_m = 150/20 = \underline{\underline{7.5\,K\Omega}}$

$$f_\beta = f_T / \beta = \frac{5 \times 10^9}{150} = \underline{\underline{33.3\,MHz}}$$

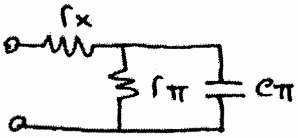
8.22

$|h_{fe}|$ becomes 20 at:

$$f_T/20 = \frac{1\times10^9}{20} = \underline{\underline{50}}\text{ MHz}$$

$$f_\beta = f_T/\beta_0 = \frac{1000\text{MHz}}{200} = \underline{\underline{5\text{MHz}}}$$

8.23



$$Z = r_x + \frac{1}{\frac{1}{r_\pi} + j\omega C_\pi}$$

$$= r_x + \frac{r_\pi}{1 + j\omega C_\pi r_\pi}$$

$$Z = r_x + \frac{r_\pi}{1 + j(\omega/\omega_\beta)}$$

$$= r_x + \frac{r_\pi\,(1 - j\,\omega/\omega_\beta)}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

$$= r_x + \frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} - j\cdot\frac{r_\pi\,(\omega/\omega_\beta)}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

$$\mathrm{Re}[Z] = r_x + \frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2}$$

For Re[Z] to be an estimate of r_x good to within 10% we must keep

$$\frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} \leq \frac{r_x}{10}$$

But $r_x \leq r_\pi/10$

Thus,

$$\frac{r_\pi}{1 + \left(\frac{\omega}{\omega_\beta}\right)^2} \leq \frac{r_\pi}{100}$$

$$1 + \left(\frac{\omega}{\omega_\beta}\right)^2 \geq 100$$

or $\omega \geq 10\,\omega_\beta$ (approx.)

8.24

	I_E (mA)	r_e (Ω)	g_m (mA/V)	r_π (kΩ)	β_0
(a)	1	25	40	2.5	100
(b)	1	25	40	3.13	125.3
(c)	0.99	25.3	39.6	2.525	100
(d)	10	2.5	400	0.25	100
(e)	0.1	250	4	25	100
(f)	1.0	25	40	0.25	10
(g)	1.25	20	50	0.20	10

CONT.	f_T (MHz)	C_π (pF)	C_μ (pF)	f_β (MHz)
(a)	400	2	13.9	4
(b)	501.3	2	10.7	4
(c)	400	2	13.8	4
(d)	400	2	157	4
(e)	100	2	4.4	1
(f)	400	2	13.9	40
(g)	800	1	9	80

8.25

$$C_{in} = C_{gs} + C_{eq}$$

$$= C_{gs} + C_{gd}\,(1 + g_m R_L')$$

$$= 0.5 + 0.1(1 + 29) = 3.5\text{ pF}$$

Neglecting R_G :

$$f_H = \frac{1}{2\pi \cdot C_{in} \cdot R_{sig}} \quad \text{i.e } R_G \text{ is very large if}$$

$$f_H > 10\text{ MHz} \Rightarrow \frac{1}{2\pi \cdot 3.5 \times 10^{-12} \times 10^6} > R_{sig}$$

$$\Rightarrow R_{sig} < 4.55\text{ k}\Omega$$

8.26

Since R_G is very large:

$$f_H = \frac{1}{2\pi \cdot C_{in} \cdot R_{sig}}$$

if $f_H \geqslant 10\,\text{MHz} \Rightarrow 10 \times 10^6 \leq \dfrac{1}{2\pi \cdot C_{in} \cdot R_{sig}}$

$$C_{in} \leq \frac{1}{2\pi \times 10 \times 10^6 \times 1 \times 10^3}$$

$C_{in} \leq 15.91$ pF

$\Rightarrow C_{gs} + C_{gd}(1 + g_m R_L') \leq 15.91$ pF

$5 \times 10^{-12} + 1 \times 10^{-12}(1 + 5 \times 10^{-3} \cdot R_L')$

$\leqslant 15.91$ pF $\Rightarrow R_L' \leq 1982\ \Omega$

Since R_G is very large: $A_M = -g_m \cdot R_L'$

$A_M \geq -5 \times 10^{-3} \cdot 1982 \Rightarrow A_M \geq -9.91$ V/V

Gain - bandwidth product : GB $\equiv |A_M| \cdot$ BW

GB $\geq 9.1 \times 10 \times 10^6$ GB $\geqslant 91$ MHz

If $f_H \geq \dfrac{10}{3}$ MHz

then : $R_L' \leq 8349\ \Omega$

$A_M \geqslant -47.75$ V/V

GB $\geqslant 139.2$ MHz

8.27

$g_m = 1\ \dfrac{\text{mA}}{\text{V}}$; $C_{gs} = 1$ pF; $C_{gd} = 0.4$ pF;

$C_{in} = 4.26$ pF; $A_M = -7\dfrac{\text{V}}{\text{V}}$; $f_H = 382$ KHz

$\Rightarrow$ GB $= 7 \times 382 \cdot 10^3 = 2.67$ MHz

We also Know that:

$$C_{gs} = \frac{2}{3} W \cdot C_{OX} + WL_{OV} \cdot C_{OX}$$

$$= WC_{OX}\left(\frac{2}{3} + L_{OV}\right)$$

$C_{gd} = WL_{OV} C_{ox}$

$\Rightarrow$ if W is reduced by half so are C_{gs} and C_{gd}.

$\Rightarrow C_{gs2} = 0.5$ pF $\quad C_{gd2} = 0.2$ pF

In saturation:

$I_D = \dfrac{1}{2} k_n' \dfrac{W}{L}(V_{OV})^2 \Rightarrow$ For I_D to remain unchanged while ω is halved

$\Rightarrow (V_{OV})^2$ is doubled

thus $V_{OV2} = \sqrt{2}\ V_{OV}$

but $g_m = \dfrac{2I_D}{V_{OV}} = g_{m2} = \dfrac{1}{\sqrt{2}} g_m$

$= 0.707$ mA / V

we can now calculate the new values of A_M, C_{in}, f_H and GB

$$A_{M2} = \frac{-R_G}{R_G + R_{sig}} \cdot g_m R_L' = -\frac{7}{\sqrt{2}} \cdot \text{V / V}$$

$= -4.9$ V / V

$C_{eq2} = (1 + 0.707 + 7.14) \cdot 0.2 \times 10^{-12}$

$= 1.21$ pF

$\Rightarrow C_{in2} = 0.5 + 1.21 = 1.71$ pF

$$f_{H2} = \frac{1}{2\pi \cdot 1.71 \times 10^{-12} \cdot (0.1 \parallel 4.7) \times 10^6}$$

$= 950$ KHz

$GB_2 = 4.9 \times 950 \cdot 10^3 = 4.65$ MHz

The ratios of new vs old values are:

$\dfrac{W_2}{W}$	$\dfrac{V_{OV2}}{V_{OV}}$	$\dfrac{g_{m2}}{g_m}$	$\dfrac{C_{gs2}}{C_{gs}}$	$\dfrac{C_{gd2}}{C_{gd}}$	$\dfrac{C_{in2}}{C_{in}}$	$\dfrac{A_{M2}}{A_M}$
$\dfrac{1}{2}$	$\sqrt{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{2}$	$\dfrac{1}{2}$	0.4	0.7

$\dfrac{f_{H2}}{f_H}$	$\dfrac{GB_2}{GB}$
2.49	1.74

8.28

$R_{sig} = 100^{K\Omega}$, $R_{in} = 100K\Omega$, $C_{gs} = 1pF$, $C_{gd} = 0.2pF$

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m (r_o \| R_D \| R_L)$$

Also $R_{in} = 100K\Omega = R_G$

$$A_M = \frac{-100}{100+100} 3(50^k \| 8K \| 10^k) = \underline{-6.1 V/_V}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} \quad \text{(Eq. 4.132)}$$

$R'_{sig} = R_{sig} \| R_G = 100 \| 100 = 50^{K\Omega}$

$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$R'_L = r_o \| R_D \| R_L = 4.1^{K\Omega}$

$C_{in} = 1 + 0.2(1 + 3 \times 4.1) = 3.66pF$

Now we can calculate f_H:

$$f_H = \frac{1}{2\pi \times 3.66 \times 10^{-12} \times 50 \times 10^3} = \underline{870^{KHz}}$$

In order to double f_H, we have to either decrease C_{in} (by reducing R_{out}) or reduce R'_{sig} by reducing R_{in}.

If we reduce $R_{out} = R_D \| r_o$:

$$\frac{f_{H2}}{f_{H1}} = \frac{C_{in1}}{C_{in2}} \Rightarrow 2 = \frac{3.66\,pF}{1 + 0.2(1 + 3 \times R'_L)}$$

$\Rightarrow R'_L = 1.27K\Omega \quad R'_L = R_{out} \| R_L = R_{out} \| 10^k$

$\Rightarrow \underline{R_{out} = 1.45K\Omega}$

Therefore in order to double f_H to $870 \times 2 = 1.74^{MHz}$, we have to reduce $R_{out} = r_o \| R_D$ to $1.45^{K\Omega}$ or equivalently reducing R_D to $1.5^{K\Omega}$. The new midband gain would be:

$$\frac{A_{M2}}{A_{M1}} = \frac{R'_{L2}}{R'_{L1}} \Rightarrow A_{M2} = -6.1 \times \frac{1.27}{4.1} = \underline{-1.9 V/_V}$$

Gain is almost reduced by a factor of $\underline{3}$.

If we reduce $R_{in} = R_G$:

$$\frac{f_{H2}}{f_{H1}} = \frac{R'_{sig1}}{R'_{sig2}} \Rightarrow 2 = \frac{50^K}{R'_{sig2}} \Rightarrow R'_{sig2} = 25^{K\Omega}$$

$\Rightarrow 25K\Omega = 100K \| R_G \Rightarrow \underline{R_G = 33^{K\Omega} = R_{in}}$

Therefore in order to double f_H, R_{in} is reduced by a factor of 3, from $100k\Omega$ to $33K\Omega$.

The new midband gain would be:

$$\frac{A_{M2}}{A_{M1}} = \frac{R_{G2}}{R_{G1}} \frac{R_{G1} + R_{sig}}{R_{G2} + R_{sig}} \Rightarrow A_{M2} = -6.1 \times \frac{1}{3} \times \frac{100+100}{33+100}$$

$\underline{A_{M2} = -3.06 V/_V}$

Gain is almost reduced by a factor of $\underline{2}$.

8.29

$R_{in} = 2M\Omega$, $g_m = 4mA/_V$, $r_o = 100^{K\Omega}$, $R_D = 10K\Omega$

$C_{gs} = 2PF$, $C_{gd} = 0.5pF$, $R_{sig} = 500K\Omega$, $R_L = 10K\Omega$

noting that $R_G = R_{in}$, we have:

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m (r_o \| R_D \| R_L) = -\frac{2 \times 4}{2 + 0.5}(100^K \| 10^K \| 10^K)$$

$\underline{A_M = -15.2 V/_V}$

$$\therefore f_H = \frac{1}{2\pi C_{in} R'_{sig}} \quad \text{where}$$

$C_{in} = C_{gs} + C_{gd}(1 + g_m(r_o \| R_D \| R_L))$, $R'_{sig} = R_{sig} \| R_G$

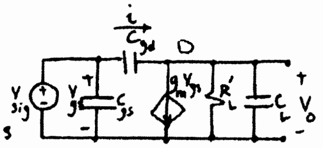
$C_{in} = 2 + 0.5(1 + 4 \times (100^K \| 10^K \| 10^K)) = \underline{12.02pF}$

$R'_{sig} = 0.5^M \| 2^{M\Omega} = 0.4M\Omega = 400k\Omega$

$$f_H = \frac{1}{2\pi \times 12.02 \times 10^{-12} \times 400 \times 10^3} = \underline{33.1^{KHz}}$$

8.30

If we write KCL at node D:



$$i = g_m V_{gs} + \frac{V_o}{R'_L} + V_o C_L s$$

then: $V_{sig} = i \times \frac{1}{C_{gd}s} + v_o$

$$V_{sig} = \left(g_m V_{gs} + \frac{V_o}{R'_L} + V_o C_L s\right)\frac{1}{C_{gd}s} + v_o\,, \quad V_{sig} = V_{gs}$$

$$v_{sig}\left(1 - \frac{g_m}{C_{gd}s}\right) = v_o\left(1 + \frac{1}{R'_L C_{gd}s} + \frac{C_L s}{C_{gd}s}\right)$$

$$\frac{v_o}{v_{sig}} = -g_m R'_L \frac{(1 - s(C_{gd}/g_m))}{R'_L C_{gd}s + 1 + R'_L C_L s}$$

$$\frac{v_o}{v_{sig}} = -g_m R'_L \frac{1 - s\, C_{gd}/g_m}{1 + s(C_L + C_{gd})R'_L}$$

$$\text{If } (g_m/C_{gd}) \gg \omega \Rightarrow \frac{v_o}{v_{sig}} = \frac{-g_m R'_L}{1 + s(C_L + C_{gd})R'_L}$$

For $C_{gd} = 0.5\,\text{pF}$, $C_L = 2\,\text{pF}$, $g_m = 4\,\text{mA/V}$, $R'_L = 5\,\text{k}\Omega$

$$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + s/\omega_H} \Rightarrow \begin{cases} A_M = -g_m R'_L = -4 \times 5 = -20\,\text{V/V} \\ f_H = \dfrac{1}{2\pi(C_L + C_{gd})R'_L} \end{cases}$$

$$\Rightarrow f_H = \frac{10^{12}}{2\pi \times (2 + 0.5) \times 5 \times 10^3}$$

$$\underline{f_H = 12.7\,\text{MHz}} \qquad g_m/C_{gd} = \frac{4}{0.5} = 8^{\text{GRad/s}} \gg \omega_H$$

8.31

If $g_m = 1\,\frac{\text{mA}}{\text{V}}$ and $r_o = 100\,\text{k}\Omega$:

$$A_M = \frac{R_G}{R_G + R_{sig}} g_m (r_o \parallel R_D \parallel R_L) \text{ where}$$

$R_G = 10\,\text{M} \parallel 47\,\text{M}$

$R_G = 8.25\,\text{M}\Omega$

$$A_M = -\frac{8.25}{8.25 + 0.1} 1(100\,\text{K} \parallel 4.7\,\text{K} \parallel 10\,\text{K})$$

$= -3.06\,\text{V/V}$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} \text{ where}$$

$R'_{sig} = R_{sig} \parallel R_G = 0.1\,\text{M} \parallel 8.25\,\text{M}\Omega$

$R'_{sig} \cong 0.1\,\text{M}\Omega$

$C_{in} = C_{gs} + C_{gd}(1 + g_m (r_o \parallel R_D \parallel R_L))$

$C_{in} = 1 + 0.2(1 + 1(100\,\text{K} \parallel 4.7\,\text{K} \parallel 10\,\text{K}))$

$= 1.82\,\text{pF}$

$$f_H = \frac{1}{2\pi \times 1.82 \times 10^{-12} \times 0.1 \times 10^6}$$

$= 875\,\text{KHz}$

8.32

$I = 2\,\text{mA}, \beta = 100, f_T = 800\,\text{MHz}$

$R_B = 50\,\text{k}\Omega, R_C = 4\,\text{k}\Omega, r_x = 50\,\Omega$

$V_A = 100, C_\mu = 1\,\text{pF}, R_{sig} = 5\,\text{k}\Omega$

$R_L = 5\,\text{k}\Omega$

$$g_m = \frac{2\,\text{mA}}{25\,\text{mV}} = 80\,\frac{\text{mA}}{\text{V}}$$

$$r_\pi = \frac{\beta_o}{g_m} = \frac{100}{80\,\text{m}} = 1250\,\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100\,\text{V}}{2\,\text{mA}} = 50\,\text{k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{80 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 16\,\text{pF}$$

$C_\mu = 1\,\text{pF} \Rightarrow C_\pi = 15\,\text{pF}$

$$A_M = \frac{-R_B}{R_B + R_{sig}} \cdot \frac{r_\pi \times g_m R_L'}{(r_\pi + r_x + (R_B \parallel R_{sig}))}$$

where $R'_L = r_o \parallel R_C \parallel R_L$

$= (50 \parallel 4 \parallel 5)\,\text{k}\Omega$

$= 2.1\,\text{k}\Omega$

$$A_M = -\frac{50}{50 + 5} \cdot \frac{1250 \times 168}{1250 + 50 + (50 \parallel 5)\,\text{k}\Omega}$$

where $168 = g_m \times R'_L$

$= 80 \times 10^{-3} \times 2.1(10^3)$

Then: $A_M = -32.6$

$20 \log|A_M| = 30.3\,\text{dB}$

$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 15 + 1(1 + 168) = 184\,\text{pF}$

$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$

$= 1250 \parallel [50 + (50\,\text{K} \parallel 5\,\text{K})] = 983\,\Omega$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} = \frac{1}{2\pi \times 184 \times 10^{-12} \times 983}$$

$= 880\,\text{KHz}$

Gain-bandwidth product

$GB = |A_M| \times f_H = 32.6 \times 880 \times 10^3$

$= 29 \times 10^6$

Previously,

$GB = 39 \times 754 \times 10^3 = 29 \times 10^6$

Thus, the designer traded gain for bandwidth by increasing I. However, by doubling I the dissipation increased by a factor of 2, since:

$$\text{Power} = \frac{I}{2I'} \times \text{Vsupply}$$

8.33

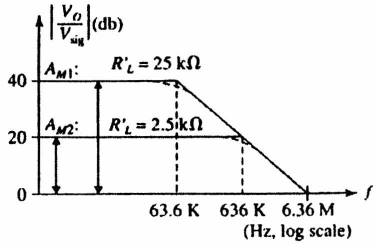
$R_B >> R_{sig},\ r_X << R_{sig}$

$R_{sig} >> r_\pi,\ g_m R_L^{'} >> 1,$

$g_m R_L^{'} C_\mu >> C\pi$

$$f_H = \frac{1}{2\pi \times 10^{-12} \times 100 \times 2.5(10^3)}$$

$f_H = 636$ KHz



$GP = 6.36 \times 10^6 = A_M \times f_H$

when $A_M = 1 \Rightarrow f_H = 6.36 \cdot 10^6$ Hz

$$R_L^{'} = \frac{1}{2\pi(6.36 \times 10^6) C_\mu \times \beta}$$

$C_\mu = 1 \times 10^{-12}$

$\beta = 100$

$R_L^{'} = 250\ \Omega$

8.34

$R_{in} = R_1 \parallel R_2 \parallel r_\pi$

where $r_\pi = \frac{\beta}{g_m}$ and $g_m = \frac{I_C}{V_T}$

$g_m = \frac{0.8}{0.025} = 32$ mA / V

$r_\pi = \frac{200}{32} = 6.25$ kΩ

$R_{in} = 68 \parallel 27 \parallel 6.25 = 4.72$ kΩ

$R_L^{'} = R_C \parallel R_L = 4.7 \parallel 10 = 3.2$ kΩ

$$A_M = \frac{R_{in}}{R_S + R_{in}} \times -g_m R_L^{'}$$

$$= \frac{-4.72}{10 + 4.72} \times 32 \times 3.2$$

$= -32.8$ V / V

$C_T = C\pi + C\mu\,(1 + g_m R_L^{'})$

where $C\pi + C\mu = \frac{g_m}{2\pi f_T} = \frac{32 \times 10^{-3}}{2\pi \times 10^9}$

$= 5.1$ pF

$C\pi = 5.1 - 0.8 = 4.3$ pF

$C_T = 4.3 + 0.8\,(1 + 32 \times 3.2)$

$= 87$ pF

The resistance seen by C_T is R_T.

$R_T = r_\pi \parallel R_1 \parallel R_2 \parallel R_3$

$= 6.25 \parallel 68 \parallel 27 \parallel 10 = 3.2$ kΩ

Thus

$$f_H = \frac{1}{2\pi C_T R_T}$$

$$= \frac{1}{2\pi \times 87 \times 10^{-12} \times 3.2 \times 10^3}$$

$= 572$ KHz

8.35

$I_C \simeq 2$ mA, $f_T = 2$ GHz, $C\mu = 1$ pF,

$r_x = 100\ \Omega,\ \beta_O = 120\ R_{sig} \simeq 0$

$g_m = \frac{I_C}{V_T} = \frac{2}{25} = 80\ \frac{\text{mA}}{\text{V}}$

$r_\pi = \frac{\beta_O}{g_m} = \frac{120}{0.08} = 1.5$ kΩ

$$f_\pi = \frac{g_m}{2\pi(C_\pi + C\mu)} \Rightarrow 2 \times 10^9$$

$$= \frac{0.08 \times 10^{12}}{2\pi(C_\pi + 1)} \Rightarrow C_\pi = 5.4 \text{ pF}$$

a) If $A_M = -10$ V / V

If $R_{sig} \simeq 0$: $A_M = \frac{-r_\pi}{r_\pi + r_x} \cdot g_m \cdot R_L^{'}$

$-10 = \frac{-1.5}{1.5 + 0.1} \cdot 0.08 \times R_L^{'} = 133.3\ \Omega.$

$C_{in} = C_\pi + C\mu(1 + g_m R_L^{'})$

$= 5.4 + 1(1 + 0.08 \times 133.3)$

$= 17$ pF

Thus $f_H = \frac{1}{2\pi \cdot C_{in}\,(R_L^{'})}$

$$= \frac{1}{2\pi \cdot 17 \times 10^{-12} \times 133.3} = 70.2 \text{ MHz}$$

b) $A_M = (-1$ V / V)

$\Rightarrow R_L^{'} = 13.3\ \Omega$

$C_{in} = 7.5$ pF

$f_H = 1.6$ GHz

$g_m = \frac{0.5 \text{ mA}}{25 \text{ mV}} = 20$ mA / V

$r_\pi = \frac{\beta_O}{g_m} = \frac{100}{20 \text{ mA / V}} = 5$ kΩ

8.36 $\frac{V_A}{I_C} = \frac{100 \text{ V}}{0.5 \text{ mA}} = 200$ kΩ

$$C\pi + C\mu = \frac{g_m}{\omega_T} = \frac{20 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 4 \text{ pF}$$

Since $C\mu = 1\text{ pF} \rightarrow C\pi = 3\text{ pF}$

The midband voltage gain is:

$$A_M = \frac{-R_B}{R_B + R_{\text{sig}}} \cdot$$

$$\frac{r_\pi}{r_\pi + r_x + (R_B \| R_{\text{sig}})} \cdot g_m \cdot R_L'$$

with $R_L' = r_O \| R_e \| R_L$

$$= (200 \| 8 \| 5)\text{ k}\Omega = 3\text{ k}\Omega$$

Then $g_m R_L' = 20 \times 3 = 60\text{ V/V}$

$$A_M = \frac{-100}{100 + 5} \cdot \frac{5}{5 + 0.05 + (100 \| 5)} \times 60$$

$$= -29.1\text{ V/V or } 29.3\text{ dB}$$

To determine f_H:

$$C_{in} = C_\pi + C\mu\,(1 + g_m R_L')$$

$$= 4 + 1 \cdot (1 + 60) = 65\text{ pF}$$

$$R'_{\text{sig}} = r_\pi \| [r_x + (R_B \| R_{\text{sig}})]$$

$$= 5 \| [0.05 + (100 \| 5)] = 2.45\text{ k}\Omega$$

Thus, $f_H = \dfrac{1}{2\pi \cdot C_{in} \cdot R_{\text{sig}}}$

$$= \frac{1}{2\pi \times 65 \times 10^{-12} \times 2.45 \times 10^3} = 999\text{ KHz}$$

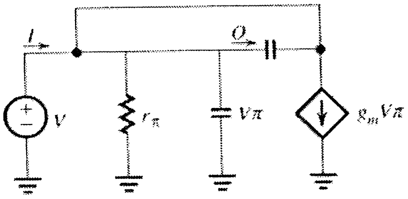
$GB = 29.1 \times 999 \times 10^3 = 29.1\text{ MHz}$

In example 9.4: $A_M = -39\text{ V/V}$,

$f_H = 754\text{ KHz} \rightarrow \text{GB} = 29.4\text{ MHz}$

Notice how operation at lower supply voltage, thus I_C reduced the mid-band gain, increased the f_H while keeping the gain-band width product constant.

8.37



$$I = \frac{V}{r_\pi} + sC\pi\,V\pi + g_m\,V\pi$$

$$Y_{in} = \left(g_m + \frac{1}{r_\pi}\right) + sC\pi$$

$$Z_i = \frac{1}{\left(g_m + \dfrac{1}{r_\pi}\right) + sC\pi}$$

$$= \frac{1}{\dfrac{1}{re} + sC\pi} = \frac{re}{1 + sC\pi r_e}$$

$$f_T = \frac{g_m}{2\pi(C\pi + C\mu)}$$

Since $C\pi$ contains a component that is proportional to the bias current, it follows that at high currents $C\pi >> C\mu$ and

$$f_T \simeq \frac{g_m}{2\pi C\pi} \simeq \frac{1}{2\pi \cdot C\pi\, r_e}$$

Thus,

$$Z_i = \frac{re}{1 + s/\omega_T} \text{ (at high currents)}$$

The phase angle will be –45° at $\omega = \omega_T$, or

$f = f_T = 400\text{ MHz}$

For a lower bias current so that $C\pi = C\mu$,

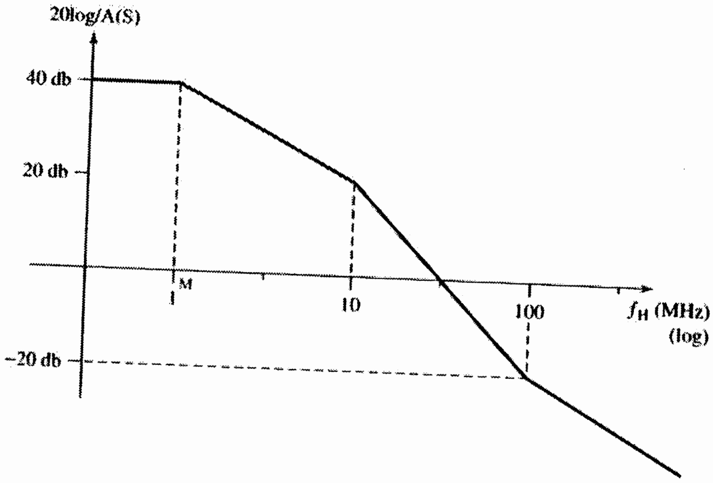
$$f_T = \frac{1}{4\pi C\pi r_e} \text{ and } Z_i = \frac{r_e}{1 + \dfrac{s}{2\omega_T}}$$

-45° angle is obtained at $\omega = 2\omega_T$ or

$f = 2f_T = 800\text{ MHz}$

(Assuming f_T remains constant which is not necessarily true)

This figure is for 8.38



8.38

$$40\text{ db} = 20\log A_O \Rightarrow A_O = 100\text{ V/V}$$

$$A(s) = +100\frac{(1 + s/100\times 10^6\times 2\pi)}{\left(1+\frac{s}{2\pi\times 10^7}\right)\left(1+\frac{s}{2\pi\times 10^6}\right)}$$

$$A(s) = +100\frac{\left(1+\frac{s}{2\pi\times 10^8}\right)}{\left(1+\frac{s}{2\pi\times 10^7}\right)\left(1+\frac{s}{2\pi\times 10^6}\right)}$$

$$\omega_H = \frac{1}{\sqrt{\left(\frac{1}{2\pi\times 10^7}\right)^2 + \left(\frac{1}{2\pi\times 10^6}\right)^2 - 2\left(\frac{1}{2\pi\times 10^8}\right)^2)}}$$

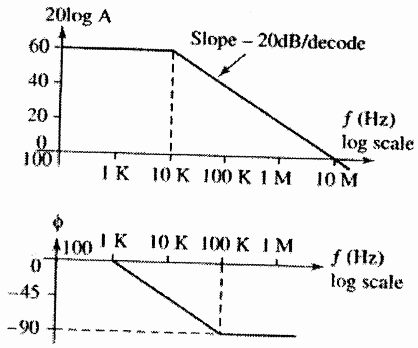
$f_H = 0.995$ MHz

8.39

a) Gain $A = 60$ dB $= 1000$

$$A(s) = \frac{1000}{\left(1+\frac{1}{2\pi\times 10\times 10^3}\right)} = \frac{1000}{\left(1+\frac{1}{2\pi\times 10^4}\right)}$$

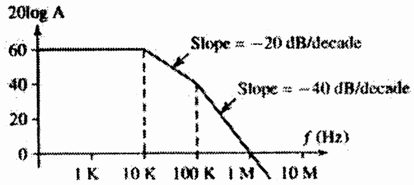
(b)



(c) Gain-bandwidth product = 1000 × 10 K
= 10 MHz

(d) From the gain plot, unity gain frequency = 10 MHz

(e) From the plot unity gain frequency = 1 MHz



8.40

$$f_H(s) = \frac{1}{\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right)}$$

$\omega_{P1} < \omega_{P2}$

using dominant-pole approximator: $\omega_H \simeq \omega_{P1}$

using the root sum of squares formula:

$$\omega_H \simeq \frac{1}{\sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2}}} = \frac{\omega_{P1}}{\sqrt{1+\left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}}$$

The difference between the two estimates for ω_H is:

$$\Delta\omega_H = \omega_{P1} - \frac{\omega_{P1}}{\sqrt{1+\left(\frac{\omega_{P1}}{\omega_{P2}}\right)^2}}$$

$$\text{If } n = \frac{\omega_{P2}}{\omega_{P1}} : \frac{\Delta\omega_H}{\omega_{P1}} = 1 - \frac{1}{\sqrt{1+\frac{1}{n^2}}}$$

$$\text{for } \frac{\Delta\omega_H}{\omega_{P1}} = 10\,\% = 0.1 \Rightarrow n = 2.07$$

$$\text{for } \frac{\Delta\omega_H}{\omega_{P1}} = 1\,\% = 0.01 \Rightarrow n = 7.02$$

8.41

$$A(s) = -100\,\frac{1+s/10^6}{(1+s/10^5)(1+s/10^7)}$$

a) $\omega_H \simeq 10^5 \text{ rad/s}$

b) $\omega_H \simeq \dfrac{1}{\sqrt{\left(\frac{1}{10^5}\right)^2 + \left(\frac{1}{10^7}\right)^2 - 2\left(\frac{1}{10^6}\right)^2}} = 101 \text{ Krad/s}$

IF the pole at 10^6 rad/s is lowered to 10^5 rad/s, the transfer function becomes:

$$A(s) = \frac{-100}{1+s/10^7} \Longrightarrow f_H = \frac{10^7}{2\pi} \text{ Hz}$$

8.42

$$30^\circ = 3\tan^{-1}\frac{\omega}{\omega_p} = 3\tan^{-1}\frac{10^6}{\omega_p} \Longrightarrow \omega_p = 5.67\times10^6 \text{ rad/s}$$

8.43

$$\omega_H \simeq \frac{1}{\tau_{gs}+\tau_{gd}} = \frac{1}{C_{gs}R_{gs}+C_{gd}R_{gd}}$$

$$\omega_H \simeq \frac{1}{C_{gs}R' + C_{gd}(R'+R_L'+g_mR_L'R')} \quad \text{(From Example 6.6)}$$

For $C_{gs} = C_{gd} = 1\text{ PF}$, $R_L' = 3.33\text{ K}\Omega$, $g_m = 4\text{ mA/V}$

$$\omega_H = \frac{1}{10^{-12}R' + 10^{-12}(R' + 3.33\times10^3 + 4\times3.33\times R')}$$

To obtain $\omega_H = 2\pi\times150\times10^3$

$$2\pi\times150\times10^3 = \frac{10^{12}}{3.33\times10^3 + 15.32R'} \Longrightarrow R' = 69.04\text{ K}\Omega$$

$$R' = R \,//\, R_{in} = R \,//\, 420^K = 69.04 \Longrightarrow \underline{R = 82.6\text{ K}\Omega}$$

8.44

$$\tau_H = \tau_{gs} + \tau_{gd}$$

where: $\tau_{gs} = C_{gs}\cdot R_{gs} = C_{gs}(R_G \| R_{sig})$

$$\tau_{gd} = C_{gd}\cdot R_{gd} = C_{gd}(R_{sig}' + R_L' + g_mR_L'\cdot R_{sig}')$$

since $R_{sig}' = R_G \| R_{sig}$

$$\tau_{gd} = C_{gd}\cdot(R_G \| R_{sig})\left[1 + \frac{R_L'}{(R_G \| R_{sig})} + g_mR_L'\right].$$

$$\Rightarrow \tau_H = (R_G \| R_{sig})\left[C_{gs} + C_{gd}\left[1 + g_mR_L' + \frac{R_L'}{R_G \| R_{sig}}\right]\right]$$

$$\tau_H = \frac{R_G\cdot R_{sig}}{R_G + R_{sig}}\cdot\left[C_{gs} + C_{gd}\left[1 + g_mR_L' + R_L'\frac{(R_G + R_{sig})}{R_G\cdot R_{sig}}\right]\right]$$

b) $\tau = C_{in} \cdot R'_{sig}$ where $R'_{sig} = R_G \| R_{sig}$ and $C_{in} = C_{gs} + C_{eq}$ where C_{eq} is the result of applying miller's thearem to reflect C_{gd} to the gate-ground nodes.

From Eq 9.76

$$Z_{eq} = \frac{Z}{1-K} = \frac{1/s\,C_{gd}}{1-(-g_m R'_L)}$$

$$= \frac{1}{sC_{gd}(1+g_m R'_L)}$$

$$\Rightarrow C_{eq} = C_{gd}(1+g_m R'_L)$$

Thus $\tau = (C_{gs} + C_{gd}(1+g_m R'_L)) \cdot \dfrac{R_G \cdot R_{sig}}{R_G + R_{sig}}$

Evalvating for: $R_G = 420\ \text{k}\Omega$

$C_{gs} = C_{gd} = 1\ \text{pF}$ $R'_L = 3.33\ \text{k}\Omega$

$R_{sig} = 100\ \text{k}\Omega$ $g_m = 4\ \text{mA/V}$

(1) For the complete expression found in part a)

$\tau_H = 1.230\ \mu\text{S} \rightarrow \omega_H = 813\ \text{K rad/s}$

(2) For the approximate expression found in part b) $\tau = 1.228\ \mu\text{S} \rightarrow \omega_H = 814\ \text{K rad/S}$

$(\tau_H - \tau) \times 100/\tau = 0.163\ \%$

8.45

If a capacitor C_L is connected in parallel with R'_L then $\omega_H \cong \dfrac{1}{\tau_{gs} + \tau_{gd} + R'_L \cdot C_L}$

the values of τ_{gs} and τ_{gd} remain unaffected since each is derived by setting the other capacitors to zero. When considering the resistances seen by C_L, $C_{gs} = C_{gd} = 0$ and $V_{gs} = 0$ ⇒ the open-circuit time constant of C_L is: $R'_L \cdot C_L$

$$\Rightarrow \omega_H = \frac{1}{(80.8 + 1160) \times 10^{-9} + 3.33 \times 10^{3} \cdot 20 \times 10^{-12}}$$

$\omega_H = 765\ \text{K rad/s}$

$f_H = \omega_H/2\pi = 121.7\ \text{MHz}$

8.46

$$A_M = \frac{V_o}{V_i} = -\frac{R_{in}}{R_{in}+R}\, g_m R'_L = -\frac{1.2}{1.2+0.1}(2\times 12)$$

$A_M = -22.2\ \text{V/V}$

$R_{gs} = R_{in} \| R = 1.2^{M} \| 0.1^{M} = 92.3\ \text{k}\Omega$

$\tau_{gs} = R_{gs} C_{gs} = 1\times 10^{-12} \times 92.3 \times 10^{3} = 92.3\ \text{ns}$

$R_{gd} = R' + R'_L + g_m R'_L R'$ where $R' = R_{in} \| R = 92.3^{\text{k}\Omega}$

$R_{gd} = 92.3 + 12 + 2 \times 12 \times 92.3 = 2.32\ \text{M}\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 1\times 10^{-12} \times 2.32 \times 10^{6} = 2320\ \text{ns}$

$$\omega_H = \frac{1}{\tau_{gs} + \tau_{gd}} = \frac{1}{(92.3 + 2320)\times 10^{-9}} = 414.5^{\text{krad/s}}$$

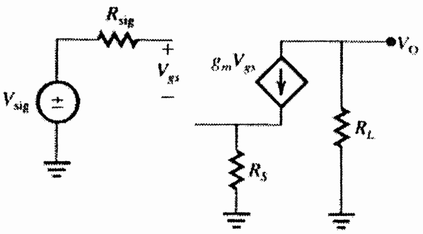
$f_H = 66\ \text{KHZ}$

8.47

a) $V_O = -g_m R_L V_{gs}$ (1)

$V_{gs} = V_{sig} - R_S \times g_m V_{gs}$

$V_{gs}(1 + g_m R_S) = V_{sig}$



$$(1) \Rightarrow v_O = \frac{-g_m R_L}{1 + g_m R_S} v_{sig} \Rightarrow \frac{v_O}{v_{sig}} \Rightarrow \frac{-g_m R_L}{1 + g_m R_S}$$

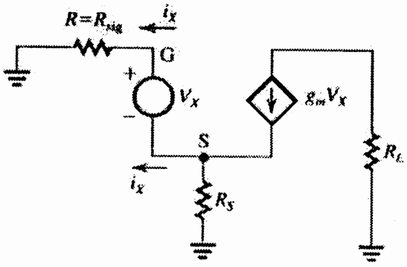
b) $V_S = (g_m V_X - i_X) R_S$

$V_G = V_S + V_X = (g_m V_X - i_X) R_S + V_X$

$$\Rightarrow i_X = \frac{V_G}{R} = \frac{(1 + g_m R_S)}{R} V_X - i_X \frac{R_S}{R}$$

$$R_{gs} = \frac{v_X}{i_X} = \frac{1 + R_S/R}{\dfrac{1 + g_m R_S}{R}} = \frac{R + R_S}{1 + g_m R_S}$$

(R is R_{sig})

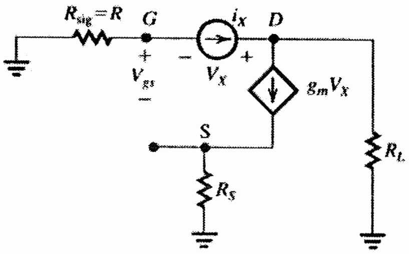


to calculate R_{gd}:

$V_G = -Ri_X$

$$\left.\begin{aligned} V_S &= -Ri_X - V_{gs} \\ V_S &= R_S \times g_mV_{gs} \end{aligned}\right\}$$

$$R_Sg_mV_{gs} = -Ri_X - V_{gs} \Rightarrow V_{gs} = \frac{-Ri_X}{1 + g_mR_S}$$



At D: $i_X = g_mV_{gs} + \dfrac{V_X - R_Xi_X}{R_L}$

substitute V_{gs} : $i_X = -\dfrac{g_mRi_X}{1 + g_mR_S} + \dfrac{V_X}{R_L} - \dfrac{Ri_X}{R_L}$

$$i_X\left[1 + \frac{g_mR}{1 + g_mR_S} + \frac{R}{R_L}\right] = \frac{V_X}{R_L}$$

$$R_{gd} = \frac{V_X}{i_X} = R_L + R + \frac{g_mRR_L}{1 + g_mR_S} \quad (R \text{ is } R_{sig})$$

c) $R_S = 0$:

$$\frac{v_O}{v_{sig}} = \frac{-4 \times 5\text{ K}}{1 + 4 \times \text{O}} = -20\text{V/V}$$

$R_{gs} = R_{sig} = 100\text{ k}\Omega$

$R_{gs} = 5\text{ K} + 100\text{ K} + 4 \times 5 \times 100 = 2105\text{ k}\Omega$

$$\omega_H \simeq \frac{1}{C_{gs}R_{gs} + C_{gd}R_{gd}}$$

$$= \frac{1}{10^{-12} \times 100 \times 10^3 + 10^{-12} \times 2105 \times 10^3}$$

$\omega_H \simeq 453.5$ K rad/s

|Gain| × Bandwidth $= 20 \times 453.5$

$= 9.07$ M rad/s

$R_S = 100\ \Omega$:

$$\frac{v_O}{v_{sig}} = \frac{-4 \times 5}{1 + 4 \times 0.1} = -14.3\text{ V/V}$$

$$R_{gs} = \frac{100 + 0.1}{1 + 4 \times 0.1} = 71.5\text{ k}\Omega$$

$$R_{gd} = 5 + 100 + \frac{4 \times 5 \times 100}{1 + 4 \times 0.1} = 1533.6\text{ k}\Omega$$

$$\omega_H = \frac{1}{10^{-12} \times 71.5 \times 10^3 + 10^{-12} \times 1533.6 \times 10^3}$$

$= 623$ K rad/s

|Gain| × Bandwidth

$= 14.3 \times 623\text{ K} = 8.91$ M rad/s

$R_S = 250\ \Omega$:

$$\frac{v_O}{v_{sig}} = \frac{-4 \times 5}{1 + 4 \times 0.25} = -10\text{ V/V}$$

$$R_{gs} = \frac{100 + 0.25}{1 + 4 \times 0.25} = 50.1\text{ k}\Omega$$

$$R_{gd} = 5 + 100 + \frac{4 \times 5 \times 100}{1 + 4 \times 0.25} = 1105\text{ k}\Omega$$

$$\omega_H = \frac{1}{10^{-12} \times 50.1 \times 10^3 + 10^{-12} \times 1105 \times 10^3}$$

$= 865.7$ K rad/s

|gain| × Bandwidth $= 10 \times 865.7$ K

$= 8.66$ M rad/s

Summary table:

$R_S(\Omega)$	Gain (V/υ)	W (K rad/s)	Gain.BW product (M rad/s)
0	−20	453.5	9.07
100	−14.3	623.0	8.91
250	−10	865.7	8.66

The Gain × Bandwidth is approximately constant.

8.48

$A_M = \frac{V_o}{V_{sig}} = -\frac{R_{in}}{R_{in}+R_{sig}}(g_m R_L') = -\frac{5}{5+1}(0.3\times 100^k)$

$A_M = -25\,V/V$ Now refer to Example 6.6.

$R_{gs} = R_{in} \| R_{sig} = 5^{M\Omega} \| 1^{M\Omega} = 0.83^{M\Omega}$

$\tau_{gs} = R_{gs} C_{gs} = 0.2\times 10^{-12} \times 0.83\times 10^{6} = 166.7\,ns$

$R_{gd} = R' + R_L' + g_m R_L' R'$

$R' = R_{in} \| R_{sig} = 0.83\,M\Omega$

$\Big\} \Rightarrow R_{gd} = 0.83 + 0.1 + 0.83 \times 0.3 \times 100$

$R_{gd} = 25.92\,M\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 25.92\times 10^{6} \times 0.1\times 10^{-12} = 2592\,ns$

$\omega_H \simeq \frac{1}{\tau_{gs}+\tau_{gd}} = \frac{1}{(166.7+2592)\times 10^{-9}} = 362.5^{Krad/s}$

$f_H = 57.7^{KHz}$

8.49

IF we assume that capacitors are perfect open circuits for midband, then:

$A_M = \frac{V_o}{V_{sig}} = \frac{-R_{in}}{R_{in}+R_{sig}}(g_m R_L') = \frac{-650}{650+150}(5\times 10) = 40.6\,V/V$

$\tau_{gs} = C_{gs} R_{gs} = C_{gs}(R_{in} \| R_{sig}) = 2^{P} \times (150^{K} \| 650K)$

$\tau_{gs} = 243.75\,ns$

$\tau_{gd} = C_{gd} R_{gd}$, Refer to Example 6.6

$R_{gd} = R' + R_L' + g_m R_L' R'$

$R' = 150^{K} \| 650K = 121.9\,K\Omega$

$\Rightarrow R_{gd} = 121.9 + 10 + 5\times 10 \times 121.9$

$R_{gd} = 6.2\,M\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 0.5^{P} \times 6.2^{M} = 3100\,ns$

$\tau_L = R_L' C_L = 10^{k} \times 3^{P} = 30\,ns$

$\omega_H = \frac{1}{\tau_{gs}+\tau_{gd}+\tau_L} = \frac{1}{30^{n} + 3100^{n} + 243.75^{n}} = 296.4\,Krad/s$

$f_H = 47.2\,KHz$

8.50

$R_{in} = \frac{R}{1-Gain} = \frac{100}{1-0.95} = 2000\,K\Omega = 2\,M\Omega$

8.51

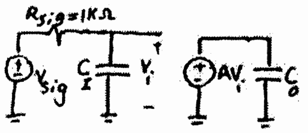
$Z_I = Z/_{1-K} \Rightarrow C_I = 0.1\times(1-(-1000))$

$\Rightarrow C_I = 100.1\,pF$

$C_o = 0.1\times(\frac{-1}{1000}+1)$

$C_o = 99.9\,fF$

(using Miller's Theorem)



$v_o = Av_i = A \times v_{sig} \frac{1/C_I s}{R_{sig}+\frac{1}{C_I s}} \Rightarrow \frac{v_o}{v_{sig}} = \frac{A}{1+C_I R_{sig} S}$

$\omega_H = \frac{1}{C_I R_{sig}} = \frac{1}{100.1^{P}\times 1^{K}} = 9.99\,Mrad/s \Rightarrow f_H = 1.59\,MHz$

To calculate unity gain frequency:

$|Gain| = 1$

$\frac{v_o}{v_i} = \frac{A}{1+C_I R_{sig} S} = \frac{-1000}{1+100.1\times 10^{-9} S}$ $(s=j\omega)$

$\frac{1000}{\sqrt{1+(100.1\times 10^{-9}\times \omega_T)^2}} = 1 \Rightarrow \omega_T \simeq 10\,Grad/s$

$f_T \simeq 1.59\,GHz$

As we can see $f_T \simeq f_H \times A$

8.52

Using Miller's Theorem, in each case the capacitance at the input is C(1–A) and the capacitance at the output is $C\left(1 - \frac{1}{A}\right)$.

Thus :

a) $A = -1000$ V/υ and $C = 1$ pF
$C_i = 1.001$ nF and $C_o = 1.001$ pF
b) $A = -10$ V/υ and $C = 10$ pF
$C_i = 110$ pF and $C_o = 11$ pF
c) $A = -1$ V/υ and $C = 10$ pF
$C_i = 20$ pF and $C_o = 20$ pF
d) $A = 1$ V/υ and $C = 10$ pF
$C_i = 0$ pF and $C_o = 0$ pF
e) $A = 10$ V/υ and $C = 10$ pF
$C_i = -90$ pF and $C_o = 9$ pF

In (e) the negative capacitance at the input can be used to cancel the effect of the input capacitance of the amplifier.

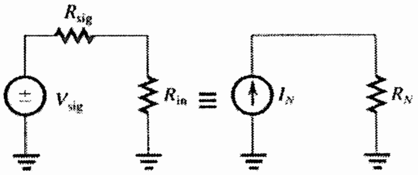
8.53

a) $R_{in} = \frac{R}{1 - A} = \frac{R}{1 - 2} = -R$

(Miller's theorem)

b) $I_N = \frac{V_{sig}}{R_{sig}}$

$R_N = R_{sig} \| R_{in}$

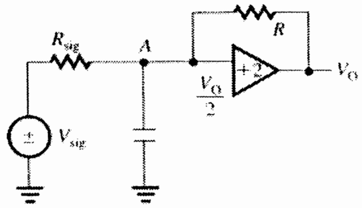


If $R_{sig} = R$ then :

$R_N = R \| (-R) = \infty \Rightarrow I_L = I_N$

$= \frac{V_{sig}}{R_{sig}} = \frac{V_{sig}}{R}$

c)



KCL at A:

$$\frac{\frac{V_O}{2} - V_{sig}}{R_{sig}} + \frac{v_O}{2} \times Cs + \frac{-v_O}{2R} = 0$$

$$\text{If } R_{sig} = R \Rightarrow \frac{+v_{sig}}{R} = \frac{v_O}{2}Cs \Rightarrow \frac{v_O}{v_{sig}} = \frac{2}{RCs}$$

8.54

$A_M = -g_m R_L' = -5 \times 20 = \underline{-100\ V/V}$

$C_{in} = C_{gs} + C_{gd}(1 + g_m R_L')$ (Eq. 6.55)

$C_{in} = 2 + 0.1(1 + 5 \times 20) = \underline{12.1\ pF}$

$f_H \simeq \frac{1}{2\pi C_{in} R_{sig}}$ (Eq. 6.54)

$f_H \simeq \frac{1}{2\pi \times 12.1 \times 10^{-12} \times 20^k} = \underline{658\ KHz}$

8.55

$\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R_{CL}$

$\tau_H = C_{gs} R_{sig} + C_{gd}[R_{sig}(1 + g_m R_L') + R_L'] + C_L R_L'$

$\tau_H = 2^p \times 20^k + 0.1^p[20^k(1 + 5 \times 20) + 20^k] + 1^p \times 20^k$

$\tau_H = 264\ ns$

$f_H \simeq \frac{1}{2\pi \tau_H} = 603\ KHz$

$A_M = -g_m R_L' = -5 \times 20 = -100\ V/V$

τ_{gs} : 15.1%
τ_{gd} : 77.3%
τ_L : 7.6%

Contribution of each time-constant to the overall τ_H.

If we compare f_H to the one obtained in Problem 6.66, we notice that Problem 6.66 has a larger f_H due to neglecting the time constants of C_L and C_{gs}.

8.56

we have : $\omega_Z = g_m / C_{gd}$

$$\Rightarrow f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{5^m}{2\pi \times 0.1^P} = \underline{7.96 GHZ}$$

f_{P1} and f_{P2} are the poles of the transfer function of equation (6.60), whose denominator is a quadratic polynomial with coefficient of s:

$$= [C_{gs} + C_{gd}[1+g_m R_L']] R_{sig} + (C_L + C_{gd}) R_L'$$
$$= [2 + 0.1 (1+5\times 20)] 20 + (1+0.1)\times 20$$
$$= 264 ns = 264 \times 10^{-9} sec$$

Coefficient of s^2:

$$= [(C_L + C_{gd}) C_{gs} + C_L C_{gd}] R_{sig} R_L'$$
$$= [(1+0.1)2 + 1\times 0.1] 20^K \times 20^K =$$
$$= 920 \times 10^{-18} (sec)^2$$

Therefore the quadratic equation is:

$$1 + 264\times 10^{-9} s + 920\times 10^{-18} s^2 = 0$$

Denoting the frequencies of the roots of this equation with ω_{P1} and ω_{P2}, we have:

$\omega_{P1} = 3.84\times 10^{6}$ rad/s $\Rightarrow f_{P_1} = \frac{\omega_{P1}}{2\pi} = 611.15 KHZ$

$\omega_{P2} = 283.12\times 10^{6}$ rad/s $\Rightarrow f_{P2} = \frac{\omega_{P2}}{2\pi} = 45.06 MHZ$

Since $f_{P_1} \ll f_{P2}$ and $f_{P_1} \ll f_Z$, a good estimate for f_H is f_{P_1}:

$$f_H \simeq f_{P_1} = 611.15 KHZ$$

Approximate value of f_{P1} obtained using (Eq. 6.66) is :

$$f_{P_1} \cong \frac{1}{2\pi[(C_{gs}+C_{gd}(1+g_m R_L'))R_{sig} + (C_L+C_{gd})R_L']}$$

$$f_{P_1} \simeq 603.16 KHZ$$

Approximate value of f_{P2} obtained using (Eq. 6.67) is:

$$f_{P_2} = \frac{[C_{gs}+C_{gd}(1+g_m R_L')]R_{sig} + (C_L+C_{gd})R_L'}{2\pi [(C_L+C_{gd})C_{gs} + C_L C_{gd}] R_L' R_{sig}}$$

$$f_{P2} = 45.67 MHZ$$

The estimate of f_{P1} using Eq. 6.66 is 1.3% lower than the exact value, while the estimate of f_{P2} is about 1.3% higher than its exact value.

8.57

$\underline{R_L' = 5K\Omega :}$

$$A_M = -g_m R_L' = -5\times 5 = -25 V/V$$

$$f_{P1} \cong \frac{1}{2\pi[(C_{gs}+C_{gd}(1+g_m R_L'))R_{sig} + (C_L+C_{gd})R_L']}$$

$$f_{P_1} = \frac{1}{2\pi[(2+0.1\times(1+5\times 5))20 + (1+0.1)\times 5]}$$

$$f_{P_1} = 1.63 MHZ$$

$$f_{P2} = \frac{[C_{gs}+C_{gd}(1+g_m R_L')]R_{sig} + (C_L+C_{gd})R_L'}{[(C_L+C_{gd})C_{gs} + C_L C_{gd}] R_L' R_{sig} \times 2\pi}$$

$$f_{P2} = \frac{(2+0.1(1+5\times 5))20 + (1+0.1)5}{((1+0.1)\times 2 + 1\times 0.1) 5 \times 20 \times 2\pi}$$

$$f_{P2} = 67.5 MHZ$$

: $S_Z = \frac{g_m}{C_{gd}} \Rightarrow f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{5^m}{2\pi 0.1^P} = 7.96 GHZ$

$f_{P_1} \ll f_{P2}$ and $f_{P_1} \ll f_Z \rightarrow f_{P_1}$ is the dominant pole.

$f_H \simeq f_{P_1} = 1.63\ \text{MHz}$

$|\text{Gain}| \times \text{Bandwidth} = 25 \times 1.63 = 40.75\ \text{MHz}$

$f_t = |A_M| f_H = 40.75\ \text{MHz}$

Since $f_{P_1} \ll f_{P_2}$ and $f_{P_1} \ll f_Z$, a dominant pole exists.

$R_L' = 10\ \text{k}\Omega$:

$A_M = -5 \times 10 = -50\ \text{V/V}$

$$f_{P1} = \frac{1}{2\pi\left[(2 + 0.1(1+5\times10))20 + (1+0.1)\times 10\right]} = 1.04\ \text{MHz}$$

$$f_{P2} = \frac{(2+0.1(1+5\times10))20 + (1+0.1)\times 10}{[(1+0.1)2 + 1\times0.1]\ 10\times20\times2\pi} = 5296\ \text{MHz}$$

$$f_Z = \frac{5}{2\pi\times0.1} = 7.96\ \text{GHz}$$

$f_{P_1} \ll f_{P2}$, $f_{P_1} \ll f_Z \Rightarrow f_{P_1}$ is the dominant pole and therefore $f_H \simeq f_{P_1} = 1.04\ \text{MHz}$

$|A_M| \cdot f_H = 50 \times 1.04 = 52\ \text{MHz}$

Since f_{P_2} is still slightly greater than $|A_M| \cdot f_H$, therfore: $f_T \simeq 52\ \text{MHz}$

$R_L' = 20\ \text{k}\Omega$:

$A_M = -5 \times 20 = -100\ \text{V/V}$, from Problem 6.68 we have:

$f_{P_1} = 603.16\ \text{KHz}$

$f_{P_2} = 45.67\ \text{MHz}$

$f_Z = 7.96\ \text{GHz}$

Again $f_{P_1} \ll f_{P_2}$ and $f_{P_1} \ll f_Z$, therefore f_{P_1} is the dominant pole and f_H can be approximated by f_{P_1}. $f_H \simeq f_{P_1} = 603.16\ \text{KHz}$

$|A_M| \cdot f_H = 60.32\ \text{MHz}$

Since $f_{P2} < |A_M| \cdot f_H$, therefore f_T is smaller than $|A_M| \cdot f_H$

The results are summarized in this table:

R_L'	$5^{\text{k}\Omega}$	$10^{\text{k}\Omega}$	$20^{\text{k}\Omega}$
$A_M^{(\text{V/V})}$	-25	-50	-100
$f_{P_1}^{(\text{MHz})}$	1.63	1.04	0.60
$\lvert A_M'\rvert \cdot f_H^{(\text{MHz})}$	40.75	52.00	60.32

8.58

$$A_M = -\frac{r_\pi}{R_{sig} + r_x + r_\pi}(g_m R_L')$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5\text{K}\Omega \Rightarrow A_M = -\frac{5}{1+0.2+5}(20\times5)$$

$\underline{A_M = 80.65\ \text{V/V}}$

Using Miller's Theorem and Eq. 6.71:

$C_{in} = C_\pi + C_\mu(1 + g_m R_L') = 10 + 0.5(1 + 20\times5) = 60.5\ \text{pF}$

Eq. 6.69: $R_{sig}' = r_\pi \| (R_{sig} + r_x) = 5^{\text{K}} \| (1^{\text{K}} + 0.2)$

$R_{sig}' = 0.97\ \text{K}\Omega$

$$\text{Eq. 6.72: } f_H \simeq \frac{1}{2\pi C_{in} R_{sig}'} = \frac{1}{2\pi\times60.5\times0.97^{\text{K}}} \Rightarrow$$

$\underline{f_H = 2.71\ \text{MHz}}$

8.59

$A_M = -139\ \text{V/V}$

Using the method of open-circuit time constants, from equation 9.100:

$$\tau_H = C_\pi \cdot R_{sig}' + C_\mu[(1 + g_m R_L') \cdot R_{sig}' + R_L'] + C_L \cdot R_L'$$

$$R_{sig}' = r_\pi \| (R_{sig} + r_x)$$

$$= 2.5\ \text{k}\Omega \| (1\ \text{k}\Omega + 0.1\ \text{k}\Omega) = 764\ \Omega$$

$$\tau_H = (10\ \text{p} \times 764) + 0.3\ \text{p}[(1 + 40 \times 5) \cdot 764 + 5\ \text{K}] + 3\ \text{p} \times 5\ \text{K}$$

$$\tau_H = 7.64\ \text{ns} + 47.57\ \text{ns} + 15\ \text{ns} = 70.21\ \text{ns}$$

$$f_H = \frac{1}{2\pi\tau_H} \simeq 2.27\ \text{MHz}$$

The % contributions to τ_H of each capacitance are: C_π : 10.8%, C_μ : 67.8%, C_L : 21.4%

f_H is 10.6% higher than the f_H obtained in this problem

8.60

we have: $r_\pi = 5k\Omega$ and

$A_M = -80.65\,V/V$, $R'_{sig} = 0.97k\Omega$

$$f_Z = \frac{g_m}{2\pi C_\mu} = \frac{20^m}{2\pi \times 0.5^P} = 6.37\,GHz$$

$$f_{P_1} \simeq \frac{1}{2\pi\left[(C_\pi + C_\mu(1+g_m R'_L))R'_{sig} + (C_L + C_\mu)R'_L\right]}$$

$$f_{P_1} \simeq \frac{1}{2\pi\left[(10 + 0.5(1+20\times5))0.97 + 2.5\times5\right]}$$

$$f_{P_1} = 2.24\,MHz$$

$$f_{P_2} = \frac{(C_\pi + C_\mu(1+g_m R'_L))R'_{sig} + (C_L + C_\mu)R'_L}{2\pi\left[C_\pi(C_L + C_\mu) + C_L C_\mu\right]R'_{sig}R'_L}$$

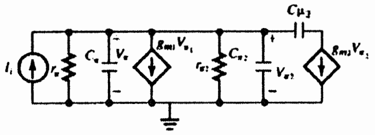
$$f_{P_2} = \frac{(10+0.5(1+20\times5))0.97 + 2.5\times5}{2\pi\left(10(2+0.5) + 2\times0.5\right)0.97\times5}$$

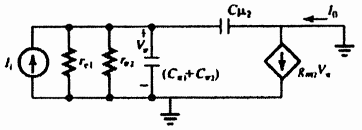
$$f_{P_2} = 89.89\,MHz$$

Since $f_{P_1} \ll f_{P_2}$ and $f_{P_1} \ll f_Z$, we can approximate f_H by f_{P_1} : $f_H \simeq f_{P_1} = 2.24\,MHz$

If we compare f_H to the results obtained from applying Miller's Theorem then our results are 17% lower.

8.61





$$V_\pi = \frac{I_i}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}}\right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})}$$

$$I_o = g_{m2}V_\pi - C_\mu s V_\pi$$

$$= \frac{(g_{m2} - C_\mu s)I_i}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}}\right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})}$$

$$\frac{I_o}{I_i} = \frac{g_{m2} - C_\mu s}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}}\right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})}$$

$I_{C1} = I_{C2} \Rightarrow r_{\pi 1} = r_{\pi 2}$,

$g_{m1} = g_{m2}$, $C_{\pi 1} = C_{\pi 2}$

$$\frac{I_O}{I_i} = \frac{g_m - C_\mu s}{\left(\frac{1}{r_e} + \frac{1}{r_\pi}\right) + (C_\mu + 2C_\pi)s}$$

$$= \frac{1 - \frac{C_\mu}{g_m}s}{\left(\frac{1}{g_m r_e} + \frac{1}{g_m r_\pi}\right) + s\frac{C_\mu + 2C_\pi}{g_m}}$$

$$g_m r_e = \frac{I_C}{V_T}\frac{V_T}{I_E} = \alpha = \frac{\beta}{\beta + 1}$$

$$g_m r_\pi = \beta$$

$$\Rightarrow \frac{I_O}{I_i} = \frac{1 - \frac{C_\mu}{g_m}s}{1 + \frac{1}{\beta} + \frac{1}{\beta} + s(2C_\pi + C_\mu)/g_m}$$

$$\frac{I_o}{I_i} = \frac{1}{1 + \frac{2}{\beta}}\;\frac{1 - s\frac{C_\mu}{g_m}}{1 + s\frac{(2C_\pi + C_\mu)}{g_m\left(1 + \frac{2}{\beta}\right)}}$$

If the circuit is biased at 1 mA and $\beta = \infty$, $f_T = 400$ MHz and $C_\mu = 2$ pF:

$$g_m = \frac{1}{0.025} = 40 \text{ mA}/v$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_\pi + C_\mu$$

$$= \frac{40 \text{ m}}{2\pi \times 400 \text{ M}} = 15.9 \text{ pF}$$

$C_\pi = 15.9 - 2 = 13.9$ pF

Pole frequency :

$$f_P = \frac{g_m}{2\pi(2C_\pi + C_\mu)} = \frac{40 \times 10^{-3}}{2\pi(2 \times 13.9 + 2)\text{pF}}$$

$f_P = 213.74$ MHz

Zero frequency:

$$f_Z = \frac{g_m}{2\pi C_\mu} = \frac{40 \text{ m}}{2\pi \times 2 \text{ p}} = 3.18 \text{ GHz}$$

8.62

$$A_M = -g_m R'_L = -5 \times 20 = \underline{-100 \, V/V}$$

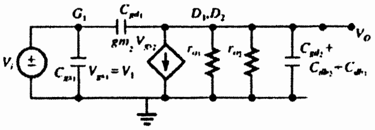
$$f_H = \frac{1}{2\pi(C_L + C_{gd})R'_L} = \frac{1}{2\pi(1 + 0.1) \times 20} = 7.23 \text{ MHz}.$$

(Note that in this case there is no Rsig and we used Eq. 6.79)

$$f_{3db} = f_H = \underline{7.23 \text{ MHz}}$$

$$f_t = |A_M| \cdot f_H = \underline{723 \text{ MHz}}$$

8.63



$$g_m = \sqrt{2\mu_n C_{Ox} \frac{W}{L} I_D} = \sqrt{2 \times 90 \times \frac{100}{1.6} \times 100}$$

$= 1060 \ \mu\text{A} / v$

$g_m = 1.06$ mA/v

$$r_{O1} = \frac{V_{A1}}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega$$

$$r_{O2} = \frac{|V_{A2}|}{|I_{D2}|} = \frac{19.2}{0.1} = 192 \text{ k}\Omega$$

DC-gain $= -g_m (r_{O1} \parallel r_{O2})$

$= -1.06 \times (128 \parallel 192)$

$= -81.4$ V / V

Total capacitance between output node and ground

$= C_{gd2} + C_{db1} + C_{db2} = 0.015 + 0.020 + 0.036$

$C_L = 0.071$ pF

Write a KCL at output:

$$sC_{gd1}(v_i - v_O) = g_m v_i + \frac{v_O}{r_{O1}} + \frac{v_O}{r_{O2}} + v_O s C_L$$

$$\frac{v_O}{v_i} = \frac{g_m - sC_{gd1}}{\frac{1}{r_{O1}} + \frac{1}{r_{O2}} + (C_L + C_{gd1})s}$$

Thus:

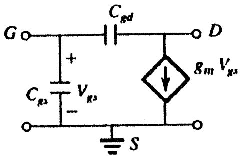
$$f_Z = \frac{g_m}{2\pi C_{gd1}} = \frac{1.06 \text{ m}}{2\pi \times 0.015 \text{ p}} = 11.3 \text{ GHz}$$

$$f_P = \frac{1}{2\pi} \frac{\frac{1}{r_{O1}} + \frac{1}{r_{O2}}}{C_L \times C_{gd1}} = \frac{\frac{1}{128 \text{ k}\Omega} + \frac{1}{192 \text{ k}\Omega}}{2\pi(0.071 + 0.015) \text{ p}}$$

$f_P = 24.1$ MHz

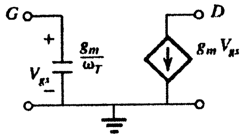
8.64

a) For small C_{gd} and low gain from G to D, we can neglect the Miller effect and C_{gd}.



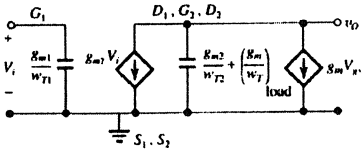
C_{gd}
G
D
+
C_{gs}
V_{gs}
−
$g_m V_{gs}$
S

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \simeq \frac{g_m}{C_{gs}} \quad \text{Thus } C_{gs} \simeq \frac{g_m}{\omega_T}$$



G
D
+
$\frac{g_m}{\omega_T}$
V_{gs}
−
$g_m V_{gs}$

b) replace the controlled source $gm_2 v_{gs2}$ with a resistance $\frac{1}{g_{m2}}$. (source absorbtion theory)



G_1
D_1, G_2, D_2
v_O
+
V_i
−
$\frac{g_{m1}}{w_{T1}}$
$g_{m1} V_i$
$\frac{g_{m2}}{w_{T2}} + \left(\frac{g_m}{w_T}\right)_{\text{load}}$
$g_m V_{gs2}$
S_1, S_2

$$V_o = -g_{m1} v_i \frac{1}{g_{m2} + s\left(\frac{g_{m2}}{\omega_{T2}} + \frac{gmload}{\omega_T \text{load}}\right)}$$

Since the load device is identical to Q_1,

$g_{\text{mload}} = g_{m1}$ and $\omega_{\text{Tload}} = \omega_{T1} = \omega_T$

Thus :

$$\frac{v_o}{v_i} = \frac{-g_{m1}/g_{m2}}{1 + \frac{s}{\omega_T}\left(1 - \frac{g_{m1}}{g_{m2}}\right)}$$

$$\frac{g_{m1}}{g_{m2}} = \frac{\mu_n C_{OX}\left(\frac{W}{L}\right)_1 V_{ov}}{\mu_n C_{OX}\left(\frac{W}{L}\right)_2 V_{ov}} = \frac{W_1}{W_2}$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{-A_o}{1 + \frac{S}{\omega_T}(1 + A_o)}$$

where $A_o = \frac{w_1}{w_2} = \frac{g_{m1}}{g_{m2}}$

c) $A_o = 3v/v$, $w_2 = 25\mu m$

$$A_o = \frac{w_1}{w_2} \Rightarrow w_1 = 3 \times 25 = 75\mu m$$

$$I_{D1} = \frac{1}{2}\mu_n C_{OX}\frac{W_1}{L_1}(V_{GS} - v_t)^2 = \frac{1}{2} \times 200^{\mu} \times \frac{75}{0.5}0.3^2$$

$$I_{D1} = 1.35mA$$

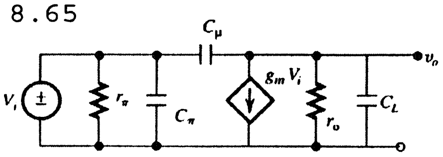
$$I_{D2} = \frac{1}{2} \times 200^{\mu A/v^2} \times \frac{25}{0.5} \times 0.3^2 = 0.45mA$$

Thus:

$$I = I_{D1} + I_{D2} = 1.35 + 0.45 = 1.8mA$$

$$f_{3db} = \frac{f_T}{1 + A_o} = \frac{12 \times 10^9}{1 + 3} = 3GHz$$

8.65



C_μ
v_o
V_i
±
r_π
C_π
$g_m V_i$
r_o
C_L

Writing a node equation at the output yields:

$$sC_\mu(v_i - v_O) = g_m \cdot v_i + \frac{v_O}{r_O} + v_O \cdot C_L \cdot s$$

$$\frac{v_O}{v_i} = \frac{C_\mu \cdot s - g_m}{\frac{1}{r_O} + (C_L + C_\mu)s}$$

$$= -g_m \cdot r_O\left[\frac{1 - sC_\mu/g_m}{1 + s(C_L + C_\mu)r_O}\right]$$

For $I_C = 200\ \mu A$, $V_A = 100\ V$:

$$g_m = \frac{200\ \mu}{0.025} = \frac{8\ mA}{V} \text{ and}$$

$$r_O = \frac{100}{200\ \mu} = 0.5\ M\Omega$$

Thus the DC-gain $= -g_m \cdot r_O$

$= -8 \times 0.5 \times 10^3 = -4000$ V/V

For $C_L = 1$ pF, $C_\mu = 0.2$ pF

$$\omega_{3dB} = \frac{1}{(C_L + C_\mu)r_O} = \frac{1}{(1 + 0.2)p \times 0.5\ M}$$

$= 1.67\text{ M rad/s}$

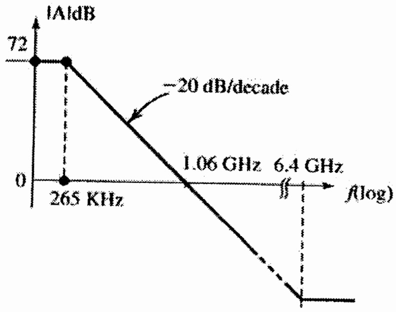
$f_{3dB} = f_H = 265.4\text{ KHz}$

$$f_Z = \frac{g_m}{2\pi \cdot C_\mu} = \frac{8\text{ m}}{2\pi \times 0.2\text{ p}} = 6.4\text{ GHz}$$

$$f_T = |A_O| \cdot f_H = 4000 \times 265.4 = 1.06\text{ GHz}$$

Bode plot for $|A|$:

4000 V/V = 720 dB



8.66

$$f_T = \frac{g_m}{2\pi(C_L + C_{gd}} \quad , g_m = 1\text{mA/V} \quad , f_T = 2\text{GHz}$$

$$\longrightarrow C_L + C_{gd} = \frac{1\times10^3}{2\pi \times 2\times10^9} = 79.61\text{ fF}$$

To have $f_{T2} = 1\text{GHz}$, we need:

$$C_L + C_{gd} = \frac{1\times10^3}{2\pi\times1\times10^9} = 159.23\text{ fF}$$

Thus we need an additional capacitance of

$$159.23 - 79.61 = \underline{79.61\text{ fF}}$$

8.67

$$f_H \simeq f_{Pi} = \frac{1}{2\pi C_{in} R'_{sig}}, \text{ where}$$

$$R'_{sig} = \frac{r_O}{2} = \frac{20\text{ k}\Omega}{2} = 10\text{ k}\Omega$$

$$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L) \text{ where}$$

$$R'_L = r_O \| r_O = \frac{r_O}{2} = 10\text{ k}\Omega$$

$$\Rightarrow C_{in} = 0.1\text{ p} + 0.1\text{ p}(1 + 2 \times 10) = 2.2\text{ pF}$$

$$\Rightarrow f_H = \frac{1}{2\pi \times 2.2\text{ p} \times 10\text{ K}} = 7.23\text{ MHz}$$

i) If the bias current I is reduced by a factor of 4:

Since $I_D \alpha V_{OV}^2 \Rightarrow$ For I_D to reduce by 1/4, V_{OV} is reduced by 1/2

$$g_{mi} = \frac{2(I_D/4)}{(V_{OV}/2)} = \frac{1}{2}\left(\frac{2I_D}{V_{OV}}\right) = \frac{1}{2} \cdot 2\text{ mA/V}$$

$$= 1\ \frac{\text{mA}}{\text{V}}$$

$$r_{oi} = \frac{V_A}{I_D/4} = 4 \times \frac{V_A}{I_D} = 4 \times 20\text{ k}\Omega = 80\text{ k}\Omega$$

8.68

then: $R'_{sig} = R'_L = \frac{r_O}{2} = \frac{80\text{ k}\Omega}{2} = 40\text{ k}\Omega$

$$C_{in} = 0.1\text{ p} + 0.1\text{ p}(1 + 1 \times 40) = 4.2\text{ pF}$$

$$\Rightarrow f_H = \frac{1}{2\pi \times 4.2\text{ p} \times 40\text{ K}} = 0.95\text{ MHz}$$

ii) if the bias current is increased by × 4:

$\rightarrow V_{OV}$ is increased by a factor of 2.

$$g_{mii} = \frac{2(4 \times I_D)}{(2 \times V_{OV})} = 2\left(\frac{2I_D}{V_{OV}}\right)$$

$$= 2 \times 2\ \frac{\text{mA}}{\text{V}} = 4\ \frac{\text{mA}}{\text{V}}$$

$$r_{O_{ii}} = \frac{V_A}{4 \times I_D} = \frac{1}{4}\left(\frac{V_A}{I_D}\right) = \frac{1}{4} \times 20\text{ k}\Omega = 5\text{ k}\Omega$$

then $R'_{sig} = R'_L = r_{O_{ii}}/2 = 2.5\text{ k}\Omega$

$$C_{in} = 0.1\text{ p} + 0.1\text{ p}(1 + 4 \times 2.5) = 1.2\text{ pF}$$

$$f_H = \frac{1}{2\pi \times 1.2\text{ p} \times 2.5\text{ K}} = 53\text{ MHz}$$

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L \cdot R_{CL}$$
$$= C_{gs} \cdot R'_{sig} + C_{gd} \cdot [R'_{sig}(1 + g_m R'_L) + R'_L]$$
$$+ C_L \cdot R'_L$$

Where $R'_{sig} = 100\text{ k}\Omega$

$$R'_L = 20\text{ k}\Omega \| 12\text{ k}\Omega = 7.5\text{ k}\Omega$$

$$\Rightarrow \tau_H = 0.2\text{ p} \times 100\text{ K}$$
$$+ 0.2\text{p}[100\text{ K}(1 + 1.5 \times 7.5) + 7.5\text{ K}]$$
$$+ C_L \cdot R'_L = 20\text{ ns} + 246\text{ ns} + C_L \cdot (7.5\text{ K})$$
$$= 266\text{ ns} + C_L \cdot (7.5\text{ K})$$

a) If $C_L = 0 \Rightarrow \tau_H = 266$ ns $\rightarrow f_H = \dfrac{1}{2\pi\tau_H}$

$= 598$ MHz

b) $C_L = 10\text{pF} \Rightarrow \tau_H = 341\text{ns} \rightarrow f_H$

$= 467$ MHz

c) $C_L = 50\text{ pF} \Rightarrow \tau_H = 641\text{ns} \rightarrow f_H$

$= 248$ MHz

Using the Miller approximation : Eq 9.80, 9.82

$C_{in} = C_{gs} + C_{gd}(1 + g_m R_L')$

$= 0.2\text{ p} + 0.2\text{ p}$

$(1 + 1.5 \times 7.5) = 2.65\text{ pF}$

$$f_H = \frac{1}{2\pi \cdot C_{in} \cdot R_{sig}'} = \frac{1}{2\pi \times 2.65\text{ p} \times 100\text{ K}}$$

$= 600$ MHz

Notice how this result is close to case a) where $C_L = 0$ and diverges further with increasing value of C_L, showing the importance of C_L in determining f_H

8.69

the low-frequency gain $\frac{v_o}{v_{sig}}$ can be written as:

$$\frac{v_o}{v_{sig}} = \frac{\frac{1}{g_m+g_{mb}}}{R_s + \frac{1}{g_m+g_{mb}}} \times (g_m+g_{mb}) \times R_L'$$

$$\frac{v_o}{v_{sig}} = \frac{(g_m+g_{mb})R_L'}{1+(g_m+g_{mb})R_{sig}} = \frac{(5+0.2\times5)\times20}{1+(5+0.2\times5)\times1} = 17.14\text{V/V}$$

$$\frac{v_o}{v_{sig}} = 17.14\text{V/V}$$

From Eq. 6.105 we have $f_{P1} = \dfrac{1}{2\pi C_{gs}(R_{sig} \| \frac{1}{g_m+g_{mb}})}$

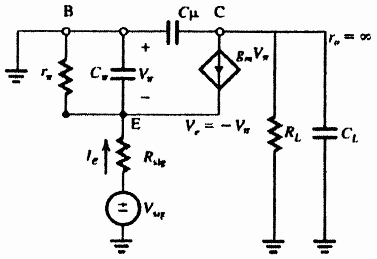
$$f_{P1} = \frac{1}{2\pi \times 2\text{pF}(1\text{K} \| \frac{1}{5+0.2\times5})} = 557\text{MHz}$$

From Eq. 6.106 we have:

$$f_{P2} = \frac{1}{2\pi(C_{gd}+C_L)R_L'} = \frac{1}{2\pi(0.1^p+2^p)20^K} = 3.79\text{MHz}$$

Since $f_{P2} \ll f_{P1}$, then f_{P2} is the dominant pole and $f_H \simeq f_{P2} = 3.79\text{MHz}$

8.70



We observe that V_e, voltage at the emitter, is equal to $-V_\pi$. We can write a node equation at the emitter:

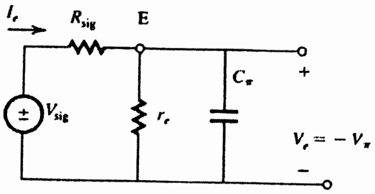
$$I_e = -V_\pi\left(\frac{1}{r_\pi} + sC_\pi\right) - g_m \cdot V_\pi$$

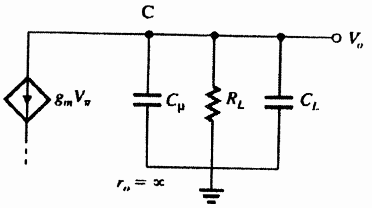
$$= V_e\left(\frac{1}{r_\pi} + g_m + sC_\pi\right)$$

Thus, the input admittance looking into the emitter is:

$$\frac{I_e}{V_e} = \frac{1}{r_\pi} + g_m + sC_\pi = \frac{1}{r_e} + sC_\pi$$

Therefore we can replace the transistor at the input of the circuit by this admittance as shown below





a) As we can see above, the circuit can be separated into two parts, each with its own pole:

$$f_{P1} = \frac{1}{2\pi \cdot C_\pi(R_{sig} \| r_e)} \text{ (input side)}$$

$$f_{P2} = \frac{1}{2\pi(C_\mu + C_L)R_L} \text{ (output side)}$$

If we compare the poles for MOSFETS, we observe that these equations are their bipolar counteparts:

$$f_{P1} = \frac{1}{2\pi \cdot C_{gs}\left(R_{\text{sig}} \| \frac{1}{g_m}\right)} \quad 9.108$$

$$f_{P2} = \frac{1}{2\pi(C_{gd} + C_L)R_L} \quad 9.109$$

b) For $C\pi = 14$ pF, $C_\mu = 2$ pF, $C_L = 1$ pF, $I_c = 1$ mA

$$R_{\text{sig}} = 1\ \text{k}\Omega,\ R_L = 10\ \text{k}\Omega \Rightarrow g_m = \frac{1}{0.025}$$

$$= 40\ \frac{\text{mA}}{\text{V}}$$

Assuming $\beta = 100$

$$f_{P1} = \frac{1}{2\pi \times 14\ \text{p}\left(1\ \text{K} \| \frac{100}{4}\right)} = 15.9\ \text{MHz}$$

$$f_{P2} = \frac{1}{2\pi \times (2\ \text{p} + 1\ \text{p})10\ \text{K}} = 5.3\ \text{MHz}$$

$$f_T = \frac{g_m}{2\pi(C\pi + C_\mu)} = \frac{40\ \text{m}}{2\pi(14\ \text{p} + 2\ \text{p})}$$

$$= 398.1\ \text{MHz}$$

$f_t >> f_{P1}$ and $f_t >> f_{P2}$

8.71

$$\text{If } f_H = 300\ \text{MHz} \Rightarrow \tau_H = \frac{1}{2\pi f_H}$$

$$= 530.5\ \text{ps}$$

$$\tau_H = C_{gs}R_{gs} + (C_{gd} + C_L')R_{gd}$$

From Example 9.12

$C_{gs} = 20$ fF, $R_{gs} = 1.38$ kΩ, $C_{gd} = 5$ fF, $R_{gd} = 18.7$ kΩ

$$\Rightarrow 530.5\ \text{p} = 20\ \text{f} \times 1.38\ \text{K} + (5\ \text{p} + C_L') \times 18.7\ \text{K}$$

Thus $C_L' = 21.9$ fF

Since the original C_L in Eq. 12 was 15 fF

$\Rightarrow$ We must add $(21.9 - 15) = 6.9$ fF at the output to reduce f_H from 396 MHz to 300 MHz

8.72

$$R_O = 2r_O + (g_m r_O)r_O = 2 \times 50\ \text{k}\Omega$$
$$+ (1 \times 50) \times 50\ \text{k}\Omega = 2.6\ \text{M}\Omega$$

$$A_V = -g_m(R_O \| R_L) = -1\ \text{m}(2.6\ \text{M} \| 2\ \text{M})$$

$$= -1130\ \frac{\text{V}}{\text{V}}$$

$$A_V = -1130\ \frac{\text{V}}{\text{V}}$$

$$R_{\text{in}2} = \frac{r_O + R_L}{g_m r_O} = \frac{50\ \text{k}\Omega + 2\ \text{M}\Omega}{1 \times 50} = 41\ \text{k}\Omega$$

$$R_{d1} = r_O \| R_{\text{in}2} = 50\ \text{k}\Omega \| 41\ \text{k}\Omega = 22.5\ \text{k}\Omega$$

$$\tau_H = R_{\text{sig}}[C_{gs} + C_{gd}(1 + g_m R_{d1})]$$
$$+ R_{d1}(C_{gd} + C_{db} + C_{gs})$$
$$+ (R_L \| R_o)(C_L + C_{db} + C_{gd})$$

$$\tau_H = 100\ \text{K} \times [30\ \text{f} + 10\ \text{f}(1 + 1 \times 22.5)]$$
$$+ 22.5\ \text{K} \times [10\ \text{f} + 10\ \text{f} + 30\ \text{f}]$$
$$+ (2\ \text{M} \| 2.6\ \text{M}) \times [40\ \text{f} + 10\ \text{f} + 10\ \text{f}]$$

$$\tau_H = 26.5\ \text{ns} + 1.125\ \text{ns} + 67.8\ \text{ns} = 95.42\ \text{ns}$$

$$f_H = \frac{1}{2\pi\tau_H} = 1.67\ \text{MHz}$$

8.73

a) $A_M = -g_m R_L' = -5 \times (20^K \| 20^K) = \underline{-50\ V/V}$

$R_{gs} = R_{sig} = 20\ \text{K}\Omega$

$R_{gd} = R_{sig}(1 + g_m R_L') + R_L' = 20^K(1 + 5 \times 20^K \| 20^K) + 20^K \| 20^K$

$R_{gd} = 1030\ \text{K}\Omega = 1.03\ \text{M}\Omega$

We use Eq. 6.57 :

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_L'$$

$$\tau_H = 2^P \times 20^K + 0.2^P \times 1030^K + 1^P \times (20^K \| 20^K) = 256\ ns$$

$$f_H = \frac{1}{2\pi\tau_H} = \underline{622\ kHz}$$

$$\underline{|A_M| \cdot f_H = 31.1\ MHz}$$

b) For the cascode amplifier:

$A_{01} = g_{m1} r_{01} = 5 \times 20 = 100\,V/V$

$A_{V02} = 1 + (g_{m2} + g_{mb2}) r_{02} = 1 + (5 + 0.2 \times 5) \times 20^K$

$A_{V02} = 121\,V/V$

$R_{out} = r_{02} + A_{V02} r_{01} = 20^K + (121 \times 20^K) = 2.44\,M\Omega$

$A_v = A_{v_o} \dfrac{R_L}{R_L + R_{out}} = -121 \times 100 \times \dfrac{20}{20 + 2440} = \underline{\underline{-98.4\,V/V}}$

Using Eq.6.137,

$$\tau_H = R_{sig}\left[C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})\right] + R_{d1}(C_{gd1} + C_{db1} + C_{gs2}) + (R_L \| R_{out})(C_L + C_{gd2})$$

$$R_{d1} = r_{01} \| \left(\frac{1}{g_{m2} + g_{mb2}} + \frac{R_L}{A_{V02}}\right)$$

$$R_{d1} = 20^K \| \left(\frac{1}{5 + 0.2 \times 5} + \frac{20}{121}\right) = 0.327\,k\Omega$$

$$\tau_H = 20^K\left[2 + 0.2(1 + 5 \times 0.327)\right] + 0.327(0.2^p + 0.2^p + 2) + (20^K \| 2.44^M)(1 + 0.2)$$

$\tau_H = 75.1\,ns$

$f_H = \dfrac{1}{2\pi \tau_H} = \underline{2.12^{MHz}}$

$\underline{|A_v| \cdot f_H = 208.61\,MHz}$

8.74

$A_v = 66dB = 1995\,V/V$

$A_v = A_{v_o} \dfrac{R_L}{R_L + R_{out}}$ and $R_L = R_{out} \Rightarrow A_v = A_{v_o} \times \dfrac{1}{2}$

$A_{v_o} = (1 + g_{m2} r_{o2}) g_{m1} r_{o1} \simeq g_m^2 r_o^2 = \left(\dfrac{2I_D}{V_{OV}} \cdot \dfrac{V_A}{I_D}\right)^2 = \left(\dfrac{2V_A}{V_{OV}}\right)^2$

$\Rightarrow 1995 = \dfrac{1}{2} \times \left(\dfrac{2 \times 10}{V_{OV}}\right)^2 \Rightarrow V_{OV} = 0.317^V$

$\Rightarrow I_D = \dfrac{1}{2} k_n' \dfrac{W}{L} V_{OV}^2 = \dfrac{1}{2} \times 200 \times 10^{-3} \times 10 \times 0.317^2 = 0.1\,mA$

Since R_{sig} is small:

$\tau_H \simeq (C_L + C_{gd})(R_L \| R_{out})$

$r_o = \dfrac{V_A}{I_D} = \dfrac{10}{0.1} = 100\,k\Omega$, $g_m = \dfrac{2I_D}{V_{OV}} = 0.631\,mA/V$

$R_{out} = A_{v_{o2}} r_{o1} + r_{o2} = (1 + g_m r_o) r_o + r_o$

$R_{out} = 6510\,k\Omega \simeq 6.5\,M\Omega$, $R_L = R_{out}$.

$\tau_H \simeq (1pF + 0.1pF)\left(\dfrac{6510^K}{2}\right) = 3580.5\,ns$

$f_H = 44.5\,KHz$

$f_t \simeq |A_v| \cdot f_H = 1995 \times 44.5 = 88.8\,MHz$

If the cascode transistor is removed, then we have a common-source configuration.

$A_M = -g_m(r_o \| R_L) = -0.637(100^K \| 6510^K)$

$A_M = -62.74\,V/V$

$f_H = \dfrac{1}{2\pi(C_L + C_{gd}) R_L'} = \dfrac{1}{2\pi(1 + 0.1)(100^K \| 6510^K)} = 1.47^{MHz}$

$f_H = 1.47\,MHz$

$|A_M| \cdot f_H = 92.2\,MHz \cong f_T$

Note that the unity-gain stays nearly unchanged. The result is the same as

8.75

$R_{sig} = 4\,k\Omega$, $R_L = 2.4\,k\Omega$,
$I = 1\,mA$, $\beta = 100$, $r_O = 100\,k\Omega$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} \times g_m(\beta r_O \| R_L)$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/0.025} = 2.5\,k\Omega$$

$$g_m = \frac{I_C}{V_r} = 40\,mA/V$$

$$A_M = \frac{2.5}{2.5 + 0.05 + 4} \times 40 \times (100 \times 100 \| 2.4\,K)$$

$$A_M = -36.6\,V/V$$

$$R_{sig}' = r_\pi \| (r_x + R_{sig}) = 2.5\,K \| (0.05 + 4\,K)$$

$$R_{sig}' = 1.55\,k\Omega = R_{\pi 1}$$

$$R_{\mu 1} = R_{sig}' = (1 + g_m R_{e1}) + R_{e1}$$

$$R_{e1} = r_{O1} \| r_{e2}\left(\frac{r_O + R_L}{r_O + R_L/\beta + 1}\right)$$

$$= 100\,K \| \frac{100\,K}{101}\left(\frac{100 + 2.4}{100 + \frac{2.4}{101}}\right)$$

$$R_{e1} = 1\,k\Omega$$

$$R_{\mu 1} = 1.55(1 + 40 \times 1) + 1 = 64.55\,k\Omega$$

$$R_{out} = \beta \cdot r_o = 10\,M\Omega$$

$$\tau_H = C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + (C_{CS1} + C_{\pi 2}) R_{e1} + (C_L + C_{CS2} + C_{\mu 2})(R_L \| R_{out})$$

$$\tau_H = 14 \times 1.55 + 2 \times 64.55 + (0 + 14) \times 1 + (0 + 2)(2.4\,K \| 10\,M)$$

$\tau_H = 169.6\,ns$ $f_H = 939\,KHz$

8.76

a) If we employ Miller's theorem to C_μ:

$$\frac{1}{C_{\mu_1}S}\,\frac{1}{1-A}=\frac{1}{C_\mu S}\,\frac{1}{1-(-1)}=\frac{1}{2C_\mu S}$$

or $2C_\mu$ appears in parallel with C_π. Thus the time constant due to $(C_\pi+2C_\mu)$ is:

$R'_{sig}(C_\pi+2C_\mu)$ which results in:

$$f_{P_1}=\frac{1}{2\pi R'_{sig}(C_\pi+2C_\mu)}$$

If we refer to Fig. 6.42, we'll see that the output pole is: $f_{P_2}=\dfrac{1}{2\pi(C_L+C_{cs2}+C_{\mu e})R_L}$

b) $R_{sig}=1\,K\Omega \Rightarrow R'_{sig}=r_\pi \| R_{sig}=\dfrac{100}{1/0.025}\| 1^K$

$\Rightarrow R'_{sig}=0.714\,K\Omega$

$$f_{P_1}=\frac{1}{2\pi\times 0.714^{k}(5+2\times 1)^{p}}=31.85\text{ MHz}$$

$$f_{P_2}=\frac{1}{2\pi(0+0+1)\times 10}=15.9\text{ MHz}$$

(Assume $R_L=10\ K\Omega$)

$$f_H=\frac{1}{\sqrt{\left(\frac{1}{f_{P_1}}\right)^2+\left(\frac{1}{f_{P_2}}\right)^2}}=14.2\text{ MHz}$$

If $R_{sig}=10\,K\Omega$:

$R'_{sig}=2.5^K \| 10^K=2K\Omega$

$$f_{P_1}=\frac{1}{2\pi(5+2)2}=11.4\text{ MHz}$$

f_{P_2} is the same: $f_{P2}=15.9\text{ MHz}$

$f_H=9.26\text{ MHz}$

8.77

$$g_m=\frac{I_C}{V_T}=\frac{0.1\text{ mA}}{25\text{ mV}}=\frac{4\text{ mA}}{\text{V}};$$

$$r_\pi=\frac{\beta}{g_m}=\frac{100}{4\text{ m}}=25\text{ k}\Omega$$

$R_{sig}=r_\pi=25\text{ k}\Omega$

$$r_O=\frac{V_A}{I_C}=\frac{100}{0.1\text{ m}}=1\text{ M}\Omega;$$

$R_L=\beta\cdot r_O=100\text{ M}\Omega$

$$f_T=\frac{1}{2\pi}\cdot\frac{g_m}{(C_\mu+C_\pi)}\Rightarrow C_\mu+C_\pi=\frac{g_m}{2\pi\cdot f_T}$$

$$\rightarrow C_\pi=\frac{4\times 10^{-3}}{2\pi\times 10^{9}}-0.1\times 10^{-12}=0.54\text{ pF}$$

To obtain the DC-gain A_M:

$$A_M=\frac{-r_\pi}{r_\pi+R_{sig}}\cdot g_m(\beta r_O \| R_L)$$

$R_{sig}=r_\pi\ \ R_L=\beta r_O$

assuming $r_x=0$

$$A_M=\frac{-1}{2}\cdot g_m\times\frac{\beta r_O}{2}$$

$$=\frac{-1}{4}\times 4\times 10^{-3}\times 100\times 1\times 10^{6}$$

$A_M=-100\text{ KV/V}$

$$R'_{sig}=r_\pi \| R_{sig}=\frac{r_\pi}{2}=12.5\text{ k}\Omega$$

$R_{\pi 1}=R'_{sig}=12.5\text{ k}\Omega$

$$r_{e2}=r_e=\frac{r_\pi}{\beta+1}=\frac{25\text{ k}\Omega}{101}=247\ \Omega$$

$$R_{C1}=r_O \| r_e\left(\frac{r_O+R_L}{r_O+\frac{R_L}{\beta+1}}\right)$$

$$=1\text{ M}\| 247\times\left(\frac{1\text{ M}+100\text{ M}}{1\text{ M}+\frac{100\text{ M}}{101}}\right)$$

$R_{C1}=12.4\text{ k}\Omega$

$R_{\mu 1}=R'_{sig}(1+g_m R_{C1})+R_{C1}$

$=12.5\text{ K}(1+4\times 12.4)+12.4\text{ k}\Omega$

$R_{\mu 1}=645\text{ k}\Omega$

$$\Rightarrow \tau_H=C_{\pi 1}R_{\pi 1}+C_{\mu 1}R_{\mu 1}+C_{\pi 2}R_{C1}+C_{\mu 2}(R_L \| R_O)$$

where

$R_O=\beta_2\cdot r_{O2}=100\times 1\text{ M}\Omega=100\text{ M}\Omega$

$\tau_H=0.54\text{ p}\times 12.5\text{ K}+0.1\text{ p}\times 645\text{ K}+0.54\text{ p}\times 12.4\text{ K}+0.1\text{ p}(100\text{ M}\| 100\text{ M})$

$\tau_H=5.08\ \mu\text{s}$

$$f_H=\frac{1}{2\pi\tau_H}=31.3\text{ KHz}$$

$f_T=|A_M|\times f_H=3.13\text{ GHz}$

8.78

a) $I = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 160^{\mu} \times 100 \times (0.5)^2 = \underline{2 mA}$

b) $g_m = \frac{2I_D}{V_{ov}} = 2 \times \frac{2}{0.5} = \underline{8 mA/V}$

$g_{mb} = \chi g_m = 0.2 \times 8 = \underline{1.6 mA/V}$

$r_o \simeq \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 2} = \underline{10 k\Omega}$

$$\therefore A_{v_o} = \frac{g_m r_o}{1 + (g_m + g_{mb}) r_o}$$

$$A_{v_o} = \frac{8 \times 10}{1 + (8 + 1.6) \times 10} = 0.82 V/V$$

If we use the approximation formula

$$A_{v_o} = \frac{1}{1 + \chi} = 0.83 V/V$$

$$R_o = \frac{1}{g_m + g_{mb}} \| r_o = \frac{1}{8 + 1.6}^{k} \| 10^{k} = \underline{103 \Omega}$$

d) with $R_L = 1k\Omega$

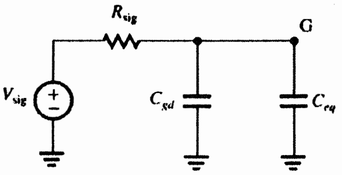
$$A_v = \frac{g_m R_L'}{1 + g_m R_L'}$$

$$R_L' = R_L \| r_o \| \frac{1}{g_{mb}} = 1^{k} \| 10^{k} \| \frac{1}{1.6}^{k} = 370 \Omega$$

$$A_v = \frac{8^{m} \times 370 \times 10^{-3}}{1 + 8 \times 0.370} = \underline{0.75 V/V}$$

8.79

Using the Miller approximation, the resulting input equivalent circuit is:



where $C_{eq} = C_{gs}(1 - K)$

and $K = \dfrac{g_m R_L'}{1 + g_m R_L'}$

$$\Rightarrow C_{eq} = C_{gs}\left[1 - \frac{g_m R_L'}{1 + g_m R_L'}\right]$$

$$= C_{gs}\left[\frac{1}{1 + gmR_L'}\right]$$

$$C_{in} = C_{gd} \| C_{eq} = C_{gd} + C_{gs}\left[\frac{1}{1 + g_m R_L'}\right]$$

$$\tau_H = R_{sig} \cdot C_{in} \Rightarrow f_H$$

$$= \frac{1}{2\pi \cdot R_{sig}\left(C_{gd} + \dfrac{C_{gs}}{1 + g_m R_L'}\right)}$$

Notice that this is the same result as obtained in problem 9.86. This estimate is higher than that obtained from the method of open-time constants since it neglects the contribution of C_L to τ_H and reduces the contribution of C_{gs} from:

$C_{gs} \cdot \dfrac{(R_{sig} + R_L)}{1 + g_m R_L'}$ to $\dfrac{C_{gs} \cdot R_{sig}}{1 + g_m R_L'}$, thus

effectively reducing the value of τ_H, and therefore increasing f_H

8.80

$$g_m = \frac{I_C}{V_T} = \frac{1}{0.025} = 40 mA/V,$$

$$r_e = \frac{\beta}{(\beta + 1) g_m} = 25 \Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = 2 GHz$$

$$\Rightarrow C_\pi + C_\mu = 3.18 pF$$

$$C_\mu = 0.1 pF \Rightarrow C_\pi = 3.08 pF$$

$$r_\pi = \frac{\beta}{g_m} = 2.5 k\Omega$$

$$r_O = \frac{V_A}{I_C} = \frac{20}{1} = 20 k\Omega$$

r_o is in effect parallel to R_L, so $R_L' = R_L \| r_O$

$R_L' = 1 k\Omega \| 20 k\Omega = 0.95 k\Omega.$

$$A_M = \frac{R_L'}{\dfrac{R_{sig} + r_\pi + r_x}{\beta + 1} + R_L'}$$

$$= \frac{0.95}{\dfrac{1 + 2.5 + 0.1}{101} + 0.95} = 0.964 V/V$$

$$R_\mu = R_{sig}' \| (r_\pi + (\beta + 1) R_L')$$

$$R_{sig}' = R_{sig} + r_x = 1 + 0.1 = 1.1 k\Omega$$

$$R_\mu = 1.1 K \| (2.5 K + 101 \times 0.95)$$

$$= 1.08 k\Omega$$

8.81

$$K_n'(W/L) = 128\frac{\mu A}{V^2} \times 25 = 3.2\,mA/V^2$$

(a) $$V_{ov} = \sqrt{\frac{I}{K_n' W/L}} = \sqrt{\frac{0.2}{3.2}} = \underline{\underline{0.25V}}$$

$$g_m = \frac{I}{V_{ov}} = \frac{0.2mA}{0.25V} = \underline{\underline{0.8}}\frac{mA}{V}$$

(b) $A_d = g_m(R_D \| r_o)$

where $r_o = \frac{V_A}{I/2} = \frac{20}{0.2/2} = 200K\Omega$

$$\Longrightarrow A_d = 0.8m \times (20K \| 200K)$$

$$= \underline{\underline{14.54}}\ V/V$$

(c) For a CS amplifier when R_{sig} is low:

$$f_H = \frac{1}{2\pi\,(C_L + C_{gd})\,R_L'}$$

where $R_L' = R_D \| r_o$

$= 20K \| 200K$

$= 18.18\,K\Omega$

and $C_L' = C_L + C_{db}$

Since for a grounded source terminal C_{db} is in parallel with the load.

$\longrightarrow C_L' = 90 + 5 = 95\,fF$

thus,

$$f_H = \frac{1}{2\pi\,(95+5)10^{-15} \times 18.18K}$$

$$= \underline{\underline{87.54}}\,MHz$$

(d) Using the open-circuit time-constants method for $R_S = 20K\Omega$

$$f_H = \frac{1}{2\pi\,\tau_H}$$

where $\tau_H = C_{gs}.R_S + C_{gd}\left[R_S(1+g_m R_L') + R_L'\right] + C_L.R_L'$

thus,

$$\tau_H = 30f \times 20K + 5f\left[20K(1+0.8\times 18.18) + 18.18K\right] + (90f + 5f) \times 18.18K$$

$$\tau_H = 0.6ns + 1.64ns + 1.72ns = 3.96ns$$

$$\Longrightarrow f_H = \frac{1}{2\pi\,3.96ns} = \underline{\underline{40.2}}\,MHz$$

8.82

$$f_z = \frac{1}{2\pi C_{ss} R_{ss}} = \frac{1}{2\pi\,(0.2p)(100K)} = \underline{\underline{7.95}}\,MHz$$

8.83

$$f_z = \frac{1}{2\pi \cdot C_{SS} \cdot R_{SS}}$$

R_{ss} is the output resistance of the current source, which for the single-transistor current source is:

$$R_{SS} = r_O = \frac{V_A}{I_D} = \frac{30\text{ V}}{100\ \mu\text{A}} = 300\text{ k}\Omega$$

$$\Rightarrow f_z = \frac{1}{2\pi \cdot 100\text{ f} \cdot 300\text{ K}} = 5.3\text{ MHz}$$

If V_{ov} is reduced from 0.5 V to 0.2 V while I is unchanged.

For the current-source transistor: when $V_{ov} = 0.5$ V

$$I = I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2 \rightarrow 100\ \mu\text{A}$$

$$= \left[\frac{1}{2}k_n'\frac{W_1}{L_1}\right] \times (0.5)^2$$

$$\rightarrow \frac{1}{2}k_n'\frac{W_1}{L_1} = 400\ \frac{\mu\text{A}}{\text{V}^2}$$

when $V_{OV} = 0.2$:

$$100\ \mu\text{A} = \left[\frac{1}{2}k_n'\frac{W_2}{L_2}\right] \times (0.2)^2$$

$$\rightarrow \frac{1}{2}k_n'\frac{W_2}{L_2} = 2500\ \frac{\mu\text{A}}{\text{V}^2}$$

Assuming that $L_1 = L_2$, (the length of the transistor is unchanged)

$$\Rightarrow \frac{W_2}{W_1} = \frac{2500}{400} \rightarrow W_2 = 6.25\ W_1$$

The width of the current-source transistor is made 6.25 times larger to operate at $V_{ov} = 0.2$ V, $I_n = 100\ \mu$A

If C_{SS} is directly proportional to W:

$$C_{SS2} = 6.25 \times 100\text{ f}$$

$$f_{Z2} = \frac{f_{Z1}}{6.25} = 848\text{ KHz}$$

8.84

(a) $V_{OV} = \sqrt{\dfrac{I}{(W/L)k_n'}}$

$= \sqrt{\dfrac{80\ \mu}{100 \times 0.2\ \frac{\text{mA}}{\text{V}^2}}} = 63.2\ \text{mV}$

$g_m = \dfrac{2(I/2)}{V_{OV}} = \dfrac{80\ \mu}{63.2\ \text{m}} = 1.27\ \dfrac{\text{mA}}{\text{V}}$

(b) $A_d = g_m(R_D \| r_O)$;

$r_O = \dfrac{V_A}{(I/2)} = \dfrac{20\ \text{V}}{40\ \mu} = 500\ \text{k}\Omega$

$\Rightarrow A_d = 1.27\ \text{m}(20\ \text{k}\Omega \| 500\ \text{k}\Omega) = 24.4\ \text{V/V}$

(c) $f_H = \dfrac{1}{2\pi(C_L + Cgd)R_L'}$

$R_L' = R_D \| r_O = 20\ \text{k}\Omega \| 500\ \text{k}\Omega = 19.23\ \text{k}\Omega$

$C_L = 100\ \text{fF} + Cdb = 110\ \text{fF}$

$\Rightarrow f_H = \dfrac{1}{2\pi(110\ \text{f} + 10\ \text{f}) \times 19.23\ \text{K}}$

$= 69\ \text{MHz}$

(d) Using the open-circuit time-constants method

for $R_{sig} = 100\ \text{k}\Omega\ \left(R_S = \dfrac{100\ \text{k}\Omega}{2} = 50\ \text{k}\Omega\right)$

$\tau_H = C_{gs} \cdot R_S$

$+ C_{gd}[R_S(1 + g_m R_L') + R_L']$

$+ C_L \cdot R_L'$

$\tau_H = 50\ \text{f} \times 50\ \text{K}$

$+ 10\ \text{f}[50\ \text{K}(1 + 1.27 \times 19.23) + 19.23\ \text{K}]$

$+ (100\ \text{f} + 10\ \text{f}) \times 19.23\ \text{K}$

$\tau_H = 2.5\ \text{ns}$

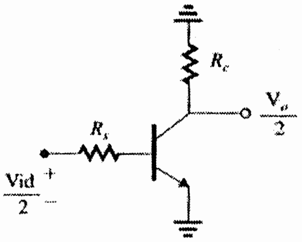
$+ 12.9\ \text{ns}$

$+ 2.11\ \text{ns} = 17.51\ \text{ns}$

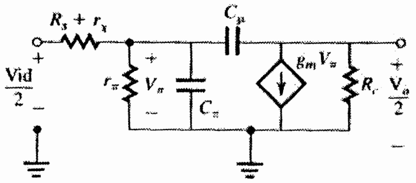
$f_H = \dfrac{1}{2\pi\tau_H} = \dfrac{1}{2\pi \times 17.51\ \text{n}} = 9.1\ \text{MHz}$

8.85

(a) Differential half-circuit:



High-frequency equivalent circuit (r_o is very large)



(b) $I = 0.5\ \text{mA} \rightarrow g_m = \dfrac{0.5}{25} = 20\ \dfrac{\text{mA}}{\text{V}}$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{20\ \text{m}} = 5\ \text{k}\Omega$

$C_\pi + C_\mu = \dfrac{20\ \text{m}}{2\pi \times 600} = 5.3\ \text{pF}$,

if $C_\mu = 0.5\ \text{pF} \Rightarrow C_\pi = 4.8\ \text{pF}$

$R_S = 10\ \text{k}\Omega,\ R_C = 10\ \text{k}\Omega,\ r_x = 100\ \Omega$

, (Notice that $R_E = \infty$,

$r_O = \infty$)

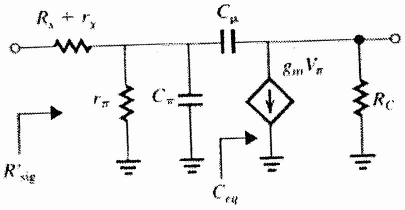
$A_M = \dfrac{-r_\pi}{r_\pi + R_S + r_X} \cdot g_m \cdot R_C$

$= \dfrac{-5\ \text{K}}{5\ \text{K} + (10\ \text{K} + 100)} \cdot 20\ \text{m} \times 10\ \text{K}$

$A_M = -66.22\ \text{V/V}$

(c) Γ

$f_H = \dfrac{1}{2_\pi C_{in} \cdot R_{sig}'}$



where:

$C_{in} = C_\pi + C_{eq}$

using Miller's approximation:

$C_{eq} = C_\mu(1 + g_m R_C)$

and $R'_{sig} = (R_S + r_x) \| r_\pi$

Thus,

$$f_H = \frac{1}{2\pi[(R_S + r_x) \| r_\pi] \cdot [C_\pi + C_\mu(1 + g_m R_C)]}$$

$$= \frac{1}{2\pi[(10\text{ K} + 100) \| 5\text{ K}] \cdot [4.8\text{ p} + 0.5\text{ p}(1 + 20 \times 10)]}$$

$= 452$ KHz

$GBW = 66.22 \times 452\text{ K} = 30$ MHz

8.86

The CMRR will have..

poles at $\underline{\underline{500\text{KHz}}}$ and at

$$\frac{1}{2\pi \times 10^6 \times 10 \times 10^{-12}} = \underline{\underline{15.9\text{ KHz}}}$$

8.87

$I = 0.6\text{mA}$

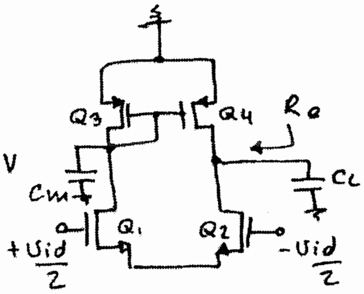
$V_{ovn} = 0.3\text{V}$

$V_{ovp} = 0.5\text{V}$

$V_{An} = |V_{Ap}| = 9\text{V}$

$C_m = 0.1\text{pF}$

$C_L = 0.2\text{pF}$



All r_o's are identical:

$$r_o = \frac{V_A}{I_D} = \frac{9}{0.3m} = 30\text{K}\Omega.$$

$$g_m = \frac{2I_D}{V_{ov}} \Rightarrow g_{m1,2} = \frac{0.6m}{0.3} = 2\frac{mA}{V}$$

$$g_{m3,4} = \frac{0.6m}{0.5} = 1.2\frac{mA}{V}$$

The low frequency differential gain is:

$$A_d = g_{m1,2}(r_{o2} \| r_{o4})$$

$$= 2\frac{mA}{V}(30\text{K} \| 30\text{K}) = \underline{\underline{30\frac{V}{V}}}$$

$$f_{P1} = 1/(2\pi C_L R_o)$$

where $R_o = r_{o2} \| r_{o4} = 15\text{K}\Omega$

$$f_{P1} = \frac{1}{2\pi \times 0.2p \times 15\text{K}} = \underline{\underline{53\text{MHz}}}$$

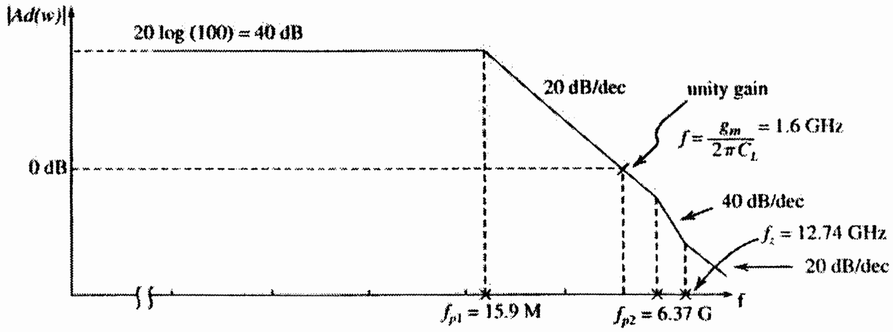
$$(7.193)\quad f_{P2} = \frac{g_{m3}}{2\pi C_m} = \frac{1.2\text{mA/V}}{2\pi \times 0.1\text{pF}}$$

$$= \underline{\underline{1.9\text{GHz}}}$$

$$(7.194)\quad f_Z = \frac{2g_{m3}}{2\pi C_m} = \frac{2 \times 1.2m}{2\pi \times 0.1p}$$

$$= \underline{\underline{3.8\text{GHz}}}$$

This figure is for 8.88



8.88

All V_{ov} are the same, all V_A are the same

$\Rightarrow$ All r_o's are identical, and all g_m's are identical

$$Ad(s) = g_m R_O \left[\frac{1 + \frac{s \cdot C_m}{2g_{m3}}}{1 + \frac{s \cdot C_m}{g_{m3}}}\right] \cdot \left(\frac{1}{1 + sC_L R_O}\right)$$

where we know that the frequencies of the zero f_z, and the pole f_P occur at very high frequencies. Thus we can assume that the pole $f_{P1} = 1/(2\pi C_L R_O)$ dominates the response of $A_d(s)$ passed the unity gain

Thus:

$$A_d(\omega) \simeq g_m R_O \left(\frac{1}{1 + j\omega C_L R_O}\right)$$

At unity gain: $|A_d(\omega_1)| = 1$

$$\Rightarrow 1 = \frac{g_m R_O}{\sqrt{1 + (\omega_1 C_L R_O)^2}}$$

$$\Rightarrow \omega_1^2 = \frac{(g_m R_O)^2 - 1}{(C_L R_O)^2}$$

Since $g_m R_O = g_m \frac{r_O}{2} >> 1 \Rightarrow \omega_1 \simeq g_m / C_L$

$$\Rightarrow f_1 = \frac{g_m}{2\pi \cdot C_L}$$

For: $V_A = 20$ V, $V_{OV} = 0.2$ V,

$I = 0.2$ mA, $C_L = 100$ fF, $C_m = 25$ fF

All r_o's are identical:

$r_O = \frac{V_A}{I_D} = \frac{20 \text{ V}}{(0.2 \text{ m}/2)} = 200$ kΩ. All g_m's are identical: $g_m = \frac{2I_D}{V_{OV}} = \frac{0.2 \text{ m}}{0.2} = \frac{1 \text{ mA}}{\text{V}}$

The low-frequency differential gain is:

$$A_{dDC} = g_m \times \frac{r_O}{2} = 1 \text{ m} \times 100 \text{ k}\Omega = 100 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi C_L \cdot R_O} \qquad R_O = \frac{r_O}{2} = 100 \text{ k}\Omega$$

$$f_{P1} = \frac{1}{2\pi \times 100 \text{ f} \times 100 \text{ K}} = 15.9 \text{ MHz}$$

$$f_{P2} = \frac{g_m}{2\pi C_m} = \frac{1 \text{ m}}{2\pi \times 25 \text{ f}} = 6.37 \text{ GHz}$$

$$f_Z = \frac{2g_m}{2\pi C_m} = 2 \times f_{PZ} = 12.74 \text{ GHz}.$$

Notice in the Bode plot the location of the unity-gain frequency.

8.89

a) $A_M = -A_{vo}\dfrac{R_L}{R_L+R_{out}} = -g_m r_o \dfrac{R_L}{R_L+r_o}$

$A_M = -5\times 40 \times \dfrac{40}{40+40} = -100\,V/V$

$R'_L = R_L \| R_{out} = R_L \| r_o = 20\,K\Omega$

$R_{gd} = R_{sig}(1+G_m R'_L)$ where $G_m = g_m$

$\Rightarrow R_{gd} = 20^K(1+5\times 20) = 2020\,K\Omega = 2.02\,M\Omega$

$R_{gd} = 2.02\,M\Omega$

$R_S = 0 \Rightarrow R_{gs} = R_{sig} = 20\,K\Omega$

$R_{C_L} = R'_L = 20\,K\Omega$

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$

$\tau_H = 2^p\times 20^K + 0.1^p\times 2.02^M + 1\times 20^K = 262\,ns$

$f_H = \dfrac{1}{2\pi\tau_H} = \underline{607.8\,KHZ}$

$|AM|\cdot f_H = +100\times 607.8 = 60.78\times 10^3\,KHz = \underline{60.78\,MHz}$

b) $R_s = 500\,\Omega$

$R_{out} = r_o[1+(g_m+g_{mb})R_s] = 40[1+(5+1)0.5] = 160\,K\Omega$

$A_M = -g_m r_o \dfrac{R_L}{R_L+R_{out}} = -5\times 40\times\dfrac{40}{40+160} = \underline{-40\,V/V}$

$R'_L = R_L \| R_{out} = 40K \| 160K = 32\,K\Omega$

$R_{gd} = R_{sig}(1+G_m R'_L)$

$G_m = \dfrac{g_m r_o}{r_o[1+(g_m+g_{mb})R_s]}$

$G_m = \dfrac{5\times 40}{40[1+(5+1)0.5]} = 1.25\,mA/V$

$R_{gd} = 20K(1+1.25^m\times 32^K) = 820\,K\Omega$

$R_{gs} = \dfrac{R_{sig}+R_s}{1+(g_m+g_{mb})R_s\dfrac{r_o}{r_o+R_L}}$

$R_{gs} = \dfrac{20^K+0.5^K}{1+(5+1)0.5\dfrac{40}{40+40}} = 8.2\,K\Omega$

$R_{C_L} = R_L \| R_{out} = R'_L = 32\,K\Omega$

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L} = 2\times 8.2 + 0.1\times 820 + 32\times 1$

$\tau_H = 130.4\,ns$

$f_H = \dfrac{1}{2\pi\tau_H} = \underline{1.22\,MHz}$

$\underline{|A_M|\cdot f_H = 48.8\,MHz}$

8.90

$$f_T = |A_M|\cdot f_H = \frac{1}{2\pi C_{gd}\times R_{sig}}$$

for $C_{gd} = 0.1$ pF and $R_{sig} = 10$ kΩ

$f_T = 159.2$ MHz

(b) If $|A_M| = 20$ V/V $\Rightarrow f_T = 159.2$ MHz

and $f_H = \dfrac{159.2\text{ M}}{20} = 7.96$ MHz

(c)

$g_m = 5\,\dfrac{\text{mA}}{\text{V}}$, $A_O = 100$ V/V, $R_L = 20$ kΩ

$$A_O = g_m\cdot r_O \Rightarrow r_O = \frac{100}{5\text{ m}} = 20\text{ k}\Omega$$

$R_L = r_O = 20$ kΩ

we can rewrite

$A_M = -G_m(R_O \| R_L)$ as

$$A_M = -(g_m r_O)\cdot\frac{R_L}{R_L+R_O}$$

$$\Rightarrow 20 = 100\times\frac{20}{R_O+20} \Rightarrow R_O = 80\text{ k}\Omega$$

Since

$R_O = r_O[1+g_m R_S] = 20\text{ K}\,[1+5\text{ m}\times R_S]$

$= 80\text{ K} \Rightarrow R_S = 600\ \Omega$

8.91

$: R_{gs} = \dfrac{R_{sig}+R_s}{1+\underbrace{(g_m+g_{mb})R_s}_{K}\dfrac{r_o}{r_o+R_L}}$

If we define:

$K = (g_m+g_{mb})R_s$ and $R_{sig} \gg R_s$

then: $R_{gs} \cong \dfrac{R_{sig}}{1+K\dfrac{r_o}{r_o+r_o}} = \dfrac{R_{sig}}{1+K/2}$

$G_m = \dfrac{g_m}{1+(g_m+g_{mb})R_s} = g_m/(K+1)$

$R'_L = R_i \| R_{out}$

$R_{out} = r_o[1+(g_m+g_{mb})R_s] = r_o(1+K)$

$R'_L = r_o \| r_o(1+K) = r_o\,\frac{(1+K)}{2+K}$

Using Eq. 6.148:

$R_{gd} = R_{sig}(1+G_m R'_L) + R'_L$

$R_{gd} = R_{sig}\left(1+\frac{g_m}{1+K}\times r_o\,\frac{1+K}{2+K}\right) + r_o\,\frac{1+K}{2+K}$

$R_{gd} = R_{sig}\left(1+\frac{A_o}{2+K}\right) + r_o\,\frac{1+K}{2+K}$

$R_{C_L} = R'_L = r_o\,\frac{1+K}{2+K}$

$\tau_H = R_{gs}C_{gs} + R_{gd}C_{gd} + R_{C_L}C_L$

$\tau_H = \frac{R_{sig}}{1+K/2}C_{gs} + R_{sig}\left(1+\frac{A_o}{2+K}\right)C_{gd} + (C_L+C_{gd})r_o\left(\frac{1+K}{2+K}\right)$

8.92

$R_{out} = r_o[1+(g_m+g_{mb})R_s] = r_o(1+K)$

$R_{out} = 40(1+K)$

$A_M = -g_m r_o\,\frac{R_L}{R_L+R_{out}} = -5\times40\times\frac{40}{40+40(1+K)}$

$\boxed{A_M = -\frac{200}{2+K}}$

$\tau_H = \frac{C_{gs}R_{sig}}{1+K/2} + C_{gd}R_{sig}\left(1+\frac{A_o}{2+K}\right) + (C_L+C_{gd})r_o\,\frac{1+K}{2+K}$

(From problem 6.111)

$\tau_H = \frac{2^{p}\,20^{k}}{1+K/2} + 0.1^{p}\times20^{k}\left(1+\frac{5\times40}{2+K}\right) + (1^{p}+0.1^{p})40^{k}\,\frac{1+K}{2+K}$

$\tau_H = \frac{80}{2+K} + 2\left(1+\frac{200}{2+K}\right) + 44\,\frac{1+K}{2+K}$

$\tau_H = \frac{528+46K}{2+K}$ nS

$f_H = \frac{1}{2\pi\tau_H} = \frac{(2+K)\times10^3}{2\pi(528+46K)}$ MHz

$f_T = |A_M|\cdot f_H$.

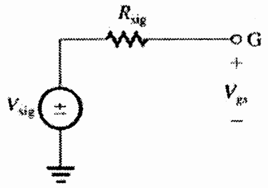
K	A_M (V/V)	f_H (MHz)	$\|A_M\|\cdot f_H$ (MHz)
0	−100	0.603	60.3
1	−66.67	0.832	55.47
2	−50.00	1.027	51.35
3	−40.00	1.195	47.8
4	−33.33	1.342	44.73
5	−28.57	1.471	42.03
6	−25.00	1.584	39.6
7	−22.22	1.686	37.46
8	−20.00	1.777	35.54
9	−18.18	1.859	33.8
10	−16.67	1.934	32.24
11	−15.38	2.002	30.79
12	−14.28	2.064	29.47
13	−13.33	2.121	28.27
14	−12.5	2.174	26.75
15	−11.76	2.223	26.14

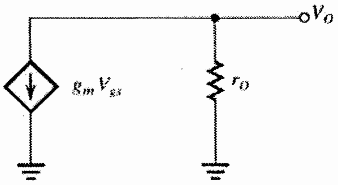
If f_H = 2MHz, then by looking at the table, K ≃ 11. Therefore: $K = 11 = (g_m+g_{mb})R_s \Rightarrow$

$R_s = \frac{11}{5+1} = \underline{1.83\,k\Omega}$

From the table : $\underline{A_M = -15.38}$

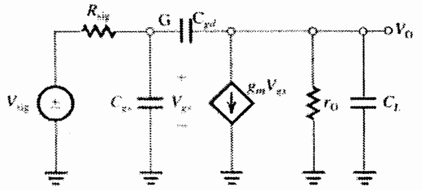
8.93


R_{sig}
G
+
V_{gs}
−
V_{sig}



then $\frac{V_O}{V_{sig}} = A_M = -g_m r_O$

The high-frequency small-signal circuit is:


R_{sig}
G
C_{gd}
V_O
V_{sig}
C_{gs}
+
V_{gs}
−
$g_m V_{gs}$
r_O
C_L

Using the open-circuit time constants method:

$\tau_H = R_{gs} \times C_{gs} + R_{gd} \times C_{gd} + R_L \times C_L$

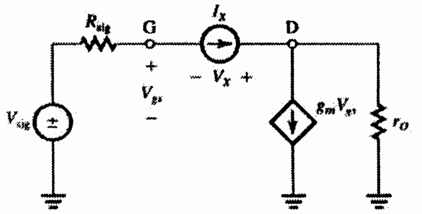
Setting $C_{gd} = C_L = 0$ we can see that

$R_{gs} = R_{sig}$

Setting $C_{gs} = C_{gd} = 0$ we can see that

$R_L = r_O$

To obtain R_{gd} set $C_{gs} = C_L = 0$ and consider the following circuit:


R_{sig}
G
I_X
D
+
V_{gs}
−
− V_X +
V_{sig}
$g_m V_{gs}$
r_O

$R_{gd} = R_X = \frac{V_X}{I_X}$

At node G: $V_{gs} = -I_X \cdot R_{sig}$ Eq 1.

At node D: $I_X = g_m V_{gs} + \frac{V_X + V_{gs}}{r_O}$ Eq 2.

Substituting V_{gs} in Eq 2 by Eq 1 and re-arranging:

$\frac{V_X}{I_X} = R_X = R_{sig}[1 + g_m r_O] + r_O$

Thus,

$\tau_H = C_{gs} \cdot R_{sig} + C_{gd}\,[R_{sig}[1 + g_m r_O] + r_O]$
$+ C_L\, r_O$

If $g_m = 1\frac{\text{mA}}{\text{V}}$, $r_O = 20\ \text{k}\Omega$, $R_{sig} = 20\ \text{k}\Omega$

$C_{gs} = 20\ \text{fF}$, $C_{gd} = 5\ \text{fF}$, $C_L = 10\ \text{fF}$

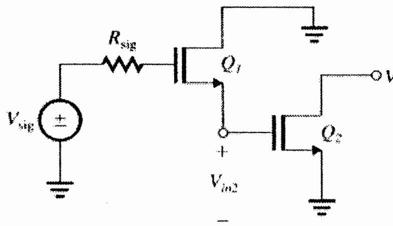
$A_M = -g_m \cdot r_O = -1 \times 20 = -20$ V/V

$\tau_H = 20\text{ f} \times 20\text{ K} + 5\text{ f}[20\text{ K}(1 + 1 \times 20) + 20\text{ K}]$
$+ 10\text{ f} \times 20\text{ K}$
$= 2.8$ ns

$f_H = \frac{1}{2\pi\,\tau_H} = 56.8$ MHz

$|A_M| \cdot f_H = 20 \times 56.8\text{ M} = 1.14$ GHz

(b) For the low-frequency analysis of the CD-CS consider the following circuit


R_{sig}
Q_1
V_O
V_{sig}
Q_2
+
V_{in2}
−

or the CS:

$V_O = V_{in2}(-g_{m2}r_{O2})$ (Eq 1)

For a CD amplifier:

$$A_M = \frac{(R_L \parallel r_O)}{(R_L \parallel r_O) + (1/g_m)}$$

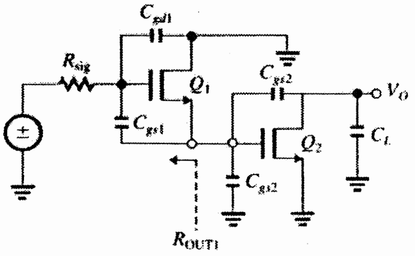
which adapted to our circuit provides:

$$V_{in2} = V_{sig}\frac{r_{O1}}{r_{O1} + 1/g_{m1}} \quad \text{(Eq. 2)}$$

Combining Eq. 1 and Eq. 2:

$$\frac{V_O}{V_{sig}} = A_M = \frac{-r_{O1}(g_{m2}\, r_{O2})}{\dfrac{1}{g_{m1}} + r_{O1}}$$

For the high-frequency analysis of the CD-CS consider the following circuit



$$\tau_H = C_{gd1} \times R_{gd1} + C_{gs1} \times R_{gs1} + C_{gs1} \times R_{gs2} + C_{gd2}R_{gd2} + C_L \cdot R_{CL}$$

To obtain the value of the resistors we must determine the corresponding equivalent circuits when all other capacitances are set to zero.

For C_{gd1}: Due to the ground connection on the drain of Q_1 C_{gd1} sees $R_{gd1} = R_{sig}$

For C_{gs1}:

$$I_X = \frac{V_X[1 + g_m\, r_O]}{R_{sig} + r_O}$$

$$\Rightarrow R_{gs1} = R_X = (R_{sig} + r_{O1})/(1 + g_{m1}r_{O1})$$

For C_{gs2}: C_{gs2} sees R_{OUT1} which for a *CD* amplifier is :

$$R_{OUT1} = R_{gs2} = \frac{1}{g_{m1}} \parallel r_{O1}$$

For C_{gd2}: Referring to the analysis of the CS amplifier

$$R_{gd} = R_{sig}(1 + g_m R_L') + R_L'$$

which adapted to our circuit becomes:

$R_L' = r_{O2}$ and

$$R_{sig} = R_{OUT1} = \frac{1}{g_{m1}} \parallel r_{O1}$$

$$\Rightarrow R_{gd2} = \left(\frac{1}{g_{m1}} \parallel r_{O1}\right) \cdot (1 + g_{m2} \cdot r_{O2}) + r_{O2}$$

For C_L: C_L sees $r_{O2} \Rightarrow R_{CL} = r_{O2}$.

Therefore:

$$\tau_H = C_{gd1} \cdot R_{sig} + C_{gs1}\left(\frac{R_{sig} + r_{O1}}{1 + g_{m1}r_{O1}}\right) + C_{gs2}\left(\frac{1}{g_{m1}} \parallel r_{O1}\right) + C_{gd2}\left[\left(\frac{1}{g_{m1}} \parallel r_{O1}\right)(1 + g_{m2}\, r_{O2}) + r_{O2}\right] + C_L \cdot r_{O2}$$

For the circuit parameters of part (a): $r_{O1} = r_{O2}$

$g_{m1} = g_{m2}$

$$A_M = -\frac{20\text{ K}}{\dfrac{1}{1\text{ m}} + 20\text{ K}} \cdot (1 \times 20) = -19\text{ V/V}$$

$$C_{gd1} \cdot R_{sig} = 5\text{ f} \times 20\text{ K} = 100\text{ ps}$$

$$C_{gs1} \cdot \frac{R_{sig} + r_{O1}}{1 + g_{m1}r_{O1}} = 20\text{ f} \times \frac{(20\text{ K} + 20\text{ K})}{(1 + 1 \times 20)} = 38.1\text{ ps}$$

$$C_{gs2}\left(\frac{1}{g_{m1}} \parallel r_{O1}\right) = 20\text{ f}(1\text{ K} \parallel 20\text{ K}) = 19.0\text{ ps}$$

$$C_{gd2}\left[\left(\frac{1}{g_{m1}} \parallel r_{O1}\right) \cdot (1 + g_{m2}r_{O2}) + r_{O2}\right] = 5\text{ f} \times \{(1\text{K} \parallel 20\text{ K})(1 + 20) + 20\text{ K}\} = 200\text{ ps}$$

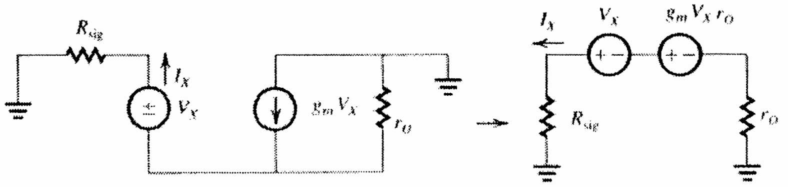
$$C_L \cdot r_{O2} = 10\text{ f} \times 20\text{ K} = 200\text{ ps}$$

$$\tau_H = 100 + 38.1 + 19 + 200 + 200 = 557.1\text{ ps}$$

$$\Rightarrow f_H = \frac{1}{2\pi\, \tau_H} = 285.7\text{ MHz}$$

$$|A_M| \cdot f_H = 19 \times 285.7\text{ M} = 5.4\text{ GHz}$$

Comparing with the stand-alone CS amplifier of part (a) we can see how A_v is approx. the same, while f_H and thus the gain-bandwidth product have increased by a factor of 5.



8.94

Each of the transistors is operating at a bias current of approximately 100 μA . Thus:

$$g_m = \frac{0.1}{0.025} = 4 \text{ mA/V},$$

$$r_\pi = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_e \simeq 250\ \Omega,\ r_O = \frac{100}{0.1} = 1 \text{ M}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{4 \text{ m}}{2\pi \times 400 \text{ M}} = 1.59 \text{ pF}$$

$$\Rightarrow C_\pi = 1.39 \text{ pF}$$

a) $R_{in} = (\beta + 1)[r_{e1} + (r_{\pi 2} \parallel r_{O1})]$

$$R_{in} = 101[250 \times 10^{-3} + 25 \text{ k}\Omega \parallel 1 \text{ M}\Omega]$$

$$\simeq 2.5 \text{ M}\Omega$$

Using Miller's Theorem for $C_{\mu 2}$:

$$A_M = -\frac{R_{in}}{R_{in} + R_{sig}} \times \frac{r_{\pi 2} \parallel r_{O1}}{r_{e1} + (r_{\pi 2} \parallel r_{O1})}$$

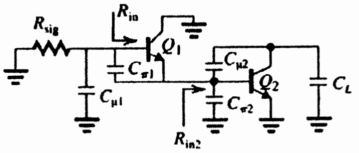
$$\times\, gm_2 r_{O2}$$

$$A_M = -\frac{2.5 \text{ M}}{2.5 \text{ M} + 0.01} \times \frac{25 \text{ K} \parallel 1 \text{ M}}{0.25 + (25 \text{ K} \parallel 1 \text{ M})}$$

$$\times 4 \times 1 \text{ M}$$

$$A_M = -3943.6 \text{ V/V}$$

b) To calculate f_H.



R_{sig}
R_{in}
Q_1
$C_{\pi 1}$
$C_{\mu 1}$
$C_{\mu 2}$
Q_2
$C_{\pi 2}$
R_{in2}
C_L

$$R_{\mu 1} = R_{sig} \parallel R_{in} = 10 \text{ k}\Omega \parallel 2.5 \text{ M}\Omega = 10 \text{ k}\Omega$$

$$R_{in2} = r_{\pi 2} \parallel r_{O1}$$

$$R_{in2} = 25 \text{ k}\Omega \parallel 1 \text{ M}\Omega$$

$$R_{in2} = 24.4 \text{ k}\Omega$$

$$R_{\pi 1} = \frac{R_{sig} + R_{in2}}{1 + \frac{R_{sig}}{r_{\pi 1}} + \frac{R_{in2}}{r_{e1}}}$$

$$R_{\pi 1} = \frac{10 + 24.4}{1 + \frac{10}{25} + \frac{24.4}{0.25}} = 0.35 \text{ k}\Omega$$

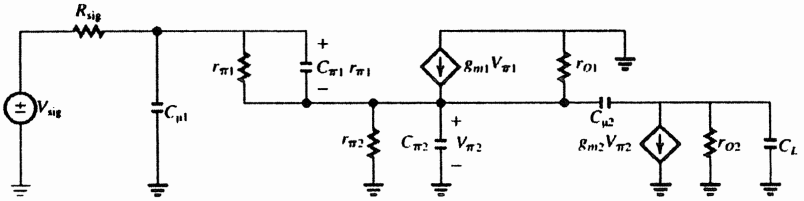
$$C_T = C_{\pi 2} + C_{\mu 2}(1 + g_{m2}\, r_{O2})$$

$$C_T = 1.39 + 0.2(1 + 4 \times 1000) = 801.6 \text{ pF}$$

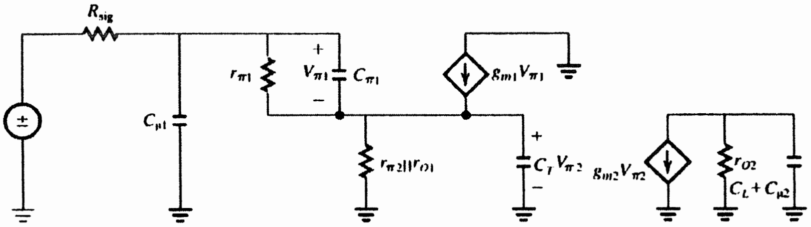
$$R_T = r_{\pi 2} \parallel r_{O1} \parallel \frac{r_{\pi 1} + R_{sig}}{\beta + 1}$$

$$= 25 \text{ K} \parallel 1000 \text{ K} \parallel \frac{25 + 10}{101}$$

$$R_T = 342\ \Omega$$



R_{sig}
V_{sig}
$C_{\mu 1}$
$r_{\pi 1}$
+
$C_{\pi 1}$ $r_{\pi 1}$
−
$g_{m1}V_{\pi 1}$
r_{O1}
$C_{\mu 2}$
$r_{\pi 2}$
$C_{\pi 2}$
+
$V_{\pi 2}$
−
$g_{m2}V_{\pi 2}$
r_{O2}
C_L



R_{sig}
$C_{\mu 1}$
$r_{\pi 1}$
+
$V_{\pi 1}$
$C_{\pi 1}$
−
$g_{m1}V_{\pi 1}$
$r_{\pi 2} \| r_{O1}$
+
C_T $V_{\pi 2}$
−
$g_{m2}V_{\pi 2}$
r_{O2}
$C_L + C_{\mu 2}$

$R_{\mu2} = r_{O2} = 1000\ \text{k}\Omega$

$\tau_H = C_{\mu1}R_{\mu1} + C_{\pi1}R_{\pi1} + C_T R_T$
$+ (C_{\mu2} + C_L) + R_{\mu2}$

$\tau_H = 0.2 \times 10 + 1.39 \times 0.35 + 801.6 \times 0.342$
$+ (0.2 + 1) \times 1000$

$\tau_H = 2 + 0.49 + 274.15 + 1200\ \text{ns}$

Thus $(C_L + C_{\mu2})R_{\mu2}$ is the dominating term,
The second most significant term is $C_T R_T$.
So $(C_L + C_{\mu2})$ dominates and then C_T or equivalently $C_{\mu2}$.

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 1476.6\ \text{n}} = 107.8\ \text{MHz}$$

8.95 Note: Although rather long, this is an excellent problem with considerable educational value.

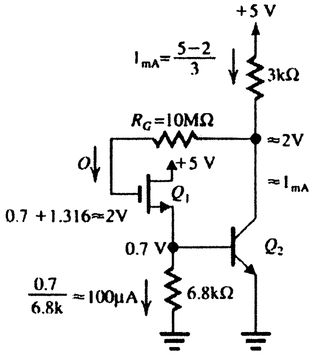
a) DC-Bias

For Q_1: $I_{D1} = \frac{1}{2}k'_n\frac{W}{L}(V_{GS} - V_t)^2$

$0.1 = \frac{1}{2} \times 2(V_{GS} - 1)^2$

$\Rightarrow V_{GS} = 1.316\ \text{V}$

$I_{D1} \simeq 0.1\ \text{mA}$
$I_{D1} \simeq 1\ \text{mA}$
see analysis



b) For Q_1:

$g_{m1} = \sqrt{2\pi k'_n\frac{W}{L}I_D} = \sqrt{2 \times 2 \times 0.1}$
$= 0.63\text{mA/V}$

For Q_2: $g_{m2} = 40\ \text{mA/V}$ $r_{\pi2} = \frac{200}{40} = 5\ \text{k}\Omega$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40 \times 10^{-3}}{2\pi \times 600 \times 10^6}$
$= 10.6\ \text{pF}$

Since $C_\mu = 0.8\ \text{pF} \Rightarrow C_\pi = 9.8\ \text{pF}$

c) at mid band :

$$\frac{V_\pi}{V_i} = \frac{6.8 \parallel 5}{(6.8 \parallel 5) + \frac{1}{g_{m1}}}$$

$$\frac{V_\pi}{V_i} = \frac{2.88}{2.88 + \frac{1}{0.63}}$$

$\frac{V_O}{V_i} = 0.64\ \text{V/V}$

$\frac{V_O}{V_\pi} = -g_{m2}V_\pi(1\ \text{K} \parallel 3\ \text{K})$ where we have neglected the effect of R_G.

$\frac{V_O}{V_\pi} = -40 \times \frac{3}{4}V_\pi = -30V_\pi \Rightarrow \frac{V_O}{V_\pi}$
$= 0.64 \times (-30) = -19.2\ \text{V/V}$

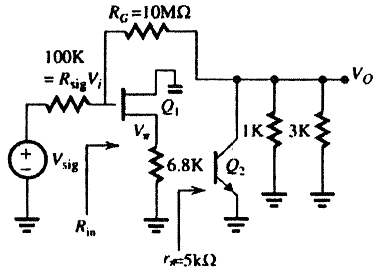
$\frac{V_O}{V_\pi} = -19.2\ \text{V/V}$

$$R_{in} = \frac{R_G}{1 - \frac{v_O}{v_i}} = \frac{10^M}{1 - (-19.2)} = 495\ \text{k}\Omega$$

$$\frac{V_O}{V_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} \times \frac{v_o}{v_i} = \frac{495}{495 + 100} \times -19.2$$
$= -16\ \text{V/V}$

d) At low frequencies :

$$C_1 \rightarrow f_{P1} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times (100 + 495) \times 10^3}$$
$= 2.7\ \text{Hz}$



$$C_2 \rightarrow f_{P2} = \frac{1}{2\pi \times 1 \times 10^{-6} \times (3 + 1) \times 10^{-3}}$$
$= 40\ \text{Hz}$

Thus $f_L \simeq 40$ Hz

e) At high frequencies :
The high frequency equivalent circuit is as follows:

$C_T = C_\pi + C_\mu(1 + g_{m2}R'_L)$

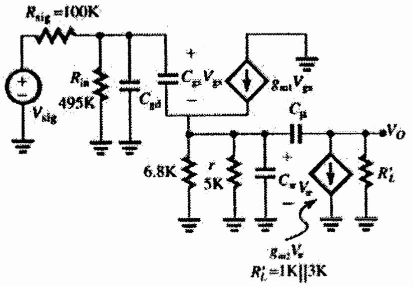
$= 9.8 + 0.8\left(1 + 40 \times \frac{3}{4}\right) = 34.6\ \text{pF}$

$$R_{gd} = R'_{sig} = 100\text{ K} \parallel 495\text{ K} = 83.2\text{ k}\Omega$$

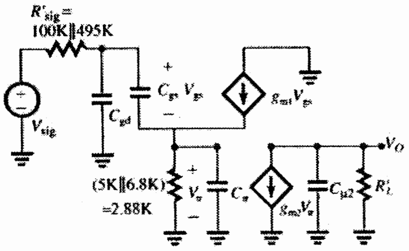
$$R_{gs} = \frac{R'_{sig} + (6.8\text{ K} \parallel 5\text{ K})}{1 + g_{m1}(6.8\text{ K} \parallel 5\text{ K})} = \frac{83.2 + 2.88}{1 + 0.63 \times 2.88}$$

$$= 30.6\text{ k}\Omega$$

$$R_T = 6.8 \parallel 5 \parallel \frac{1}{g_m} = 1\text{ k}\Omega$$


R_{sig}=100K
V_{sig}
R_{in}
495K
C_{gd}
+
C_{gs}
V_{gs}
−
$g_{m1}V_{gs}$
C_μ
V_O
6.8K
r
5K
+
C_π
V_π
−
R'_L
$g_{m2}V_\pi$
R'_L=1K||3K

⇓


R'_{sig}=
100K||495K
V_{sig}
C_{gd}
+
C_{gs} V_{gs}
−
$g_{m1}V_{gs}$
V_O
(5K||6.8K)
=2.88K
+
V_π
−
C_π
$g_{m2}V_\pi$
$C_{\mu2}$
R'_L

$$R'_L = 0.75\text{ k}\Omega$$

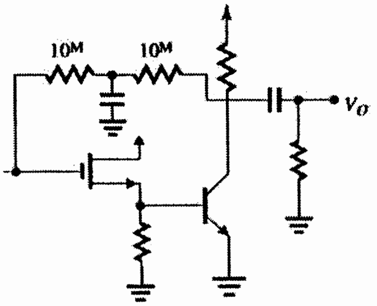
$$\tau_H = C_{gd}R_{gd} + C_{gs}R_{gs} + C_r R_r + C_\mu R'_L$$

$$\tau_H = 1 \times 83.2 + 1 \times 30.6 + 34.6 \times 1 + 0.8 \times 0.75 = 149\text{ nS}$$

$$f_H \simeq \frac{1}{2\pi\tau_H} = 1.07\text{ MHz}$$

f) There will no longer be a signal feedback. The lefthand side 10 MΩ Resistor will in effect appear between the input terminal and ground. Thus: R_{in} = 10 MΩ (a factor of 20 increase) and correspondingly A_M becomes:

$$A_M = \frac{10}{10.1} \times (-19.2) = -19\text{ V/V}$$



(an increase from −16 V/V) Now R'_{sig} becomes approximately 100 kΩ, as compared to 83.2 kΩ, and correspondingly R_{gd} becomes 100 kΩ, and R_{gs} becomes 36.6 kΩ while R_r and R'_L remain practically unchanged. Thus τ_H becomes 172.5 nS and f_H decreases from 1.07 MHz to 0.92 MHz.

8.96

$$V_{G1} = V_S \cdot \frac{2/g_m}{2/g_m + R_S} \qquad I = \frac{V_{G1}}{2/g_m}$$

$$V_O = I R_D = \frac{V_{G1}}{2/g_m} \times R_D$$

$$= V_S \times \frac{2/g_m}{2/g_m + R_S} \cdot \frac{R_D}{2/g_m}$$

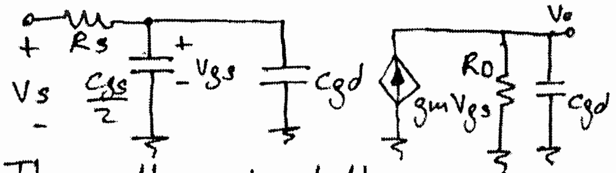
$$= \frac{V_S \cdot R_D}{2/g_m + R_S}$$

$$\Longrightarrow A_o = \frac{V_o}{V_S} = \frac{g_m R_D}{2 + g_m R_S}$$

$$g_m = \frac{200\,\mu A}{0.25V} = 0.8\,\frac{mA}{V}$$

$$\Longrightarrow A_o = \frac{0.8 \times 50}{2 + 0.8 \times 200} = 0.24\ V/V$$

The high-frequency equivalent circuit is:


R_S
+
V_S
−
$\frac{C_{gs}}{2}$
+
V_{gs}
−
C_{gd}
g_mV_{gs}
R_D
C_{gd}
V_o

Thus, the pole at the input has a frequency f_{p1}:

CONT.

$$f_{P1} = \frac{1}{2\pi R_S \times \left(\frac{C_{gs}}{2} + C_{gd}\right)}$$

$$= \frac{1}{2\pi \times 200K \times \left(1/2 + 1\right)p}$$

$$= \underline{\underline{530\,KHz}}$$

and the pole at the output has a frequency f_{P2}:

$$f_{P2} = \frac{1}{2\pi R_O C_{gd}} = \frac{1}{2\pi \times 50K \times 1p}$$

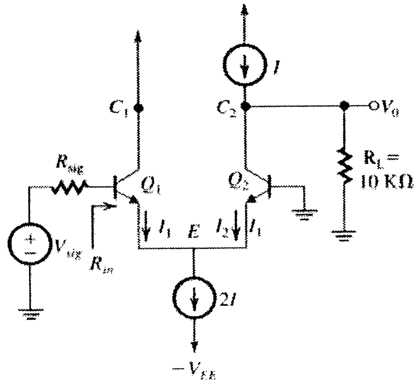
$$= \underline{\underline{3.18\,MHz}}$$

$$\text{Thus } f_H \simeq \frac{1}{\sqrt{\left(\frac{1}{530K}\right)^2 + \left(\frac{1}{3.18M}\right)^2}}$$

$$= \underline{523\,KHz}$$

Notice that this low value of f_H is due to the large value of R_S.

8.97



$$I_1 = I_2 = I = 1\text{ mA}$$

$$g_m = \frac{I}{V_T} = \frac{1\text{ mA}}{25\text{ mV}} = 40\text{ mA/V}$$

$$\gamma_e = \frac{V_T}{I_E} = 25\ \Omega$$

$$\gamma_\pi = (\beta + 1)\gamma_e \simeq 3\text{ k}\Omega$$

$$2\pi f_T = \frac{g_m}{C_\pi + C_\mu}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40 \times 10^{-3}}{2\pi \times 700 \times 10^{6}}$$

$$\simeq 9.1\text{ pF}$$

$$C_\pi = 9.1 - C_\mu = 8.6\text{ pF}$$

$$R_{in} = (\beta + 1)(2\gamma_e) = 2\gamma_\pi$$

$$= 6\text{ k}\Omega$$

$$A_M = \frac{1}{2}\left(\frac{R_{in}}{R_{in} + R_{sig}}\right) \times g_m R_L$$

$$= \frac{1}{2}\left(\frac{6}{6 + 20}\right) \times 40 \times 10$$

$$A_M = 46.15\text{ V/V}$$

The pole at the input side is

$$f_{p1} = \frac{1}{2\pi\left(\frac{c_\pi}{2} + c_\mu\right)(R_{sig} \parallel 2\gamma_\pi)}$$

$$= \frac{1}{2\pi\left(\frac{8.6}{2} + 0.5\right) \times 10^{-12} \times (20\text{ K} \parallel 6\text{ K})}$$

$$= 7.18\text{ MHz}$$

$$f_{p2} = \frac{1}{2\pi C_\mu R_L} = \frac{1}{2\pi \times 0.5 \times 10^{-12} \times 10\text{ K}}$$

$$= 31.8\text{ MHz}$$

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}}$$

$$\simeq 7\text{ MHz}$$

8.98

$I_1 = I_2 = I = 1mA \implies g_m = 40\ mA/V \quad , r_\pi = \frac{120}{40} = 3K\Omega$

$r_e = \frac{3}{121} \simeq 25\Omega \quad , C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40^m}{2\pi \times 700^M} = 9.1pF$

$\Downarrow$

$\underline{C_\pi = 8.6pF}$

Using Eq. 6.185:

$A_M = \frac{v_o}{v_{sig}} = \frac{1}{2}\left(\frac{R_{in}}{R_{in} + R_{sig}}\right) g_m R_L$

$R_{in} = 2r_\pi = 2 \times 3K\Omega = 6K\Omega$

$A_M = \frac{1}{2} \times \frac{6}{6+20} \times 40 \times 10^K = 46.15\ V/V$

$f_{p1} = \frac{1}{2\pi\left(\frac{C_\pi}{2} + C_\mu\right)\left(R_{sig} \| 2r_\pi\right)} = \frac{1}{2\pi\left(\frac{8.6}{2} + 0.5\right)\left(20^k \| 6^K\right)}$

$f_{p1} = 7.19MHz$

$f_{p2} = \frac{1}{2\pi C_\mu R_L} = \frac{1}{2\pi \times 0.5^p \times 10^K} = 31.8\ MHz$

$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 7.01MHz$

8.99

All the transistors in this problem are operating at a bias current of 0.5 mA and thus have :

$r_e = 50\Omega,\ g_m = 20\ \text{mA}/\text{V},\ r_\pi = 5\ \text{k}\Omega$

$$C_\pi + C_\mu = \frac{20\ \text{m}}{2\pi \times 400\ \text{m}} = 8\ \text{pF}$$

since $c_\mu = 2\ \text{pF} \Rightarrow C_\pi = 6\ \text{pF},\ r_o = \infty,$

$r_x = 0$

a) Common-Emitter amplifier:

$R_{sig} = 10\ \text{k}\Omega,\ R_C = 10\ \text{k}\Omega$

$$A_M = -\frac{r_\pi}{R_{sig} + r_\pi} g_m R_C = -\frac{5}{10+5} 20 \times 10$$

$$= -66.7\text{V}/\text{V}$$

$$f_H = \frac{1}{2\pi(R_{sig} \| r_\pi)[C_\pi + (1 + g_m R_c)C_\mu]} \Rightarrow$$

$$f_H = \frac{1}{2_\pi(10^k \| 5^k)[6^P + (1 + 20 \times 10)^2]}$$

$$= 117\ \text{KHz}$$

b) Cascode :

$$A_M = -\frac{\beta_1 \alpha_2 R_c}{R_{sig} + r_\pi} = -\frac{100 \times 0.99 \times 10}{10+5}$$

$$= -66\ \text{V}/\text{V}$$

Input pole

$$= f_{p1} = \frac{1}{2\pi(R_{sig} \| r_{M1})(C_{\pi 1} + 2C_{\mu 1})}$$

$$f_{p1} = \frac{1}{2_m(10^k \| 5^k)(6+4)^P} = 4.77\ \text{MHz}$$

output pole :

$$f_{p3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2^P \times 10^k} = 7.96\ \text{MHz}$$

pole at midband node :

$$f_{p2} = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi \times 6^P \times 50} = 530.5\text{MHz}$$

very high

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 4.1\text{MHz}$$

c) CC−CB Cascade (modified diff. amplifier)

$$A_M = \frac{\beta R_C}{R_{sig} + 2r_\pi} = \frac{100 \times 10}{10 + 10} = 50\ \text{V}/\text{V}$$

Input pole

$$: f_{p1} = \frac{1}{2\pi(R_{sig} \| 2r_\pi)(C_{\pi/2}/2 + C_\mu)}$$

$$f_{p1} = \frac{1}{2\pi(10^k \| 10^k)(3+2)^P} = 6.4\ \text{MHz}$$

Output pole:

$$f_{p2} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2^P \times 10^k} = 7.96\ \text{MHz}$$

$$\text{Thus: } f_H = \frac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 5\ \text{MHz}$$

d) CC−CE Cascade :

$$A_M = -\frac{(\beta_1 + 1)\beta_2 R_C}{R_{sig} + r_{\pi 1} + (\beta_1 + 1) r_{\pi 2}}$$

$$= -\frac{101 \times 100 \times 10}{10 + 5 + 101 \times 5} = -194\ \text{V}/\text{V}$$

Refer to Example 6.13 in :

$$R_{\mu 1} = (R_{sig} \| R_{in}) = 10^k \| (\beta + 1)[r_{e1} + r_{\pi 2}]$$

$$Ri_{\mu 1} = 10^k \| 101 \times [0.05 + 5] = 9.81\ \text{k}\Omega$$

$$R_{\pi 1} = r_{\pi 1} \| \frac{R_s + r_{\pi 2}}{1 + g_{m1} r_{\pi 2}} = 5 \| \frac{10 + 5}{1 + 20 \times 5} = 144\ \Omega$$

$$R_T = r_{\pi 2} \| \frac{r_{\pi 1} + R_{sig}}{\beta + 1} = 5^k \| \frac{5 + 10}{101} = 144\ \Omega$$

where

$$C_T = C_{\pi 2} + C_{\mu 2}(1 + g_{m2} R_C) = 6 + 2(1 + 200)$$

$$C_T l = 408\ \text{pF}$$

$$R_{\mu 2} = R_C = 10\ \text{k}\Omega$$

$$\tau_H = C_{\mu 1} R_{\mu 1} + C_{\pi 1} R_{\pi 1} + C_T R_T + C_{\mu 2} R_{\mu 2}$$

$$\tau_H = 2 \times 9.81 + 6 \times 0.144 + 408 \times 0.144 + 2 \times 10$$

$$\tau_H = 19.62 + 0.86 + 58.75 + 20 = 99.2\text{ns}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 99.2^n} = 1.6\ \text{MHz}$$

e) Folded Cascode :

$$A_M = -\frac{\beta_1 \alpha_2 R_C}{R_{sig} + r_{\pi 1}} = -\frac{100 \times 0.99 \times 10}{10 + 5}$$

$$= -66(\ \text{V})/\text{V}$$

Input pole :

$$f_{p1} = \frac{1}{2\pi(R_{sig} \| r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$$

$$= \frac{1}{2\pi(10 \| 5)(6 + 4)}$$

$$f_{p1} = 4.77\text{MHz}$$

At middle:

$$f_{p2} = \frac{1}{2\pi C_{\pi 2} r_{e2}} = \frac{1}{2\pi \times 6^P \times 0.05}$$

$= 530\text{MHz}$ very high!

At output:

$$f_{p3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2 \times 10}$$

$$f_{p3} = 7.96\text{MHz}$$

$$\text{Thus : } f_H \simeq \frac{1}{\sqrt{\frac{1}{4.77^2} + \frac{1}{7.96^2}}} = 4.1\ \text{MHz}$$

f) CC−CB Cascade :

$$A_M = \frac{(\beta_1+1)\alpha_2 R_C}{R_{sig} + (\beta_1+1)2r_e} = \frac{101 \times 0.99 \times 10}{10 + 101 \times 0.1}$$

$\simeq 50$ V/V

$$\text{Input pole}: f_{p1} = \frac{1}{2\pi(R_{sig} \| 2r_\pi)(C_{\pi/2} + C_\mu)}$$

$$f_{p1} = \frac{1}{2\pi(10^k \| 10^k)(3^p + 2^p)} = 6.4 \text{ MHz}$$

Output pole:

$$f_{p2} = \frac{1}{2\pi R_C C_\mu} = \frac{1}{2\pi \times 10^k \times 2^p} = 7.96 \text{ MHz}$$

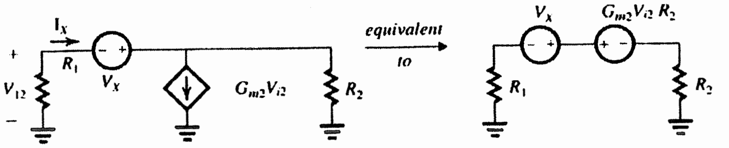
$$f_H \simeq \frac{1}{\sqrt{\frac{1}{6.4^2} + \frac{1}{7.96^2}}} = 5 \text{ MHz}$$

Summary of results :

ConFiguration	A_M (V/V)	f_H (MHz)	G.B (MHz)
a)CE	−66.7	0.117	7.8
b) Cascode	−66	4.1	271
c) CC_CB Cascade	+50	5.0	250
d)CC_CE Cascade	−194	1.6	310
e) Folded Cascode	−66	4.1	271
f) CC_CB Cascade	+50	5.0	250

8.100

(a) To obtain the DC-gain: for s = 0 in



$$\text{DC-gain} = (G_{m1}R_1) \times (G_{m2}R_2)$$

$$= (1 \times 100) \times (2 \times 50) = 10 \text{ K V/V} \rightarrow 80 \text{ dB}$$

(b) From Eq (9.175) if C_C is not connected:

$$\omega_{P1} = \frac{1}{C_1 \cdot R_1 + C_2 \cdot R_2}$$

$$= \frac{1}{0.1 \text{ p} \times 100 \text{ K} + 2 \text{ p} \times 50 \text{ K}}$$

$$= 9.1 \text{ M}\frac{\text{rad}}{\text{s}}$$

$$\Rightarrow f_{P1} = 1.45 \text{ MHz}$$

To obtain ω_{P2} we equate the coefficients of s^2 in Eq (9.171) to $1/(\omega_{P1} \cdot \omega_P)$

Thus, for C_C not connected.

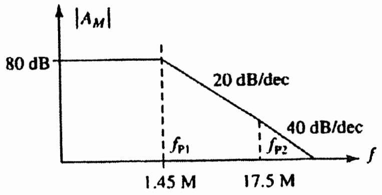
$$C_1 C_2 R_1 R_2 = \frac{1}{\omega_{P1} \cdot \omega_{P2}}$$

$$\Rightarrow \omega_{P2} = \frac{C_1 R_1 + C_2 R_2}{C_1 C_2 \cdot R_2 \cdot R_2}$$

$$\omega_{P2} = \frac{0.1 \text{ p} \times 100 \text{ K} + 2 \text{ p} \times 50 \text{ K}}{0.1 \text{ p} \times 2 \text{ p} \times 100 \text{ K} \times 50 \text{ K}}$$

$$= 110 \text{ MHz} \Rightarrow f_{P2} = 17.5 \text{ MHz}$$

The Bode plot for the gain magnitude is



(c) Since ($C_1 = 0.1$ pF) << ($C_2 = 2$ pF) and if $C_1 << C_C$ then from Eq (9.177)

$$\omega_{P2} \simeq \frac{G_{m2}}{C_2} \rightarrow \omega_{P2} \simeq \frac{2 \text{ m}}{2 \text{ p}} \simeq 1 \text{ G rad/s}$$

$$\rightarrow f_{P2} = 159 \text{ MHz}$$

two octaves below are = $\omega_{P2}/4$ = 250 M rad/s → 40 MHz then, from Eq 9.178:

$$250 \text{ M} \le \frac{G_{m1}}{C_C} \Rightarrow C_C \le \frac{1 \text{ m}}{250 \text{ M}} \Rightarrow C_C \le 4 \text{ pF}$$

For $C_C = 4$ pF and Eq (9.176)

$$\omega_{P1} \simeq \frac{1}{R_1 C_C G_{m2} R_2}$$

$$\simeq \frac{1}{100 \text{ K} \times 4 \text{ p} \times 2 \text{ m} \times 50 \text{ K}} = 25 \text{ K}\frac{\text{rad}}{\text{s}}$$

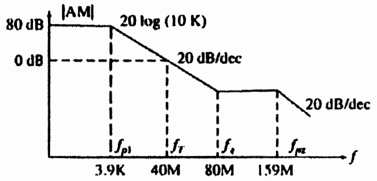
$$\Rightarrow f_{P1} = 3.9\ \text{KHz}$$

From Eq (9.173):

$$\omega_Z = \frac{G_{m2}}{C_C} = \frac{2\ \text{m}}{4\ \text{p}} = 500\ \text{M}\frac{\text{rad}}{\text{s}}$$

$$\Rightarrow f_Z = 79.6\ \text{MHz} \sim 80\ \text{MHz}$$

The Bode plot for the gain magnitude is



8.101

$G_{m1} = g_{m1} = g_{m2}$; transconductance of input stage.
$G_{m2} = g_{m6}$; transconductance of second stage.
$C_1 = C$ at node $D_2 = 0.2$ pF
$C_2 = C$ at node $D_6 = 3$ pF

$$\text{For } f_T = 50\ \text{MHz} = \frac{G_{m1}}{2\pi \times C_C}$$

$$\Rightarrow C_C = \frac{1\ \text{m}}{2\pi \times 50\ \mu} = 3.2\ \text{pF}$$

$$f_Z = \frac{G_{m2}}{2\pi C_C} = \frac{3\ \text{m}}{2\pi \times 3.2\ \text{P}} \cong 149\ \text{MHz}$$

$$f_2 = \frac{G_{m2}}{2\pi C_2} = \frac{3\ \text{m}}{2\pi \times 3\ \text{P}} \cong 159\ \text{MHz}$$

$$f_T(50\ \text{MHz}) < f_z\ (149\ \text{MHz}) < f_2(159\ \text{MHz})$$

8.102

For both transistors:
$V_{OV} = 0.2$ V $C_{gs} = 20$ fF
$I = 0.1$ mA $C_{gd} = 5$ fF
$|V_A| = 10$ V $C_{db} = 5$ fF

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1\ \text{mA}}{0.2\ \text{V}} = 1\ \frac{\text{mA}}{\text{V}}$$

$$r_O = \frac{V_A}{I_D} = \frac{10}{0.1\ \text{mA}} = 100\ \text{k}\Omega$$

(a) DC - Voltage gain

$V_O = -g_{m2} r_{O2} \times V_{gs2}$ and

$$V_{gs2} = -g_{m1} r_{O1} V_{sig} \Rightarrow A_M = \frac{V_O}{V_{sig}}$$

$$= (g_m r_O)^2 = (1 \times 100)^2 = 10\ \text{KV/V}$$

(b) Using the Miller approximation at node $G1$

$$C_{eq} = C_{gd}(1 + g_m R_L)\ \text{Eq (9.46)}$$

$$\Rightarrow C_{eq1} = C_{gd1}(1 + g_{m1} r_{O1})$$

$$= 5\ \text{f}\ (1 + 100) = 50\ \text{fF}$$

$$C_{in1} = C_{gs1} + C_{eq1} = 20\ \text{f} + 505\ \text{f} = 525\ \text{fF}$$

(c) $R_{sig} = 10\ \text{k}\Omega$

The pole caused by C_{in1} at node G_i is

$$f_{P1} = \frac{1}{2\pi .\ R_{sig} . C_{in1}} = \frac{1}{2\pi \times 10\ \text{K} \times 525\ \text{f}}$$

$$= 30.3\ \text{MHz}$$

(d) Using the Miller approximation at node G_2

$$C_{eq2} = C_{gd2}(1 + g_{m2} r_{O2}) = 505\ \text{fF}$$

$$C_{in2} = C_{db_1} + C_{gs2} + C_{eq2} = 5\ \text{f} + 20\ \text{f} + 505\ \text{f}$$

$$= 530\ \text{fF}$$

(e) At node $G2$ a pole is caused by C_{in2} and r_{O1}

$$f_{P2} = \frac{1}{2\pi \times 530 f \times 100\ \text{K}} = 3\ \text{MHz}$$

(f) The total capacitance at the output node is

$$C_{out} = C_{db2} + C_2$$

where, using the Miller theorem, C_2 is

$$C_2 = C_{gd2}\left(1 + \frac{1}{g_{m2} r_{O2}}\right) = 5\ \text{f}\left(1 + \frac{1}{100}\right) = 5.05\ \text{fF}$$

$$\Rightarrow C_{out} = 5\ \text{f} + 5.05\ \text{f} = 10.05\ \text{fF}$$

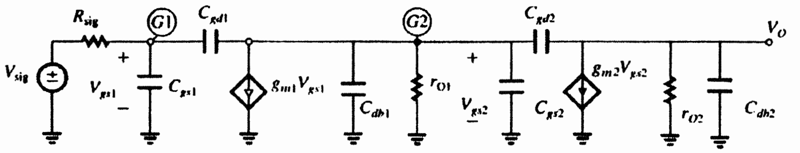
Thus a third pole is caused by C_{out} and r_{O2}

$$f_{P3} = \frac{1}{2\pi C_{out} .\ r_{O2}} = \frac{1}{2\pi \times 10.05 f \times 100\ \text{K}}$$

$$= 158.4\ \text{MHz}$$

From the 3 poles: $f_{P1} = 30.3$ MHz, $f_{P2} = 3$ MHz, $f_{P3} = 158.4$ MHz f_{P2}, the pole formed at the interface of Q_1 and Q_2 is dominant.

(g) The pole formed at the interface of Q_1 and Q_2 is dominant pole. It is at the frequency of 3 MHz.



9.1

$$A_F = \frac{A}{1+A\beta} = 100$$

$$A\beta = \frac{10^5}{100} - 1 = 999$$

$$\Rightarrow \beta = \frac{999}{10^5} = 9.99\times10^{-3}$$

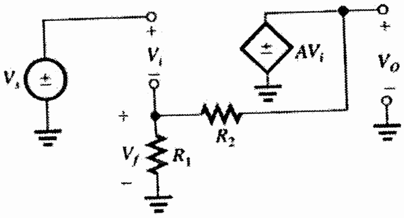
$$A = 10^3, \quad A_F = \frac{10^3}{1+10^3(9.99\times10^{-3})}$$

$$= 90.99$$

$$\frac{\Delta A_f}{A_f} = \frac{90.99-100}{100} \Rightarrow -9\%$$

9.2

(a) Replacing the op-amp with its equivalent circuit model:



$$V_f = \beta V_O = \frac{R_1}{R_1+R_2}\cdot V_O$$

$$\Rightarrow \beta = \frac{R_1}{R_1+R_2}$$

(b) $R_1 = 10\text{ k}\Omega$, $A_f = 10$ V/V, what is R_2 if:

(i) $A = 1000$ V/V

$$A_f = \frac{A}{1+\beta A} \Rightarrow \beta = \frac{1}{A_f} - \frac{1}{A}$$

$$= \frac{1}{10} - \frac{1}{10^3} = 0.099$$

$$\beta = \frac{R_1}{R_1+R_2} \Rightarrow R_2 = R_1\left(\frac{1-\beta}{\beta}\right)$$

$$= 10\text{ K}\frac{(1-0.099)}{0.099} = 91.01\text{ k}\Omega$$

(ii) $A = 100$ V/V

$$\beta = \frac{1}{10} - \frac{1}{100} = 0.09;$$

$$R_2 = 10\text{ K}\frac{(1-0.09)}{0.09} = 101.11\text{ K}$$

(iii) $A = 12$

$$\beta = \frac{1}{10} - \frac{1}{12} = 0.0167;$$

$$R_2 = 10\text{ K}\frac{(1-0.0167)}{0.0167} = 588.8\text{ k}\Omega$$

(c) if A decreases by 20%

(i) $A = 0.8\times1000 = 800$ V / V

$$A_f = \frac{800}{1+(0.099)(800)} = 9.975$$

$$\frac{\Delta A_f}{A} = \frac{9.975-10}{10} = -0.25\%$$

(ii) $A = 0.8\times100 = 80$ V / V

$$A_f = \frac{80}{1+(0.09)(80)} = 9.756$$

$$\frac{\Delta A_f}{A} = \frac{9.756-10}{10} = -2.44\%$$

(iii) $A = 0.8\times12 = 9.6$ V / V

$$A_f = 9.6/(1+(0.0167)(9.6)) = 8.27$$

$$\frac{\Delta A_f}{A} = \frac{8.27-10}{10} = -17.26\%$$

9.3

All output voltage is fed back $\therefore \beta = 1$

$$A_f = \frac{100}{1+100\times1} = 0.99$$

$$1+A\beta = 1+100\times1 = 101 \equiv 40.1\text{ dB}$$

$$V_o = 0.99V_s = 0.99\text{V}$$

$$V_i = V_s - V_o\beta = 1-0.99 = 10\text{mV}$$

$$A = 90 \Rightarrow A_f = \frac{90}{1+90\times1} \approx 0.989$$

$$\frac{\Delta A_f}{A_f} = \frac{0.989-0.99}{0.99} \cong -0.1\%$$

9.4

$$A_F = \frac{A_0}{1+A_0\beta} = \frac{1}{1/A_0+\beta} = \frac{1}{\beta(1+1/A_0\beta)}$$

so $A_f \to 1/\beta$ will be within $x\%$ when

$$1/(A_0\beta) = 0.01\times x$$

(a) For 1%: $A_0\beta = 1/0.01 = 100$

Many possible solutions.

Let $A_0 = 10^5$ + $A_0\beta = 100 \Rightarrow \beta = 10^{-3}$

(b) For 5%: $A_0\beta = 1/0.05 = 20$

Let $A_0 = 10^5$ + $A_0\beta = 20 \Rightarrow \beta = 2\times10^{-4}$

(c) For 10%: $A_0\beta = 1/0.1 = 10$

Let $A_0 = 10^5$ + $A_0\beta = 10 \Rightarrow \beta = 10^{-4}$

(d) For 50%: $A_0\beta = 1/0.5 = 2$

Let $A_o = 10^5$: $A_o\beta = 2 \Rightarrow \beta = 2\times10^{-5}$

%error	A_o	$A_o\beta$	$1+A_o\beta$
1	10^5	100	101
5	10^5	20	21
10	10^5	10	11
50	10^5	2	3

9.5

$0 \le p \le 1$ linear

(a) For $A_O = 1$:

$$A_{f1} = \frac{A_O}{1 + A_O\beta} = \frac{1}{1+0} = 1 \text{ V/V}$$

$$A_{f2} = \frac{1}{1 + 1 \times 0.5} = 0.667 \text{ V/V}$$

$$A_{f3} = \frac{1}{1 + 1 \times 1} = 0.5 \text{ V/V}$$

(b) For $A_o = 10$: $A_{f1} = \frac{10}{1+0} = 10 \text{ V/V}$

$$A_{f2} = \frac{10}{1 + \frac{10}{2}} = 1.6 \text{ V/V}$$

$$A_{f3} = \frac{10}{1 + 10 \times 1} = 0.909 \text{ V/V}$$

(c) For $A_O = 100$: $A_{f1} = \frac{100}{1+0} = 100 \text{ V/V}$

$$A_{f2} = \frac{100}{1 + \frac{100}{2}} = 1.96 \text{ V/V}$$

$$A_{f3} = \frac{100}{1 + 100} = 0.99 \text{ V/V}$$

(d) For $A_o = 10^4$: $A_{f1} = \frac{10^4}{1+0} = 10^4 \text{ V/V}$

$$A_{f2} = \frac{10^4}{1 + 10^4/2} = 1.99 \text{ V/V}$$

$$A_{f3} = \frac{10^4}{1 + 10^4} = 0.9999 \text{ V/V}$$

9.6

A_o : 2mV $\rightarrow$ 10v

$A_D = 10\text{v}/(2\times10^{-3})\text{V} = 5000 \doteq 74 \text{ dB}$

A_F : 200mV $\rightarrow$ 10V

$A_F = (10^{5}/(200) = 500 \doteq 54\text{dB}$

$$A_F = \frac{A_o}{1+\beta A_o} = \frac{5000}{1+5000\beta} = 500$$

$\Rightarrow 1 + 5000\beta = 10$

$\Rightarrow \beta = 9/5000 = 0.0018 \doteq -54\text{dB}$

$(1+A_o\beta) = 10 \equiv 20\text{dB}$

$A_o\beta = 5000(9/5000) = 9 \equiv 19.08\text{dB}.$

9.7

$$\frac{dA_f}{A_f} = \frac{1}{1+A\beta} \cdot \frac{dA}{A}$$

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{1+A\beta} \equiv -20\text{dB}$$

$\Rightarrow 1 + A\beta = +20\text{dB} \equiv 10$

$\therefore A\beta = 9$

Require $\frac{1}{1+A\beta} = \frac{1}{2} \Rightarrow A\beta = 1$

9.8

$$A_f = 25;\ \frac{\partial A_f}{A_f} = 1\%;\ \frac{\partial A}{A} = 10\%$$

$$\frac{\partial A_f}{A_f} = \frac{1}{1+A\beta} \cdot \frac{\partial A}{A} \Rightarrow 1 = \frac{10}{1+A\beta} \Rightarrow A\beta = 9$$

Since

$$A_f = \frac{A}{1+\beta A} \Rightarrow 25 = \frac{A}{1+9} \rightarrow A = 250 \text{ V/V}$$

thus $\beta = \frac{9}{250} = 0.036$

9.9

$$A_f = 25;\ \frac{\partial A_f}{A_f} = 1\%;\ \frac{\partial A}{A} = 10\%$$

$$\frac{\partial A_f}{A_f} = \frac{1}{1+A\beta} \cdot \frac{\partial A}{A} \Rightarrow 1 = \frac{10}{1+A\beta} \Rightarrow A\beta = 9$$

Since:

$$A_f = \frac{A}{1+A\beta} \Rightarrow 25 = \frac{A}{1+9} \rightarrow A = 250 \text{ V/V}$$

The lowest gain is $A - 10\%\ A = 250 - 25$

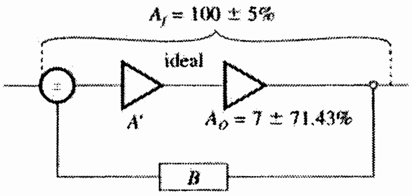
$= 225$ V/V and $\beta = \frac{9}{250} = 0.036$

9.10

For an output stage with gain varying between 2 and 12

$A_O = 7 \pm 5\text{V/V}$ i.e $7\ \text{V/V} \pm 71.43\%$

$A_f = 100 \pm 5\%$



Referring to the schematic the total open-loop gain is: $A = A' \times 7$

Since the first stage is ideal the total open-loop gain variation is 71.43%

Thus: $\dfrac{\partial A_f}{A_f} = \dfrac{1}{1+\beta A} \cdot \dfrac{\partial A}{A} \to 5\% = \dfrac{71.43\%}{1+\beta A}$

$\to \beta A = 13.286$

Since

$$A_f = 100 = \frac{A}{1+\beta A} = \frac{A}{14.286}$$

$\Rightarrow A = 1428.6\ \text{V/V}$

Thus, $\beta = \dfrac{13.286}{1428.6} = 0.0093$

if $\dfrac{\partial A_f}{A_f} = 0.5 = \dfrac{71.43}{1+\beta A} \to \beta A = 141.86$

$\Rightarrow A_f = 100 = \dfrac{A}{142.86} \to A = 14286\ \text{V/V}$

and $\beta = \dfrac{141.86}{14286} = 0.00993$

Following the same procedure:

if $A_f = 10$ and $\partial A_f / A_f = 5\%$

$\beta A = 13.286$

$A = 142.86 \Rightarrow A' = \dfrac{142.86}{7} = 20.41\ \text{V/V}$

if $A_f = 10$ and $\partial A_f / A_f = 0.5\%$

$\beta A = 141.86$

$A = 1428.6 \Rightarrow A' = \dfrac{1428.6}{7} = 204.09\ \text{V/V}$

9.11

$$A(S) = Am\frac{S}{S+W_L}$$

$$A_f(S) = \frac{Am\dfrac{S}{S+W_L}}{1+\dfrac{Am\,S}{S+W_L}\beta} = \frac{Am\,S}{S+W_L+Am\beta S}$$

$$= \frac{Am}{1+Am\beta} \cdot \frac{S}{S+\dfrac{W_L}{1+Am\beta}}$$

Thus

$$Am_f = \frac{Am}{1+Am\beta}$$

$$W_{Lf} = \frac{W_L}{1+Am\beta}$$

Both decreased by same amount

9.12

Worst case: A_{F1}

$$= \frac{A_O}{1+A_O\beta} = 9.8 \text{ (down 2\%)}$$

full battery: $A_{F2} = \dfrac{2A_O}{1+2A_O\beta} = 10$

from A_{F1}: $1 + A_O\beta = A_O/9.8$

$\therefore \beta = \dfrac{1}{9.8} - \dfrac{1}{A_O}$

Then $A_{F2} = \dfrac{2A_O}{1+2A_O\left[\dfrac{1}{9.8}-\dfrac{1}{A_O}\right]} = 10$

$\Rightarrow 1 + 2A_O\left[\dfrac{1}{9.8}-\dfrac{1}{A_O}\right] = \dfrac{2A_O}{10}$

$\Rightarrow 2A_O\left[\dfrac{1}{9.8}-\dfrac{1}{10}\right] = 2-1$

$\therefore 2A_O = 490$

[Check $\dfrac{2A_O}{1+2A_O\left[\dfrac{1}{9.8}-\dfrac{2}{2A_O}\right]\beta_{\text{const}}} = 10$

$\dfrac{A_O}{1+A_O\left[\dfrac{1}{9.8}-\dfrac{1}{A_O}\right]} = 9.8]$

If β varies by ±1% the worst case for A_f is if β by 1%

$$A_{f1} = 9.8 = \frac{A_0}{1+A_0\beta_{new}}$$

$$= \frac{A_0}{1+A_0\beta 1.01} \quad \beta_{new} \overset{\Delta}{=} 1.01\beta$$

$$\beta = \frac{1}{9.8} - \frac{1}{A_0} = \frac{1}{9.8} - \frac{2}{490} = \frac{48}{490}$$

$$9.8(1 + A_0\beta 1.01) = A_0$$

$$9.8\left(1 + A_0\frac{48}{490}1.01\right) = A_0$$

$$\Rightarrow 2A_0 = 645\frac{\text{V}}{\text{V}}$$

9.13

$$A_f = \frac{A_o}{1+A_o\beta} = 10 = \frac{100}{1+100\beta}$$

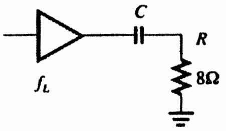
$$\therefore (1+A_o\beta) = 100/10 = 10$$

$$f_L' = f_L/(1+A_o\beta) = 100/10 = 10\text{ Hz}$$

$$f_h' = f_H(1+A_o\beta) = 10\text{K}\times 10 = 100\text{kHz}$$

9.14

For an 8Ω loudspeaker and f_L = 100 Hz



$$f_L = \frac{1}{2\pi RC} \Rightarrow C = \frac{1}{2\pi \times 100 \times 8} = 199\ \mu\text{F}$$

If feed-back is used and: A_f = 10 V/V,
A = 1000 V/V

$$A_f = \frac{A}{1+\beta A} \Rightarrow 1+\beta A = \frac{1000}{10} = 100$$

$f_{Lf} = f_L/(1 + \beta A) = 100/100 = 1$ Hz

Since feed-back reduces the effective fL_f, then a smaller capacitor C can be chosen for a larger value of f_L.

If fL_f must now be 50 Hz:

$$50 = \frac{f_L}{100} \Rightarrow f_L = 5\text{ KHz} = \frac{1}{2\pi \times 8 \times C}$$

$\rightarrow C = 3.98\ \mu\text{F}$

9.15

$$V_O = \frac{V_S \cdot A_1A_2}{1 + A_1A_2\beta} + \frac{V_N \cdot A_1}{1 + A_1A_2\beta}$$

$= \text{V}_{Sf} + \text{V}_{Nf}$

Closed loop again

is: $\dfrac{V_O}{V_S} = \dfrac{A_1 \cdot A_2}{1 + A_1A_2\beta} = 10\dfrac{\text{V}}{\text{V}}$ (1)

if the output ripple V_{Nf} is ±100 mV

$$\Rightarrow 100\text{ mV} = \frac{1 \times 0.9}{1 + 0.9A_2\beta} \rightarrow A_2\beta = 8.88$$

Substituting in (1):

$$\frac{0.9\text{A}_2}{1 + 0.9 \times 8.88} = 10 \rightarrow \text{A}_2 = 100\frac{\text{V}}{\text{V}}$$

thus $\beta = \dfrac{8.88}{100} = 0.0888$

Using the same procedure:

If V_{Nf} = ±10 mV, then $A_2\beta = 98.89$ and

$A_2 = 10\dfrac{\text{V}}{\text{V}}$

$\beta = 0.09899$

If V_{Nf} = ±1 mV, then $A_2\beta = 998.89$, and

$\text{A}_2 = 10\text{ K}\dfrac{\text{V}}{\text{V}}$, $\beta = 0.099889$

9.16

Nominal $\dfrac{A}{1+\beta A} = 100$, when A reduces

to $\dfrac{1}{10} \Rightarrow \dfrac{\text{A}/10}{1+\beta\text{A}/10} = 99$

Compare these two:

$\beta A = 890\ \ \beta\text{A} + 1 = 891$

$\Rightarrow A = 100(1 + \beta A) = 89.1 \times 10^3$

$$\beta = \frac{890}{89.1 \times 10^3} = 0.01$$

when A increased 10 times

$$A_f = \frac{10A}{1 + 10\beta A} = \frac{89.1 \times 10^4}{1 + 8900} = 100.10$$

when $A \rightarrow \infty$ $A_f = \dfrac{1}{\beta} = 100$

9.17

A_1 has f_H high, A_2 has A_M = 10 V/V with f_L = 80 Hz, f_H = 8 KHz.

$$A_F = \frac{A_1A_2}{1 + A_1A_2\beta} = 100$$

Require f_{Hf} = 40 KHz = $8(1 + A_1A_2\beta)$

$\therefore 1 + A_1A_2\beta = 40/8 = 5$

and $A_F = \dfrac{A_1A_2}{5} = 100 \Rightarrow A_1A_2 = 500$

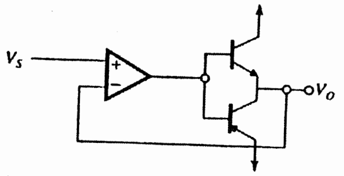
9.18

$\Rightarrow A_1 = 500/A_2 = 500/10 = 50$

$1 + A_1A_2\beta = 5 \Rightarrow \beta = 4/A_1A_2 = 4/500$

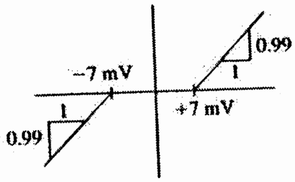
$\therefore \beta = 0.008$

$f_{Lf} = f_L/(1 + A_1A_2\beta) = 80/5 = 16$ Hz



Dead band with be narrowed by the factor $1 + A\beta = 1 + A$ since $\beta = 1$ and since $A >> 1$, $1 + A \rightarrow A$

$\therefore$ new limits are $\pm\dfrac{0.7}{A} = \pm\dfrac{0.7}{100} = \pm 7$ mV



New slope = gain $= A_f = \dfrac{A}{1+A}$

$$\Rightarrow \frac{100}{1+100} = 0.99$$

9.19

For $A = v_O/v_I = 10^2$ (select lowest A_o) to reduce · % change in gain by factor of 10

$$1 + A\beta = 10 \Rightarrow \beta = 9/10^2$$

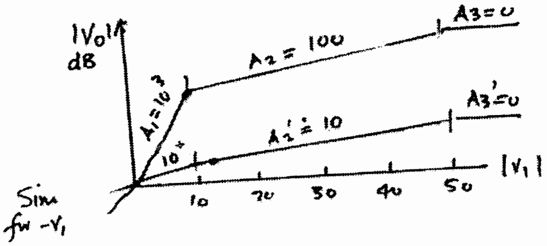
For $A_2 = 10^2$: $A_{2F} = 10^2/10 = 10$

For $A_1 = 10^3$: $A_{1F} = \dfrac{A}{1+A\beta}$

$$\therefore A_{2F} = \frac{10^3}{1+10^3(9/10^2)} = \frac{10^3}{91} = 10.98$$

For $A_3 = 0$: stays saturated

[For 10mV in and $A_1 = 10^3$, $V_O = 10$ V ?
For 10mV in and $A_2 = 10^2$. $V_O = 1$ V]



9.20

$\beta = \dfrac{V_f}{V_O} = \dfrac{R_1}{R_1+R_2}$ and open-loop gain 5 A

$$\Rightarrow A_f = \frac{A}{1+\beta A} = \frac{A}{1+\left(\frac{R_1 A}{R_1+R_2}\right)}$$

when: $A >> 1 \rightarrow A_f \simeq \dfrac{R_1+R_2}{R_1}$

$$A_f \simeq 1 + \frac{R_2}{R_1}$$

For: $A_f = 100$ V / V, $A = 10^4$, $R_1 = 1$ kΩ

$$100 = \frac{10^4}{1 + 10^4 \times \frac{1\text{ K}}{1\text{ K} + R_2}}$$

$$\rightarrow 1 + \frac{10^7}{10^3 + R_2} = 10^2$$

$$\rightarrow R_2 = 100.01\text{ k}\Omega$$

If we use the approximate result for $A >> 1$

$$100 = 1 + \frac{R_2}{1\text{ K}} \rightarrow R_2 = 99\text{ k}\Omega$$

If R_1 is removed:

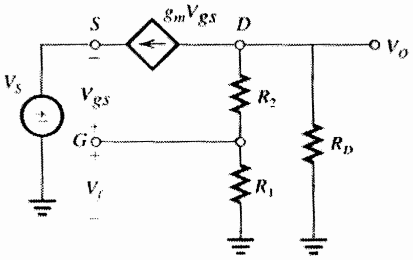
$$V_f = V_O \rightarrow \beta = 1 \rightarrow A_f = \frac{A}{1+A} \simeq 1$$

9.21

(a) If R_2 and R_1 are removed and the transistor gate is grounded then we have a *CG* amplifier

Thus: $A = g_m \cdot R_D$

Referring to Exercise 10.6, the equivalent small-signal circuit for Fig 10.7 c is:



$$\beta = \frac{V_f}{V_O} = \frac{R_1}{R_1+R_2}$$

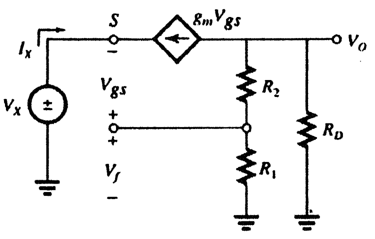
The amount of feed-back $1 + \beta A$ is:

$$1 + \frac{g_m R_D \cdot R_1}{R_1 + R_2}$$

(b) For a *CG* amplifier (with no feed-back)

$R_{in} = 1/g_m$ and $R_O = R_D$

(c) To obtain R_{inf} consider the following circuit



$$R_{inf} = \frac{V_X}{I_X}$$

$I_X = -g_m V_{gs}$ (Eq 1)

$V_O = I_X \cdot R_D$

$$V_f = \frac{V_O R_1}{R_1 + R_2} = (I_X R_D)\frac{R_1}{R_1 + R_2} \quad \text{(Eq 2)}$$

$V_X = -V_{gs} + V_f$ (Eq 3)

Sustituting Eq 1 and 2 into Eq 3:

$$V_X = \frac{I_X}{g_m} + \frac{I_X R_D R_1}{R_1 + R_2}$$

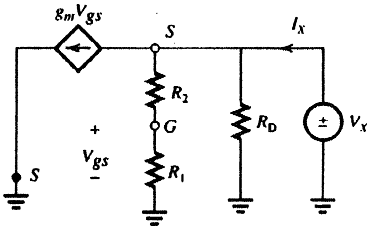
$$\Rightarrow \frac{V_X}{I_X} = \left(\frac{1}{g_m} + \frac{R_D R_1}{R_1 + R_2}\right)$$

Rearranging: $R_{inf} = \left(\frac{1}{g_m}\right)\left(1 + \frac{g_m R_D R_1}{R_1 + R_2}\right)$

Thus $R_{inf} = R_{in}(1 + A\beta)$

The input impedance is increased by a factor of $1 + A\beta$

To obtain R_{of} consider the following circuit:



$$R_{of} = \frac{V_X}{I_X}$$

$$I_X = g_m V_{gs} + \frac{V_X}{R_1 + R_2} + \frac{V_X}{R_D}$$

but $V_{gs} = \frac{R_1 \cdot V_X}{R_1 + R_2}$

$$\Rightarrow I_X = \frac{g_m R_1 V_X}{R_1 + R_2} + \frac{V_X}{R_1 + R_2} + \frac{V_X}{R_D}$$

$$= V_X\left\{\frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_D}\right\}$$

$$\Rightarrow R_{of} = \frac{V_X}{I_X} = \frac{1}{\frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_D}}$$

to re-arrange let's multiply by $\frac{R_D}{R_D}$

$$\Rightarrow R_{of} = \frac{R_D}{\frac{g_m R_1 R_D}{R_1 + R_2} + 1 + \frac{R_D}{R_1 + R_2}}$$

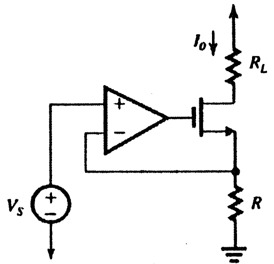
Since $R_1 + R_2 >> R_D \rightarrow \frac{R_D}{R_1 + R_2} \simeq 0$

$$R_{of} = \frac{R_D}{1 + \frac{g_m R_1 R_D}{R_1 + R_2}} = \frac{R_O}{1 + A\beta}$$

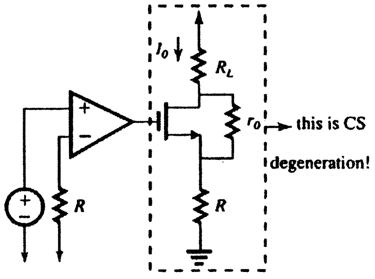
The output impedance is reduced by a factor of $1 + A\beta$

9.22

(a) A positive change in V_S results in a positive change at the gate of Q, which in turn will cause I_O to increase, causing a positive change is V_f



⇓ open circuit



$$\text{Gm} = \frac{g_m}{1 + g_m R + \frac{R}{r_o}}$$

b) $A = \mu \cdot G_m$

$$G_m = \frac{g_m}{1 + g_m R + \frac{R}{r_0}} \simeq \frac{g_m}{1 + g_m R}$$

c) $\beta = R$

d) $A_f = \dfrac{A}{1 + \beta A} = \dfrac{\mu \cdot \dfrac{g_m}{1 + g_m R}}{1 + R\mu \dfrac{g_m}{(1 + g_m R)}}$

e) when $\beta A = \dfrac{R\mu g_m}{1 + g_m \mathrm{R}} >> 1$

$$A_f = \frac{1}{\beta} = \frac{1}{R}$$

9.23

$$\beta = \frac{V_f}{I_O}$$

$$V_f = \left(I_O \times \frac{R_M}{R_1 + R_2 + R_M}\right) \times R_1$$

$$\Rightarrow \beta = \frac{V_f}{I_O} = \frac{R_M R_1}{R_1 + R_2 + R_M}$$

To obtain A, remove R_1, R_2 and R_M and ground the negative input of the OP-AMP

$V_O = I_O \cdot R_L$

$V_O = \mu V_i \Rightarrow \mu V_i = I_O R_L$

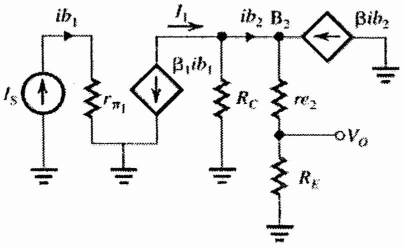
$$\to A = \frac{I_O}{V_i} = \frac{\mu}{R_L}$$

$$A_f = \frac{A}{1 + \beta A} = \frac{(\mu/R_L)}{1 + \left(\frac{R_M R_1}{R_1 + R_2}\right)(\mu/R_L)}$$

if βA is $>>1 \Rightarrow A_f \approx \dfrac{R_1 + R_2}{R_M \cdot R_1} \simeq \dfrac{1}{\beta}$

9.24

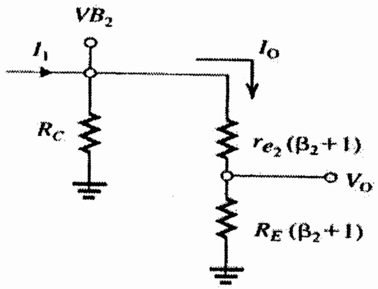
The equivalent small-signal circuit without the feed-back resistor R_F is:



$I_l = -\beta_1 I_S$

Reflecting r_{e2} and R_E towards the base B_2

But: $\beta_2 R_E >> R_C$ Therefore most of I_l will flow thru R_C and V_{B2} will be:



$V_{B2} = I_l R_C = -\beta_1 I_S \cdot R_C$

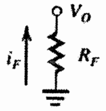
$$\text{Thus: } V_O = \frac{-\beta_1 I_S \cdot R_C \cdot R_E\,(\beta_2 + 1)}{(R_E + r_{e2})(\beta_2 + 1)}$$

we can also assume $R_E >> r_{e2}$ (e.g for $I_C = 1$ mA, $\beta = 100 \to r_e = 25\ \Omega$)

Then: $V_O \simeq -\beta_1 R_C I_S$

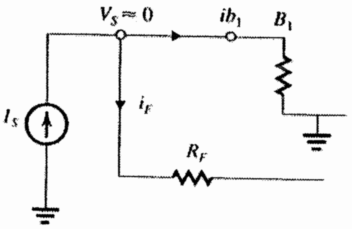
$$\Rightarrow A = \frac{V_O}{I_S} \equiv -\beta_1 \cdot R_C$$

To obtain β: if the signal voltage at the input is nearly zero:



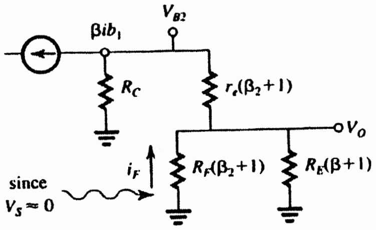
$$\Rightarrow \beta = \frac{I_f}{V_O} = \frac{-1}{R_F}$$

To obtain A_f: At the input side:



$I_S = ib_1 + i_F \qquad ib_1 = I_S - i_F$

At the output ride, after reflecting the emitter resistors towards the base:



Recall that: $R_C << \beta_2 R_E$

$R_E << R_F \rightarrow R_E\beta_2 << R_F\beta_2 \rightarrow R_C << R_F\beta_2$

Thus: $R_C \parallel \{(\beta+1)(r_e + R_F \parallel R_E)\} \approx R_C$

Therefore:

$V_{B2} \simeq -\beta_1 ib_1 \times R_C = -\beta_1(I_S - i_F)R_C$

again, since we can neglect r_e

$V_o \simeq V_{B2} = -\overline{\beta_1}(I_S - i_F)R_C$

and $V_O = -R_F i_F \rightarrow i_F = \dfrac{-V_O}{R_F}$

$$V_O = -\beta_1 I_S R_C - \beta_1 \frac{R_C}{R_F} V_O$$

$$\Rightarrow V_O\left(1 + \beta_1 \frac{R_C}{R_F}\right) = -\beta_1 R_C \cdot I_S$$

$$\Rightarrow A_f = \frac{V_O}{I_S} = \frac{-\beta_1 R_C}{1 + \beta_1 \frac{R_C}{R_F}}$$

which is the name result we would obtain from substituting A and β into: $A_f = A/(1 + A\beta)$

If $\beta_1 = 100$, $R_C = R_E = 10\ \text{k}\Omega$

$R_F = 100\ \text{k}\Omega$

$A = -100 \times 10\ \text{K} = -1 \times 10^6\ \text{V/A}$;

$$A_F = \frac{-10^6}{1 + 10^6 \times 10^{-5}} = -90.9\ \text{KV/A}$$

$\beta = -1/100\ \text{K} = -1 \times 10^{-5}$

9.25

To obtain A remove R_F and consider the small-signal response of the resulting CE:

$$V_O = -\beta I_i \cdot R_C \Rightarrow A = \frac{V_O}{I_i} = -\beta(R_C \parallel r_O)$$

if $r_O >> R_C \Rightarrow A \simeq -\beta R_C$

If the voltage at the input is near to cero volts

$$\Rightarrow I_f = -\frac{V_O}{R_F} \rightarrow \beta_F = \frac{I_C}{V_O} = \frac{-1}{R_F}$$

$$A_f = \frac{A}{1 + \beta_F A} = \frac{-\beta R_C}{1 + \frac{\beta R_C}{R_F}}$$

For $\beta = 100$, $R_C = 10\ \text{K}$ and $R_F = 100\ \text{K}$

$$A_f = \frac{-100 \times 10\ \text{K}}{1 + (100 \times 10\ \text{K})/100\ \text{K}}$$

$$= -90.9 \times 10^3\ \text{V/A}$$

9.26

$$A_F = \frac{A}{1 + A\beta} = \frac{10^3 \times 2}{1 + 2.10^3 \times 0.1} = 9.95\ \text{V}$$

$R_{if} = R_i(1 + A\beta) = 1(201) = 201\ \text{K}\Omega$

$R_{of} = R_o/(1 + A\beta) = 1/201 = 4.975\ \text{K}\Omega$

9.27

Here R_O is lowered by amount of feedback

i.e. $(1 + A\beta) = 80$

$\Rightarrow A\beta = 79$

$R_o = R_{of}(1 + A\beta) = 100 \times 80 = 8\ \text{K}\Omega$

9.28

The derivations are also valid for the case when A is a function of frequency

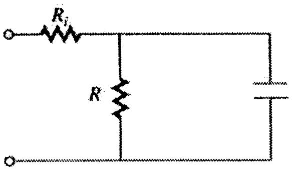
To obtain Z_{if} and Z_{of} we must replace A by its $A(S)$ form:

$$Z_{if} = R_i \cdot \left(1 + \frac{A_O}{\left(1 + \frac{S}{\omega_H}\right)} \cdot \beta\right)$$

$$Z_{of} = \frac{R_O}{1 + \frac{A_o}{1 + \frac{S}{\omega_H}} \beta}$$

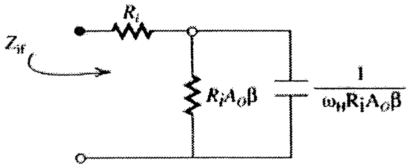
To obtain the equivalent circuits:

$$Z_{if} = R_i + \frac{R_i A_O \beta}{1 + \frac{S}{\omega_H}} \text{ which corresponds to:}$$



where $R \parallel C = \frac{R}{1 + RC_S}$; thus: $R = R_i A_O \beta$

$$RC = \frac{1}{\omega_H} \rightarrow C = \frac{1}{\omega_H \cdot R_i A_O \beta}$$

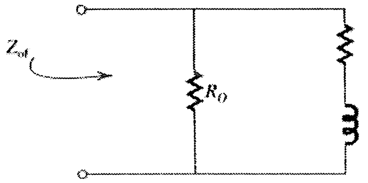


$$Z_{of} = \frac{R_O}{1 + \frac{A_O \beta}{\left(1 + \frac{S}{\omega_H}\right)}} = \frac{1}{\frac{1}{R_O} + \frac{(A_O \beta / R_O)}{1 + \frac{S}{\omega_H}}}$$

$$= \frac{1}{\frac{1}{R_O} + \frac{1}{\frac{1}{K} + \frac{S}{\omega_H K}}}$$

where $K = \frac{A_O \beta}{R_O}$

This is equivalent to a circuit of the form:



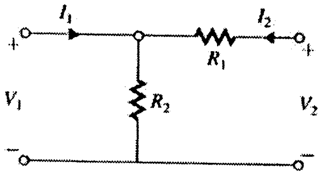
$$R = \frac{1}{K} = \frac{R_O}{A_O \beta}$$

$$L = \frac{1}{W_H \cdot K} = \frac{R_O}{W_H A_O \beta}$$

9.29

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$



(a) $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}\ \Omega$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} = \frac{R_2}{R_1 + R_2}$ V/V

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} = \frac{-R_2}{R_1 + R_2}$ A/A

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} = \frac{1}{R_1 + R_2}\ \mho$

(b) $\beta = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} = h_{12} = \frac{R_2}{R_1 + R_2}$

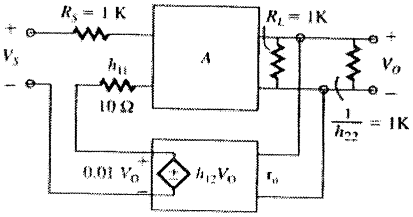
$\Rightarrow \frac{R_1}{R_2} = \frac{1}{\beta} - 1 = 99$

$\Rightarrow R_2 = R_1 / 99 = 10.1\ \Omega$

Thus $h_{11} = 10\ \Omega$; $h_{12} = 0.01$ V/V

$h_{21} = -0.01$ V/V; $h_{22} = 0.99 \times 10^{-3}\ \mho$

(c)



9.30

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{1000}{1 + A\beta} = 100\ \Omega$$

$$\Rightarrow 1 + A\beta = 1000/100 = 10$$

$$\Rightarrow A_F = \frac{A}{1 + A\beta} = \frac{100}{10} = 10 \text{ V/V}$$

if $\beta = 1$: $R_{of} \Rightarrow \frac{R_o}{1 + A} = \frac{1000}{1 + 10^4} = 9.9\ \Omega$

9.31

(a) If the loop gain is large $\frac{V_O}{V_S} \approx \frac{1}{\beta}$

$$\beta = \frac{V_f}{V_O} = \frac{R_1}{R_1 + R_2} = \frac{1}{1 + 10}$$

$$= \frac{1}{11} \rightarrow \frac{V_O}{V_S} = \frac{1}{\beta} = 11 \text{ V/V}$$

(b) To solve for i_{e_1} are i_{e_2}:

At the base of Q_2: $iR_f + iB_1 = i_{e2} - I_2$

$$\left(\frac{i_{e1}R_S + 0.7}{R_1}\right) + \frac{i_{e1}}{(\beta + 1)} = i_{e2} - I_2$$

Substituting and re-organizing:

$$(0.1099)\, i_{e1} + 1.7.10^{-3} = i_{e2}$$

$$i_a + i_{b2} = I_1$$

$$\Rightarrow \frac{\beta_1}{\beta_1 + 1} i_{e1} + \frac{1}{\beta_2 + 1} i_{e2} = I_1$$

$$\Rightarrow i_{e2} = (\beta_2 + 1)\left(I_1 - \frac{\beta_1}{\beta_1 + 1} i_{e1}\right)$$

$$= 101\left(0.1\text{m} - \frac{100}{101} i_{e1}\right)$$

$$0.1099\, i_{e1} + 1.7 \times 10^{-3} = 10.1 \times 10^{-3} - 100 i_{e1}$$

$$\Rightarrow i_{e1} = 83.08\ \mu\text{A}$$

and: $i_{e2} = 1.708$ mA

At the base of Q_1:

$$V_{B1} = i_{e1} \cdot R_S + 0.7 = 0.708 \text{ V}$$

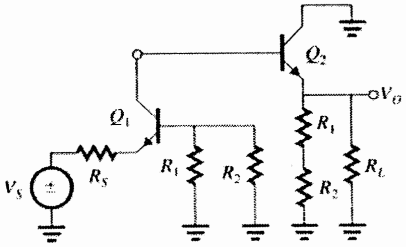
$$V_{R2} = V_{E2-B1} = R_2(i_{e2} - I_2)$$

$$= 10\text{ K}(1.708\text{ m} - 1\text{ m}) = 7.08 \text{ V}$$

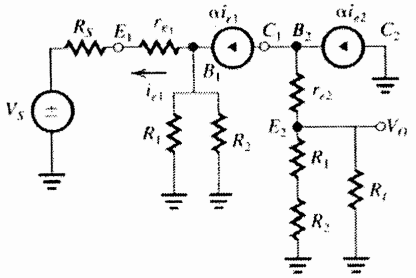
$$V_{E2} = V_{B1} + V_{E2-B1} = 0.708 + 7.08 = 7.788 \text{ V}$$

c) A-circuit

(Figure a)



(Figure b)



$$R_i = R_S + r_{e1} + \frac{R_1 \| R_2}{\beta + 1}$$

$$r_{e1} = \frac{V_T}{i_{e1}} = \frac{25 \text{ mV}}{83.08\ \mu\text{A}} = 301\ \Omega$$

$$R_i = 100 + 300 + \frac{1\text{ K} \| 10\text{ K}}{101}$$

$$R_i = 409\ \Omega$$

$$R_O \approx R_L \| (R_1 + R_2)$$

$$= 1\text{K} \| (1\text{ K} + 10\text{ K}) = 916\ \Omega$$

To obtain A: (see figure on previous page)

$$V_O = -\alpha i_{e1} \times (\beta + 1)\{(R_1 + R_2) \| R_L\} \quad (1)$$

$$i_{e1} = -V_S / \{R_S + r_{e1} + (R_1 \| R_2)/(\beta + 1)\} \quad (2)$$

Combining (1) & (2):

$$\Rightarrow V_O = \frac{\alpha V_S(\beta + 1)\{(R_1 + R_2) \| R_L\}}{R_S + r_{e1} + (R_1 \| R_2)/(\beta + 1)}$$

$$A = \frac{V_O}{V_S} = \frac{\beta\{(R_1 + R_2) \| R_L\}}{R_S + r_{e1} + (R_1 \| R_2)/(\beta + 1)}$$

$$= \frac{\beta R_O}{R_i} = 100 \times \frac{916}{409}$$

$A \simeq 224$ V/V

d) As in part a) $\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{11} = 0.0909$

e) $A_f = \frac{A}{1 + \beta A} = \frac{224}{1 + 224 \times 0.0909}$

$$= 10.48$$

$$R_{if} = R_i(1 + A\beta) = 409 \times 21.36 = 8.7 \text{ k}\Omega$$

$$R_{of} = \frac{R_O}{1 + A\beta} = \frac{916}{21.36} \simeq 43\ \Omega$$

$$R_{in} = R_{if} - R_S = 8.6 \text{ k}\Omega$$

$$R_{out} = 1/(1/43 - 1/1\text{ K}) = 44.9\ \Omega$$

$$\frac{\Delta A_f}{A_f} = \frac{10.48 - 11}{11} = -4.73\%$$

9.32

$Q_3 + Q_4$ form current multiplier

$\times\, 120/40 = \times\, 3$

$g_{m1} = 2\sqrt{\tfrac{1}{2}120\,(20/1)\,100} \doteq 693\,\mu A/V$

$g_{m5} = 2\sqrt{\tfrac{1}{2}\,60\,(20/1)\,1000} \doteq 1550\,\mu A/V$

$g_{m3} = 2\sqrt{\tfrac{1}{2}\,60\,(40/1)\,100} \doteq 693\,\mu A/V$

$g_{m4} = 2\sqrt{\tfrac{1}{2}\,60\,(120/1)\,300} \doteq 2078\,\mu A/V$

$g_{m2} = g_{m1} \qquad = 693\,\mu A/V$

$r_{o1} = 24/100 \Rightarrow 240\,K\Omega$

$r_{o2} = 24/100 \Rightarrow 240\,K\Omega$

$r_{o3} = 24/100 \Rightarrow 240\,K\Omega$

$r_{o4} = 24/300 \Rightarrow 80\,K\Omega$

$r_{o5} = 24/1000 \Rightarrow 24\,K$

(c) Openloop gain $A\beta \simeq \underline{g_{m1}(r_{o1}\|r_{o3})} \times 1$

$\left[\ \underline{(3\times g_{m1})}\left(\frac{r_{o4}}{3}\right) \equiv g_{m1}r_{o1}\right]$

$\beta = 1: \quad \therefore A \simeq g_{m1}(r_{o2}\|r_{o3})$

$\Rightarrow A \simeq 693 \times 120 \times 10^{-3} = 84$

(d) $A_F = \dfrac{A}{1+A\beta} = \dfrac{84}{1+84} = 0.988\ V/V$

$R_o = r_{o5}\,\|\,r_{o5} = 12\,K$

$R_{of} = R_o/(1+A\beta) = 12/85 = 140\,\Omega$

(e) To obtain $V_o/V_s = 5$ we could change direct connection from Q_{5S} to Q_{2G} by voltage divider $R_1/(R_1+R_2)$ to change β from 1 to $1/5.3$ then

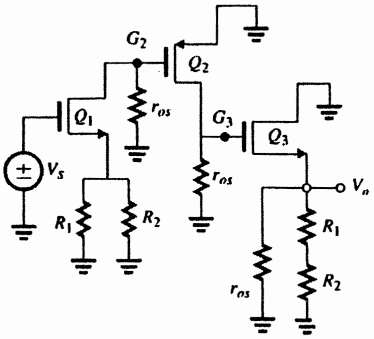
$A_F = \dfrac{84}{1+84\times 1/5.3} = \dfrac{84}{16.8} = 5$

Now $1+A\beta = 16.8$

$R''_{of} = R_o/(1+A\beta') = 12/16.8 = 714\,\Omega$

9.33

a) Transistors Q_1 and Q_2 are used in CS configuration. Therefore an increase in V_s causes the small-signal drain voltage of Q_1 to increase, followed by a voltage increase at the drain of Q_2. Transistor Q_3 is used in CD configuration. An increase in gate voltage at Q_3 results in an increase at the output V_o (source of Q_3) which through the voltage dividing feed-back causes V_f to increase. The feed-back is indeed negative.



b) If the loop gain $1 + A\beta$ is large then $A\beta >> 1$

$$A_f = \frac{A}{1+A\beta} \simeq \frac{1}{\beta} = \frac{R_1+R_2}{R_1} = 10$$

c) Find DC voltages : $V_{GS} - V_T = V_{OV} \rightarrow V_{GS} = V_{ov} + V_t$ for all transistors $|V_{GS}| = 0.2 + 0.5 = 0.7\,V$.

Then:

$V_{S1} = V_{DC} - V_{GS1} = 0.9 - 0.7 \rightarrow V_{S1} = 0.2\,V$

$V_{GS} = 0.2\,V$

$V_{G3} = V_{GS3} + V_{S3} = 0.7 + 0.2 = 0.9V$

$V_{G2} = V_{DD} - V_{SG2} = V_{DD} - 0.7$

For all current sources to operate in saturation

$|V_{DS}| \geq |V_{OV}| \qquad |V_{OV}| = 0.2\,V$

For source I_1 : $|V_{DS}| = V_{DD} - V_{G2} = 0.7\,V$

I_2: $V_{DS} = V_{G3} = 0.9\,V$

I_3: $V_{DS} = V_{S3} = 0.2\,V$.

d) obtain the A-circuit

Load of feed-back network at the input: $R_1 \,\|\, R_2$

Load of feed-back network at the output: $R_1 + R_2$

The A-circuit is:

Where $r_{os} = r_o$ is the output resistance of the current sources

Gain of each stage:

$$\begin{cases} \text{All } g_m\text{'s} = 2I_D/V_{OV} = 2 \times 0.1\ \text{m}/0.2 \\ \qquad = 1\ \text{mA/V} \\ \text{all } r_o\text{'s} = V_A/I_D = 10/0.1\ \text{m} = 100\ \text{k}\Omega \end{cases}$$

For Q_1: $A_{V1} = \dfrac{V_{G2}}{V_S} = g_{m\,eff}\,(r_{Oeff} \parallel r_{OS})$

$\dfrac{V_{G2}}{V_S} = \dfrac{g_{m1}}{1 + g_{m1}R_S} \cdot (r_{O1}(1 + g_{m1}R_S) \parallel r_{OS})$;

$R_S = R_1 \parallel R_2$

$R_S = 2\text{ K} \parallel 18\text{ K} = 1.8\text{ K}$ and $1 + g_m R_s$

$= 1 + 1.8 = 2.8$

$\Rightarrow \dfrac{V_{G2}}{V_S} = \dfrac{1\text{ m}}{2.8}[(100\text{ K} \times 2.8) \parallel 100\text{ K}]$

$= 26.3$ V/V

For Q_2:

$A_{V2} = \dfrac{V_{G3}}{V_{G2}} = g_{m2}(r_{O2} \parallel r_{OS})$

$r_{O2} = r_{OS}$

$g_{m2} = g_m$

$\dfrac{V_{G3}}{V_{G2}} = g_m\dfrac{r_O}{2} = 1\text{ m} \times \dfrac{100\text{ K}}{2} = 50$ V/V

For Q_3:

$r_{OS} \parallel (R_1 + R_2) = 100\text{ K} \parallel (18\text{ K} + 2\text{ K})$

$= 16.7\ \text{k}\Omega$

For a common-drain amplifier:

$$A_V = \frac{r_O \parallel R_L}{(r_O \parallel R_L) + \dfrac{1}{gm}}$$

where $R_L = r_{OS} \parallel (R_1 + R_2)$

$\Rightarrow A_{V3} = \dfrac{V_O}{V_{G3}} = \dfrac{100\text{ K} \parallel 16.7\text{ K}}{(100\text{ K} \parallel 16.7\text{ K}) + 1/1\text{ m}}$

$= 0.93$

Find the overall voltage-gain:

$A = A_{Vi} \cdot A_{V2} \cdot A_{V3} = 26.3 \times 50 \times 0.93$

$= 1223$ V/V

(e) Find β: $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{2}{2 + 18} = 0.1$

(f) $A_f = \dfrac{V_O}{V_S} = \dfrac{A}{1 + A\beta} = \dfrac{1223}{1 + 0.1 \times 1223}$

$= 9.92$ V/V

which is $\approx \dfrac{1}{\beta} = 10$ as found in (b)

(g) For the common-drain stage:

$R_o \simeq \dfrac{1}{g_m} \parallel (r_{OS} \parallel R_1 + R_2) = 1\text{ K} \parallel 16.7\text{ K}$

$= 944\ \Omega$

$R_{Of} = \dfrac{R_O}{1 + A\beta} = \dfrac{944}{1 + 0.1 \times 1223} = 7.66\ \Omega$

Since $R_L = \infty \Rightarrow R_{out} = R_{of}$

9.34

(a) $i_{D6} = I_{ref}$,

$i_{D1} = i_{D2} = i_{D3} = i_{D4} = 50\ \mu\text{A} \Rightarrow i_{D7} = 100\ \mu\text{A}$

$i_{D8} = i_{D5} = 250\ \mu\text{A}$

For Q_6: $2.5\text{ V} - I_{REF} \times 80\text{ K} = -2.5\text{ V} + V_{GS6} =$
$-1.5V$, $V_{OV} = 0.25$ V

$V_{GS6} = V_{tn} + V_{OV} = 0.75 + 0.25 = 1$ V

$\text{Iref} = \dfrac{4}{800\text{ K}} = 0.05\text{ mA} = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{\omega}{L}\right)_6 V_{OV}^2$

$= \dfrac{1}{2}100\mu\left(\dfrac{W}{L}\right)_6 (0.25)^2$

$\Rightarrow \left(\dfrac{W}{L}\right)_6 = \dfrac{16}{1}$

For Q_7 and Q_8:

$I_{D7} = 100\ \mu\text{A} \rightarrow \left(\dfrac{W}{L}\right)_7 = \dfrac{32}{1}$

$I_{D8} = 250\ \mu\text{A} \rightarrow \left(\dfrac{W}{L}\right)_8 = \dfrac{80}{1}$

For Q_1 and Q_2:

$I_{D1,2} = 50\ \mu \rightarrow (W/L)_{1,2} = 16/1$

For Q_3 and Q_4: Since $\mu_p C_{ox} = \dfrac{1}{2}\mu_n C_{ox}$ and

$I_{D1} = I_{D2} = I_{D3} = I_{D4}$

$\left(\dfrac{W}{L}\right)_{3,4} = 2 \times \left(\dfrac{W}{L}\right)_{1,2} = \dfrac{32}{1}$

For Q_5: Transistor Q_5 must be sized such as
$V_{ODC} = 0$ V

Since: $V_{D4} = V_{D3} = V_{G3,4}$

$\Rightarrow (2.5)\text{ V} - V_{SG3} - V_{GS5} = 0\text{ V}$

$V_{GS3} = V_{tp} = +0.75\text{ V} + 0.25\text{ V} = 1\text{ V}$

$\Rightarrow V_{GS5} = 2.5 - V_{SG3} = 1.5\text{ V}$

$\Rightarrow V_{ov5} - V_{DS5} - V_{th} = 1.5 - 0.75 = 0.75\text{ V}$

$\Rightarrow 250\mu = \dfrac{1}{2}100\mu\left(\dfrac{W}{L}\right)_5 (0.75)^2$

$\Rightarrow \left(\dfrac{W}{L}\right)_5 \approx 9$

(b) The common-range is the range of common-mode input voltage in which Q_1, Q_2 and Q_7 remain in saturation.

Q_1 and Q_2 will enter the triode region when:
$V_{G1,2} = V_{D1,2} + V_{th} = 1.5 + 0.75 = 2.25$ V

Q_7 will enter the triode region when:
$V_{G1,2} = V_{G7,2} - V_{th} + V_{GS1,2} = 1 - 0.75 + 1 = 1.25$ V

Thus, the common-mode input range is:
1.25 to 2.25 V

(c) Find g_m : $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{100\ \mu}{0.25} = 0.4\dfrac{\text{mA}}{\text{V}}$

Q_5: $V_{OV} = V_{GS5} - V_{th} = V_{G5} - V_{S5} - V_{th} = 2.5\text{ V}$
$- 1\text{ V} - 0\text{ V} - 0.75\text{ V} - 0.75\text{ V}$

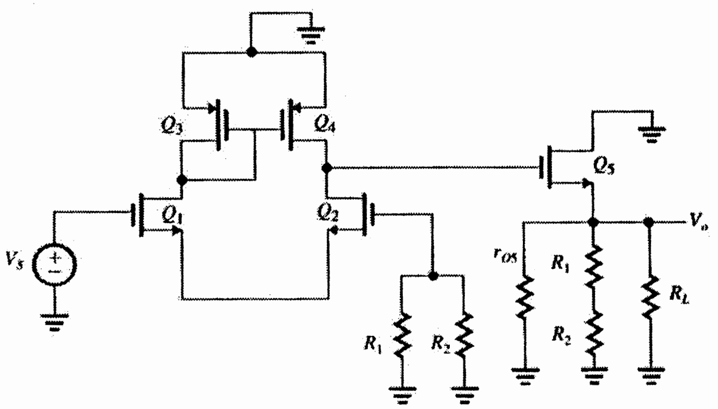
$g_{m3} = \dfrac{500\ \mu}{0.75} = 0.67\dfrac{\text{mA}}{\text{V}}$

(d) $r_O = V_A/I_O$

$r_{O1} = r_{O2} = r_{O3} = r_{O4} = \dfrac{10}{50\ \mu} = 200\ \text{k}\Omega$

$r_{O5} = r_{O8} = \dfrac{10}{250\ \mu} = 40\ \text{k}\Omega$

(e) A-circuit:



Gain: For the active-loaded differential path:

$$A_1 = g_m(r_{o2} \| r_{o4}) = 0.4\text{ m} \times \frac{200\text{ K}}{2} = 40\frac{\text{V}}{\text{V}}$$

For the common drain

stage: $A_1 = \dfrac{r_{o5} \| R'_L}{(r_{o5} \| R'_L) + \dfrac{1}{g_{m5}}}$

where $R'_L = r_{o5} \| R_L\,(R_1 + R_2) = 22.2\text{ k}\Omega$

$$\Rightarrow A_2 = \frac{40\text{ K} \| 22.2\text{ K}}{(40\text{ K} \| 22.2\text{ K}) + \dfrac{1}{0.67\text{ m}}} = 0.91$$

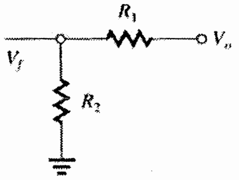
Total gain:

$$A = A_1 \times A_2 = 40 \times 0.91 = 36.4\frac{\text{V}}{\text{V}}$$

If

$$A_f = 10\frac{\text{V}}{\text{V}} = \frac{A}{1 + A\beta} \rightarrow \beta = \frac{1}{A_f} - \frac{1}{A} - \frac{1}{10} - \frac{1}{36.4}$$

$= 0.0725$



$$\beta = \frac{R_2}{R_1 + R_2}$$

and $R_1 + R_2 = 100\text{ k}\Omega$

$\Rightarrow R_2 = 0.0725 \times 100\text{ K} = 7.25\text{ k}\Omega$

$R_1 = 100\text{ K} - 7.25\text{ K} = 92.25\text{ k}\Omega$

(f) For the common drain stage

$$R_O = \frac{1}{g_{m5}} \| R'_L = 14.3\text{ K} \| 1.5\text{ K} = 1\text{ k}\Omega$$

$$R_{of} = \frac{R_O}{1 + A\beta} = \frac{1\text{ K}}{1 + 36.4 \times 0.0725} = 275\ \Omega$$

Excluding R_L:

$$R_{\text{OUT}} = 1 \Big/ \left(\frac{1}{275} - \frac{1}{100\text{ K}}\right) = 276\ \Omega$$

9.35

(a) V_f is taken across R_1.

If V_s increases, so does the current at the output of A_1 and the voltage at the output of A_2. It follows that V_o increases and a portion of it is sampled by the Resistor divider $R_1 / (R_1 + R_2)$.

(b)Refer to circuit diagram.

(c) A-circuit

$$A = \frac{V_S}{V_O} = \left(\frac{82\text{ K}}{82\text{ K} + 9\text{ K} + 10\text{ K} \| 90}\right) \times \left(\frac{20 \times 5\text{ K}}{3.2\text{ K} + 5\text{ K}}\right)$$

$$\times\,(20 \times (20 \| 20)) \times \left(\frac{100\text{ K} \| 1\text{ K}}{100\text{ K} \| 1\text{ K} + 1\text{ K}}\right)$$

$A = (0.82) \times (12 \cdot 19) \times (200) \times (0.5)$

$A = 1000$ V/V

(d) $\beta = \dfrac{10\text{ K}}{10\text{ K} + 90\text{ K}} = 0.1$

$1 + A\beta = 1 + 0.1 \times 1000 = 101$

(e) $A_f = \dfrac{1000}{1 + 100} = 9.9$ V/V

(f) $R_i = 82\text{ K} + (10\text{ K} \| 90\text{ K}) + 9\text{ K}$

$= 100\text{ K}$

$R_{if} = R_i \cdot (1 + A\beta) = 100\text{ K}(101)$

$= 10.1\text{ M}\Omega$

$R_{in} = 10.1\text{ M} - 9\text{ K} \simeq 10.1\text{ M}\Omega$

(g) $R_O = (1\text{ K} \parallel 100\text{ K} \parallel 1\text{ K}) = 500\ \Omega$

$R_{of} = \dfrac{R_O}{1 + A\beta} = \dfrac{500}{101} \simeq 5\ \Omega$

W/O R_L : $R_{OUT} = \dfrac{1}{\dfrac{1}{5} - \dfrac{1}{1\text{ K}}} = 5.02\ \Omega$

(h) If $f_H = 100\text{ Hz} \rightarrow f_{Hf} = 100 \times 101$
$= 10.1$ KHz

(i) if $A_1' = \dfrac{1}{2} A_1 \Rightarrow A = \dfrac{1000}{2} = 500$

$A_f = \dfrac{500}{1 + 50} = 9.8$

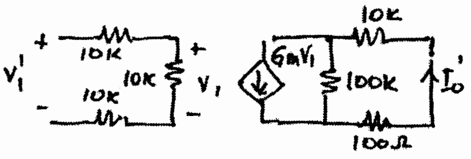
$\dfrac{\Delta A_f}{A_f} = \dfrac{9.8 - 9.9}{9.9} = -1.01\%$

9.36

$A = G_m = 100\text{mA/V}$ $\qquad \beta = 0.1\text{ V/mA}$

$r_{in} = 10K$, $r_{out} = 100K$

A-circuit



$V_i = V_1' \dfrac{10}{10+10+10} = V_1'/3$

$I_o' = \dfrac{G_m V_1\, 100}{100 + 10 + 0.1} = 30.28 V_1' \text{ mA}$

$A = \dfrac{I_o'}{V_i'} = 30.28\text{ mA/V}$

$A_F = \dfrac{A}{1+A\beta} = \dfrac{30.28}{1+30.28(0.1)} = 7.52 \frac{\text{mA}}{\text{V}}$

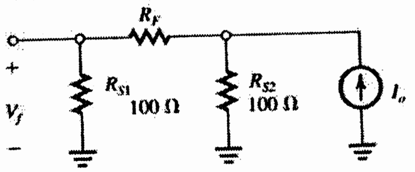
$R_i = R_S + R_{id} + R_1 = 30\text{K}\Omega$
$R_{if} = R_1(1+A\beta) = 120.8\text{ K}\Omega$
$R_{in} = R_{if} - R_S = 110.8\text{ K}\Omega$

$R_O = R_L + R_{op} + R_2 = 110.1\text{ K}\Omega$
$R_{of} = R_O(1+A\beta) = 443.4\text{K}\Omega$
$R_{out} = R_{of} - R_L = 433.4\text{K}\Omega$

9.37

(a) $A_f = \dfrac{A}{1\ 1\ A\beta}$ if A is large then $A_f \simeq \dfrac{1}{\beta}$
$= 0.1$ A/V

To obtain β:



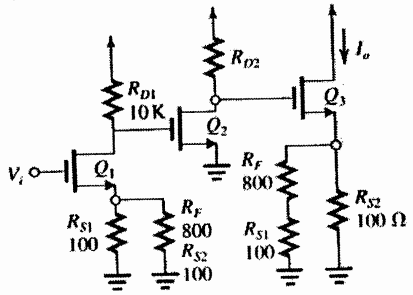
$\beta = \dfrac{V_f}{I_O}$

$\beta = \dfrac{R_{S2} \cdot R_{S1}}{R_{S2} + R_F + R_{E1}} = \dfrac{100 \times 100}{100 + R_F + 100}$

$= \dfrac{10^4}{200\ 1\ R_F}$

If $\dfrac{1}{\beta} = 0.1 \rightarrow R_F = 800\ \Omega$

(b) A-circuit:



To obtain A:
For the first stage:

$\dfrac{V_{D1}}{V_i} = \dfrac{-g_m \cdot R_{D1}}{1 + g_m(R_S \parallel R_F\ 1\ R_{S2})}$

Substituting: $g_m = 4\text{ mA / V}$

$R_{D1} = 10\text{ K } (R_{S1} \parallel R_F\ 1\ R_{S2}) = 90\ \Omega$

$\dfrac{V_{D1}}{V_i} = -29.4$ V/V

For the second stage:

$\dfrac{V_{D2}}{V_{D1}} = -g_m R_{D2} = -4 \times 10 = -40\text{ V / V}$

For the third stage: $\dfrac{I_O}{V_{D2}} = \dfrac{I_{D3}}{V_{G3}}$

$= \dfrac{1}{1/g_m\ 1\ R_{S2} \parallel (R_{S1}\ 1\ R_F)}$

Substituting

$g_m = 4\text{ mA / V } (R_{S2} \parallel R_{S1}\ 1\ R_F) = 90\ \Omega$

$\dfrac{I_O}{V_{D2}} = 3\text{ mA / V}$

Combining the gain of the three stages:

$$A = \frac{I_O}{V_i} = -29.4 \times -40 \times 3 \times 10^{-3}$$

$$= 3.53 \text{ A/V}$$

(c) $1 + A\beta = 1\ 1\ 3.53 \times 10 = 36.3$

$$A_f = \frac{V_O}{I_S} = \frac{A}{1\ 1\ A\beta} = \frac{3.53}{36.3} = 0.097 \text{ A / V}$$

The design value for A_f is 0.1 A/V

$$\frac{\Delta A_f}{A_f} = -\frac{0.003}{0.1} = -3\%$$

To change A_f such as it gets closer to 0.1 A/V R_f must be reduced, this will increase both the values of A and B

(d) $R_{D3} = r_{O3} + [R_{S2} \| (R_F + R_{S1})]$

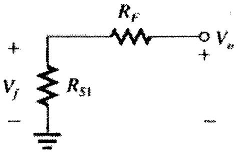
$$= 20 \text{ K} + 90 \simeq 20 \text{ k}\Omega$$

$$R_{out} = R_{of} = (1 + A\beta)R_{o3} = 36.3 \times 20 \text{ K}$$

$$= 726 \text{ k}\Omega$$

(e) If the output is taken at V_o:

The feed-back network changes to:

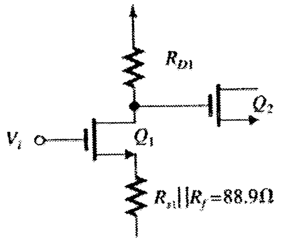


$$\beta = \frac{100}{800 + 100} = 0.111$$

Therefore the A-circuit changes at the source of Q_1 to:

$$R_{S1} \| R_f = 100 \| 800$$

$$= 88.9 \ \Omega$$



For the first stage:

$$\frac{V_{D1}}{V_i} = \frac{-g_m R_{D1}}{1 + g_m((R_{S1} \| R_f))}$$

$$= -29.5 \text{ V / V}$$

For the second stage: $\dfrac{V_{D2}}{V_{D1}}$

$$= -40 \text{ V / V (unchanged)}$$

For the third stage:

$$\frac{V_{O2}}{V_{D2}} = \frac{V_{S3}}{V_{G3}}$$

$$= \frac{R_{S2} \| (R_F + R_{S1})}{1/g_m + R_{S2} \| (R_F + R_{S1})}$$

$$= 0.265 \text{ V / V}$$

Combining the gains of all three stages:

$$A = \frac{V_O}{V_S} = -29.5 \times -40 \times 0.265$$

$$= 312.7 \text{ V / V}$$

(f) $1 + A\beta = 1 + 312.7 \times 0.111 = 35.74$

$$R_{out2} = (1 + A\beta)R_o$$

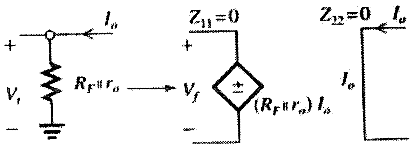
$$= (1 + A\beta)\,(R_{S2} \| R_{S1} + R_F) \| r_O \| \frac{1}{g_m}$$

$$= 35.74 \times \{90 \| 20 \text{ K} \| 250\}$$

$$= 2.35 \text{ k}\Omega$$

9.38

(a) The β circuit:



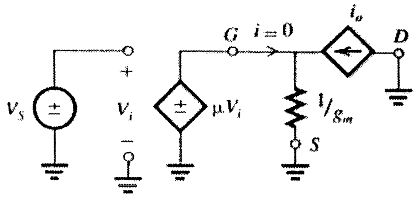
$$\beta = \frac{V_f}{I_O} = \frac{R_f r_O}{R_f + r_O}$$

$A_f = \dfrac{A}{1 + A\beta}$ for $A\beta >> 1$, $A_f \simeq \dfrac{1}{\beta}$ if

$$A_f = 10 \frac{\text{mA}}{\text{V}} \Rightarrow \beta = 100 \ \Omega$$

$$R_F = 100.5 \ \Omega$$

(b) The A-circuit:



$\mu = 1000$ V / A

$g_m = 2$ mA / V

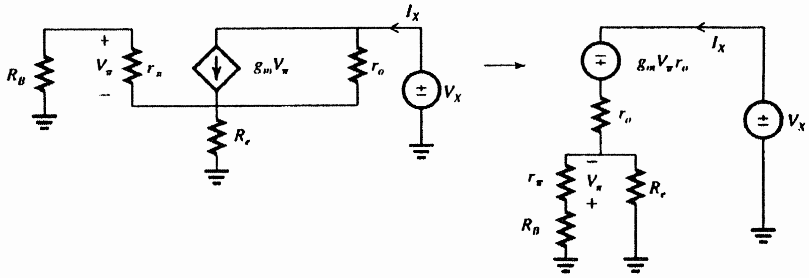
$r_O = 20$ K

$$A = \frac{I_O}{V_S} = \mu g_m = 2\text{A / V}$$

(c) $A\beta = \mu g_m(R_F \| r_O) = 2 \times 100 = 200$

$1 + A\beta = 201$

This figure is for 9.39


I_X
R_B
$+$
V_π
$-$
r_π
g_mV_π
r_o
$\pm$
V_X
R_e
$g_mV_\pi r_o$
I_X
r_o
r_π
$-$
V_π
$+$
R_e
R_B
V_X

(d) $A_f = \dfrac{A}{1 + A\beta} = \dfrac{2}{201} = 9.95$ mA/V

(e) $R_O = r_O \Rightarrow R_{of} = r_O(1 + A\beta)$

$= 20\text{ K} \times 201 = 4.02\text{ M}\Omega$

9.39

$$V_\pi = -\frac{r_\pi I_X \cdot R_e}{R_e + R_B + r_\pi}$$

$$V_X + g_m V_\pi r_o = I_X\{r_o + (R_e \| r_\pi + R_B)\}$$

$$V_X - \frac{r_\pi \cdot R_e}{R_e + R_B + r_\pi} g_m r_O I_X$$

$$\frac{V_X}{I_X} = r_O + (R_e \| r_\pi + R_B)$$

$$= I_x\{r_o + R_e \| (r_\pi + R_B)\}$$

$$\frac{V_X}{I_X} = r_O + \left(R_e \| (r_\pi + R_B) + g_m r_O \frac{r_\pi R_e}{R_e + R_B + r_\pi}\right)$$

$$\left\{1 + g_m r_O \cdot \frac{r_\pi \cdot R_e}{R_e + R_B + r_\pi} \times \frac{R_e + R_B + r_\pi}{R_e(r_\pi + R_B)}\right\}$$

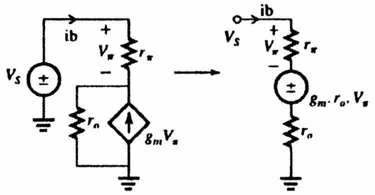
$$R_O = r_O + R_e \| (r_\pi + R_B)\left\{1 + g_m r_O \frac{r_\pi}{r_\pi + R_B}\right\}$$

If $R_B = 0$ $R_O = r_O + (R_e \| r_\pi)\{1 + g_m r_O\}$

R_O is maximum when $R_e >> r_\pi$ then:

$R_O = r_O + r_\pi\{1 + g_m r_O\}$

If $R_B = 0$ and $R_e = \infty$:


ib
V_S
$\pm$
$+$
V_π
$-$
r_π
r_o
g_mV_π
ib
V_S
$+$
V_π
$-$
r_π
$\pm$
$g_m \cdot r_o \cdot V_\pi$
r_o

$V_\pi = ib \cdot r_\pi$

$V_S - g_m r_O V_\pi = ib(r_\pi + r_O) \Rightarrow V_S = g_m r_O r_\pi ib$
$+ ib(r_\pi + r_O)$

$$\Rightarrow ib = \frac{V_S}{g_m r_O r_\pi + r_\pi + r_O}$$

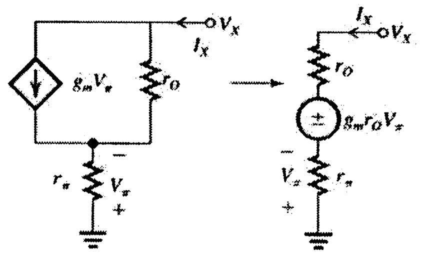
and:

$$g_m V_\pi = g_m r_\pi ib = \frac{g_m r_\pi V_S}{g_m r_O r_\pi + r_\pi + r_O}$$

and $ir_O = ib + g_m V_\pi$

$$= (1 + g_m r_\pi) \cdot \frac{V_S}{g_m r_O r_\pi + r_\pi + r_O}$$

9.40



$V_\pi = -I_Xr_\pi$

$V_X - g_mr_Or_\pi I_X = I_X(r_O + r_\pi)$

$\frac{V_X}{I_X} = R_O = r_O + r_\pi + g_mr_Or_\pi$

$= r_\pi + r_O(1 + g_mr_\pi)$

$= r_\pi + r_O(1 + h_{fe})$

$\approx h_{fe}r_O$

9.41

For: $g_{m1} = g_{m2} = 5\text{ mA/V},\ r_{O2} = 20\text{ K}$,

assume $R_D = 1\text{ k}\Omega$

$A = \frac{I_0}{V_S} = G_{m1}R_{Dgm2},\ G_{m1} = \frac{g_{m1}}{1 + g_mR_F}$

$= \frac{g_{m1}g_{m2}R_D}{1 + g_{m1}R_F}$

$= 16.67 \times 10^{-3}\frac{\text{A}}{\text{V}} = 16.67\text{ mS}$

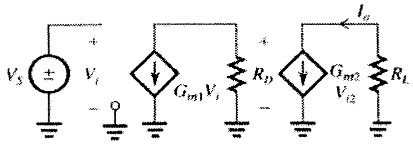
$\beta = R_f = 100\ \Omega$

$1 + A\beta = 1 + 16.67 \times 10^{-3} \cdot 100 = 2.67$

$A_f = \frac{A}{1 + A\beta} = \frac{16.67\text{mS}}{267} = 6.25\text{ mS}$

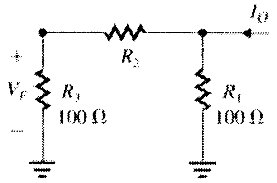
$R_o = R_F//(r_{O2}R_L) = 99.5\ \Omega$

$R_{of} = R_o(1 + A\beta) = 99.5\ \Omega \cdot 2.67 = 266\ \Omega$



9.42

Feed-back circuit:



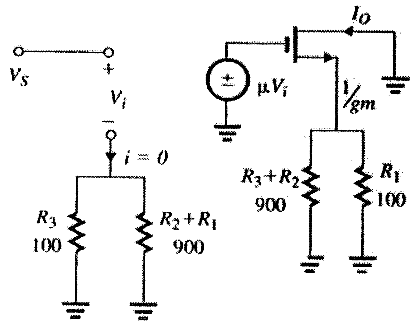
$\beta = \frac{V_f}{I_O} = \frac{R_1 \cdot R_3}{R_1 + R_2 + R_3}$

if $A\beta >> 1 \Rightarrow A_f \simeq \frac{1}{\beta} = \frac{R_1 + R_2 + R_3}{R_1R_3}$

If $A_F = 100\text{ m}\frac{\text{A}}{\text{V}}\ 0.1 = \frac{200 + R_2}{100 \times 100}$

$\rightarrow R_2 = 800\ \Omega\quad \beta = 10$

A-circuit:



$I_O = \frac{\mu V_S}{1/g_m + R_1 \parallel (R_1 + R_2)}$

$A = \frac{I_O}{V_S} = \frac{\mu}{1/g_m + R_1 \parallel (R_1 + R_2)}$

For $gm = 1\text{ mA}\quad A = \frac{\mu}{1\text{ K} + 90} = \frac{\mu}{1090}$

Amount of feed-back: $1 + A\beta = 1000$
i.e. 60 dB

and $\beta = 10 \Rightarrow A = \frac{999}{10} = 99.9$

$\mu = 1090 \times 99.9 \simeq 109 \times 10^3\text{ V/V}$

R_o is degenerated by $R_1//(R_2 + R_3)$

$R_O = r_0 + g_mr_0[R_1//R_2 + R_3] + [R_1//R_2 + R_3]$

$= 50\text{ k}\Omega + 90\ \Omega + 4.5\text{ k}\Omega = 54.59\text{ k}\Omega$

$R_{out} = R_0(1 + A\beta) = 54.59\text{ M}\Omega$

9.43

$I_{D1} = I_{D2} = I_{D3} = I_{D4} = 0.1\text{ mA}$

$I_{D5} = 0.8\text{ mA}$

$g_{m1} = g_{m2} = g_{m3} = g_{m4} = \frac{2 \times 0.1\text{ m}}{0.2}$

$= 1\text{ mA/V}$

$gm_5 = \frac{2 \times 0.8\text{ m}}{0.2} = 8\text{ mA/V}$

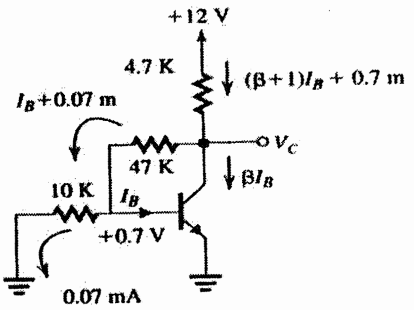
$r_{O2} = \frac{|V_A|}{I_D} = \frac{20}{0.1\text{ m}} = 200\text{ k}\Omega$

$r_{O5} = \frac{20}{0.8\text{ m}} = 25\text{ k}\Omega$

$|V_{G3}| = 0.2 + 0.4 = 0.6\text{ V}$

9.44

(a)



$V_C = 0.7 + (I_B + 0.07)47 = 3.99 + 47\, I_B$

and $\dfrac{12 - V_C}{4.7} = (\beta + 1)I_B + 0.07$

Solving both equations results in: $I_B \simeq 0.015$ mA

$I_C \simeq 1.5$ mA and $V_C = 4.7$ V

$V_\pi = I_i(R_S \| R_f \| r_\pi)$

$V_O = -g_m V_\pi (R_f \| R_c)$

$\Rightarrow A = \dfrac{V_o}{I_i} = -g_m(R_f \| R_c)(R_s \| R_f \| r_\pi)$

$g_m = \dfrac{I_C}{V_T} = \dfrac{1.5\text{ m}}{25\text{ m}} = 60\ \dfrac{\text{mA}}{\text{V}}$ and

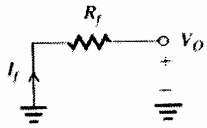
$r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{60\text{ m}} = 1.67\text{ k}\Omega$

Substituting: $A = -358.7\text{ k}\Omega$

$R_i = R_S \| R_f \| r_\pi = 1.4\text{ k}\Omega$

$R_O = R_C \| R_f = 4.27\text{ k}\Omega$

(d) To determine β:



$\beta = \dfrac{I_f}{V_O} = \dfrac{-1}{R_f} = \dfrac{-1}{47\text{ k}\Omega}$

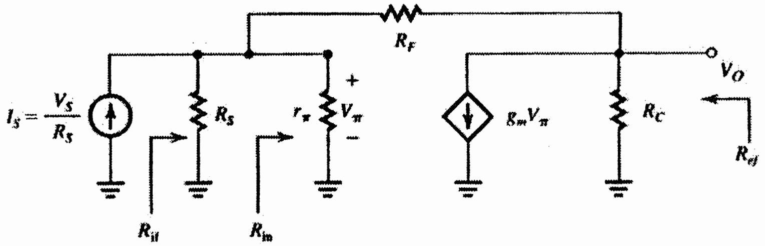
$A\beta = \dfrac{358.7}{47} = 7.63 \quad 1 + A\beta = 8.63$

(e) $A_f = \dfrac{V_O}{I_S} = \dfrac{A}{1 + A\beta} = \dfrac{-358.7}{8.63}$

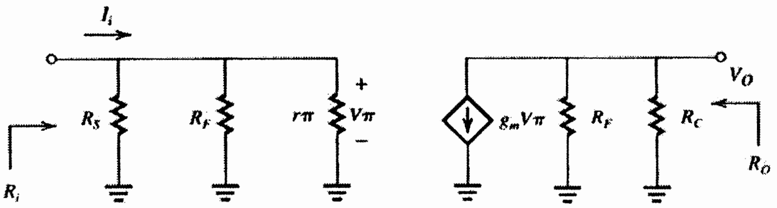
$= -41.6\text{ k}\Omega$

$R_{if} = \dfrac{R_i}{1 + A\beta} = \dfrac{1.4\text{ K}}{8.63} = 162.2\ \Omega$

(b)



(c) A-circuit:



To find A:

$R_{of} = \dfrac{R_O}{1 + A\beta} = \dfrac{4.27\text{ K}}{8.63} = 495\ \Omega$

$R_{in} = \dfrac{1}{\dfrac{1}{R_{if}} - \dfrac{1}{R_S}} = \dfrac{1}{\dfrac{1}{162.2} - \dfrac{1}{10\text{ K}}} = 164.87\ \Omega$

$R_{out} = \dfrac{1}{\dfrac{1}{R_{of}} - \dfrac{1}{R_L}} = \dfrac{1}{\dfrac{1}{495} - \dfrac{1}{\infty}} = 495\ \Omega$

$V_S = I_S \cdot R_S$

$\dfrac{V_O}{V_S} = \dfrac{V_O}{I_S} \cdot \dfrac{1}{R_S} = \dfrac{-41.6\text{ K}}{10\text{ K}} = -4.16$ V/V

if $A\beta >> 1$

$\Rightarrow A_f \simeq -47\text{ k}\Omega$

$\rightarrow \dfrac{V_O}{V_S} = \dfrac{-47}{10} = -4.7$ V/V

9.45

(a) if $A\beta >> 1 \Rightarrow A_f \simeq \dfrac{1}{\beta}$ with $\beta = \dfrac{-1}{R_F}$

$A_f = \dfrac{V_O}{I_S} = -R_F$

Since $V_S = I_S \cdot R_S \Rightarrow \dfrac{V_O}{V_S} = \dfrac{A_F}{R_S} = \dfrac{-R_F}{R_S}$

For $\dfrac{V_O}{V_S} = -10\ \dfrac{\text{V}}{\text{V}} = \dfrac{-R_F}{1\text{ k}\Omega} \Rightarrow R_F = 10\text{ k}\Omega$

(b) $\mu = 10^3$ V / V $R_{id} = 100\text{ k}\Omega$

$r_O = 1\text{ k}\Omega$

$R_L = \infty$, $R_f = 10$ K, $R_S = 1$ K

$$R_i = R_{id} \| R_F \| R_S = 100 \| 10 \| 1 = 901\ \Omega$$

$$A = \frac{V_o}{I_i} = -\mu R_i \frac{(R_F \| R_L)}{r_O + (R_F \| R_L)}$$

$$= \frac{-10^3 \times 901(10\text{ K})}{1\text{ K} + 10\text{ K}} = -819\text{ k}\Omega$$

$$A\beta = \frac{819}{10} = 81.9$$

$$1 + A\beta = 82.9$$

$$A_f = \frac{A}{1 + A\beta} = \frac{-819 \times 10^3}{82.9} = -9.88\text{ k}\Omega$$

$$\frac{V_O}{V_S} = \frac{-9.88}{1} = -9.88\text{ V/V}$$

To find R_{in}: Since R_{id} is large

(Eq 10.58) $R_{in} \simeq \dfrac{R_F}{\mu'}$

$$\mu' = \frac{\mu(R_F \| R_L)}{r_O + (R_F \| R_L)} = 909$$

$$R_{in} = \frac{10\text{ K}}{909} = 11\ \Omega$$

Since: $R_r >> \dfrac{r_o}{1 + \mu \dfrac{R_i}{R_F}}$

$$R_{out} \simeq \frac{R_F}{R_i} \cdot \frac{r_O}{\mu} = \frac{10}{0.901} \times \frac{1}{1} = 11.1\ \Omega$$

$r_0 = 1000\Omega \quad \mu = 1000$

(e) $f_H = 1\text{ kHz} \Rightarrow f_{Hf} = 1\text{ K}(1 + A\beta)$

$= 1\text{ K} \times 82.9 = 82.9\text{ kHz}$

9.46

(a) $V_{OV} = V_{GS} - V_t \Rightarrow V_{GS_{1,2}} V_{OV} + V_t$

$= 0.2 + 0.5 = 0.7\text{ V}$

$V_{G1} = V_{GS1} = 0.7\text{ V}$

$V_O = V_{GS1} = 0.7\text{ V}$

$V_{G2} = V_O + V_{GS2} = 0.7 + 0.7 = 1.4\text{ V}$

(b) $I_{D1} = I_{D2} = 0.5\text{ mA}$

$$g_m = \frac{2I_D}{V_{OV}} \to g_{m1,2} = \frac{2 \times 0.5\text{ m}}{0.2} = 5\,\frac{\text{mA}}{\text{V}}$$

$$r_O = \frac{V_A}{I_D} \to r_{O1,2} = \frac{10}{0.5\text{ m}} = 20\text{ k}\Omega$$

(c) A-circuit:

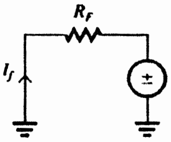
$V_1 = I_S R_F \quad V_2 = -g_{m1} r_{O1}(I_S R_F)$

$$V_O = \frac{V_2 \cdot (R_F \| r_{O2})}{1/g_{m2} + (R_F \| r_{O2})}$$

$$= \frac{-g_{m1} \cdot r_{O1}(I_S R_F)(R_F \| r_{O2})}{1/g_{m2} + (R_F \| r_{O2})}$$

$$A = \frac{V_O}{I_S} = \frac{-g_{m1} \cdot r_{O1} \cdot R_F\,(R_F \| r_{O2})}{1/g_{m2} + (R_F \| r_{O2})}$$

(d)



$$\beta = \frac{I_f}{V_O} = \frac{-1}{R_F}$$

$$A\beta = \frac{1 \cdot r_{O1} \cdot (R_F \| r_{O2})}{1/g_{m2} + (R_F \| r_{O2})}$$

$$1 + A\beta = \frac{1/g_{m2} + (1 + g_{m1} r_{O1})(R_f \| r_{O2})}{1/g_{m2} + (R_f \| r_{O2})}$$

(e) $A_f = \dfrac{A}{1 + A\beta}$

$$= \frac{-g_{m1} \cdot r_{O1} \cdot R_F\,(R_F \| r_{O2})}{1/g_{m2} + (1 + g_{m1} \cdot r_{O1})(R_F \| r_{O2})}$$

(f) $R_i = R_F$

$R_O = R_F \| r_{O2} \| 1/g_{m2}$

$$R_{if} = \frac{R_i}{1 + A\beta} \Rightarrow \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{A\beta}{R_i} = \frac{1}{R_i} + \frac{A\beta}{R_F}$$

$$\frac{1}{R_i} + \frac{g_{m1} \cdot r_{O1} \cdot (R_F \| r_{O2})}{(1/g_{m2} + (R_F \| r_{O2})) \cdot R_F}$$

If we define: $\mu = \dfrac{g_{m1} \cdot r_{O1}(R_F \| r_{O2})}{(1/g_{m2} + (R_F \| r_{O2}))}$

$$\Rightarrow \frac{1}{R_{if}} = \frac{1}{R_i} + \frac{\mu}{R_f} \Rightarrow R_{if} = R_i \,\Big\|\, \frac{R_f}{\mu}$$

$$= R_f \,\Big\|\, \frac{R_F}{\mu} = \frac{R_F}{1 + \mu}$$

Since $R_{if} = R_S \| R_{in}$ and $R_S = \infty$

$$\Rightarrow R_{in} = \frac{R_F}{1 + \mu} = \frac{R_F}{1 + \dfrac{g_{m1} r_{O1}(R_F \| r_{O2})}{\dfrac{1}{g_{m2}} + (R_F \| r_{O2})}}$$

$R_{of} = \dfrac{R_O}{1 + A\beta}$ Since: $R_{of} = R_L \| R_{out}$ and

$R_L = \infty \quad \Rightarrow R_{of} = R_{out}$

$$R_{out} = \frac{(R_f \| r_{O2} \| 1/g_{m2})(1/g_{m2} + (R_f \| r_{O2}))}{1/g_{m2} + (1 + g_{m1} r_{O1})(R_f \| r_{O2})}$$

but: $1/g_{m2} << R_f \| r_{o2} \Rightarrow R_{out} = \dfrac{(1/g_{m2})}{(1 + g_{m1} \cdot r_O)}$

(g) $g_{m1} = g_{m2} = 5\text{ mA/V}$

$r_{O1} = r_{O2} = 20\text{ k}\Omega$

$R_f = 10\text{ K}$

$R_F \| r_{O2} = 10 \| 20 = 6.67\text{ k}\Omega$

$\dfrac{1}{g_{m2}} + R_F \| r_{O2} = 200 + 6.67\text{ K} = 6.87\text{ k}\Omega$

$$A = \frac{-5 \times 20 \times 10\text{ K} \times 6.67}{6.87} = -970.8\text{ k}\Omega$$

$$\beta = \frac{-1}{10\text{ K}} = -100\,\mu\text{A/V}$$

$A\beta = 97.08$

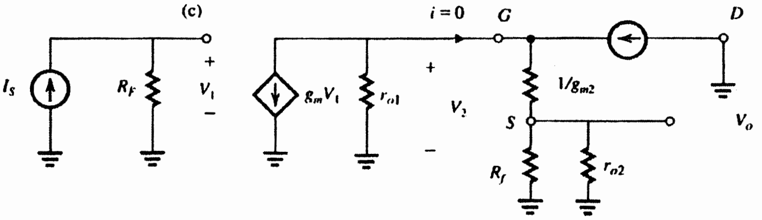
$A_f = -970.8\text{ K}/(98.08) = -9.9\text{ k}\Omega$

$R_i = 10\text{ K};\ R_O = 6.67\text{ K} \| 200 = 194.2\ \Omega$

$$R_{in} = \frac{10\text{ K}}{1 + \dfrac{5 \times 20 \times 6.67}{6.87}} = 102\ \Omega$$

$$R_{out} = \frac{200}{1 + 5 \times 20} \cong 1.98\ \Omega$$

(c)

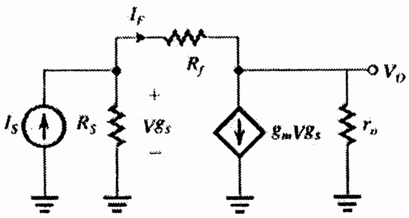


9.47

$$A_f = \frac{V_O}{I_S}$$

Across R_f: $I_F = \frac{V_{gs} - V_O}{R_F} 5 \frac{V_{gs}}{R_F} - \frac{V_O}{R_F}$ (1)

At the output: $I_F = g_m V_{gs} + \frac{V_o}{r_o}$ (2)



Combining (1) and (2):

$$\frac{V_{gs}}{R_F} - \frac{V_O}{R_F} = g_m V_{gs} + \frac{V_O}{r_O}$$

$$\Rightarrow V_{gs}\left(\frac{1}{R_F} - g_m\right) = V_O\left(\frac{1}{r_O} + \frac{1}{R_F}\right)$$

$$= \frac{V_O}{(r_O \parallel R_F)} \quad (3)$$

At the input: $I_F = I_S - \frac{V_{gs}}{R_S} \Rightarrow$ Combining with (1)

$$\frac{V_{gs}}{R_F} - \frac{V_O}{R_F} = I_S - \frac{V_{gs}}{R_S} \Rightarrow V_{gs}\left(\frac{1}{R_S} + \frac{1}{R_F}\right)$$

$$= I_S + \frac{V_{gs}}{R_S} \Rightarrow \frac{V_{gs}}{(R_F \parallel R_S)} = I_S + \frac{V_O}{R_f}$$

$$\Rightarrow V_{gs} = I_S(R_F \parallel R_S) + V_O\frac{(R_F \parallel R_S)}{R_F} \quad (4)$$

To simplify let's call: $K_1 = \frac{1}{R_F} - g_m$

$K_2 = R_F \parallel R_S$ and $K_3 = r_O \parallel R_F$

The Eq (3) and (4) become:

$$V_{gs}K_1 = \frac{V_O}{K_3} \qquad V_{gs} = I_S K_2 + V_O\frac{K_2}{R_f}$$

Combining: $I_S K_2 \cdot K_1 + V_O\frac{K_2 \cdot K_1}{R_f} = \frac{V_O}{K_3}$

$$I_S K_1 \cdot K_2 = V_O\left(\frac{1}{K_3} - \frac{K_1 \cdot K_2}{R_f}\right)$$

$$\rightarrow \frac{V_O}{I_S} = \frac{K_1 \cdot K_2}{\frac{1}{K_3} - \frac{K_1 \cdot K_2}{R_f}}$$

$\frac{V_O}{I_S} = \frac{K_1 \cdot K_2 \cdot K_3}{1 - \frac{K_1 \cdot K_2 \cdot K_3}{R_f}}$ where we recognize an Eq of the form: $\frac{A}{1 + A\beta}$ with $\beta = \frac{-1}{R_f}$

Substituting for K_1, K_3, K, and re-arrenging the signs:

$$\frac{V_O}{I_S} = \frac{-\left(g_m - \frac{1}{R_F}\right)(R_f \parallel R_S)(r_O \parallel R_f)}{1 + \left(g_m - \frac{1}{R_F}\right)(R_f \parallel R_S)\frac{(r_O \parallel R_f)}{R_f}}$$

For the feed-back analysis method to be accurate

$$g_m >> \frac{1}{R_F}$$

9.48

A-circuit:

$V_i = g_{m1}V_{gs} \cdot R_{D1} \quad V_{gs} = -I_S(R_f \parallel 1/g_{m1})$

$\Rightarrow V_i = g_{m1} \cdot R_{D1}(R_f \parallel 1/g_{m1})$

$V_O = -g_{m2}V_i(R_{D2} \parallel R_f)$

$$\Rightarrow \frac{V_O}{I_S} = -g_{m2}(R_{D2} \parallel R_f) \times g_{m1}R_{D1}(R_f \parallel 1/g_{m1})$$

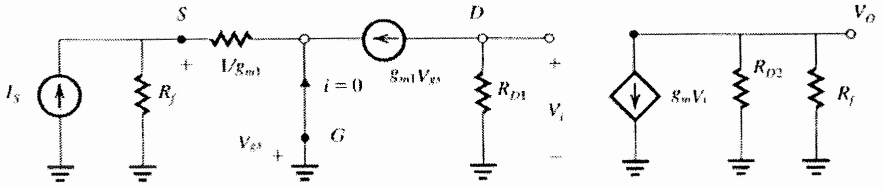
$$= g_{m1} \cdot g_{m2} \cdot R_{D1} \cdot (R_{D2} \parallel R_f)(R_f \parallel 1/g_{m1})$$

$$\beta = \frac{-1}{R_f} \Rightarrow A_f = \frac{A}{1 + A\beta}$$

$$= \frac{-g_{m1} \cdot g_{m2} \cdot R_{D1}(R_{D2} \parallel R_f)(R_f \parallel 1/g_{m1})}{1 + \frac{g_{m1}g_{m2}R_{D1}}{R_f}(R_{D2} \parallel R_f)(R_f \parallel 1/g_{m1})}$$

$R_i = \left(R_f \parallel \frac{1}{g_m}\right)$ and $R_{if} = \frac{R_i}{1 + A\beta}$

$R_O = (R_{O2} \parallel R_f)$ and $R_{of} = R_O/(1 + A\beta)$



This figure is for 9.48

9.49

. (a) Due to the feed-back we can assume that I_f is very small.

$\Rightarrow V_O \simeq V_{B1} = 0.7\text{ V}$

$V_{B2} = V_O + 0.7 = 1.4\text{ V}$

$$I_{RE} = \frac{0.7+5}{10\text{ K}} = 0.57\text{ mA} \simeq I_{E2}$$

$$I_{RC} = \frac{5-1.4}{10\text{ K}} = 0.36\text{ mA}$$

$$I_{C1} = I_{RC} - \frac{I_{C1}}{\beta} = 0.36\text{ mA} - \frac{0.56\text{ mA}}{100}$$

$= 0.35\text{ mA}$

(b) A-circuit: neglecting r_{e1} and r_{e2}

$$g_{m1} = \frac{0.35\text{ m}}{25\text{ m}} = 14\,\frac{\text{mA}}{\text{V}}$$

$$r_{\pi 1} = \frac{100}{14\text{ m}} = 7.14\text{ k}\Omega$$

$$r_{e2} = \frac{25\text{ m}}{0.58\text{ m} \times 0.99} = 43.5\ \Omega$$

$V_{\pi 1} = I_S\,(r_{\pi 1} \parallel R_f) = I_S\,(7.14\text{ K} \parallel 10\text{ K})$

$= I_S \times 4.16\text{ K}$

$$V_O = \frac{-g_{m1} \cdot V_{\pi 1} \cdot R_C \cdot R_F \parallel R_E(\beta+1)}{R_C + (\beta+1)(r_{e2} + R_F \parallel R_E)}$$

$$= -\frac{14\text{ m} \times 10\text{ K} \times 5\text{K} \times (101)}{10\text{ K} + 101(43.5 + 5\text{ K})}\,V_{\pi 1}$$

$= -136\,V_{\pi 1}$

$V_O = (-136 \times 4.16\text{ K})I_S$

$$\Rightarrow A = \frac{V_O}{I_S} \cong -566\text{ k}\Omega$$

$R_i = R_f \parallel r_\pi = 4.16\text{ k}\Omega$

$$\left(\frac{R_C + r_{e2}}{\beta+1}\right) \parallel R_E \parallel R_f$$

$$= \left[\frac{10\text{ K} + 43.5}{101}\right] \parallel 5\text{ K} = 97.5\ \Omega$$

$$\text{(C)}\ \beta = -\frac{1}{R_F} = \frac{-1}{10\text{ K}} = -100\,\mu\frac{\text{A}}{\text{V}}$$

$A\beta = 100\,\mu \times 566\text{ k}\Omega = 56.6$

$1 + A\beta = 57.6$

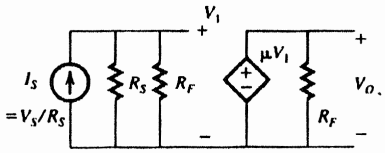
$$\text{(d)}\ A_f = \frac{A}{1+A\beta} = 9.8\text{ k}\Omega$$

$$R_{if} = \frac{R_i}{1+A\beta} = \frac{4.16\text{ K}}{58.4} = 73.14\ \Omega$$

$$R_{of} = \frac{R_O}{1+A\beta} = \frac{140.5}{58.4} = 2.4\ \Omega$$

9.50

A-circuit



$\beta = -1/R_F$

$V_O = -\mu V_1$

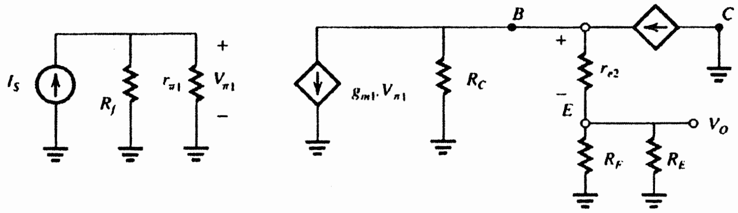
$V_1 = I_S\,(R_S \parallel R_F)$

$\therefore V_O = -\mu\,I_S\,(R_S \parallel R_F)$

$$A \equiv \frac{V_O}{I_S} = -\mu\frac{R_S R_F}{(R_S \parallel R_F)}$$

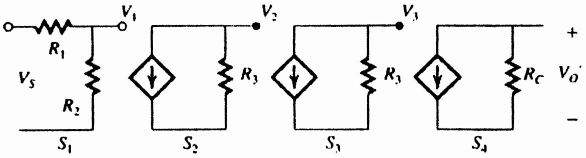
$$A_F = \frac{A}{1+A\beta} = \frac{-\mu(R_S \parallel R_F)}{1 + \mu(R_S \parallel R_F)R_F}$$

$$= \frac{-\mu(R_S \parallel R_F)}{1 + \mu\left(\dfrac{R_S \parallel R_F}{R_F}\right)}$$



This figure is for 9.49

This figure is for 9.50(b)



$$\frac{-\mu R_S R_F}{R_S + R_F + \mu R_S} = \frac{-R_F}{1 + \dfrac{R_F + R_S}{\mu R_S}} \approx -R_F \text{ if } \mu \text{ is}$$

large

$$\frac{V_O}{V_S} = \frac{V_O}{I_S}\frac{I_S}{V_S} = -R_F\frac{1}{R_S} = \frac{-R_F}{R_S}$$

(b) For circuit of Fig p8.46 (b)

$$I_B = \frac{V_{CC}\dfrac{10}{10+15} - V_{BE}}{[(15 \parallel 10) + (4.7 \times 101)] \times 10^3}$$

$$= \frac{15(10/25) - 0.7}{(6 + 474.4) \times 10^3} = \frac{5.3\text{ V}}{480.7\text{ K}} \approx 0.011\text{ mA}$$

$I_C = 100\, I_B = 1.1$ mA

$r_e = V_T / I = 22.6\ \Omega$

$r_\pi = (\beta + 1)r_e = 2.286$ K

$g_m = \beta / r_\pi = 100/2.286\text{ K} \rightarrow 43.7\text{ mA/V}$

$R_{in} = (15 \parallel 10 \parallel 2.286)\text{ K} = 1.5\text{ K}$

$R_C \parallel R_B \parallel r_\pi = (7.5 \parallel 6 \parallel 2.286) = 1.35\text{ K}$

For S_1: $R_l = R_S$, $R_2 = R_{in} \parallel R_F$

$\therefore V_1 = 1.5/11.5\, V_S = 0.13\, V_S$

For S_2: $R_3 = R_C \parallel R_B \parallel r_\pi$

$$\therefore \frac{V_2}{V_1} = -g_m R_3 = -43.7 \times 1.35$$

$= -59$ V/V

For S_3: Same as S_2 $\therefore \dfrac{V_3}{V_2} = -59$ V/V

For S_4:

$$\frac{V_O}{V_3} = -g_m R_C = -43.7 \times 7.5 = -327.75\text{ V/V}$$

$$\frac{V_O}{V_S} = -0.13 \times 59 \times 59 \times 327.75$$

$$\rightarrow \frac{V_O}{V_S} = -1.488 \times 10^5$$

Because we have ignored r_o, etc let us estimate $V_O/V_S = -1 \times 10^5$ which is quite large.

Then $A_F = \dfrac{A}{1 + A\beta} \approx 100$ needed

$$= \frac{10^5}{1 + 10^5\beta} \approx \frac{1}{\beta} = 100$$

Select R_F so that

$R_F / R_S = 100 \rightarrow R_F = 100 \times 10\text{ K} = 1\text{ M}\Omega$

We can ignore leading effect of R_F in A-circuit. R_L will cause loading of R_C

$$V_L = \left(\frac{R_L}{(R_C + R_L)}\right)V_o$$

$V_L = (1/8.5) = 0.11\, V_O$

Now $A_O \approx 1.65 \times 10^4$

$$A_F^{'} = \frac{10^4}{1 + 10^4/100} = 99.4$$

9.51

(a) To lower R_{in} and raise R_{out} SHUNT - SERIES

(b) To raise R_{in} and R_{out} SERIES - SERIES

(c) To lower R_{in} and R_{out} SHUNT - SHUNT

9.52

$A_f = -100$ A/A and $1 + A\beta$ is 40 dB

$\Rightarrow 1 + A\beta = 100$

$A\beta = 99$

and since $A_F = A/(1 + A\beta) \Rightarrow$

$A = -100 \times 100 = -10.000$ A/A

and $\beta = -0.0099$

$R_i = R_S \parallel R_{id} \parallel (R_l + R_2)$ and

$R_S = R_{id} = \infty \Rightarrow R_i = R_1 + R_2$

$$R_{in} \cong \frac{R_2}{\mu} \Rightarrow 1\text{ K} = \frac{R_2}{\mu}$$

If we assume that $\frac{1}{g_m} << (R_1 \parallel R_2 \parallel r_{O2})$ then we can use eq (10.73)

$$A\beta = \frac{\mu R_l}{R_2} \rightarrow A\beta = \frac{(R_1 + R_2)}{(R_2 / \mu)}$$

$$\rightarrow 99 = \frac{R_1 + R_2}{1\text{ K}} \qquad (1)$$

$$\beta = \frac{-R_1}{R_1 + R_2}$$

$\Rightarrow$ For $\beta = -0.0099$

$R_2 = (100.01)R_1 \qquad (2)$

Combining (1) and (2): $R_1 + R_2 = 99\text{ K}$

$R_2 = (100.01)R_1$

$\Rightarrow R_1 = 980.1\ \Omega$

$R_2 = 98.02\text{ k}\Omega$

$$99 = \frac{\mu .99\text{ K}}{98.02\text{ K}} \rightarrow \mu = 98.02\text{ V / V}$$

Given $g_m = 5\text{ m}\frac{\text{A}}{\text{V}}$ and $r_O = 20\text{ K}$ we

observe that the assumption $\frac{1}{g_m} <<$

$(R_1 \parallel R_2 \parallel r_{o2})$ is not valid

$\left(\frac{1}{5\text{ m}} = 200\ \Omega\right)$

$\approx ((980.1 \parallel 98.02\text{ K} \parallel 20\text{ K}) = 925.5)$

, cannot be used.

Instead we use:

$$-10.000 = \frac{-\mu \cdot 99\text{ K}}{200 + 925.5}\left(\frac{20\text{ K}}{20\text{ K} + 970.4}\right)$$

$\Rightarrow \mu = 119.2$

To calculate R_{out} we cannot use .

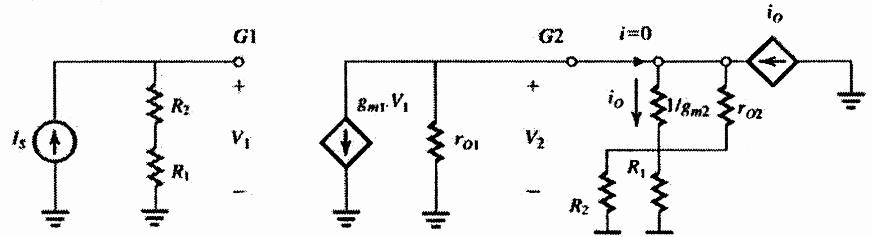
$R_o = r_{O2} + g_{m2}r_{O2}(R_1 \parallel R_2) + (R_1 \parallel R_2) = 119\text{ k}\Omega$

$R_{of} = R_O(1 + A\beta) = 11.950$

Since: $R_{out} = R_{of} - R_L$ and $R_L = 0$

$\Rightarrow R_{out} = R_{of} = 9.7\text{ M}\Omega$

This figure is for 9.53(c)



9.53

(a) $V_{GS} = V_{OV} + V_t$

$= 0.2 + 0.5 = 0.7$ V

$\Rightarrow V_{GS1} = V_{G1} = 0.7$ V

Since $I_{G1} = 0 \rightarrow V_{S2} = 0.7$

$$I_{D2} = \frac{V_{S2}}{R_1} = \frac{0.7}{3.5\text{ K}} = 0.2\text{ mA}$$

$$\text{(b)}\,g_m = \frac{2I_D}{V_{OV}} = \frac{0.4\text{ m}}{0.2} = 2\text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{0.2\text{ m}} = 50\text{ k}\Omega$$

(c) The A-circuit:

$V_1 = I_S(R_1 + R_2)$

$V_2 = -g_{m1}\,r_{O1}V_1 = -g_{m1}\,r_{O1}(R_1 + R_2)I_S$

$$i_O = \frac{V_2}{1/g_{m2} + R_2 \parallel R_1 \parallel r_{O2}}$$

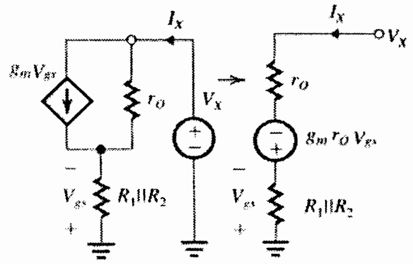
$$= \frac{-g_{m1}\,r_{O1}(R_1 + R_2)I_S}{1/g_{m2} + R_2 \parallel R_1 \parallel r_{O2}}$$

$$\Rightarrow A = \frac{i_O}{I_S} = \frac{-g_m \cdot r_O \cdot (R_1 + R_2)}{1/g_m + (R_2 \parallel R_1 \parallel r_O)}$$

$$= \frac{-2 \times 50(3.5 + 14)\text{ K}}{500 + (3.5 \parallel 14 \parallel 50)\text{ K}} = -555.3\ \frac{\text{A}}{\text{A}}$$

$R_i = R_2 + R_1 = 3.5 + 14 = 17.5$ kΩ

To get R_O:



$V_{gs} = -I_X(R_1 \parallel R_2)$

$V_X + gm\,r_O V_{gs} = I_X(r_O + R_1 \parallel R_2)$

$$R_O = \frac{V_X}{I_X} = r_O + R_1 \parallel R_2 + gm\,r_O\,(R_1 \parallel R_2)$$

$R_O = 50\text{ K} + (3.5 \parallel 14)\text{ K} + 2 \times 50(3.5 \parallel 14)\text{ K}$

$R_O = 332.8$ kΩ

$$\text{(d)}\ \beta = \frac{-R_1}{R_1 + R_2} = \frac{-3.5}{17.5} = -0.2$$

(e) $A\beta = 555.3 \times 0.2 = 111.06$

$$A_f = \frac{A}{1 + A} = \frac{-555.3}{112.06} = -5\text{ A/A}$$

(f) $R_{in} = R_{if} \parallel R_S$ since $R_S = \infty \Rightarrow R_{in} = R_{if}$

$$R_{in} = \frac{R_i}{1 + A\beta} = \frac{17.5\text{ K}}{112.06} = 156.2\ \Omega$$

$R_{out} = R_{of} - R_L$ but $R_L = 0 \Rightarrow R_{out} = R_{of}$

$R_{out} = R_o(1 + A\beta)$

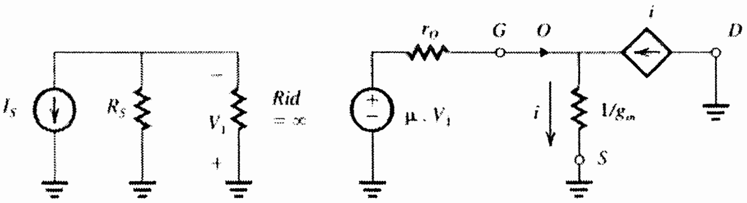
$= 332.8\text{ K} \times 112.06 = 37.3$ MΩ.

9.54

(a) if μ is large, the loop gain is large and the current at the negative input is $\sim 0 \Rightarrow V_{in} \sim -R_S \cdot I_S$

Since $I_S \simeq I_O \Rightarrow \frac{I_O}{I_S} \simeq 1$ A/A

(b) A-circuit:



$V_1 = I_S \cdot R_S$

$\Rightarrow I_O = i = \mu \cdot R_S g_m \cdot I_S$

$i = \dfrac{(\mu \cdot V_1)}{1/g_m}$

$\Rightarrow A = \dfrac{I_O}{I_S} = \mu \cdot R_S \cdot g_m$

$R_i = R_S \; R_O = r_{02}$

(c) $\beta = 1$

(d) $A\beta = \mu \cdot R_S \cdot g_m$

$A_f = \dfrac{A}{1 + A\beta} = \dfrac{\mu \cdot R_S \cdot g_m}{1 + \mu \cdot R_S \cdot g_m}$

If $r >> 1 \Rightarrow A_f \simeq 1$

(e) $R_{if} = \dfrac{R_i}{1 + A\beta} = \dfrac{R_S}{1 + \mu R_S \cdot g_m} \Rightarrow \dfrac{1}{R_{if}}$

$= \dfrac{1}{R_S} + \mu \cdot g_m$

$R_{if} = R_S \parallel 1/\mu \cdot g_m$

since $R_{if} = R_S \parallel R_{in}$

$\Rightarrow R_{in} = \dfrac{1}{\mu \cdot g_m}$ if $r >> 1 \rightarrow R_{in} = 0$

$R_{of} = R_O(1 + A\beta) = r_{02}(1 + \mu R_S g_m)$

$R_{of} = R_{out} + R_L$ and

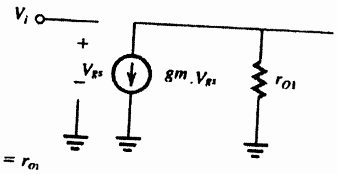
$R_L = 0 \Rightarrow R_{out} = \dfrac{1}{g_m} + \mu R_S$

if $\mu >> 1 \Rightarrow R_{out} = \infty$

For Q_1:

This is equivalent to circuit with:

$I_s = g_m V_i$



$R_s = r_{o1}$

$I_O = V_2 g_{m1} \dfrac{\mu g_{m2} r_{01}}{1 + \mu g_{m2} r_{01}}$

$V_1 g_{m1}$ if $\mu g_{m2} r_{01} > 1$

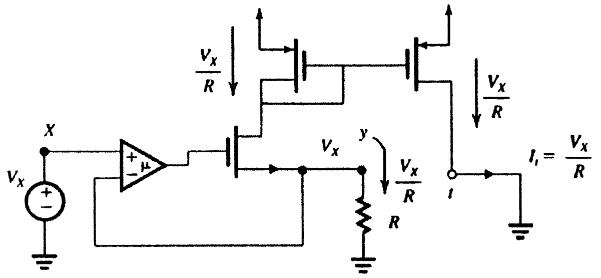
$R_{out} = r_{02}(1 + \beta A)$

$= r_{02}(1 + \mu r_{01} g_{m2})$

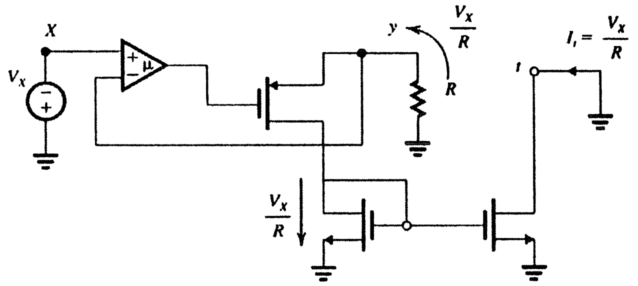
$\approx \mu(r_{02} g_{m2} r_0)$

9.55

(a) When V_X is positive:


$\frac{V_X}{R}$
X
V_X
+
−
μ
V_X
y
$\frac{V_X}{R}$
R
t
$I_t = \frac{V_X}{R}$
$\frac{V_X}{R}$

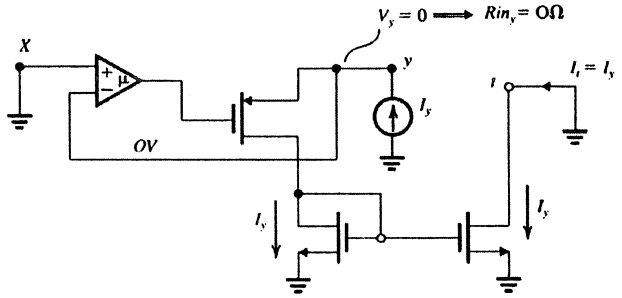
When V_X is negative:

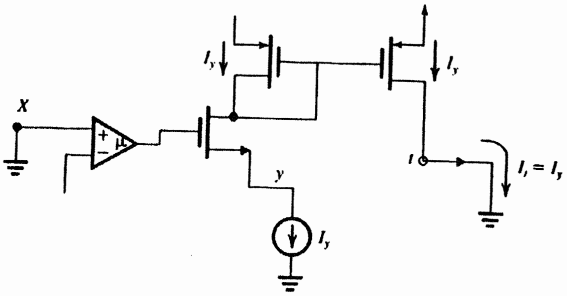

X
V_X
−
+
μ
y
$\frac{V_X}{R}$
R
t
$I_t = \frac{V_X}{R}$
$\frac{V_X}{R}$

(b) When I_y is positive:

$$R_{in} = \frac{1}{\mu(g_{mp} + g_{mn})} \sim 0 \text{ (if } \mu >> 1)$$

When I_y is negative:


$V_y = 0 \Longrightarrow Rin_y = 0\Omega$
X
+
−
μ
OV
y
I_y
t
$I_t = I_y$
I_y
I_y



(c) When R_{zout} is $r_{O2} \parallel r_{O4}$

9.56

$\underline{\text{Neglect } I_{B2}}$; $I_{B1} \approx \frac{200}{100} = 2\mu A$

$V_{BE} = 0.7V \quad \therefore V_{B1} = +0.7V$

But no d.c. component in V_s

$\therefore I_{RS}$ (into V_s) $= 0.7/10K = 0.07\mu A$

Thus $I_F = I_{RS} + I_{B1} = 0.07 + 0.002$

$= 0.072\, mA$

$V_{E2} = 0.7 + 10 \times 0.072 = 0.7 + 0.72 = 1.42V$

$I_{C2} = 1.42/140 + 0.072 = 10.2mA$

$I_{B2} = I_{E2}/(\beta+1) = 0.1\,\mu A \approx \frac{1}{2}\, 200\mu A$

Iterate:

$I_{B1} = \frac{200\mu A - 100\mu A}{100} = 0.001\mu A$

$V_{E2} = 0.7 + 10 \times 0.073 = 1.41V$

$I_{E2} = 1.41/0.140 + 0.071 = 10.1\, mA$

$I_{B2} = 10.1/101 = 100\mu A \quad \therefore I_{C2} = 10mA$

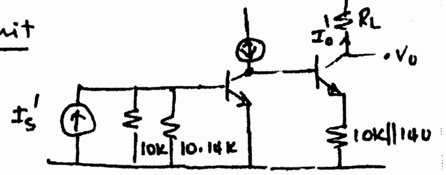
$V_{B2} = V_{E2} + V_{BE} = 1.41 + 0.7 = 2.11V$

$V_D = 10 - 10 \times 500\Omega = +5V$

$r_{e1} = \frac{V_T}{I_E} = \frac{25mV}{0.1mA} = 250\Omega, \quad r_{e2} = 25\Omega$

$\beta = \frac{R_E}{R_F + R_E} = \frac{140}{10K + 140} \approx 0.0138$

$\underline{\text{A-circuit}}$



$V_{B1} = 10.14k \parallel 10k \parallel \beta(250) I_s'$

$= 4.2 \times 10^3\; I_s'$

$\Rightarrow \frac{I_o'}{I_s'} = 4.2 \times 10^3 \frac{(\beta+1)(r_{e2} + 10K \parallel 140)}{250}$

$\times \frac{1}{(r_{e2} + 10K \parallel 140)}$

$= 1.69 \times 10^3\; A/A$

$A_F \equiv \frac{I_o}{I_s} = \frac{A}{1 + A\beta} = \frac{1.69 \times 10^3}{1 + 1.69 \times 10^3 \times (0.0138)}$

$= 69.6$

$\Rightarrow \frac{V_o}{V_s} = \frac{I_o R_L}{I_s R_s} = \frac{500}{10^4} \cdot 69.6 = 3.5 V/V$

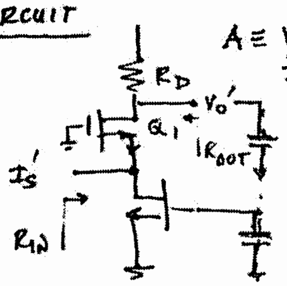
$R_i = 10K \parallel 10.14K \parallel 25K = 4.2\, K\Omega$

$R_{if} = R_i/(1 + A\beta) = \frac{4.2}{1 + 23.3} = 172.8\Omega$

$\Rightarrow = \underline{R_{IN}} \parallel R_s \quad \Rightarrow R_{IN} = 175.8\Omega$

9.57

A-CIRCUIT

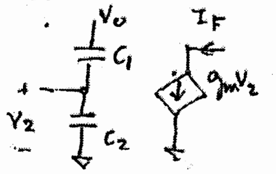


$A \equiv \frac{V_o'}{I_s'} = [R_D \| X_C] = R_D$ kΩ

Neglecting loading of C's

$A_F = \frac{A}{1+A\beta}$

B-CIRCUIT



$\beta = \frac{C_1}{C_1+C_2} g_{mf}$

$= \frac{1}{10} \times 1 \frac{mA}{V}$

Here. $g_{m1} = 5 mA/V$ $\quad g_{mf} = 1 mA/V$

$R_D = 10K$

Thus $A = 10 K\Omega$

$A_F = \frac{10K}{1 + 10K(0.1)} = 5 K\Omega$

$R_{IN} = (R_D \| r_o) \longrightarrow R_D$

$R_{if} = R_D/(1+A\beta)$ shunt $= \frac{R_D}{2} = 5K\Omega$

$R_{out} = R_D/(1+A\beta) = \frac{R_D}{2} = 5K\Omega$

9.58

(a) $V_{B1} \simeq 12 \frac{15}{100+15} = 1.57$ V

$V_{E1} \simeq 1.57 - 0.7 = 0.87$ V

$I_{E1} = 0.87 / 0.87 = 1$ mA $\rightarrow g_{m1}$

$= \frac{\alpha I_{E1}}{V_T} = \frac{0.99 \times 1}{25} \simeq 40 \frac{mA}{V}$

$V_{C1} \simeq 12 - 10 \times 1 = 2$ V

$V_{E2} \simeq 2 - 0.7 = 1.3$ V

$I_{E2} \simeq 1.3 / 3.4 \simeq 0.4$ mA $\rightarrow g_{m2}$

$= \frac{0.99 \times 0.4}{25} \simeq 16 \frac{mA}{V}$

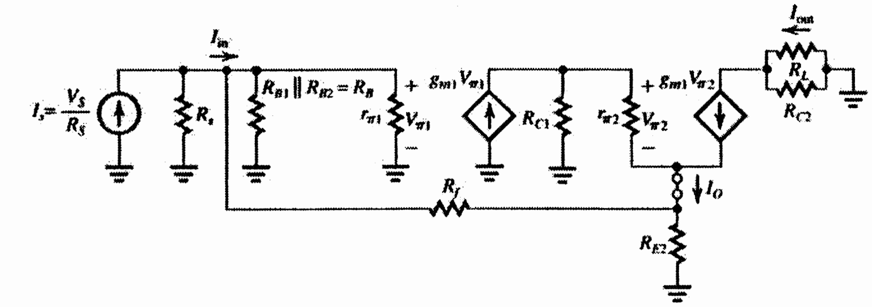
$V_{C2} \simeq 12 - 0.4 \times 8 = 8.8$ V

$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{40 m} = 2.5$ kΩ

$r_{\pi 2} = \frac{100}{16 m} = 6.25$ kΩ

$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25}{0.4} = 62.5$ Ω

(b)



I_{in}
$I_s = \frac{V_S}{R_S}$
R_s
$R_{B1} \| R_{B2} = R_B$
$r_{\pi1}$
$V_{\pi1}$
$g_{m1}V_{\pi1}$
R_{C1}
$r_{\pi2}$
$V_{\pi2}$
$g_{m1}V_{\pi2}$
I_{out}
R_L
R_{C2}
R_f
I_O
R_{E2}

To obtain A:

$R_B = R_{B1} \| R_{B2}$

$V_{\pi1} = I_i[R_S \| (R_{E2} + R_f) \| R_B \| r_{\pi1}]$

$= 1.535\,k \times I_i$

$V_{b2} = -g_{m1}, V_{\pi1}$

$\{R_{C1} \| [r_{\pi2} + (\beta + 1)(R_{E2} \| R_f)\}$

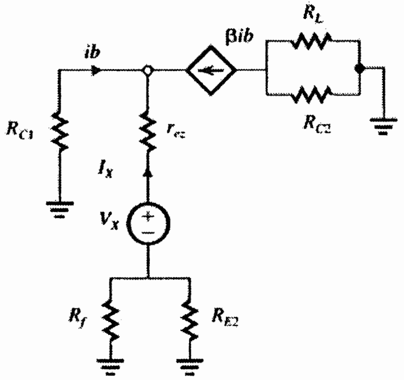
$$= I_O \simeq \frac{V_{b2}}{r_{e2} + (R_{E2} \| R_f)}$$

Combining these equations we obtain:

$$A = \frac{I_O}{I_S} = -201.45\text{ A/A}$$

$R_i = R_S \| (R_{E2} + R_f) \| R_B \| r_{\pi1} = 1.535\text{ k}\Omega$

R_O is obtained by looking into nodes Y and Y', with I_i set to zero



R_L
βib
ib
R_{C1}
r_{e2}
R_{C2}
I_X
V_X
+
−
R_f
R_{E2}

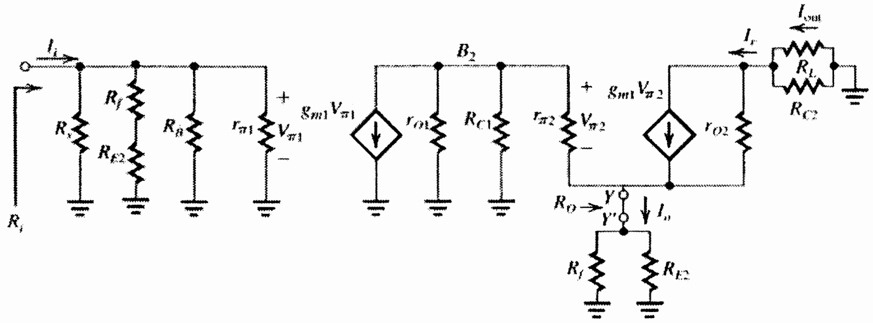
$$R_O = \frac{V_x}{I_x}$$

$I_x = -(\beta + 1)ib$

$-R_{C1} \times ib = (\beta + 1)ib[r_{e2} + R_f \| R_{E2}] + V_x$

Replacing $-ib$ by $I_x/(\beta + 1)$

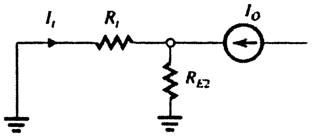
See Figure below.



I_i
R_i
R_s
R_f
R_{E2}
R_B
$r_{\pi1}$
$V_{\pi1}$
$g_{m1}V_{\pi1}$
r_{o1}
R_{C1}
B_2
$r_{\pi2}$
$V_{\pi2}$
$g_{m1}V_{\pi2}$
r_{o2}
I_r
I_{out}
R_L
R_{C2}
R_O
Y
Y'
I_o
R_f
R_{E2}

$$R_O = \frac{V_x}{I_x} = \frac{R_{C1}}{\beta + 1} + r_{e2} + (R_f \parallel R_{E2}) = 2.69\ \text{k}\Omega$$

(d)



$$\beta = \frac{I_f}{I_O} = -\frac{R_{E2}}{R_{E2} + R_f} = -\frac{3.4}{13.4} = -0.254$$

(e) $A\beta = 51.1$

$1 + A\beta = 52.1$

$$R_{if} = \frac{R_i}{1 + A\beta} = 29.5\ \Omega$$

$$A_f = \frac{I_O}{I_i} = \frac{A}{1 + A\beta} = -3.87\ \text{A/A}$$

$$R_{of} = R_O(1 + A\beta) = 140.1\ \text{k}\Omega$$

(f) $\frac{I_{out}}{I_{in}} \simeq \frac{I_{out}}{I_S} = \frac{R_{C2}}{R_L + R_{C2}} \cdot \frac{I_C}{I_S} \simeq \frac{R_{C2}}{R_L + R_{C2}} \cdot \frac{I_O}{I_S}$

$$\rightarrow \frac{I_{out}}{I_{in}} = -3.44\ \text{A/A}$$

$$R_{in} = \frac{1}{\frac{1}{R_{if}} \cdot \frac{1}{R_S}} \simeq 29.5\ \Omega$$

Notice that $R_{in} = R_{if}$

$$A_f = \frac{I_O}{I_i} = \frac{I_O}{I_S}$$

To obtain R_O: recall from problem 10.49 that:

$$R_{out} = r_{o2}[1 + g_{m2}(r_{\pi 2} \parallel R_{of})]$$

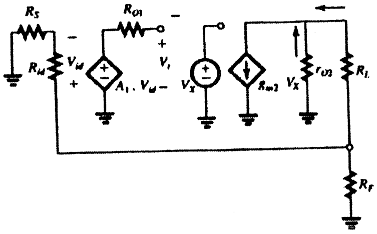
$$r_{O2} = \frac{75}{0.4} = 187.5\ \text{k}\Omega \quad r_{\pi 2} = 6.25\ \text{k}\Omega$$

$g_{m2} = 16$ mA/V

$R_{of} = 140.1$ kΩ

$\Rightarrow R_{out} = 18.1$ MΩ

9.59



$$\frac{-V_t}{V_x} = \frac{g_{m2} \cdot r_{O2}}{r_{O2} + R_L + R_F \parallel (R_{id} + R_S)} \times \frac{R_F}{R_F + R_{id} + R_S}$$

$\times R_{id} \times A_1$

Re-grouping to put this equation in the form of

$$A\beta = (A_1 g_{m2}) \cdot \left(\frac{R_{id}}{R_{id} + R_S + R_F}\right) \cdot$$

$$\frac{r_{O2}}{(r_{O2} + R_L + R_F \parallel (R_{id} + R_S)} \cdot R_F$$

Since R_F is usually << R_{id}

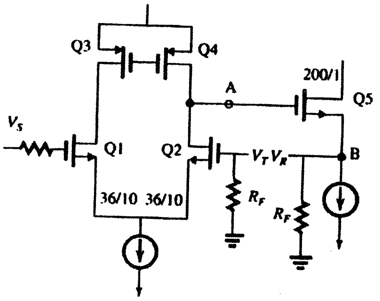
$$A\beta \simeq \left(A_1 \cdot g_{m2}\left(\frac{R_{id}}{R_{id} + R_S + R_F}\right)\right.$$

$$\left.\left(\frac{r_{O2}}{r_{O2} + R_L + R_F}\right) \cdot R_F\right)$$

where the term R_F is the only difference recall that

$\beta = R_F$ thus R_F is small

9.60



$V_T = V_{G2}$ and $V_S \rightarrow 0$

$$V_A = -g_{m2}(r_{O2} \parallel r_{O4})$$

$$V_R = V_A \frac{(R_F \parallel r_{O2})}{(R_F \parallel r_{O5}) + 1/g_{m5}}$$

$$A\beta = \frac{-V_T}{V_R} = 1\text{gm}_2 \frac{(r_{O2} \parallel r_{O4})(R_F \parallel r_{O5})}{(R_F \parallel r_{O5}) + 1/g_{m5}}$$

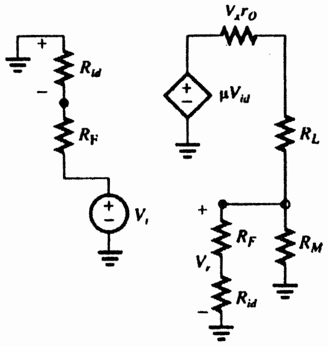
9.61

$$\frac{V_x}{V_t} = \mu \frac{R_{id}}{R_F + R_{id}}$$

$$\frac{V_r}{V_x} = \frac{(R_F + R_{id}) \parallel R_M}{(R_F + R_{id}) \parallel R_M + R_L + r_O}$$

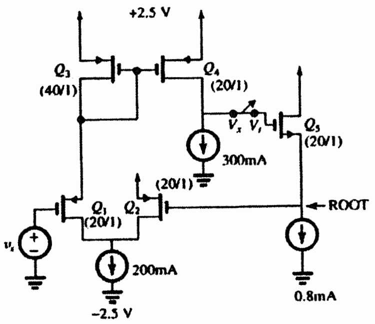
$$A\beta = \mu \frac{R_{id}}{R_F + R_{id}} \frac{(R_F + R_{id}) \parallel R_M}{(R_F + R_{id}) \parallel R_M + R_L + r_O}$$

$$\approx \mu \frac{(R_F + R_{id}) \parallel R_M}{(R_F + R_{id}) \parallel R_M + R_i + r_O} \approx \mu \frac{R_M}{R_M + R_L + r_O}$$



R_{id}
R_F
V_t
$V_A r_O$
μV_{id}
R_L
R_F
R_M
V_r
R_{id}

9.62



+2.5 V
Q_3
(40/1)
Q_4
(20/1)
V_x V_t
Q_5
(20/1)
300mA
(20/1)
Q_1
(20/1)
Q_2
ROOT
v_s
200mA
0.8mA
−2.5 V

Breaking at the gate of QS:

$$\frac{V_r}{V_t} = \left(\frac{g_{m5}r_{O5}}{1 + g_{m5}r_{O5}}\right)\left(\frac{1}{2}g_{m2}r_{O4}\right)$$

$$= \frac{2 \times 10^{-3} \cdot 30 \times 10^{3}}{1 + 2 \times 10^{-3} \cdot 30 \times 10^{3}}$$

$$\left(\frac{1}{2} \cdot 0.693 \times 10^{-3} \cdot 80 \times 10^{3}\right)$$

$$= 27.26$$

$$1 + A\beta = 28.26$$

$$R_{out} = \frac{1}{g_{m5}}\frac{1}{1 + A\beta} = 17.7\ \Omega$$

$$k'n = 2k'_p = 120\ \mu A / V^2$$

$$V_t = 0.7V$$

$$V_A = 24V / \mu m$$

$$r_{O5} = \frac{V_A}{I_{DS}} = 30\ k\Omega$$

$$r_{O4} = \frac{V_A}{I_{DS}} = \frac{24}{0.3 \times 10^{-3}} = 80\ k\Omega$$

$$g_{m2} = \sqrt{2k'n\left(\frac{W}{L}\right)I_{DS2}}$$

$$= \sqrt{2.120 \times 10^{-6} \cdot 20 \cdot 100 \times 10^{-6}}$$

$$= 0.693 \times 10^{-3} = 0.693\ mS$$

$$g_{m5} = \sqrt{2k'_n\left(\frac{W}{L}\right)I_{DS5}}$$

$$= 2\ mS$$

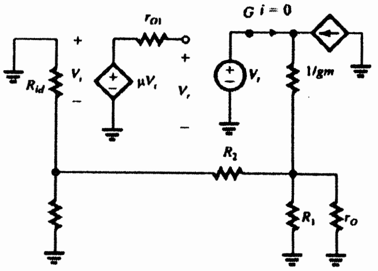
9.63

$$\frac{-V_r}{V_t} = \frac{(R_2 + R_S \parallel R_{id}) \parallel (R_1 \parallel r_O)}{\frac{1}{g_m} + (R_2 + R_S \parallel R_{id}) \parallel (R_1 \parallel r_O)}$$

$$\cdot \frac{R_S \parallel R_{id}}{R_S \parallel R_{id} + R_2} \cdot \mu$$

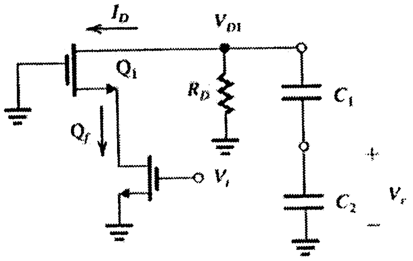
Since $R_{id} = R_S = \infty \Rightarrow$ The expression reduces to

$$A\beta = \frac{-V_r}{V_t} = \mu\frac{R_1 \parallel r_O}{\frac{1}{g_m} + (R_1 \parallel r_O)} = 997$$



r_{O1}
$G\ i = 0$
+
V_t
−
R_{id}
μV_t
+
V_r
−
V_t
$1/gm$
R_2
R_1
r_O

9.64



Assume that C_1 and C_2 are small and do not load the output.

Neglect r_{of} and r_{o1}

Since $I_D = I_{D1} = I_{Df}$

$$V_{D1} = -gm_f V_i \cdot R_D$$

$$V_r = V_{D1} \cdot \frac{1/SC_2}{\frac{1}{SC_2} + \frac{1}{SC_1}}$$

$$\Rightarrow \frac{-V_r}{V_i} = \frac{C_1}{C_2 + C_1} \cdot gmf \cdot R_D$$

$$A\beta = \frac{0.9}{0.1 + 0.9} \cdot 1 \times 10 = 9$$

9.65

$$A(S) = \frac{10^5}{1 + 5/100}$$

$$\text{Ang}(A) = -\tan^{-1}\frac{\omega}{100} - 2\tan^{-1}\frac{\omega}{10^4}$$

at ω_{180} : Ang$(A) = -180°$ for $\omega_{180} >> 100$

$$\Rightarrow 18° = 90° + 2\tan^{-1}\left[\frac{\omega_{180}}{10^4}\right]$$

$$\text{hence } \tan^{-1}\frac{\omega_{180}}{10^4} = \frac{90°}{2}$$

$$\text{i.e. } \frac{\omega_{180}}{10^4} = \tan(45°) = 1$$

$$\therefore \omega_{180} = 10^4 \text{ rad/s}$$

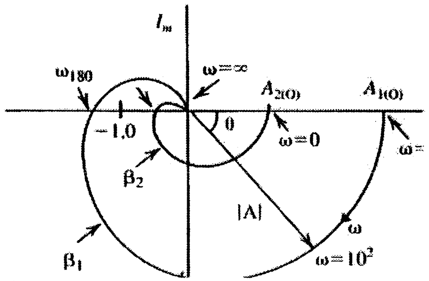
$$|A\beta| = \frac{10^5\beta}{\sqrt{1 + (10^4/10^2)^2}} \cdot \frac{1}{(\sqrt{1+1})^2} = 1$$

$$\Rightarrow \beta = 0.002$$

$$A_f(0) = \frac{10^5}{1 + 10^5(0.002)} = 500 \text{ V/V}$$

9.66

ω	Ang(A)	$\lvert A\rvert B_1$	$\lvert A\rvert B_2$
0	0	10^5	10^2
10^2	45	7.07×10^4	70.7
10^3	95.7	9.85×10^3	9.85
10^4	180	500	0.5
∞	0	0	0



(not to scale)

9.67

$$A(S) = \frac{10^3}{1 + 5/10^4}$$

$$\beta(S) = \frac{K}{(1 + 5/10^4)^2}$$

$$\text{Ang}(A\beta) = -\tan^{-1}\frac{\omega}{10^4} - 2\tan^{-1}\frac{\omega}{10^4}$$

$$= -3\tan^{-1}\frac{\omega}{10^4}$$

For 180° $\omega_{180} = \sqrt{3} \times 10^4$ rad/s

For $|A\beta(\omega_{180})| < 1$

$$\frac{10^3}{\sqrt{1 + (\sqrt{3})^2}} \cdot \frac{K}{1 + (\sqrt{3})^2} < 1$$

$$\Rightarrow K < 0.008$$

9.68

$$A(S) = \frac{1000}{(1 + S/10^4)(1 + S/10^5)^2}$$

and β is independent of frequency

$$\text{Ang}(A) = -\tan^{-1}\frac{\omega}{10^4} - 2\tan^{-1}\frac{\omega}{10^5}$$

try $\omega = 10^4$: θ 5 45° + 2 × 5.7 = 56.4°

try $\omega = 10^5$: θ 5 84.2° + 2 × 45 = 174.2°

Iteraction yields $\omega = 1.1 \times 10^5$ rad/s

For oscillations: $|AB(\omega_{180})| \geq 1$

$$\frac{\beta\ 10^3}{(\sqrt{1 + 11^2})(\sqrt{1 + 1.1^2})} \geq 1$$

$$\Rightarrow \beta \geq 0.0244$$

9.69

$$A(jf) = \frac{(10 \times 10^6)/10^4}{1 + jf/10^4}$$

$$\therefore \; A(jf) = \frac{10^3}{1 + jf/10^4}$$

$\beta = 0.1$ independent of frequency

$$A_f(jf) = \frac{10^3}{1+10^3(0.1)} \cdot \frac{1}{1 + \frac{jf}{10^4(1+10^3(0.1))}}$$

$$= \frac{9.9}{1 + jf/(101 \times 10^4)}$$

$A_f(0) = 9.9$ V/V

$f_{Pf} = 10^4 (101) = 1.01$ MHz

for $\frac{f}{f_{Pf}} \gg 1$: $A_f \approx 9.9 \frac{10^4(101)}{f}$

for $A_f = 1$: $f = f_T = 10$ MHz

Pole is shifted by $(1 + A(0)\beta) = 101$

9.70

$$A(j_f) = \frac{10^3}{(1 + j_f/10^4)(1 + j_f/10^5)}$$

(a) closed-loop poles given by

$1 + A(f)\,B = 0$

using $P = j_f$

$P^2 + P(10^4 + 10^5) + (1 + 10^3\beta)10^4 \cdot 10^5 = 0$

i.e. $P^2 + (1.1 \times 10^5)P + 10^9(1 + 10^3\beta) = 0$

compare terms with

$(P + f_{Pf})^2 = P^2 + 2f_{Pf} \cdot P + f_{Pf}^2$

$2f_{Pf} = (1.1 \times 10^5)$

$(1 + 10^3\beta) \times 10^9 = f_{Pf}^2$

$\Rightarrow f_{Pf} = 5.5 \times 10^5$

and $(1 + 10^3\beta) = 3.025 \Rightarrow$

$\beta \; 5 \; 2.025 \times 10^{-3}$

(b) At 55 kHz

$$A(f) = \frac{10^3}{\left(1 + j\frac{55 \times 10^3}{10^4}\right)\left(1 + j\frac{55 \times 10^3}{10^5}\right)}$$

$$= \frac{10^3}{(1 + j5.5)(1 + j0.55)}$$

$$= \frac{10^3}{1 + j6.05 - 3.025} = \frac{10^3}{-2.025 + j6.05}$$

$$= -24.75(2.025 + j6.05) = -49.75 - j149.74$$

$|A(55\text{ kHz})| = 157.7$

$$A_f(55\text{ kHz}) = \frac{-49.75 - j149.74}{1 - (49.75 + j149.74)2.025 \times 10^{-3}}$$

$$= -\frac{49.75 + j149.74}{0.9}(0.9 + j0.3) = 0.16 - j166.3$$

$|A_f(55\text{ KHz})| = 166.3$

(c) from $S^2 + (\omega_0/Q)S + \omega_0^2$ cf above

$$Q = \frac{f_{Pf}}{2f_{Pf}} = \frac{1}{2}$$

(d) $P^2 + 1.1 \times 10^5 P + (1 + 10^3\beta) = 0$

$\Rightarrow P^2 + 1.1 \times 10^5\,P + 21.25 \times 10^9 = 0$

$$P = \frac{-1.1 \times 10^5 \pm \sqrt{(1.1 \times 10^5)^2 - 4(21.25 \times 10^9)}}{2}$$

$$= \frac{-1.1 \times 10^5 \pm j\,2.7 \times 10^5}{2}$$

$$= -5.5 \times 10^4 \pm j\,1.35 \times 10^5 \text{ Hz}$$

$$Q = \frac{|P|}{2(5.5 \times 10^4)}$$

$$= \frac{\sqrt{(5.5 \times 10^4)^2 + (1.35 \times 10^5)^2}}{1.1 \times 10^5}$$

$= 1.33$

9.71

$$A(jf) = \frac{10^3}{(1 + jf/10)(1 + jf/f_P)}$$

$$A_f(0) = \frac{10^3}{1 + 10^3 \beta} = 100$$

$\Rightarrow \beta = 9 \times 10^{-3}$ V/V

Maximally flat when $Q = 0.707 = 1/\sqrt{2}$

from

$$p^2 + p(f_1 + f_2) + (1 + A_0\beta)(f_1 \times f_2) = 0$$

$$Q = \frac{\sqrt{(1 + A_0\beta)f_1 f_2}}{f_1 + f_2}$$

$$\Rightarrow \frac{\sqrt{(1 + 10^3\beta)\,10^3 f_{Pf}}}{10^3 + f_{Pf}} = \frac{1}{\sqrt{2}}$$

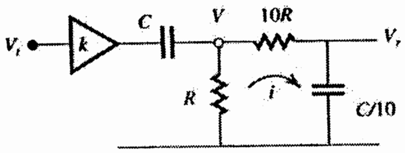
$$\Rightarrow f_{Pf}^2 + (2 \times 10^3)f_{Pf} + 10^6$$

$$= 2(1 + 10^3\beta)10^6$$

$$\Rightarrow f_{Pf} = \frac{18 \times 10^3 \pm \sqrt{(18 \times 10^3)^2 - 4(10^6)}}{2}$$

$= 17.94$ KHz.

9.72



$$i = \frac{SCV_r}{10} \Rightarrow V = V_r + SCRV_r$$

$$= V_r(1 + \mathrm{SCR})$$

By KCL : $\Sigma_i = 0 \; a + V$

$$\Rightarrow \frac{(KV_i - V)}{1/SC} = \frac{V}{R} + i = \frac{V}{R} + \frac{SCV_r}{10}$$

$KSCV_i - SCV_r(1 + \mathrm{SCR})$

$$= V_r\left(\frac{1 + SCR}{R} + \frac{SC}{10}\right)$$

Collecting terms:

$K\,SCV_i R = V_r[(SCR)^2 + 2.1\,SCR + 1]$

Thus $L(S) \triangleq -V_r/V_i$

$$= \frac{-K/CR\;S}{S^2 + \frac{2.1\,S}{CR} + \frac{1}{CR^2}}$$

$$\equiv \frac{-K/CR\;S}{S^2 + \left(\frac{\omega_O}{Q}\right)S + \omega_O^2}$$

from which $\omega_o = 1/CR$

$$Q = \frac{1}{2.1 - K}$$

Poles coincide when $Q = 1/2$

$\Rightarrow K = 2.1 - 2 = 0.1$

maximally flat when $Q = 1/\sqrt{2}$

$\Rightarrow K = 2.1 - 1.414 = 0.686$

Oscillates when $Q \to \infty$

$\Rightarrow K = 2.1$

9.73

$$A(f) = \frac{-K}{1 + j_f/10^5}$$

$$\text{for } \beta = 1 : A\beta = \frac{K^3}{\left(1 + \frac{j_f}{10^5}\right)^3}$$

For oscillations to occur : $|A\beta| \geq 1$ at

$\phi(A\beta) = 180°$

$3\tan^{-1}$

$$\left(\frac{f_{180°}}{10^5}\right) = 180° \Rightarrow f_{180°} = \sqrt{3} \times 10^5 \text{ Hz}$$

$f_{180°} = 173.2$ KHz

Amplifier is unstable if $|A\beta| \geq 1$ at $f_{180°}$

$$\left[\frac{K}{\sqrt{1 + (\sqrt{3})^2}}\right]^3 \geq 1 \to K \geq 2$$

9.74

$$A(f) = \frac{10^5}{1 + j_f/10}$$

$$\text{for } \beta = 1\text{: } A(f)B = \frac{10^5}{1 + j_f/10}$$

$$\text{for } f >> 10\text{: } |A\beta| \approx 10^5 \cdot \frac{10}{f_1}$$

$\Rightarrow f_1 = 1$ MHz

$$\text{at } f_1\text{: phase margin} = 180° - \tan^{-1}\frac{10^6}{10}$$

$= 90°$

9.75

$$A(f) = \frac{10^5}{1 + j_f/10\,(1 + j_f/10^4)}$$

$A\beta(0) = 10^5\beta$

$$A_F(0) = 100 = \frac{10^5}{1 + 10^5\beta} \Rightarrow \beta \approx 0.01$$

$|A\beta| = 1 \Rightarrow |1 + j_f/10| \cdot |1 + j_f/10^4|$

$= 10^5\beta = 10^3$

$(1 + f^2/10^2)(1 + f^2/10^8) = 10^6$

$f^4 + f^2(10^8 + 10^2) - (10^8)(10^2)10^6 = 0$

$$f \simeq \frac{-10^8 + \sqrt{10^{16} + 4 \times 10^6}}{2}$$

$\Rightarrow 61.8 \times 10^6 \Rightarrow f = 7.86$ KHz

Phase margin

$$= 180 - \left(\tan^{-1}\frac{7.86 \times 10^3}{10} + \tan^{-1}\frac{7.86}{10}\right)$$

$\approx 180° - 90° - 38.16° = 51.8°$

$$\text{For } PM \geq 45°\text{: } \tan^{-1}\frac{f_1}{10^4} \leq 45°$$

$\Rightarrow f_1 \leq 10^4$

thus

$$|A\beta| = 1 = \frac{10^5\beta}{\sqrt{1 + (10^3)^2} - \sqrt{2}}$$

$$\Rightarrow \beta = \frac{\sqrt{2}}{100} = 0.0141$$

9.76

$$|1 + e^{-j\theta}| = |1 + \cos\theta - j\sin\theta|$$

$= [(1 + \cos\theta)^2 + (\sin\theta)^2]$

$= [1 + 2\cos\theta + \cos_2\theta + 1 - \cos_2\theta]^{1/2}$

$= \sqrt{2}(1 + \cos\theta)^{1/2}$

$$\text{for 5\%: } 1 + \cos\theta = \frac{1}{1.05^2(2)} = 0.4535$$

$\theta = 123.13°$ and $PM = 180 - \theta = 56.87°$

for 10%: $1 + \cos\theta = \frac{1}{1.1^2(2)} = -0.586$

$\theta = 125.93°$ and $PM = 54.07°$

for 0.1 dB $\equiv 10^{0.1/20} = 1.0116$

$\cos\theta = \frac{1}{2(1.0116)^2} - 1 = -0.5114$

$\theta = 120.76°$ and $PM = 59.24°$

for 1 dB $\equiv 10^{1/20} = 1.122$

$\cos\theta = \frac{1}{2(1.122)^2} - 1 = -0.6028$

$\theta = 127.07°$ and $PM = 52.93°$

9.77

$$A(j_f) = \frac{10^5}{\left(1 + \frac{j_f}{10^5}\right)\left(1 + \frac{j_f}{3.16 \times 10^5}\right)\left(1 + \frac{j_f}{10^6}\right)}$$

Assume β independent of frequency

For 45° PM: $\theta = 180 - 45$

$$\tan.^{-1}\frac{f_1}{10^5} + \tan.^{-1}\frac{f_1}{3.16 \times 10^5} + \tan.^{-1}\frac{f_1}{10^6} = 135°$$

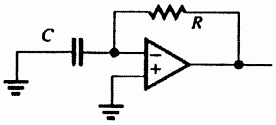
solve $\Rightarrow f_1 = 3.16 \times 10^5$ Hz

$$|A\beta(f_1)| = 1 = \frac{10^5\beta}{\sqrt{1 + (3.16)^2} \cdot \sqrt{2} \cdot \sqrt{1 + (0.316)^2}}$$

$\Rightarrow \beta = 49 \times 10^{-6}$

$$Af(o) = \frac{10^5}{1 + 10^5(4.9 \times 10^{-6})} = 16.9 \times 10^3$$

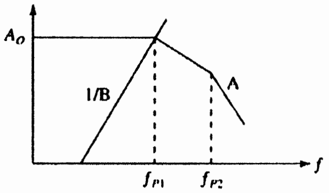
9.78



$$\beta = \frac{1/SC}{R + 1/SC} = \frac{1}{1 + SCR}$$

$$\beta(f) = \frac{1}{1 + j2\pi fCR}$$

$$A(j_f) = \frac{10^3}{\left(1 + \frac{j_f}{10^6}\right)\left(1 + \frac{j_f}{10^7}\right)}$$



From sketch, we need

$$A_O \frac{1}{2\pi f_{\phi 1} CR} = 1 = A\beta$$

$$\Rightarrow RC = \frac{A_0}{2\pi f_{\phi 1}} = \frac{10^3}{2\pi \times 10^6} = 159.2\ \mu s$$

At 1 MHz Ang(β) = −90°

$\text{Ang}(A) = -\tan.^{-1}1 - \tan.^{-1}0.1$

$= -45 - 5.7 = -50.7°$

$\therefore PM = 180 - (90 + 50.7) = 39.3°$

Gain margin exists at ω_{180}

then $\tan.^{-1}\frac{f_1}{10^6} + \tan.^{-1}\frac{f}{10^7} = 90°$

$\therefore f_{180} = \sqrt{10^6 \cdot 10^7}$ = geometric mean

= 3.16 MHz

$A\beta(f_{180}) = 20\log|A| - 20\log|1/\beta|$

$|A|$ has fallen 10 db, $|\beta|$ has risen 10 dB

thus $GM = 1\,(10) - (-10) = 20$ dB

9.79

For 90° PM:

$$\tan.^{-1}\frac{f_1}{10^5} + \tan^{-1}\frac{f_1}{10^6} + \tan.^{-1}\frac{f_1}{10^7} = 90°$$

From graph $f_1 = 3 \times 10^5$ Hz

thus 71.6 + 16.7 + 1.72 = 89.9° (close)

$$|A(f_1)| = \frac{10^5}{\sqrt{1 + 3^2} \cdot \sqrt{1 + 0.3^2} \cdot \sqrt{1 + 0.03^2}}$$

$|A\beta| = 1 \Rightarrow B = 33.0 \times 10^{-6}$

$$\therefore Af(0) = \frac{10^5}{1 + 10^5\beta} = 2.32 \times 10^4$$

For $PM = 45°$ $f_1 \approx 10^6$ Hz from graph

thus 84.3 + 45 + 5.7 = 135° (ok)

$$|A(f_2)| = \frac{10^5}{\sqrt{1 + 10^2} \cdot \sqrt{2} \cdot \sqrt{1 + 0.1^2}} = 7 \times 10^3$$

$|A\beta| = 1 \Rightarrow \beta = 1.43 \times 10^{-4}$

$$\therefore Af(o) = \frac{10^5}{1 + 10^5\beta} = 6.54 \times 10^3$$

9.80

$f_1 = 2\text{MHz}$

$A_o = 80\text{dB} \equiv 10^4$

$\Rightarrow f_p = f_1/A = (2\times10^6)/10^4 = 200\text{Hz}$

9.81

$f_{p1} = 2\text{MHz}, \quad f_{p2} = 10\text{MHz}$

$A_o = 80\text{dB} \equiv 10^4$

$f_D = \frac{f_P}{A_o} = \frac{10\times10^6}{10^4} = 10^3\text{Hz}$

$f_D' = 1/(C_x + C_c)2\pi R_x \longrightarrow C \times \frac{2\times10^6}{10^3} = 2000C$

9.82

$$R_1 = R_2 = R$$

$$C_2 = \frac{C_1}{10} = C$$

$$C_f >> C; \; g_m = \frac{100}{R}$$

$$: \omega_1 = \frac{1}{C_1R_1} = \frac{1}{10\cdot C\cdot R}$$

$$\omega_2 = \frac{1}{C_2R_2} = \frac{1}{C\cdot R}$$

$$\omega'_{P1} = \frac{1}{g_mR_2\cdot C_fR_1} = \frac{1}{100\cdot C_f\cdot R}$$

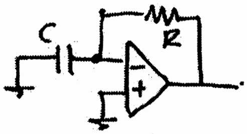
$$\omega'_{P2} = \frac{g_mC_f}{C_1C_2 + C_f(C_1 + C_2)}$$

$$= \frac{g_mC_f}{C_1(C_2 + C_f) + C_fC_2}$$

for $C_f >> C \Rightarrow \omega'_{PS} = \frac{g_m}{C_1 + C_2}$

$$= \frac{100}{11\times CR} = \frac{9.1}{CR}$$

9.83



$A_o = 10^4$

poles at $10^5, 10^6, 10^7$ Hz

For $\beta = 1$, f_P must be kept $\times10^4$ lower than lowest amplifier pole at 10^5Hz

$\Rightarrow f_P = \frac{10^5}{10^4} = 10\text{Hz}$

$f_P = \frac{1}{2\pi CR}$ and $R = 1\text{M}\Omega$

$\Rightarrow C = \frac{1}{2\pi\, 10^6\,(10)} = 15.9\text{nF}$

9.84

$$A_O = 80\text{ dB} \equiv 10^4$$

$$f_{P1} = 10^5 = \frac{1}{2\pi C_1R_1} \Rightarrow R_1 = \frac{1}{2\pi f_{P1}C_1}$$

$$\Rightarrow R_1 = \frac{1}{2\pi\cdot10^5\cdot(150\times10^{-12})} = 10.62\text{ k}\Omega$$

$$f_{P2} = 10^6 = \frac{1}{2\pi C_2R_2}$$

$$\Rightarrow R_2 = \frac{1}{2\pi\,10^6\,(5\times10^{-12})} = 31.85\text{ k}\Omega$$

Assuming $f_{P2} >> f_{P3}$

$$f'_{P1} = \frac{f_{P3}}{10^4} = \frac{2\times10^6}{10^4} = 200\text{ Hz}$$

and $f'_{P1} = \frac{1}{2\pi g_mR_1R_2C_f}$

$$\Rightarrow C_f = \frac{1}{2\pi g_mR_1R_2f'_{P1}}$$

$\therefore C_f$

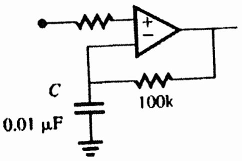
$$= \frac{1}{2\pi(40\times10^{-3})(10.62\times10^3)(31.85\times10^3)200}$$

$= 58.8\ P_f$

$$f'_{P2} = \frac{1}{2\pi}\frac{g_mC_f}{C_1C_2 + C_f(C_1 + C_2)}$$

$$= \frac{1}{2\pi}\frac{40\times10^{-3}(58.8\times10^{-12})}{(150\times5)10^{-24} + 58.8(155)10^{-24}} = 37.95\text{ MHz}$$

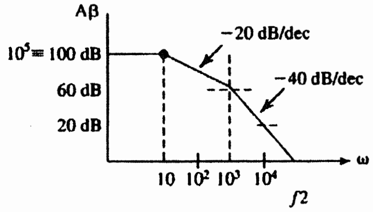
9.85



$$\beta(S) = \frac{1}{1 + SCR}$$

$$= \frac{1}{1 + 10^{-3}S}$$

(a) $$A\beta(S) = \frac{10^5}{1 + S/10} \cdot \frac{1}{1 + S/10^3}$$



(b) From plot $|A\beta| = 20$ dB at $10^4 = \omega$

Hence $|A\beta| = 1$ at 31.6 Krad/s $\left(\frac{1}{2}\text{dec}\right)$

At $\omega = 10^4$ the phase is $-180°$ decreasing at a rate of 45°/dec, at 31.6 $\text{K}\frac{\text{rad}}{\text{S}}$ $\frac{1}{2}$ (dec above $\omega = 10^4$) the phase margin is $-22.5°$.
The circuit will oscillate.

(c) $$A_f(S)\frac{\dfrac{10^5}{1 + S/10}}{1 + \dfrac{10^5}{1 + S/10} \cdot \dfrac{1}{1 + S/10^3}}$$

$$= \frac{10^5(1 + S/10^3)}{(1 + S/10)(1 + S/10^3) + 10^5}$$

$$\therefore A_f(S) = \frac{1 + S/10^3}{1 + S/10^6 + S^2/10^9} = \frac{10^6S + 10^9}{S^2 + 10^3S + 10^9}$$

Zero at $S = -10^{-3}$ rad/s

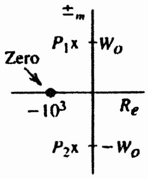
poles at $$\frac{-10^3 \pm \sqrt{10^6 - 4 \times 10^9}}{2}$$

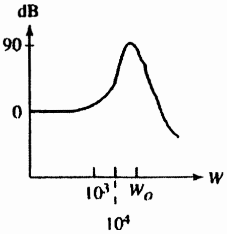
$$= \frac{-10^3 \pm j\,63.2 \times 10^3}{2}$$

$$= -500 \pm j\,31.6 \times 10^3 \text{ rad/s}$$

$\omega_0 = 31.6$ Krad/s

$Q = 31.6$





10.1

$$V_{ICM(max)} \leq V_{DD} - |V_{tp}| - |V_{OV1}| - |V_{OV5}|$$
$$\leq +2.5 - 0.7 - 0.3 - 0.3$$
$$\leq +1.2V$$

$$V_{ICM(min)} \geq -V_{SS} + V_{OV3} + V_{tn} - |V_{tp}|$$
$$\geq -2.5 + 0.3 (+0.7 - 0.7)$$
$$\geq -2.2V$$

$$-V_{SS} + V_{OV6} \leq V_O \leq V_{DD} - |V_{OV7}|$$
$$-2.5 + 0.3 \leq V_O \leq +2.5 - 0.3$$
$$-2.2V \leq V_O \leq +2.2V$$

10.2

$V_A' = 25V/\mu m$, $|V_P'| = 20V/\mu m$, $L = 0.8\mu m$

Hence $V_A = 20V$ and $|V_P| = 16V$

For all devices $V_{OV} = 0.25V$

$$A = A_1 A_2 = G_{m1}(r_{o2} \| r_{o4}) G_{m2}(r_{o6} \| r_{o7})$$

$$[r_{op} \| r_{on}] = \left[\frac{V_A}{I} \times \frac{V_P}{I}\right] \times \frac{I}{V_A + V_P} = \left[\frac{V_A \| V_P}{I}\right]$$

$$\text{For } A_2: R_o = \frac{8.89}{I} \rightarrow \frac{8.89V}{0.4\text{ mA}} = 22.2K\Omega$$

To avoid systematic output dc. offset

$$\frac{(W/L)_6}{(W/L)_4} = \frac{2(W/L)_7}{(W/L)_5}$$

Since Q_5, Q_6, Q_7 carry I and Q_4 only $I/2$ satisfy requirement by making Q_4 have $(W/L)/2$

Since $g_m = \sqrt{2(\mu C_{ox})(W/L) I} = 2K(V_{OV})$

$$g_{m1} = 2I_1/V_{OV} = 0.4\text{mA}/0.25V = 1.6\text{mA/V}$$
$$g_{m6} = 2I_6/V_{OV} = 3.2\text{mA/V}$$

$$\therefore A = (1.6)(44.4)(3.2)(22.2) = 5047 \text{ V/V}$$

For unity gain amplifier

$$A_F = \frac{A}{1 + A\beta} = \frac{5047}{1 + 5047\beta} \doteq 1$$

Thus $(1 + A\beta) = 5047$

Then $R_{of} = R_o/(1 + A\beta)$

$$= 22.2K/5047 \approx 4.4\Omega$$

10.3

$$A = A_1 A_2 = G_{m1}(r_{o2} \| r_{o4}) G_{m2}(r_{o6} \| r_{o7})$$
$$= \frac{2I_1}{V_{OV}} \cdot \frac{1}{2} \frac{V_A}{I_1} \cdot \frac{2I_2}{V_{OV}} \cdot \frac{1}{2} \frac{V_A}{I_2}$$
$$= \left[\frac{V_A}{V_{OV}}\right]^2 = 2500$$

where $V_A = 10V/\mu m \times 1\mu m = 10V$

Hence $V_{OV} = V_A/50 = 10/50 = 0.2V$

10.4

$CMRR = g_{m1}(r_{o2} \| r_{o4}) \times 2 g_{m3} R_{SS}$ with

$R_{SS} = r_{o5}$

$$80dB = \frac{2I_1}{V_{OV}}\left(\frac{1}{2} \times \frac{V_A}{I_1}\right) \times 2 \times \frac{2I_3}{V_{OV}}\left(\frac{V_A}{I_5}\right)$$

$$= \left(\frac{V_A}{V_{OV}}\right)^2 \times 2 \text{ (Note } I_5 = 2I_3)$$

$$10000 = 2\left(\frac{V_A}{V_{OV}}\right)^2.$$

$$V_A = V_{ov}\sqrt{\frac{10,000}{2}}$$

$$V_A = .15 \times \frac{100}{\sqrt{2}}$$

$$V_A = V_A' \times L \Rightarrow 20 \frac{\text{V}}{\mu\text{m}} \times L$$

$$L = \frac{0.15 \times 100}{20 \times \sqrt{2}} \Rightarrow L = \frac{15}{20\sqrt{2}} = 0.53\ \mu\text{m}$$

10.5

$G_{m1} = 0.3\text{ mA/v}\, G_{m2} = 0.6\text{ mA/V}\ C_2 = 1\text{p}$

$r_{02} = r_{04} = 222\text{ k}\Omega\ r_{06} = r_{07} = 111\text{ k}\Omega$

(a) $f_{p2} = \dfrac{G_{m2}}{2\pi C_2} = \dfrac{0.6\times10^{-3}}{2\pi 10^{-12}} = 95.5\text{ MHz}$

(b) $R = \dfrac{1}{G_{m2}} = \dfrac{1}{0.6} = 1.66\text{ k}\Omega$

(c) For $pm = 80^\circ$: $\tan^{-1}\dfrac{f_t}{f_{P2}} = 10^\circ$

$f_t = f_{p2}\tan 10^\circ$

$= 95.5\times0.176 = 16.84\text{ MHz}$

$C_C = \dfrac{G_{m1}}{2\pi f_t} = \dfrac{0.3\times10^{-3}}{2\pi 16.84\times10^{6}} \Rightarrow 2.83\text{ pF}$

$A = A_1A_2 = G_{m1}(r_{o2}\,\|\,r_{o4})G_{m2}(r_{o6}\,\|\,r_{o7})$

$= 0.3\times111\times0.6\times55.5$

$= 1109 \equiv 60.8\text{ dB}$

Donamant pole $f_{p1} = f_t/|A|$

Thus fp1 is approx 3 decades below ft i.e. at 16.84 KHz providing uniform 20 dB/dec slope drawn to ft.

(d) $f_t = \dfrac{G_{m1}}{2\pi C_C}$ $\therefore$ to double f_t, halve C_C

$C_{c(new)} = 1.4\text{pF}$

$\tan^{-1}\dfrac{f_t}{f_P} = \tan^{-1}\dfrac{33.7}{95.5} = 19.4^\circ$

The zero must be moved to reduce the

$19.1-10 = 9.4^\circ$

$\tan^{-1}\dfrac{f_t}{f_z} = 9.4^\circ \rightarrow \dfrac{f_t}{f_z} = 0.16$

$\Rightarrow f_z = 0.16\ f_t = 0.16\times33.7$

$= 5.6\text{ MHz}$

$$f_z = \frac{1}{2\pi C_C\left[R-\frac{1}{G_m}\right]} \rightarrow \left[R-\frac{1}{G_m}\right] = \frac{1}{2\pi f_z C_C}$$

Hence $[R - 1/G_m] = \dfrac{10^{12}\cdot10^{-6}}{2\pi\ 5.6\times1.4}$

$R = 1.67 + 20.3 = 21.97\text{ k}\Omega$

10.6

Two-stage amp with $C_2 = 1\text{pF}$

$f_t = 100\text{MHz}$, $PM = 75^\circ$

For $PM = 75^\circ$: $\tan^{-1}\dfrac{f_t}{f_{P2}} = 15^\circ$

$\therefore f_{P_2} = f_t\tan 15^\circ = 3.73 f_t = 373\text{MHz}$

$f_{P2} = \dfrac{G_{m6}}{2\pi C_2} = \dfrac{1}{2\pi R_2 10^{-12}} = 373\text{MHz}$

$\Rightarrow R_2 = \dfrac{10^{12}}{2\pi(373\times10^6)} = 426\Omega$

$\Rightarrow G_{m6} = \dfrac{1}{R_2} = 2.35\times10^{-3}\text{mA/V}$

To move zero to infinity $R = \dfrac{1}{G_{m6}} = 426\Omega$

$SR = \dfrac{I}{C_c} = \dfrac{200\mu A}{C_c}$

$SR = 2\pi f_t V_{ov1} = 2\pi 10^8\times0.2 = 1.26\times10^8\text{ V/s}$

$\Rightarrow C_c = \dfrac{200\times10^{-6}}{1.26\times10^8} \Rightarrow 1.6\text{pF}$

10.7

$G_{m1} = 1\text{ mA/V}, G_{m2} = 2\text{ mA/V},$

$\text{R} = 500\ \Omega$

$f_t \approx \dfrac{G_{m1}}{2\pi C_c} \Rightarrow C_c = \dfrac{G_{m2}}{2\pi f_t}$

$= \dfrac{1\times10^{-3}}{2\pi100\times10^6} \Rightarrow 1.59\text{pF}$

For $\dfrac{1}{G_{m2}} - R = \dfrac{10^3}{2} - 500 = 0$

Zero has been moved to ∞

For $PM = 60$: $f_t = f_{p2}\tan(90-60)^\circ$

$\Rightarrow f_{P2} = f_t/\tan 30^\circ = 173\text{ MHz}$

$C_2 \approx \dfrac{G_{m2}}{2\pi f_{P2}} = \dfrac{2\times10^{-3}}{2\pi(173\times10^6)} = 1.84\text{ pF}$

10.8

$SR = 60\text{ V}/\mu\text{s}$

$f_t = 50\text{ MHz}$

(a) $SR = 2\pi f_t V_{ov1}$

$\Rightarrow V_{ov1} = SR/2\pi f_t$

$= \dfrac{60\times10^6}{2\pi(50\times10^6)} \approx 0.2\text{ V}$

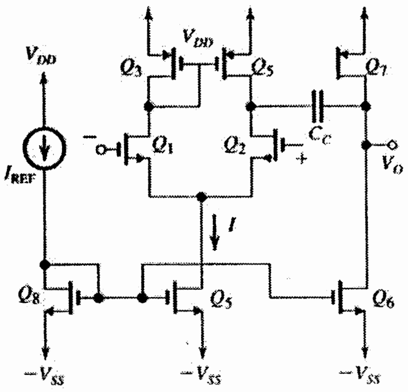
(b) $SR = \dfrac{I}{C_C} \Rightarrow C_C = \dfrac{100\ \mu A}{60\times10^6} = 1.67\text{ pF}$

(c) $I = \frac{1}{2}\mu C_{ox}(W/L)[V_{ov}]^2$

$\Rightarrow\left(\dfrac{W}{L}\right) = \dfrac{2I}{50[0.2]^2} = \dfrac{100}{1}$

10.9

Invert circuit leaving V_{DD} × V_{SS} and neverse all arrows on FETS.



10.10

a)

$$PSRR = g_{m1}(r_{o2} \| r_{o4})g_{m6}r_{o6}$$

$$= \frac{2 \times I/2}{V_{ov}}\left(\frac{1}{2}\frac{V_A}{(I/2)}\right)\frac{2I}{V_{ov}}\frac{V_A}{I} = 2\left(\frac{V_A}{V_{ov}}\right)^2$$

b) $|V_{ov}| = 0.2\text{V}$, $PSRR = 80\text{dB}$,

$|V'_A| = 20V/\mu\text{m}$

$$PSRR = 2\left|\frac{V_A}{V_{ov}}\right|^2 \Rightarrow 80 \text{ dB} = 10000$$

$$= 2\left|\frac{20 \times L}{0.2}\right|^2 \Rightarrow \frac{100}{\sqrt{2}} = 100L \Rightarrow L = 0.7\mu\text{m}$$

10.11

V_{BIAS1}: V_S can rise to $V_{DD} + V_T - V_{ov}$

V_{D3} can rise to $V_{S3} - V_{ov}$

$\therefore V_{BIAS1} = V_{DD} - V_{ov10} - V_{ov4} + V_T$

$= 1.65 - 0.2 - 0.2 + 0.5$

$= 1.75\text{V}$

$V_{BIAS2} = V_{DD} - V_{ov10}$

$= +1.65 - 0.2 = +1.45\text{V}$

$V_{BIAS3} = -V_{SS} + V_{ov11}$

$= -1.65 + 0.2 = -1.45\text{V}$

$V_{ICM(max)} = V_{DD} - |V_{ov9}| + V_{tn}$

$= +1.65 - 0.2 + 0.5$

$= +1.75\text{V}$

$V_{ICM(min)} = -V_{SS} + V_{ov11} + V_{ov1} + V_T$

$= -1.65 + 0.2 + 0.2 + 0.5$

$= -0.75\text{V}$

$-V_{SS} + 2V_{ov} + V_t \leq V_O \leq +V_{DD} - 2V_{ov}$

$-1.65 + 0.4 + 0.5 \leq V_o \leq +1.65 - 0.4$

$-0.75\text{V} \leq V_O \leq +1.25\text{V}$

10.12

$I = 125\mu A$, $I_B = 150\mu A$. $V_t = 0.2\text{V}$

For Q_9, Q_{10}: $I_B = 150\mu A$

$I = \frac{1}{2}(\mu C_{ox})(W/L)(V_{ov})^2$

$150 = \frac{1}{2}\ 90\ (W/L)(0.2)^2$

$\Rightarrow (W/L)_{9,10} = (83.33/1)$

For Q_1, Q_2: $I = 125\mu A/2$

$\frac{1}{2}.125 = \frac{1}{2}\ 250\ (W/L)(0.2)^2$

$\Rightarrow (W/L)_{1,2} = (12.5/1)$

For Q_{11}: $I = 125\mu A$

$125 = \frac{1}{2}\ 250\ (W/L)(0.2)^2$

$\Rightarrow (W/L)_{11} = (25/1)$

For $Q_3\ Q_4$: $I = 125\mu A/2$

$\frac{1}{2}125 = \frac{1}{2}\ 60\ (W/L)(0.2)^2$

$\Rightarrow (W/L)_{3,4} = (52/1)$

For $Q5, Q_6, Q_7, Q_8$: $I = 125\mu A/2$

$\frac{1}{2}125 = \frac{1}{2}\ 250\ (W/L)(0.2)^2$

$\Rightarrow (W/L)_{5,6,7,8} = (12.5/1)$

10.13

$$G_m = \frac{I}{V_{ov}} \qquad r_o = \frac{V_A}{I}$$

$$R_o = R_{o4} \| R_{o6}$$

$$R_{o4} = g_{m4} r_{o4} (r_{o2} \| r_{o10})$$

$$R_{o6} = g_{m6} (r_{o6} \| r_{o8})$$

$$A = G_{m1} R_o$$

$$Q_{10}: \ I = I_B \Rightarrow g_{m10} = 0.75\ \text{mA/V}$$

$$r_{o10} = 66.6\ \text{K}\Omega$$

$$Q_x: \ I = 125\mu A \Rightarrow g_x = 0.31\ \text{mA/V}$$

$$r_{ox} = 160\ \text{K}\Omega$$

$$\therefore R_{o4} = 0.31 \times 160 \times 47\text{K} = 2359\ \text{K}$$

$$R_{o6} = 0.31 \times 160 \times 160\text{K} = 8000\ \text{K}$$

$$R_o = 2.35 \| 8.0 = 1.8\ \text{M}\Omega$$

$$A = g_{m1} R_o = 0.31 \times 1800 = 558$$

$$\frac{v_o}{v_i} = 1 + \frac{(1/sC)}{(1/59C)} = 1 + \frac{9sC}{sC} = 10$$

$$\therefore \beta = 1/10 = 0.1$$

$$A_F = \frac{A}{1+A\beta} = \frac{558}{1+558\times 0.1} = 9.8$$

$$R_{oF} = \frac{R_o}{1+A\beta} = \frac{1.8}{56.8} = 31.69\ \text{K}\Omega$$

10.14

$$SR = \frac{I}{C_L} \Rightarrow I = SR \times C_L$$

$$= 10 \times 10^6 \times 10 \times 10^{-12}$$

$$= 100\ \mu A$$

(1) $I_B = 1.2I = 120\ \mu A$

(2) $f_p = \frac{1}{2\pi C_L R_o} \cdot ft = \frac{Gm}{2\pi C_L}$

$$G_m = \frac{2I/2}{V_{ov}} = \frac{100\ \mu A}{0.2\ V} = 0.5\ mA/V$$

$$f_t = \frac{0.5 \times 10^{-3}}{2\pi 10 \times 10^{-12}} = 7.96\ MHz$$

(4) $R_{of} \simeq \frac{1}{G_m} = \frac{1000}{0.5} = 2\ k\Omega$

$$A_V = f_t / f_p = G_m R_o$$

But $R_{of} = \frac{R_o}{1 + G_m R_o} \Rightarrow R_o = 2\ m\Omega$

$$\therefore A_V = 0.5 \times 10^{-3} \times 2 \times 10^6 = 1000$$

$$f_p = f_t/1000 = 7.96\ KHz$$

(5) $\theta = -\tan^{-1}\frac{f_t}{f_p} - 2\left(\tan^{-1}\frac{f_t}{f_p^2}\right)$

$Pm@\ f_p^2 = 90^\circ - \tan^{-1}\frac{f_t}{f_p^2}$

$$= 90^\circ - \tan^{-1}\left[\frac{7.96\ MHz}{25\ MHz}\right]$$

$$= 90^\circ - 17.7^\circ = 72.3^\circ$$

(4) For $pm = 75^\circ$: $\tan^{-1}\frac{f_t}{f_p^2} = 15^\circ$

Thus $f_t^* = f_p^2 \tan 15^\circ$

$$= 25\ MHz \times 0.27 = 6.7\ MHz$$

(5) $\frac{ft^*}{ft} = \frac{C_L}{C_L^*} \Rightarrow \frac{6.7}{7.96} = \frac{10\ pF}{C_L^*}$

$$\Rightarrow C_L^* = C_L\frac{7.96}{6.7} = C_L \times 1.19$$

$\therefore$ Increase C_L by 19%

(6) $SR^* = \frac{I}{C_L^*} \Rightarrow \frac{SR}{1.19} = 8.4\ V/\mu s$

10.15

$A = 80\ dB \quad f_t = 10\ MHz \quad C_L = 10\ pF$

$I_B = I$ All same $|V_{ov}|$, $L = 0\mu m$

$|V_A| = 20V$

$g_m = \frac{2I}{V_{ov}}$ and $f_t = \frac{g_m}{2\pi C_L}$

$$A = g_{m1}[g_{m4} r_{04}(r_{02} \| r10)] \| [g_{m6}\ r_{06}\ r_{08}]$$

Consider $Q_1 : I_1 = \frac{1}{2}kn[W/L]_1[V_{ov}]^2$

$$= \frac{1}{2}200[W/L]_1[V_{ov}]^2$$

$Q_1\ Q_2\ Q_5\ Q_6\ Q_7\ Q_8$ are same

$g_{m1} = \frac{2I_1}{V_{ov}}$ and $r_{o1} = \frac{V_A}{I_1}$

Consider $Q_3, Q_4 : I_3 = \frac{1}{2}kp[W/L]_3[V_{ov}]^2$

$$I_1 = \frac{1}{2}\frac{200}{2.5}[W/L]_3[V_{ov}]^2$$

$$\Rightarrow [W/L]_3 = 2.5[W/L]_1$$

$g_{m3,4} = g_{m1}$ and $r_{03,4} = r_{o1}$

Consider $Q_9\,Q_{10}: I_{10} = \frac{1}{2}kp[W/L]_{10}[V_{ov}]^2$

$$2I_1 = \frac{1}{2}\frac{200}{2.5}[W/L]_{10}[V_{ov}]^2$$

$\Rightarrow [W/L]_{10} = 5[W/L]_1$

$g_{m10} = 2g_{m1}$ and $r_{09,10} = r_{01/2}$

Consider $Q_{11}: I_{11} = \frac{1}{2}kn[W/L]_{11}[V_{ov}]^2$

$$2I_1 = \frac{1}{2}200[W/L]_{11}[V_{ov}]^2$$

$\Rightarrow [W/L]_{11} = 2[W/L]_1$

$g_{m11} - 2g_{m1}$ and $r_{011} = r_{01/2}$

Thus

$$A = g_{m1}\left[g_{m1}r_{01}\left(r_{01} \parallel \frac{r_{01}}{2}\right)\right] \parallel [g_{m1}\ r_{01}\ r_{01}]$$

$$= g_{m1}[g_{m1}r_{01}]\left(r_{01} \parallel \frac{r_{01}}{2} \parallel r_{01}\right)$$

$$10^4 = \frac{1}{4}g_{m1}\ g_{m1}\ r_{01}\ r_{01}$$

$\Rightarrow g_{m1}\ r_{01} = 200$

Non $g_{m1}\ r_{01} = \frac{2I}{V_{ov}} \cdot \frac{V_A}{I}$

$\Rightarrow V_{ov} = 2VA/200 = 2(20)/200 = 0.2\text{V}$

Hence $g_{m1} = \frac{1}{2\pi f_t C_L} = 0.628\ \text{mA/V}$

$\rightarrow r_{01} = 200/g_{m1} = 318\ \text{k}\Omega$

$g_m = \frac{2I}{V_{ov}} \Rightarrow I = \frac{g_m V_{ov}}{2}$

$\rightarrow I_1 = \frac{g_{m1}V_{ov}}{2} = \frac{0.678\ \text{mA}/V \times 6.7\text{V}}{2}$

$= 62.8\ \mu\text{A}$

$\text{SR} = 2\pi f_t \text{V}_{ov} = 2\pi 10 \times 10^6 \times 0.2 = 12.5\text{V}/\mu\text{s}$

$Q_1\,Q_2\,Q_5\,Q_6\,Q_7\,Q_8$:

$I = \frac{1}{2}kn[W/L][V_{ov}]^2$

$62.8 = \frac{1}{2}200[W/L][V_{ov}]^2$

$\Rightarrow [W/L]_1 = 15.7$

For $Q_3, Q_4: I = \frac{1}{2}\frac{200}{2.5}[W/L][V_{ov}]^2$

$$62.8 = \frac{1}{2}\frac{200}{2.5}[W/L][0.2]^2$$

$\Rightarrow [W/L]_3 = 2.5[W/L]_1 = 39.25$

For Q_9, Q_{10}:

$[W/L]_9 = 5[W/L]_1 = 78.5$

For $Q_{11}: [W/L]_{11} = 2[W/L]_1 = 31.4$

For $L = 1\ \mu\text{m}$: $W_x = [W/L]_x\ \mu\text{m}$

$\therefore$ width for $Q_1, Q_2, Q_5, Q_6, Q_7, Q_8 = 15.7\ \mu\text{m}$

for $Q_3, Q_4 = 39.25\ \mu\text{m}$

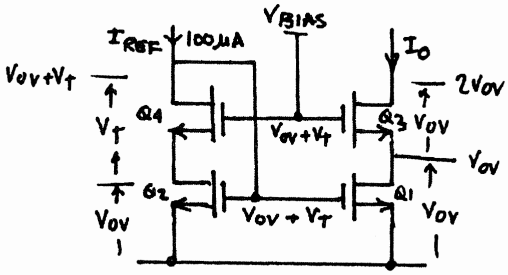
for $Q_9, Q_{10} = 78.5\ \mu\text{m}$

for $Q_{11} = 31.4\ \mu\text{m}$

10.16

Simply invert circuit relative to V_{DD}, V_{ss} and reverse all arrows on FETS

10.17



All same $k(W/L)$ $\therefore I_o \approx I_{REF}$

All same $r_o = V_A/I = 10\text{V}/100\mu\text{A} = 100\text{k}\Omega$

$I = \frac{1}{2}k[W/L][V_{ov}]^2$

$V_{D3} = V_o$: $V_o(\text{min}) = 2V_{ov}$

R_o (looking into Q3D and assuming I_o current source is ideal). $r_o' = \infty$)

$= r_{o3}(1 + g_{m3}r_{o1}) \approx g_m r_o^2$

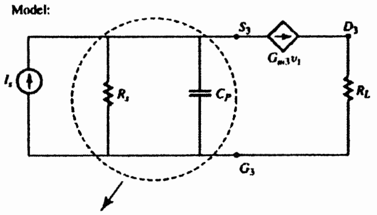
$g_m = \frac{2I}{V_{ov}} = \frac{2 \times 100}{0.2} \Rightarrow 1\text{mA/V}$

Then $R_o \approx g_m r_o^2 = 1 \times 10^5 \times 10^5$

$\approx 10^4\ \text{M}\Omega$

10.18

Model:



$$\boxed{Z} \Rightarrow R_S \parallel \frac{1}{C_pS} = \left[\frac{\frac{1}{R_SC_pS}}{R_S + \frac{1}{C_pS}}\right]\frac{C_pS}{C_pS} = \frac{R_s}{R_SC_pS + 1}$$

Summing currents at node S_3:

$$-I_S + g_{m3}V_1 + V_1\frac{(1 + R_SC_pS)}{R_S} = 0$$

$$\frac{V_1}{I_S} = \frac{1}{g_{m3} + \frac{(1 + R_SC_pS)}{R_S}} = \frac{1}{\left(g_{m3} + \frac{1}{R_S}\right) + C_pS}$$

$$W_{3\text{ dB}} = \frac{\left(g_{m3} + \frac{1}{R_S}\right)}{C_P} \text{ so}$$

$$f_{3\text{ dB}} = \frac{\left(g_{m3} + \frac{1}{R_S}\right)}{2\pi C_P} \approx \frac{g_{m3}}{2\pi C_P}$$

$$p\mu = 180 - \phi_{\text{total}} = 90° - \tan^{-1}\left(\frac{f_t}{f_{p2}}\right)$$

For $Pm = 75°$: $\frac{f_t}{f_{p2}} = \tan 15° = 0.27$

$$\frac{f_t}{f_{p2}} = \frac{C_P}{C_L} = 0.27$$

$$\therefore C_P = 0.27C_L$$

10.19

$$I_3 = I_1\sqrt{\frac{I_{S3}\,I_{S4}}{I_{S1}\,I_{S2}}}$$

$$= 154\sqrt{\frac{10^{-14}\cdot 10^{-14}}{3\times 10^{-14}\cdot 6\times 10^{-14}}}$$

$$= 36.3\,\mu\text{A}$$

10.20

$I_{E_{TOT}} = 0.73$ mA

$I_{EA} = 0.25(0.73) = 0.1825$ mA

$I_{EB} = 0.75(0.73) = 0.5475$ mA

$$V_{EB_A} = V_T \ln\frac{0.1835\times 10^{-3}}{0.25\times 10^{-14}} = 0.625\text{V}$$

$$g_{mA} = \frac{I_C}{V_T} = \frac{I_E}{V_T} = 7.3\text{ mA/V}$$

$$r_{eA} = \frac{\alpha}{g_{mA}} = 134.3\ \Omega$$

$$r_{\pi A} = (\beta + 1)r_{eA} = 6.85\text{ k}\Omega$$

$$r_{oA} = \frac{V_A}{I_{CA}} = 274\text{ k}\Omega$$

$$V_{EB_B} = V_{EB_A} = 0.625\text{V}$$

$$g_{mB} = \frac{0.5475}{25} = 21.9\text{ mA/V}$$

$$r_{e_B} = \frac{\alpha}{g_{mB}} = 44.7\ \Omega$$

$$r_{\pi B} = (\beta + 1)r_{e_B} = 2.28\text{ k}\Omega$$

$$r_{oB} = 91.3\text{ k}\Omega$$

10.21

Let $V_{BE} = 0$

For breakdown $V_{ID} = V_{B1} - V_{B2}$

$> V_{BE1} + V_{BE2} + 7 + 50$

or $V_{ID} \geq 58.4$ V

10.22

$$V_{SG1} + V_{GS2} = V_{SG4} + V_{GS3}$$

since V_t's are equal

$$\sqrt{\frac{I_1}{K_1}} + \sqrt{\frac{I_1}{K_2}} = \sqrt{\frac{I_3}{K_4}} + \sqrt{\frac{I_3}{K_3}}$$

$$\sqrt{I_1}\left[\frac{1}{\sqrt{K_1}} + \frac{1}{\sqrt{K_2}}\right] = \sqrt{I_3}\left[\frac{1}{\sqrt{K_4}} + \frac{1}{\sqrt{K_3}}\right]$$

or

$$\sqrt{\frac{I_1}{I_3}} = \frac{\sqrt{\frac{1}{K_3}} + \frac{1}{\sqrt{K_4}}}{\sqrt{\frac{1}{K_1}} + \sqrt{\frac{1}{K_2}}}$$

$K_1 = K_2$, $K_3 = K_4 = 16\,K_1$

$$\sqrt{I_1} = \sqrt{I_3}\sqrt{\frac{K_2}{K_4}}$$

or $I_1 = I_3\frac{K_2}{K_4}$

$$= \frac{I_3}{16} = \underline{\underline{100\,\mu\text{A}}}$$

10.23

As $V_{BE} = 0.7$

$$I_{ref} = \frac{5 - 14 - (-5)}{R_s}$$

$= 220.5\ \mu A$

At this current level

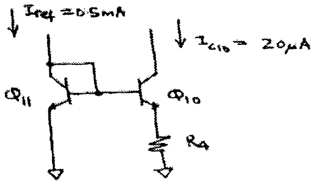
$$V_{BE} = V_T \ln\frac{220.6 \times 10^{-3}}{10^{44}} = 595\ \text{mV}$$

$$\Rightarrow I_{ref} = \frac{10 - 2(0.595)}{39\ \text{k}} = 226\ \mu A$$

For $I_{ref} = 0.75$ mA $V_{BE} = 0.625$ V

$$R_5 = \frac{10 - 2(0.625)}{0.73 \times 10^{-3}} = 12\ \text{k}\Omega$$

10.24



$$I_{C10} R_4 = V_{BE11} - V_{BE10}$$

$$= V_T \ln \frac{I_{ref}}{I_{C10}}$$

$$\therefore\ R_4 = \frac{25 \times 10^{-3}}{20 \times 10^{-6}} \ln \frac{0.5 \times 10^{-3}}{20 \times 10^{-6}} = \underline{\underline{4.02\ \text{k}\Omega}}$$

$$V_{BE11} = V_T \ln \frac{0.5 \times 10^{-3}}{10^{-14}} = \underline{\underline{616\ \text{mV}}}$$

$$V_{BE10} = V_T \ln \frac{20 \times 10^{-6}}{10^{-14}} = \underline{\underline{535\ \text{mV}}}$$

10.25

Assume $\beta_p >> 1$

$$I_{C10} = \frac{2I}{1 + 2/\beta_p} + \frac{2I}{\beta_p}$$

$$\simeq 2I\left(1 - \frac{2}{\beta_p} + \frac{1}{\beta_p}\right)$$

$$= 2I(1 - 1/\beta_p)$$

$$\Rightarrow I \simeq \frac{I_{C10}}{2}\left(1 + \frac{1}{S_p}\right)$$

Thus $\frac{1}{\beta_p} = 0.1 \Rightarrow \beta_p = 10$

Without the above assumption and using the exact relationship $\beta_p = 7.79$.

10.26

In this case

$$\frac{4I}{1 + 2/\beta_p} + \frac{2I}{\beta_p} = I_{C10}$$

For $\beta_p >> 1$

$I_{C10} \simeq 4I$ or $I = 4.75\ \mu A$

To correct we need $I_{C10} = 38\ \mu A$

$$\Rightarrow R_4 = \frac{V_T}{I_{C10}} \ln\frac{0.73\ \text{mA}}{I_{C10}} = 1.94\ \text{k}\Omega$$

10.27

At $I = 9.5\ \mu A$

$V_{BE5} = V_{BE6} = 517$ mV

and $V_{B6} = V_{BE6} + IR_2$

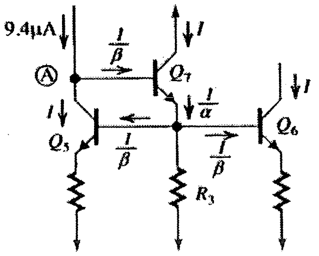
$= 526.5$ mV

If R_2 is shorted $V_{BE6} = V_{BG} = 526.5$ mV

and $I_{C6} = I_S\ e^{V_{BE6}/VT}$

$= 14\ \mu A$

10.28



ΣI @A $I = I + I/\beta = 9.4\ \mu A$

$$\Rightarrow I = \frac{9.4}{I + r\beta} = 9.353\ \mu A$$

$$I_{R3} = \frac{I}{\alpha} = \frac{2I}{\beta} = 9.307\ \mu A$$

$$V_{B5} = I_{R3}R_3 = V_{BE5} + \frac{IR}{\alpha}$$

$$V_{BE5} = V_T \ln\frac{9.353\ \mu}{10^{-14}} = 516.4\ \text{mV}$$

Thus $V_{B5} = 525.8$ mV

and $R_3 = \frac{V_{B5}}{I_{R3}} = 56.5\ \text{k}\Omega$

10.29

Assume equal collector current

$I_{C1} = I_{C2} = 9.5\ \mu A$

$$I_{os} = I_{B1} - I_{B2} = \frac{I_{C1}}{\beta_1} - \frac{I_{C2}}{\beta_2}$$

$= 15.7$ mA

$$I_B = \frac{1}{2}(I_{B1} - I_{B2})$$

$= 55.4$ mA

10.30

$I_B = 40\,nA$, $I_{OS} = 4\,nA$

Thus, base currents are

$I_{B1} = (I_B \pm \frac{I_{OS}}{2})$

$= \frac{9.5}{\beta_N}\,\mu A$

$\hat{\beta}_N = \frac{9.5\,\mu A}{38\,nA} = \underline{\underline{250}}$

$\check{\beta}_N = \frac{9.5\,\mu A}{42\,nA} = \underline{\underline{226}}$

$\Rightarrow \Delta\beta_N = 24$

$\overline{\beta}_N = \frac{\hat{\beta}_N + \check{\beta}_N}{2} = 238$

10.31

$I_{C1} + I_{C2} = 19\,\mu A$

Mirror forces $I_{C2} = 0.9\,I_{C1}$

Thus $I_{C1} = \frac{19}{1.9}\,\mu A = 10\,\mu A$

and $I_{C2} = 9\,\mu A$

$V_{OS} = \Delta V_{BE}$

$= V_{BE1} - V_{BE2}$

$= V_T \ln \frac{10}{9} = \underline{\underline{2.63\,mV}}$

10.32

At $I_{C17} = 550\ \mu A$, $V_{BE17} = 618\ mV$

$I_{B17} = \frac{550}{200} = 2.75\ \mu A$

$\Rightarrow I_{C16} = 9.5\ \mu A = I_{B17} + \frac{I_{B17}R_8 + V_{BE17}}{R_9}$

or $R_9 = 99.7\ k\Omega$

10.33

Neglecting base currents

$I_{C18} = I_{C19} = \frac{180}{2} = 90\ \mu A$

$V_{BE18} = V_T \ln \frac{90 \times 10^{-6}}{10^{-14}} = 573\ mV$

Thus $R_{10} = \frac{V_{BE18}}{I_{C18}} = 6.37\ k\Omega$

$I_{C14} = 3 \times 10^{-14}\, e^{573/25} = 270\ \mu A = I_{C20}$

10.34

$I_{V_{CC}} = I_{C12} + I_{C13A} + I_{C13B} + I_{C14} + I_{C9} + I_{C8}$

$+ I_{C7} + I_{C16}$

$= (730 + 180 + 550 + 154 + 19 + 19 + 10.5$

$+ 162)\ \mu A$

$= 1.68\ mA$

$P_{Diss} = P_Q = I_{V_{CC}}(V_{CC} + V_{EE})$

$= 1.68(15 + 15)\ mW$

$= 50.4\ mW$

10.35

Series connection of devices assures the same bias currents.

$R_{id} = (\beta + 1)(6r_e)$

$r_e = \frac{V_T}{9.5\mu A} = 2.63\ k\Omega$

$R_{id} = 3.17\ M\Omega$

$r_e = \frac{v_{id}}{6r_e}, i_o = 2i_e$

$\Rightarrow G_{m1} = \frac{i_o}{v_{id}} = \frac{2}{6r_e} = \frac{1}{3r_e}$

$= 127\ \mu A / V$

$R_{o4} = r_o(1 + gm(R_E \| r_e\ ))$

$g_m = 1 / r_e$

$R_E = 2r_e = 5.36\ k\Omega$

$r_\pi = (\beta_p + 1)r_e = 134\ k\Omega$

Thus$R_{o4} = 15.4\ M\Omega$

$R_{o6} = 18.2\ M\Omega$ (from text)

$R_{o1} = R_{o4} \| R_{o6} = 8.34\ M\Omega$

$G_{m1}R_{o1} = 127 \times 8.34 = 1059\ V / V$

See gain decreases due to negative feedback

10.36

$R_o = r_{o6}(1 + g_{m6}(R_2 \| r_{\pi6}\ ))$

need to double the second factor

Since $r_{\pi6} >> R_2$

$R_{o6} \simeq r_{o6}(1 + g_{m6}R_2)$

Thus

$1 + g_{m6}R'_2 = 2(1 + g_{m6}R_2)$

$g_{m6} = \frac{1}{2.63\ k\Omega}$, $R_2 = 1\ k\Omega$

$R'_2 = 4.63\ k\Omega$

10.37

$I_{c5} = I_{c6} = I_{c7}$

$\Rightarrow r_{e5} = r_{e6} = r_{e7} = 2.63\text{ k}\Omega$

(a) $V_{b6} = (r_{e6} + R_2)i_e = 4.63\text{ k}\Omega \times i_e$

(b) $R_B = (50\text{k} \parallel r_{\pi 5} \parallel r_{\pi 6})$

$= 45.1\text{ k}\Omega$

$\Rightarrow i_{c7} = \frac{V_{b6}}{R_B} = 0.103 i_e$

(c) $i_{b7} = \frac{i_{e7}}{\beta + 1} = \frac{0.103}{201} i_e = 510\ \mu\text{A} \times i_e$

(d) $V_{b7} = V_{b6} + r_{e7}\, i_{e7}$

$= (4.63\text{ k}\Omega + 2.63\text{ k}\Omega \times 0.103)i_e$

$= 4.9\text{ k}\Omega \times i_e$

(e) $R_{in} = \frac{V_{b7}}{i_e} = 4.9\text{ k}\Omega$

10.38

$$\frac{\Delta I}{I} = \frac{\Delta R}{R + \Delta R + r_e}$$

$$\Delta I = cami\ V_{DS} = \frac{V_{DS}}{2r_e}$$

Thus

$$\frac{V_{DS}}{2r_{eI}} = \frac{\Delta R}{R + \Delta R + r_e};\ r_e I = V_T$$

$$\frac{V_{DS}}{2r_e I} = \frac{\Delta R}{R}\left[\frac{1}{1 + \frac{r_e}{R} + \frac{\Delta R}{R}}\right] (*)$$

$$\frac{V_{DS}}{2V_T}\left(1 + \frac{r_e}{R}\right) = \frac{\Delta R}{R}\left(1 - \frac{V_{DS}}{2V_T}\right)$$

$$\frac{\Delta R}{R} = \frac{V_{DS}}{2V_T} \frac{1 + (r_e/R)}{1 - \frac{V_{DS}}{2V_T}}$$

(b) $V_{DS} = 5\text{ mV},\ r_e = 2.63\text{k}\Omega,\ R = 1\text{ k}\Omega$

$$\frac{\Delta R}{R} = \frac{5}{2(25)} \frac{1 + 2.63}{1 - \frac{5}{2(25)}} = 0.40$$

(c) R_2 completely shorted

$$\Rightarrow \frac{\Delta R}{R} = -1$$

From (*) $\frac{V_{DS}}{2V_T} = -1\frac{1}{r_e/R}$

$\Rightarrow V_{DS} = -19\ mV$ (or $19mV$)

10.39

Current in the collector of Q_3 remains unchanged at 9.5 μA

Thus $I_{E3} = I_{E4} = \frac{51}{50} 9.5\ \mu\text{A} = 9.69\ \mu\text{A}$

$I_{C4} = \frac{20}{21} \cdot I_{E4} = 9.23\mu\text{A}$

$\Delta I = 9.5 - 9.23 = 0.27\mu\text{A}$

$$V_{OS} = \frac{\Delta I}{G_{m1}} = 2r_e\Delta I$$

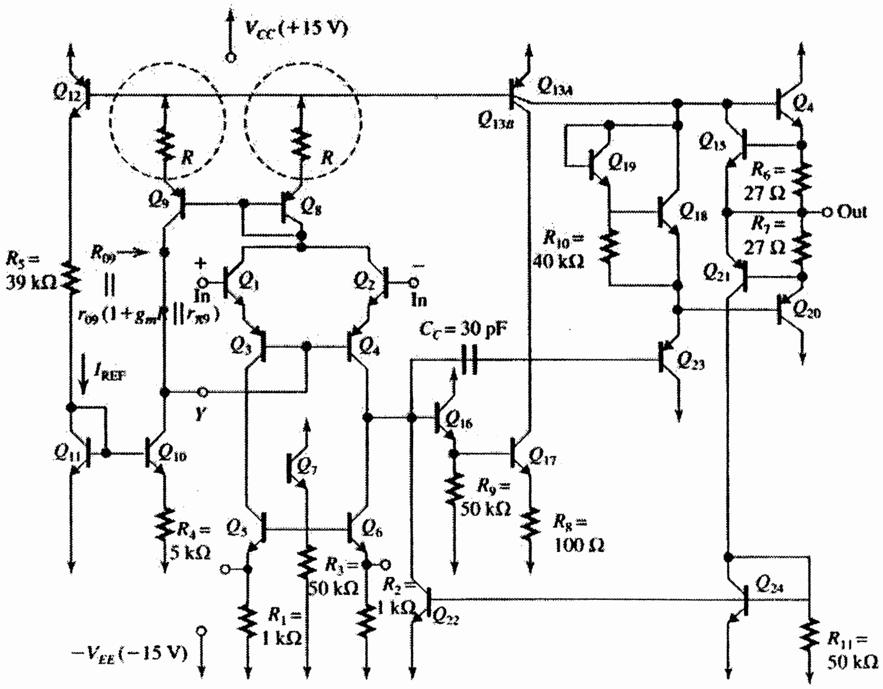
with

$r_e = 2.63\text{ k}\Omega$:

$V_{OS} = 2 \times 2.63 \times 10^3 \times 0.27 \times 10^{-6} = 1.4\text{ mV}$

10.40

Ignoring body effect


VCC(+15 V)
Q12
Q13A
Q13B
Q14
R
R
Q9
Q8
Q19
Q15
R6 = 27 Ω
Q18
Out
R7 = 27 Ω
R10 = 40 kΩ
Q21
Q20
R5 = 39 kΩ
Ro9
||
ro9(1 + gmR || rπ9)
+
In
Q1
Q2
−
In
CC = 30 pF
Q3
Q4
Q23
IREF
Y
Q16
Q11
Q10
Q7
Q17
R9 = 50 kΩ
R8 = 100 Ω
R4 = 5 kΩ
Q5
Q6
R3 = 50 kΩ
R2 = 1 kΩ
Q22
Q24
R1 = 1 kΩ
−VEE(−15 V)
R11 = 50 kΩ

$R = 5\text{ k}\Omega$ to make $R_{o9} = R_{o10}$ since I is the same for Q_9 & Q_{10}

$R_O(\text{left of Y}) = R_{o9} \| R_{o10} = \frac{1}{2} r_O [1 + g_m(5\text{ K} \| r_\pi)]$

10.41

Following the instruction of the problem, the resistance seen between the common base connection of Q_3 and Q_4
and ground is :

$$R_f = (1 + A\beta)R_O = (1 + \beta_P)R_O$$

Since the loop is broken, we have :

$$\frac{i}{\beta_P} + \frac{i}{\beta_P} \approx \frac{v_{icm}}{R_f} \Rightarrow$$

$$\frac{2i}{\beta_P} \approx \frac{v_{icm}}{(1+\beta_P)R_O} \Rightarrow i = \frac{\beta_P}{1+\beta_P}\frac{v_{icm}}{2R_O} = \frac{v_{icm}}{2R_O}$$

ave

$$i_o = E_{mi} \Rightarrow G_{mcm} \equiv \frac{i_o}{v_{icm}} = \frac{E_{mi}}{v_{icm}} = \frac{E_m}{2R_o}$$

$$CMRR = \frac{G_{m1}}{G_{mcm}} = \frac{g_{m1}}{\frac{E_m}{2R_o}}$$

$$= \frac{2g_{m1}R_o}{E_m} = \frac{2g_{m1}(R_{09} \| R_{o10})}{E_m}$$

10.42

$$R_{i2} = (\beta + 1)[r_{e16} + (R_{i17} \| R_9)]$$

$$r_{e16} = 1.54\ \text{k}\Omega$$

$$r_{e17} = 45.5\ \Omega$$

$$R_{i17} = 201(45.5 + 50) = 19.2\ \text{k}\Omega$$

$$\Rightarrow R_{i2} = 201[(1.54 + 19.2 \| 50)]\ \text{k}\Omega = 3.1\ \text{M}\Omega$$

$$v_{b17} = \frac{(R_{i17} \| R_9)}{r_{e16} + (R_{i17} \| R_9)} v_{i2}$$

$$= 0.9 v_{i2}$$

$$i_{c17} = \frac{\alpha}{r_{e17} + R_8} 0.9 v_{i2}$$

$$\Rightarrow G_{m2} = \frac{\alpha(0.9)}{45.5 + 50} = 9.38\ \text{mA/V}$$

10.43

$$R_{o17} = 787\ \text{k}\Omega$$

$$i_{c13B} = 550\ \mu\text{A}$$

$$g_{m13B} = 22\ \text{mA/V};$$

$$r_{\pi 13B} = (\beta + 1)/g_m = 2.32\ \text{k}\Omega$$

$$r_o = \frac{50}{550\mu\text{A}} = 90.9\ \text{k}\Omega$$

$$R_{o13B} = r_o(1 + g_m(R_e \| r_\pi))$$

$$= 90.9[1 + 22(R_E \| 2.32)]$$

$$= 787$$

$$\Rightarrow R_E \| 2.32 = 0.348$$

$$\text{and}\frac{1}{R_E} = 2.44 \text{ or } R_E = 0.410\ \text{k}\Omega$$

$$R_E = 410\ \Omega$$

$$\text{Current}\frac{R_{E12}}{R_E} = \frac{550\ \mu\text{A}}{730\ \mu\text{A}} \Rightarrow R_{E12} = 309\ \Omega$$

$$\frac{R_{E13A}}{R_E} = \frac{550}{180} = 1.25\ \text{k}\Omega$$

10.44

$$\hat{V}_o = V_{CC} - V_{CEsat18A} - V_{BE14}$$

$$= 4.2\ \text{V}$$

$$\hat{V}_o \approx -V_{EE} + V_{CEsat17} + V_{BE23} + V_{BE20}$$

$$= -5 + 0.2 + 0.6 + 0.6$$

$$= -3.6\ \text{V}$$

10.45

With Q_{23} removed, current in Q_{17} increases to 730μA. This changes G_{m2}

$$r_{e17} = \frac{V_T}{730\ \mu\text{A}} = 34.2\ \Omega$$

$$\Rightarrow G_{m2} \approx 0.923\frac{\alpha}{100 + 34.2} = 6.8\ \text{mA/V}$$

Because $r_{o17} >> r_{o13B}$ R_{o2} remains Virtually unchanged at 81kΩ

$$R_{i3} = (\beta + 1)(R_L \| r_{o13A}) = 74\ \text{k}\Omega$$

$$\Rightarrow A_z = -6.8(81)\frac{74}{74 + 81} = -263\ \text{V/V}$$

10.46

Ignore base current of Qs

$$180\ \mu\text{A} = I_{C15} + \frac{I}{\beta_{T1}},$$

where $I = I_{R6}$

$$I_{C15} = I_S e^{V_{BE}/VT}$$

where $V_{BE} = IR_6$

$$\text{Thus } I = \frac{V_T}{27}\ln\left[\frac{180\mu\text{A} - \frac{I}{201}}{I_S}\right]$$

$$= 191{,}422\ \text{V/V}$$

$$= 105.6\text{dB}$$

Output current is limited to ± 20 mA
(see problem 37 and 38)

$\Rightarrow |V_d| < 20\text{ mA}(200)$

$|V_d| < 4\text{ V}$

To obtain a seed solution, let $I = 0$ right hand side

$$\Rightarrow I = \frac{V_T}{27}\ln\frac{180\ \mu A}{10^{-14}} = 21.9\text{ mA}$$

Iterating I = 21.0 mA

10.47

Maximum output current of the 1st stage = 19μA

$\Rightarrow I_{C22} = 19\mu A \Rightarrow V_{BE22} = V_{BE24} = 534\text{ mV}$

$\Rightarrow I_{R11} = \frac{534}{50} = 10.7\mu A$

$\therefore I_{c21} = (19 + 10.7)\mu A = 29.7\ \mu A$

and $V_{BE21} = 545.3\text{ mV}$

$V_{BE21} = IR_7 \Rightarrow I = 20.2\text{ mA}$

A simple doubling of R_7

10.48

$$\frac{v_o}{v_i} = \frac{243{,}147}{0.97} = 250{,}667\text{ V/V} \cong \underline{\underline{108\text{ dB}}}$$

$$\frac{R_L}{R_o + R_L} = 0.9 \implies R_o = R_L\left(\frac{1}{0.97} - 1\right)$$

$$\text{or}\quad R_o = \underline{\underline{61.9\Omega}}$$

$$\left.\frac{v_o}{v_i}\right|_{R_L = 200} = 250{,}667 \cdot \frac{200}{200 + 61.9}$$

10.49

80° PM says that 2nd pole introduces 10° of phase shift at 1MHz

$$\text{i.e}\quad \tan^{-1}\frac{f_t}{f_{P2}} = 10^\circ$$

$$\text{or}\quad f_{P2} = \underline{\underline{5.67\text{ MHz}}}$$

10.50

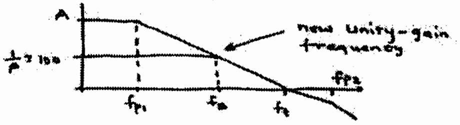
Each pole adds 5° of phase shift

$$\tan^{-1}\frac{10^6}{f_{2,3}} = 5^\circ$$

$$\Rightarrow f_{2,3} = \underline{\underline{11.4\text{ MHz}}}$$

10.51

Consider Bode plot



85° of closed-loop phase margin

$$\Rightarrow \tan^{-1}\frac{f_B}{f_{P2}} = 5^\circ$$

$$\text{or}\quad f_B = \underline{\underline{437\text{ kHz}}}$$

Recalling the "broadbanding" effect of negative feedback, we get

$$f_B = f_{P1}(1 + A\beta) \simeq f_{P1}A\beta$$

Loop gain $A\beta = 2.43\times10^5 \cdot \frac{1}{100} = 2.43\times10^3$

$$\Rightarrow f_{P1} = \underline{\underline{180\text{ Hz}}}$$

$$f_B = \frac{G_{m1}}{2\pi C_c} = 437\text{kHz}$$

$$\Rightarrow C_c = \frac{1}{5.26\times10^5\,(2\pi)\,437\times10^3}$$

$$= \underline{\underline{0.69\text{ pF}}}$$

10.52

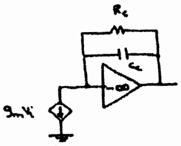
dominant pole $f_P = \frac{1}{2\pi R(AC_c)}$; $A = 1000$

with single pole response

$$A_o f_P = f_t \Rightarrow f_P = \frac{5\times10^6}{10^6} = 5\text{ Hz}$$

$$\Rightarrow R = \frac{1}{2\pi(5)\,1000\,(50\text{pF})} = \underline{\underline{637\text{ k}\Omega}}$$

10.53

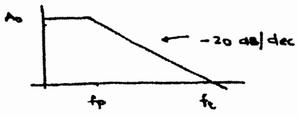


DC, gain is

$$A_0 = G_{m1} R_c$$
$$= 10 \times 10^{-3} \times 10^8$$
$$= \underline{\underline{10^6 \text{ V/V}}}$$

$$f_p = \frac{1}{2\pi R_c C_c} = \frac{1}{2\pi \times 10^8 \; 50 \times 10^{-12}}$$
$$= 31.8 \text{ Hz}$$

$$f_t = A_0 f_p$$
$$= 31.8 \text{ mHz}$$



$$SR = \frac{2I}{C_c}$$

$$G_{m1} = \frac{I}{2V_T} \Rightarrow 2I = 4G_{m1}V_T$$

$$SR = \frac{4G_{m1}V_T}{C_c} = \frac{4(10 \times 10^{-3})(25 \times 10^{-3})}{50 \times 10^{-12}}$$
$$= \underline{\underline{20 \text{ V}/\mu\text{s}}}$$

10.54

$$\underline{\underline{I_{E1} = I_{E2}}} = 50\,\mu\text{A} \simeq \underline{\underline{I_{E3} = I_{E4}}}$$

$$\underline{\underline{I_{E5} = 1\text{mA}}} \;;\; V_{BE5} = V_{BE6}$$

$$\therefore \; \underline{\underline{I_{E6} = 1\text{mA} = I_{E7}}}$$

$$r_{e1} = r_{e2} = 500\,\Omega$$
$$r_{e5} = r_{e6} = r_{e7} = 25\,\Omega$$
$$G_{m1} = 2\left(\frac{1}{2r_{e1}}\right) = 2\text{mA/V}$$
$$R_{o1} = (\beta+1)(r_{e5} \| r_{e6})$$
$$= 1.25 \text{ k}\Omega$$
and $A_1 = G_{m1}R_{o1} = 2.5$ V/V

10.55

$I = 10\,\mu\text{A}, \; \dfrac{I_{S2}}{I_{S1}} = 2,$

$R_2 = 1.73 \text{ k}\Omega, R_3 = R_4 = 20 \text{ k}\Omega.$

$I_5 = 10\,\mu\text{A}, \; I_6 = 40\,\mu\text{A}$

From $V_{BE5} = V_{BE1} \Rightarrow V_T \ln \dfrac{I_5}{I_{S5}} = V_T \ln \dfrac{I_1}{I_{S1}}$

or $\dfrac{I_5}{I_{S5}} = \dfrac{I_1}{I_{S1}}$

Since $I_5 = 10\,\mu\text{A} = I_1$, then $\dfrac{I_{S5}}{I_{S1}} = 1$ or equivalently Q_1 and Q_5 have the same emitter area.

For Q_6: $I_6 = 40\,\mu\text{A}$ or $I_6 = 4I_1$. Similar to Q_5:

$V_{BE6} = V_{BE1}$, therefore: $\dfrac{I_{S6}}{I_{S1}} = 4$. If a resitor R_6 is connected to the emitter of Q_6 and the current I_6 is reduced to 10µA, then we can write:

$V_{BE1} - V_{BE6} = R_6 I_6$ or

$$V_T \ln \frac{I_1}{I_{S1}} - V_T \ln \frac{I_6}{I_{S6}} = R_6 I_6$$

$$\Rightarrow V_T \ln \frac{I_1}{I_{S1}} \frac{I_{S6}}{I_6} = R_6 I_6$$

$$\therefore 25 \times 10^{-5} \times \ln 4 = R_6 \times 10 \times 10^{-6}$$

$$\Rightarrow R_6 = 3.5 \text{ k}\Omega$$

The output resistance of Q_5 is simply r_{o5}:

$$R_{o5} = r_o = \frac{V_A}{I_C} = \frac{30}{10\,\mu\text{A}} = 3 \text{ M}\Omega$$

For Q6, the output resistance is increased by a factor of $(1 + g_m R'_E)$ where

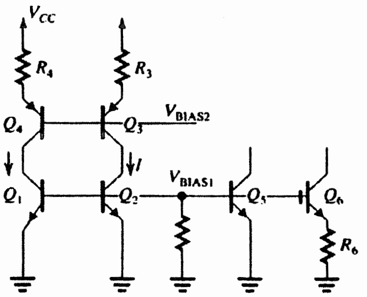
$R'_E = R_6 \| r_{\pi 6}$ (See Eq. on page of the Text.)

$$R_{o6} = (1 + g_m R'_E) r_{o6}$$

$$g_{m6} = \frac{I_{C6}}{V_T} = \frac{10}{25} = 0.4 \text{ mA/V}$$

$$r_\pi = \frac{\beta_n}{g_m} = \frac{40}{0.4} = 100 \text{ k}\Omega, \; r_{o6} = 3 \text{ M}\Omega$$

$$R_{o6} = (1 + 0.4 \times (3.5 \text{ k}\Omega \| 100 \text{ k}\Omega)) \times 3 \text{ M}\Omega$$
$$= 7 \text{ M}\Omega$$



10.56

a) $V_{CC} = 3$ V $V_{BIAS} = 2.3$ V

The minimum allowed value of V_{ICM} in the circuit of Fig. 12.40(a) is limited by the need to keep Q_1 in the active mode. Since the collector of Q_1 is at a voltage $V_{BE3} \simeq 0.7$V, we see that the voltage applied to the base of Q_1 cannot go lower than 0.1 V. Thus $V_{ICMIN} = 0.1$ V

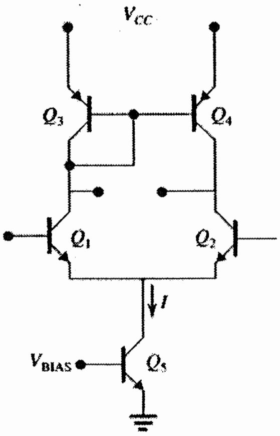
For V_{ICMmax}, see Eq. on page of the Text:

$V_{ICMmax} = V_{CC} - 0.8\text{V} = 3\text{V} - 0.8\text{V} = 2.2\text{V}$

b) Similarly, V_{ICMmax} is limited by the need to keep Q_1 in the active mode. $V_{ICMmax} = V_{CC} - 0.1\text{V} = 2.9\text{V}$

The lower end of the input common-mode range is achieved when the voltage across Q_5, V_{CE5}, does not fall below 0.1V. Hence:

$V_{ICMmin} = 0.1 + 0.7 = 0.8\text{V}$



10.57

$V_{CC} = 3\text{ V}, V_{BIAS} = 2.3\text{ V},$

$I = 20\ \mu\text{A}, R_C = 20\ \text{k}\Omega$

From Equations we have :

$$V_{ICM\,\min} = V_{RC} - 0.6\text{ V} = \frac{I}{2} \times R_C - 0.6\text{ V}$$

$$= \frac{20}{2} \times 10^{-6} \times 20 \times 10^3 - 0.6 = -0.4\text{ V}$$

$$V_{ICM\,\max} = V_{CC} - 0.8\text{ V} = 2.2\text{ V}$$

$$-0.4\text{V} \le V_{ICM} \le 2.2\text{V}$$

$$\frac{v_o}{v_{id}} = -g_{m1,2}R_C = -\frac{I/2}{V_T}R_C$$

$$= -\frac{V_{RC}}{V_T} = -\frac{20/2 \times 10^{-6} \times 20 \times 10^3}{25 \times 10^{-3}} = 8\text{ V/V}$$

10.58

$$\frac{v_o}{v_{id}} = -g_{m3,4}R_C.$$

With $A_V = 10\text{V/V}$ and $I_6 = 10\ \mu\text{A}$, we have :

$$g_{m3} = \frac{I_{C3}}{V_T} = \frac{10/2 \times 10^{-6}}{25 \times 10^{-3}}$$

$$\Rightarrow g_{m3,4} = 0.2\text{ mA/V}$$

Then $10\text{V/V} = -0.2R_C \Rightarrow R_C = 50\ \text{k}\Omega$

$$R_{id} = 2r_{\pi 3,4} = 2\frac{\beta_n}{g_{m3}} = 2 \times \frac{40}{0.2} = 400\ \text{k}\Omega$$

To find the common-input range, we calculate V_{RC}: $V_{RC} = 50\ \text{k}\Omega \times \frac{10}{2}\ \mu\text{A} = 0.25\text{ V}$ From discussion on page of the Text :

$V_{ICMmin} = 0.1 + 0.7 = 0.8\text{V},$

$V_{ICMmax} = V_{CC} - V_{RC} - 0.1 + 0.7$

$= 3 - 0.25 - 0.1 + 0.7$

$= 3.35\text{ V}$

$0.8\text{ V} \le V_{ICM} \le 3.35\text{V}$

As we can see, in the circuit in Fig. 12.41, the input common-mode range is extended above V_{CC}.

10.59

$$R_{id} = 2r_{\pi 1} = 2p_P/g_{m1} = 2 \times 10 \times \frac{25 \times 10^{-3}}{\frac{6}{2} \times 10^{-6}}$$

$= 167\ \text{k}\Omega$

To find the short-circuit trans conductance, we short the output to ground as shown in Fig. 12.43(b) on the same page of the Text:

$$G_{m1} = \frac{i_{C7}}{V_{id}/2}\ \text{(see Example 12.5)}$$

$$r_{o1} = \frac{|V_{AP}|}{I_{C1}} = \frac{20}{3} = 6.7\text{ M}\Omega$$

$$r_{o7} = \frac{|V_{An}|}{I_{C7}} = \frac{30}{3} = 10\text{ M}\Omega$$

$$r_{e7} = \frac{1}{g_{m7}} = \frac{V_t}{I_{C7}} = \frac{25}{3} = 8.3\ \text{k}\Omega$$

$R_7 = 22\ \text{k}\Omega$ and if we neglect r_{o1} and r_{o7} as they are large, we can write :

$$i_{c7} = g_{m1}\left(\frac{v_{id}}{2}\right)\frac{R_7}{r_{e7} + R_7} = \frac{3 \times 10^{-6}}{25 \times 10^{-3}}$$

$$\left(\frac{v_{id}}{2}\right)\frac{22\ \text{k}\Omega}{8.3\ \text{k}\Omega + 22\ \text{k}\Omega} = 87.1 \times 10^{-6}\frac{v_{id}}{2}$$

$$G_{m1} = \frac{i_{c7}}{v_{id}/2} = 0.087\ \text{mA/V}$$

Now to calculate $R_o : R_o = \left(R_{o9} \| R_{o7} \| \frac{R_L}{2}\right)$

$R_{o9} = r_{o9} + (R_9 \| r_{\pi 9})(1 + g_{m9} r_{o9})$,

$$r_{o9} = \frac{V_{AP}}{I_{en}} = \frac{20}{6} = 6.7\ \text{M}\Omega,$$

$$r_{M9} = \frac{\beta_P}{g_{m9}} = \frac{10}{6/25} = 41.7\ \text{k}\Omega$$

$R_{o9} = 6.7\ \text{k}\Omega + (33\ \text{k}\Omega \| 41.7\ \text{k}\Omega)$

$$\left(1 + \frac{6 \times 10^{-6}}{25 \times 10^{-3}} \times 6.7 \times 10^{6}\right)$$

$\Rightarrow R_{o9} = 36.3\ \text{M}\Omega$

$R_{o7} = r_{o7} + (R_7 \| r_{\pi 7})(1 + g_{m7} r_{o7})$,

$$r_{o7} = \frac{V_{AN}}{I_{C7}} = \frac{30}{3} = 10\ \text{M}\Omega,$$

$$r_{\pi 7} = \frac{30}{6/25} = 5\ \text{k}\Omega$$

$$g_{m7} = \frac{6}{25} = 0.24\ \text{mA/V}$$

$R_{o7} = 10\ \text{k}\Omega + (22\ \text{k}\Omega \| 5\ \text{k}\Omega)$

$(1 + 0.24 \times 10^{-3} \times 10 \times 10^{6})$

$\Rightarrow R_{o7} = 19.8\ \text{M}\Omega$

$\therefore R = 36.3 \| 19.8 \| 1.3/2 = 0.62\ \text{M}\Omega$

$A_d = G_{m1} R = 0.087 \times 10^{-3} \times 0.62 \times 10^{6}$

$= 54\ \text{V/V}$

$$A_d = 54\ \frac{\text{V}}{\text{V}}$$

10.60

$$A_{\text{open}} = G_{m1} \cdot R_{\text{out}}$$

$$G_{m1} = \frac{i_O}{V_{id}/2} \text{ where}$$

$$i_O \approx i_{e7} = g_{m1} = \left(\frac{V_{id}}{2}\right)\left[\frac{R_7}{R_7 + r_{e7}}\right]$$

$$R_7 = \frac{0.2\text{ V}}{(2I + I)} = \frac{0.2\text{ V}}{3I} \text{ and}$$

$$r_{e7} \approx \frac{1}{g_{m7}} = \frac{V_T}{I_{C7}} = \frac{25\text{ m}}{2I}$$

$$\therefore G_{m1} = \left[\frac{I_{C1}}{V_T}\right]\left[\frac{\left(\frac{0.2}{3I}\right)}{\left(\frac{25\text{ m}}{2I}\right) + \left(\frac{0.2}{3I}\right)}\right]$$

$$= \frac{I}{25\text{ m}}\left[\frac{\frac{0.2}{3I}}{\frac{75\text{ m}}{6I} + \frac{0.4}{6I}}\right] = \frac{I}{25\text{ m}}\left[\frac{0.2 \times 6I}{0.475 \times 3I}\right]$$

$$G_{m1} = 33.7I$$

$$R_{\text{out}} = R_{O7} \parallel R_{O9}$$

$$R_{O7} = r_{O7} + (R_7 \parallel r_{\pi7})(1 + g_{m7}r_{O7})$$

using $\beta_n = 40,\ V_{An} = 30:$

$$r_{O7} = \frac{V_{An}}{I_{C7}} = \frac{30}{2I}$$

$$r_{\pi7} = \frac{\beta_n}{g_{m7}}$$

$$g_{m7} = \frac{I_{C7}}{V_T} = \frac{2I}{25\text{ m}}$$

$$r_{\pi7} = \frac{40}{2I} \times 25\text{ m} = \frac{1}{2I}$$

$$R_{O7} = \frac{30}{2I} + \frac{\left[\frac{0.2}{3I} \times \frac{1}{2I}\right]}{\left(\frac{0.2}{3I} + \frac{1}{2I}\right)} \times \left[1 + \frac{2I}{25\text{ m}} \times \frac{30}{2I}\right]$$

$$R_{O7} = \frac{30}{2I} + \frac{\left[\frac{0.2}{6I}\right]}{\left[\frac{0.4 + 3}{6I}\right]} \times [1 + 1,200]$$

$$= \frac{30}{2I} + \frac{0.2}{3.4I} \times 1,201$$

$$R_{O7} = \frac{102 + 240.2}{6.8I} = \frac{50.3}{I}$$

$$R_{O9} = r_{O9} + (R_9 \parallel r_{\pi9})(1 + g_{m9}r_{O9}) \text{ where}$$

$$R_9 = \frac{0.3}{2I}$$

using $\beta_p = 10$ and $|V_{A_p}| = 20\text{ V}$

$$r_{O9} = \frac{|V_{A_p}|}{I_{C9}} = \frac{20}{2I}$$

$$g_{m9} = \frac{I_{C9}}{V_T} = \frac{2I}{25\text{ m}}$$

$$r_{\pi9} = \frac{\beta_p}{g_{m9}} = \frac{10 \times 25\text{ m}}{2I}$$

$$R_{O9} = \frac{20}{2I} + \frac{\left[\frac{0.3}{2I} \times \frac{0.25}{2I}\right]}{\frac{(0.3 + 0.25)}{2I}} \times \left[1 + \frac{2I}{25\text{ m}} \times \frac{20}{2I}\right]$$

$$= \frac{20}{2I} + \frac{0.075}{(0.55)2I} \times [801]$$

$$R_{O9} = \frac{(20 + 109.2)}{2I} = \frac{129.2}{2I} =$$

$$R_{\text{out}} = R_{O7} \parallel R_{O9} = \frac{\frac{50.3}{I} \times \frac{64.6}{I}}{\left(\frac{50.3 + 64.6}{I}\right)} = \frac{3,249.4}{114.9I}$$

$$= \frac{28.3}{I}$$

$$A_{\text{open}} = 33.7I \times \frac{28.3}{I} = 0.84\text{ V/V}$$

$$R_{\text{out}} = R_{O7} \parallel R_{O9} \parallel \left(\frac{R_L}{2}\right) = \frac{\frac{28.3}{I} \cdot \frac{R_L}{2}}{\frac{28.3(2) + IR_L}{2I}}$$

$$= \frac{28.3(R_L)}{56.6 + IR_L}$$

$$A_d = 33.7I \times \frac{28.3(R_L)}{56.6 + IR_L} = \frac{953.7IR_L}{56.6 + IR_L}$$

$$\text{For } A_d = 160\text{ V/V} = \frac{953.7 \times 2 \times 10^6 \times I}{56.6 + I \times 2 \times 10^6}$$

$$160(56.6) + [I \times 2 \times 10^6 \times 160 - I \times 953.7 \times 2 \times 10^6] = 0$$

$$I(320 \times 10^6 - 1,907 \times 10^6) = -9056$$

$$I = \frac{-9056}{-1,587\text{ M}} = 5.7\ \mu\text{A}$$

$$\text{For } A_d = 320\text{ V/V} = \frac{1,907\text{ M} \times I}{56.6 + I(2\text{ M})}$$

$$320(56.6) = [1,907\text{ M} - 320(2\text{ M})]I$$

$$I = \frac{18,112}{1,267\text{ M}} = 14.3\ \mu\text{A}$$

10.61

(a)To find the loopgain of the common-mode feedback loop for the circuit in Fig. 12.44 of the Text, we set the input voltage to zero (that is, I_1, and I_2 are zero), break the loop at the input of the common-mode feedback circuit block, apply a test voltage v_t to the input of common-mode feedback circuit and find the output voltage v_o at the output of the amplifier (where the loop was broken), and then $A\beta = -\frac{v_o}{v_t}$. Looking at half circuit and assuming that r_{o7} is relatively large, we have : $i_{07} \approx \frac{v_t}{(\beta + 1)(r_{e7} + R_7)}$. Note that $(\beta + 1)(r_{e7} + R_7)$ is the small-signal input resistance seen at the base of Q_7.

Thus, $i_o = i_{c7} = \beta i_{b7} \approx \frac{\beta v_t}{(\beta + 1)(r_{e7} + R_7)}$

$$= \frac{v_t}{\frac{\beta + 1}{\beta}(r_{e7} + R_7)} \approx \frac{v_t}{r_{e7} + R_7}$$

$$v_o = -i_o \times (R_{o7} \| R_{o9}) \approx -\frac{v_t(R_{o7} \| R_{o9})}{r_{e7} + R_7}$$

$$\Rightarrow A\beta = -\frac{v_o}{v_t} \approx \frac{R_{o7} \| R_{o9}}{r_{e7} + R_7}$$

(b) From Example 12.6 of the Text, we have :

$R_{o7} = 23\ \text{M}\Omega$ and

$R_{o9} = 12.9\ \text{M}\Omega \Rightarrow R_{o7} \| R_{o9} = 8.3\ \text{M}\Omega$

$r_{e7} = \frac{25\ \text{mV}}{10\ \mu\text{A}} = 2.5\ \text{k}\Omega$ and $R_7 = 20\ \text{k}\Omega$

Thus, $A\beta \approx \frac{R_{o7} \| R_{o9}}{r_{e7} + R_7} = \frac{8.3\ \text{M}\Omega}{2.5\ \text{k}\Omega + 20\ \text{k}\Omega}$

$= 368.9 \Rightarrow A\beta \approx 368.9$

(c) With the CMF present, we have

$$\Delta V_{CM} = \frac{\Delta V_{CM} \text{ when CMF is absent}}{1 + A\beta}$$

$$\Rightarrow \Delta V_{CM} = \frac{2.5\text{V}}{1 + 368.9} = 6.76\ \text{mV}$$

Note that the corresponding value for ΔV_{CM} found by a different approach in Example 12.6 is 6.75mV which is only 0.1% off from the calculated value in this problem.

10.62

$I_Q = 0.4\ \text{mA}$ $i_{LMax} = 10\ \text{mA}$

a) The output voltage v_o can swing as low as 0.1V when Q_P is in active, and Q_N supplies the load current : $v_{omin} = 0.1\ \text{V}$

v_o can go up as high as $V_{CC} - 0.1\ \text{V}$ when Q_N is inactive and Q_P supplies the load current :

$0.1\text{V} \leq v_o \leq V_{CC} - 0.1\ \text{V}$

b) $i_L = 0$ $R_o = R_{oN} \| R_{op}$

$i_L = 0 \Rightarrow i_p = i_N = I_Q = 0.4\ \text{mA}$

$$r_{oN} = \frac{V_{AN}}{I_N} = \frac{30}{0.4} = 75\ \text{k}\Omega$$

$$r_{op} = \frac{V_{AP}}{I_P} = \frac{20}{0.4} = 50\ \text{k}\Omega$$

$R_O = 30\ \text{k}\Omega$

c) $R_{op} = \frac{R_o}{1 + A\beta} = \frac{30\ \text{k}\Omega}{1 + 10^5 \times 1} = 0.3\ \Omega$

d) $i_L = 10\text{mA}$ Since i_L is at its max, then Q_N is inactive mode. Hence: $i_N = \frac{I_Q}{2} = 0.2\ \text{mA}$ and since we have: $i_p = i_N + i_L \Rightarrow i_p = 10.2\ \text{mA}$

e) $i_L = -10\text{mA}$ then Q_P is inactive and

$i_P = 0.2\text{mA}$.

For Q_N: $i_P - i_L$ $i_N = 0.2 - (-10) = 102\ \text{mA}$

10.63

$$v_{B7} = v_{BEN} = V_T \ln\frac{i_N}{I_{SN}}$$

$v_{B6} = R_5 i_4 + v_{BE5} = R_5 i_4 + V_T \ln\frac{i_4}{I_{S5}}$ Note that $i_4 \approx i_5$

IF we substitute for $i_4 = \frac{v_{BEP} - v_{BE4}}{R_4}$, then :

$v_{B6} = \frac{R_5}{R_4}(v_{BEP} - v_{BE4}) + V_T \ln\frac{i_4}{I_{S5}}$ Note that $R_5 = R_4$

$$v_{B6} = V_T \ln\frac{i_P}{I_{SP}} - V_T \ln\frac{i_4}{I_{S5}} + V_T \ln\frac{i_4}{I_{S5}}$$

$$V_T \ln\frac{i_P}{I_{SP}} \times \frac{I_{S4}}{i_4} \times \frac{i_4}{I_{S5}}$$

Now substitute for $I_{S5} = I_{S4}\frac{I_{SN}}{I_{SP}}$, then we have:

$$v_{B6} = V_T \ln\frac{i_P}{I_{SP}}\frac{I_{S4}}{I_{S4}}\frac{I_{SP}}{I_{SN}} = V_T \ln\frac{i_P}{I_{SN}}$$

Now if we consider V_E and write :

$$v_E = v_{B6} + v_{BE6} = v_{B7} + v_{BE7}$$

$$\Rightarrow V_T \ln\frac{i_P}{I_{SN}} + v_T \ln\frac{i_6}{I_{S6}} = V_T \ln\frac{i_N}{I_{SN}} + V_T \ln\frac{i_7}{I_{S7}}$$

$$\Rightarrow \frac{i_P}{I_{SN}} \times \frac{i_6}{I_{S6}} = \frac{i_N}{I_{SN}} \times \frac{i_7}{I_{S7}}$$

$\Rightarrow i_7 = \frac{I_{S7}}{I_{S6}} \frac{i_P}{i_N} \times i_6$ Note that $I_{S7} = I_{S6}$, hence

$$i_7 = \frac{i_P}{i_N} i_6$$

We can write : $i_6 + i_7 = I$,hence :

$$i_6 + \frac{i_P}{i_N} i_6 = I \Rightarrow i_6 = \frac{I i_N}{i_P + i_N}$$

$$i_7 = \frac{i_P}{i_N} \times I \frac{i_N}{i_N + i_P} \Rightarrow i_7 = I \frac{i_P}{i_P + i_N}$$

10.64

For Q_7 we can write : $v_{B7} = v_{BEN} = V_T \ln \frac{i_N}{I_{SN}}$

At node E, we have :

$$v_E = v_{EB7} + v_{B7} = V_T \ln \frac{i_7}{I_{S7}} + V_T \ln \frac{i_N}{I_{SN}}$$

$$= V_T \ln \frac{i_7}{I_{S7}} \frac{i_N}{I_{SN}}$$

$$i_{C7} = I \frac{i_P}{i_P + i_N}$$

$$\therefore v_E = V_T \ln \left[\frac{i_P i_N}{i_N + i_P} \frac{I}{I_{SN} I_{S7}} \right]$$

10.65

$I_Q = 0.36$ mA , $I = 10$ μA , $I_{SN} = 8 I_{s10}$,

$I_{S7} = 4 I_{S11}$

From Eq. 12.138, we have :

$$I_Q = 2\left(\frac{I_{REF}^2}{I}\right) \frac{I_{SN}}{I_{S10}} \frac{I_{S7}}{I_{S11}} \Rightarrow 0.36 \times 10^{-3}$$

$$= 2\left(\frac{I_{REF}^2}{10 \times 10^{-6}} \times 8 \times 4\right)$$

$$I_{REF}^2 = \frac{0.36 \times 10^{-8}}{64} \Rightarrow I_{REF} = 7.5 \ \mu\text{A}$$

The minimum current in the inactive output transistors, Q_N and Q_P is $\frac{1}{2} I_Q$ or 0.18 mA.

11.1

$$T(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{1}{s^3+2s^2+2s+1}$$

$$T(j\omega) = [j(2\omega-\omega^3)+(1-2\omega^2)]$$

$$|T(j\omega)| = [(2\omega-\omega^3)^2+(1-2\omega^2)^2]^{-1}$$

$$= [4\omega^2-4\omega^4+\omega^6+1-4\omega^2+4\omega^4]$$

$$= [1+\omega^6]^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{1+\omega^6}}$$

For phase Angle:

$$\phi(\omega) = \tan^{-1}\left[\frac{Im[T(j\omega)]}{Re[T(j\omega)]}\right]$$

$$= -\tan^{-1}\left[\frac{2\omega-\omega^3}{1-2\omega^2}\right]$$

For $w = 0.1$:

$$|T(j\omega)| = (1+0.1^6)^{-1/2} \cong 1$$

$$\phi(\omega) = -11.5^\circ = -0.20 \text{ rad}$$

For $\omega = 1$ rad/s:

$$|T(j\omega)| = (1+1^6)^{-1/2} = 1/\sqrt{2} = 0.707$$

$$\phi = -\tan^{-1}\left(\frac{1}{-1}\right) = -135^\circ = 2.356 \text{ rad}$$

Note: $G = -3$ dB

Also: $\tan^{-1}(-1) = -45^\circ$ or -135°

$$\tan^{-1}\left(\frac{-1}{1}\right) = -45^\circ$$

$$\tan^{-1}\left(\frac{1}{-1}\right) = -135^\circ$$

For $w = 10$ rad/s:

$$|T(j\omega)| = (1+10^6)^{-1/2} = 0.001$$

$$\phi = -\tan^{-1}\left[\frac{2(10)-10^3}{1-2(10^2)}\right]$$

$$= -\tan^{-1}\left[\frac{-980}{-199}\right]$$

$$= -\left[180^\circ+\tan^{-1}\left(\frac{980}{199}\right)\right]$$

$$= -258.5^\circ$$

$$= 4.512 \text{ rad}$$

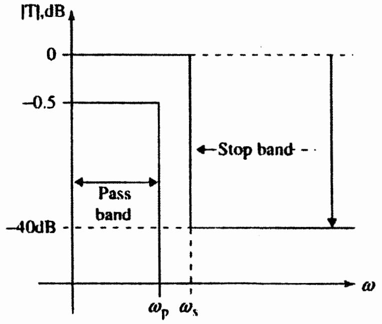
Now consider an input of $A \sin wt$ to $T(s)$. The output is then given by:

$$A|T(j\omega)|\sin(\omega t+\phi(\omega))$$

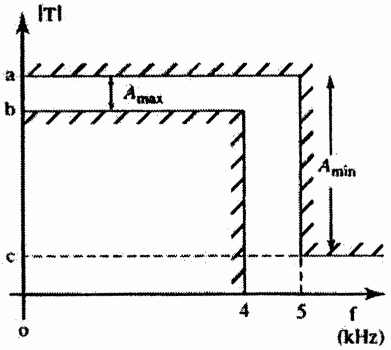
Using this result, the output to each of the following inputs will be:

INPUT	OUTPUT
$2\sin(0.1t)$	$2\sin(0.1t-0.2)$ ie $2 \times 1 = 2$
$2\sin(1t)$	$\sqrt{2}\sin(t-2.356)$ i.e. $2 \times 1/\sqrt{2} = \sqrt{2}$
$2\sin(10t)$	$2 \times 10^{-3}\sin(10t-4.512)$

11.2



11.3


|T|
a
b
c
A_{max}
A_{min}
o
4
5
f
(kHz)

Note $|T|$ is shown in a linear scale but A_{max} and A_{min} are in dB

From the problem

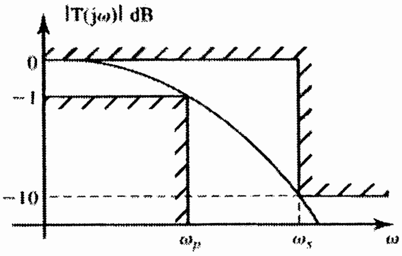
$$\frac{a}{b} = 1.1,\ C = 0.1\%\ a \text{ or } \frac{C}{a} = 0.001$$

$$A_{max} = 20\log_{10}a - 20\log_{10}b$$
$$= 20\log_{10}a/b$$
$$= 20\log_{10}(1.1)$$
$$= 0.83 \text{ dB}$$

$$A_{min} = 20\log_{10}a - 20\log_{10}c$$
$$= 20\log_{10}\left(\frac{a}{c}\right)$$
$$= 20\log_{10}(0.001)$$
$$= 60 \text{ dB}$$

$$\text{Selectivity} = \frac{w_s}{w_p} = \frac{2\pi 5}{2\pi 4} = 1.25$$

11.4


|T(jω)| dB
0
−1
−10
ω_p
ω_s
ω

$T(s) = \frac{k}{1 + s\tau}$ If $\tau = 1$s & the DC gain $= 1$

$$= \frac{1}{1 + 1s}$$

then $k = 1$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \omega^2}}$$

At the passband edge:

$$|T(j\omega_p)| = \frac{1}{\sqrt{1 + \omega_p^2}} = 10^{-1/20}$$

$\therefore \omega_p = 0.5088$ rad/s

At the stopband edge:

$$|T(j\omega_s)| = \frac{1}{\sqrt{1 + \omega_s^2}} = 10^{-10/20}$$

$\therefore \omega_s = 3$ rad/s

$$\text{Selectivity} = \frac{\omega_s}{\omega_p} = \frac{3}{0.5088} = 5.9$$

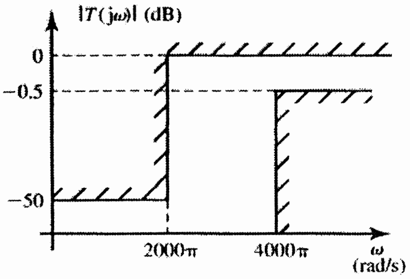
11.5

Passband is defined by: $f \geq 2$ kHz

$\Rightarrow w_p = 2\pi(2000)$ rad/s

Stopband is defined by: $f \leq 1$ kHz

$\Rightarrow w_s = 2\pi(1000)$ rad/s


|T(jω)| (dB)
0
−0.5
−50
2000π
4000π
ω
(rad/s)

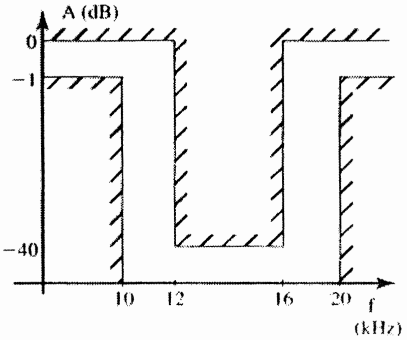
Note we assumed a maximum transmission of 0 dB.

11.6

Passband: $f \in \{[0, 10 \text{ kHz}] \cup [20 \text{ kHz}, \infty]\}$

Stopband: $f \in [12 \text{ kHz}, 16 \text{ kHz}]$

$A_{max} = 1$ dB, $A_{min} = 40$ dB


A (dB)
0
−1
−40
10
12
16
20
f
(kHz)

11.7

Poles at -1 and $-0.5 \pm j0.8$ gives a denominator:

$$D(s) = (s+1)(s+0.5-j0.8)(s+0.5+j0.8)$$

$$=(s+1)(s^2+2(0.5)s+0.5^2+0.8^2)$$

$$=(s+1)(s^2+s+0.89)$$

Zeros at ∞ and $\pm jz$ give a numerator:

$$N(s) = k(s+jz)(s-jz) = k(s^2+4)$$

Note there is one zero at ∞ because Degree $(D(s))$ − Degree $(N(s))$ − 1

$$T(s) = \frac{k(s^2+4)}{(s+1)(s^2+s+0.89)}$$

$$|T(jo)| = \frac{k(4)}{0.89} = 1 \qquad \therefore \text{DC gain} = 1$$

$$\Rightarrow k = 0.225$$

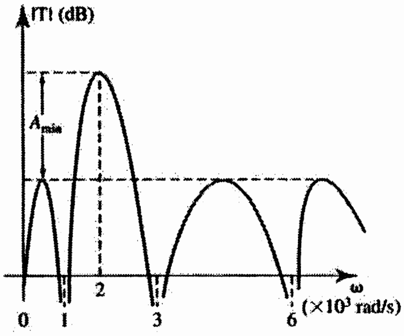
$$\therefore T(s) = \frac{0.2225(s^2+4)}{(s+1)(s^2+s+0.89)}$$

$G_{m1} = 1$ mA/V , $G_{m2} = 2$ mA/V ,

$R = 500\ \Omega$

$$f_t \approx \frac{G_{m1}}{2\pi C_C} \Rightarrow C_C = \frac{G_{m1}}{2\pi f t}$$

11.8



Numerator is given by

$a_7(s-0)(s^2+(10^3)^2)(s^2+(3\times10^3)^2)3$

$(s^2+(6\times10^3)^2)$

$=a_7s(s^2+10^6)(s^2+9\times10^6)(s^2+36\times10^6)$

Degree of Numerator $\triangleq m=7$

Degree of Denominator $\triangleq N$

Given that there is one zero at ∞:

$N-M=1\Rightarrow N=8$

$$\therefore T(s)=\frac{a_7s(s^2+10^6)(s^2+9\times10^6)(s^2+36\times10^6)}{s^8+b_2s^7+b_6s^6+\ldots+b_0}$$

From circuit : current drawn from V_{DD} rail = 2 IB

= current returnen to V_{SS} rail

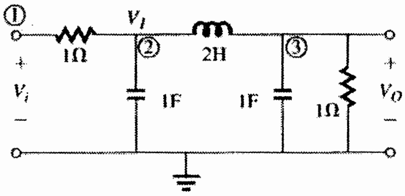
$\therefore Power=(V_{DD}+V_{SS})\times2I_B\Rightarrow$

$1\text{ mW}=(1.65+1.65)\times2I_B$

$$\therefore I_B=\frac{1\text{ mW}}{4\times1.65\text{ V}}=151.5\ \mu\text{A}$$

$\Rightarrow I=I_B/1.2=126.3\ \mu\text{A}$

11.9



The easiest way to solve the circuit is to use nodal analysis at nodes (1), (2), (3)

At node (3) $\Sigma I=0$

$$\frac{V_O}{1}+\frac{V_O}{1/s}+\frac{V_O-V_1}{2s}=0$$

$\therefore V_1=V_O(2s^2+2s+1)$ Eq. (a)

At node (2) $\Sigma I=0$

$$\frac{V_1-V_i}{1}+\frac{V_1}{1/s}+\frac{V_1-V_O}{2s}=0$$

$\therefore V_1(2s^2+2s+1)=V_O+2sV_i$ Eq. (b)

(a) → (b)

$V_O(2s^2+2s+1)^2=V_O+2sV_i$

$V_O(4s^4+s^3(4+4)+s^2(2+4+2)+s(2+2)+1)$

$=V_O+2sV_i$

$$\frac{V_O(s)}{V_i(s)}\triangleq T(s)\frac{2s}{4s^4+8s^3+8s^2+4s}$$

$$T(s)=\frac{0.5}{s^3+2s^2+2s+1}$$

Poles are given by:

$s^3+2s^2+2s+1=0$

$(s+1)(s^2+s+1)=0$

$\therefore$ Poles are $s=-1$ and $s=-\frac{1}{2}\pm j\sqrt{3}/2$

11.10

$A_{max}=1$ dB, $A_{min}=20$ dB, $w_s/w_p=1.3$

$$\text{Using: }A(\omega_s)=10\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

$$=A_{min}$$

$\epsilon=[10^{1/10}-1]^{1/2}=0.5088$

$$A_{min}=10\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

$$10^{A_{min}/10}-1=\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}$$

$$\log(10^{A_{min}/10}-1)=\log\left(\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right)$$

$$N=\frac{\log[(10^{A_{min}/10}-1)/\epsilon^2]}{2\log(\omega_s/\omega_p)}$$

$=11.3\Rightarrow$ choose $N=12$

The actual value of stopband attenuation can be calculated using the integer value of N:

$$A(\omega_s)=10\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right];N=12$$

$=27.35$ dB actual attenuation

If the stopband specs are to be met exactly we need to find A_{max}.

$$\epsilon^2=\frac{10^{A_{min}/10}-1}{(\omega_s/\omega_p)^{2N}}$$

$A_{min}=20$

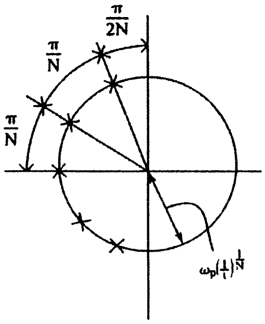
$N=12$

$=0.1824$

$\therefore A_{max}=10\log(1+\epsilon^2)$

$=0.73$ dB

11.11



$\omega_p = 10^3$ rad/s $N = 5$

$A_{max} = 1$ dB $\Rightarrow \epsilon = 0.5088$

find solution graphically

$$P_1 = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N} \measuredangle\left(\frac{\pi}{2} + \frac{\pi}{2N}\right)$$

$$= 873.59 \measuredangle\left(\frac{6\pi}{10}\right)$$

$$= 873.59\left[\cos\left(\frac{6\pi}{10}\right) \pm j\sin\left(\frac{6\pi}{10}\right)\right]$$

$$= -269.96 \pm j830.84$$

$$P_2 = 873.59 \measuredangle\left[\frac{\pi}{2} + \frac{\pi}{2N} + \frac{\pi}{N}\right]$$

$$= -706.75 \pm j\,513.49$$

$$P_3 = 873.59 \measuredangle \pi = -873.59$$

11.12

$f_p = 10$ kHz $\quad \frac{\omega_s}{\omega_p} = 1.5 \quad A_{min} = 15$ dB

$f_s = 15$ kHz $\quad A_{max} = 2$ dB

$$\epsilon^2 = 10^{A_{max}/10} - 1 \Rightarrow \epsilon = 0.76478$$

Manipulation Eq (16.15) we get:

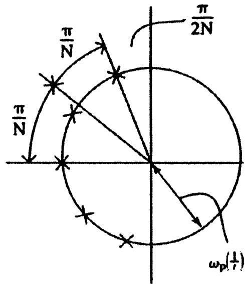
$$N \doteq \frac{\log[(10^{A_{min}/10} - 1)/\epsilon^2]}{2\log(\omega_s/\omega_p)} = 4.88$$

$\therefore$ use $N = 5$

finding natural modes graphically:-

$$\text{radius} = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N} \Delta\omega_0$$

$$\omega_0 = 6.629 \times 10^4$$



$$P_1 = \omega_0 \measuredangle\left(\frac{\pi}{2} + \frac{\pi}{2N}\right) = \omega_0 \measuredangle\left(\frac{6\pi}{10}\right)$$

$$= \omega_0\left(\cos\left(\frac{6\pi}{10}\right) \pm j\sin\left(\frac{6\pi}{10}\right)\right)$$

$$= \omega_0(-0.309 \pm j0.951)$$

$$P_2 = \omega_0\left(\cos\frac{8\pi}{10} \pm j\sin\frac{8\pi}{10}\right)$$

$$= \omega_0(-0.809 \pm j0.588)$$

$$P_3 = \omega_0(\cos\pi \pm j\sin\pi) = -\omega_0$$

Given a natural mode $-\alpha \pm j\beta$, the following term results

$$(s + \alpha + j\beta)(s + \alpha - j\beta)$$

$$= s^2 + 2\alpha s + \alpha^2 + \beta^2$$

$$= s^2 + 2Re[P]s + |P|^2$$

Also, note that for a Butterworth, all natural modes have a magnitude of w_0.

P_1 yields: $s^2 + 0.618\omega_0 s + \omega_0^2$

P_2 yields: $s^2 + 1.618\omega_0 s + \omega_0^2$

P_3 yields: $s + \omega_0$

$$\therefore T(s) = \frac{k}{(s + \omega_0)(s^2 + 0.618\omega_0 s + \omega_0^2)}$$

$$\times \frac{1}{s^2 + 1.618\omega_0 s + \omega_0^2}$$

for unity dc gain

$$|T(jo)| = \frac{k}{\omega_0^5} = 1 \Rightarrow k = \omega_0^5$$

$$\therefore T(s) = \frac{\omega_0^5}{(s + \omega_0)(s^2 + 0.618\omega_0 s + \omega_0^2)}$$

$$= \frac{1}{(s^2 + 1.618\omega_0 s + \omega_0^2)}$$

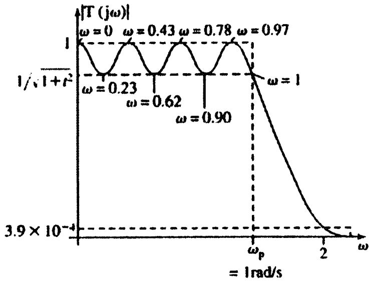
for attenuation at 20 kHz

$$\frac{\omega_s}{\omega_p} = \frac{20}{10} = 2$$

$$A(\omega_s) = 10\log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

$$= 27.8 \text{ dB}$$

11.13



Given $A_{max} = 1\text{dB} \Rightarrow \epsilon = 0.5088$

$$|T(j\omega)| = \left[1 + \epsilon^2\text{Cos}^2\left(N\text{Cos}^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{-1/2}$$

for $\omega \le \omega_p$

If $|T(j\omega)| = 1$

$1 = 1 + \epsilon^2\text{Cos}^2(N\text{Cos}^{-1}(\omega/\omega_p))$

$\omega_p = 1$

$N\text{Cos}^{-1}(\omega/1) = \text{Cos}^{-1}(0)$

$$\text{Cos}^{-1}(\omega) = \frac{2i+1}{2N}\pi$$

$$\therefore \omega_i = \text{Cos}\left[\frac{2i+1}{2N}\pi\right]$$

ω's repeat after this value $i = 0, 1, \ldots, \frac{N-1}{2}$

$\omega_0 = 0.9749$

$\omega_1 = 0.7818$

$\omega_2 = 0.4339$

$\omega_3 = 0$

ω values at which $|T| = 1$

note $\omega_4 = -0.4339$

$= -\omega_2!$

If $|T| = 1/\sqrt{1+\epsilon^2}$, then

$$1/\sqrt{1+\epsilon^2} = [1 + \epsilon^2\text{Cos}^2(N\text{Cos}^{-1}(\omega/\omega_p))]^{-1/2}$$

$$1 = \text{Cos}\left(N\text{Cos}^{-1}\left(\frac{\omega}{\omega_p}\right)\right)$$

$N\text{Cos}^{-1}(\omega) = \text{Cos}^{-1}(0)$

$= i\pi \qquad i = 0, 1, 2\ldots$

$$\omega_i = \text{Cos}\left[\frac{i\pi}{N}\right] \qquad i = 0, 1, 2, \ldots\frac{N}{2}$$

$\omega_0 = 1.0$

$\omega_1 = 0.9010$

$\omega_2 = 0.6235$

$\omega_3 = 0.2252$

ω values at which

$|T| = (1+\epsilon^2)^{-1/2}$

Note $\omega_4 = 0.2252$

$= -\omega_3!$

To find $|T(jz)|$ since $\omega > \omega_p$

$$|T(j\omega)| = \left[1 + \epsilon^2\text{Cosh}^2\left(N\text{Cosh}^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{\frac{-1}{2}}$$

$= [1 + 0.5088^2\text{Cosh}^2(7\text{Cosh}^{-1}2)]^{\frac{-1}{2}}$

$= 3.898 \times 10^{-4}$ V/V

$|T|_{ab} = -68.2$ dB

For roll-off consider

$$T(s) = \frac{k}{s^7 + b_6s^6 + \ldots b_0}$$

for $\omega \gg \omega_p$ $|T(j\omega)| \cong \frac{k}{\omega^7}$

$\therefore$ Roll-off is $\frac{1}{2^7}$ or $20\log\left(\frac{1}{2^7}\right)$

per octave $= -42$ dB / octave.

11.14

$\omega_S/\omega_p = 2$

$$A_{max} = 1\text{dB} \Rightarrow \epsilon = \sqrt{10^{\frac{A_{max}}{10}} - 1}$$

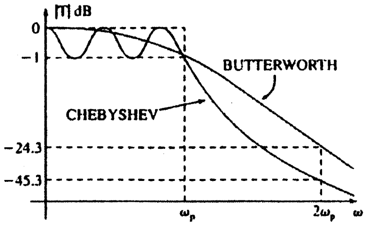
$= 0.5088$

$|T_B| = [1 + \epsilon^2(\omega_S/\omega_p)^{2N}]^{-1/2}$

$$T_C| = \left[1 + \epsilon^2\text{Cosh}^2\left(N\text{Cosh}^{-1}\left[\frac{\omega_S}{\omega_p}\right]\right)\right]^{-1/2}$$

$|T_B| = 6.13 \times 10^{-2} \Rightarrow -24.3$ dB

$|T_C| = 5.43 \times 10^{-3} \Rightarrow -45.3$ dB



11.15

(a) $f_P = 3.4$ kHz

$A_{max} = 1$ dB $\Rightarrow \epsilon = 0.5088$

$f_S = 4$ kHz $A_{min} = 35$ dB

$\omega_S / \omega_P = 1.176$

Using Eq (16.22):

$$A(\omega S) = 10 \log\left[1 + \epsilon^2 \text{Cosh}^2\left(N \text{Cosh}^{-1}\left(\frac{\omega_S}{\omega_P}\right)\right)\right]$$

& trying different values for N

N	$A(\omega_s)$
8	28.8 dB
9	33.9 dB
10	38.98 dB

∴ Use N = 10

Excess attenuation $= 39 - 35 = 4$dB

(b) Poles are given by:

$$P_K = -\omega_P \text{Sin}\left(\frac{2k-1}{N} \cdot \frac{\pi}{2}\right) \text{Sinh}\left(\frac{1}{N}\left(\text{Sinh}^{-1}\left(\frac{1}{\epsilon}\right)\right)\right)$$

$$+ j\omega_P \text{Cos}\left(\frac{2k-1}{N} \cdot \frac{\pi}{2}\right) \text{Cosh}\left(\frac{1}{N} \text{Sinh}^{-1}\left(\frac{1}{\epsilon}\right)\right)$$

for $k = 1, 2, \dots, N$.

Since $\epsilon = 0.5088$ and $N = 10$

$\text{Sinh}(1/N \text{Sinh}^{-1}(1/\epsilon)) = 0.1433$

$\text{Cosh}(1/N \text{Sinh}^{-1}(1/\epsilon)) = 1.010$

$$\therefore P_1 = \omega_P\left[-0.1433 \text{Sin}\left(\frac{\pi}{20}\right) + j1.010 \text{Cos}\left(\frac{\pi}{20}\right)\right]$$

$$= \omega_P(-0.0224 + j0.9978)$$

$P_2 = \omega_P(-0.0650 + j0.900)$

$P_3 = \omega_P(-0.1013 + j0.7143)$

$P_4 = \omega_P(-0.1277 + j0.4586)$

$P_5 = \omega_P(-0.1415 + j0.1580)$

Now it should be realized that the remaining poles are complex conjugates of the above.

pole-pair P_1 & P_1^* give a factor:

$$S^2 + 2(0.0224)\omega_P S + \omega_P^2(0.0224^2 + 0.9978^2)$$

$$= S^2 + 0.0448\omega_P S + 1.023\omega_P^2$$

i.e. this factor is from $(S - P_1)(S - P_1^*)$

$P_{2\text{yields}}$: $S^2 + 0.130\omega_P S + 0.902\omega_P^2$

$P_{3\text{yields}}$: $S^2 + 0.203\omega_P S + 0.721\omega_P^2$

$P_{4\text{yields}}$: $S^2 + 0.255\omega_P S + 0.476\omega_P^2$

$P_{5\text{yields}}$: $S^2 + 0.283\omega_P S + 0.212\omega_P^2$

Now $T(S)$ is given by

$$T(S) = \frac{k\omega_P^{10}}{E2^9(S - P_1)(S - P_1^*)\dots(S - P_5)(S - P_5^*)}$$

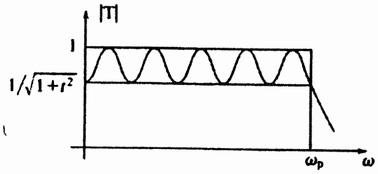
where the second order terms of the denominator are given above.

k is the dc gain

∴ we want the dc gain to be

$$k = \frac{1}{1 + \epsilon^2} = 0.8913$$

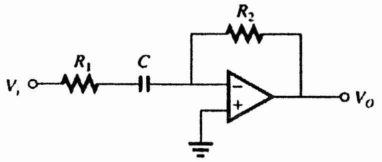
$\omega_P = 2\pi \times 3400$



11.16

$f_O = 100$ KHz $R_i(\infty) = 100$ kΩ

$|T(\infty)| = 1$



$R_i(\infty) = R_1 = 100$ kΩ

$|T(\infty)| = R_2/R_1 = 1$

$R_2 = R_1 = 100$ kΩ

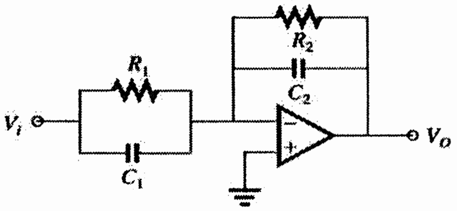
$CR_1 = 1/W_O$

$$C = \frac{1}{W_O R_1} = \frac{1}{2\pi\ 100 \times 10^3 \times 100 \times 10^3}$$

$= 15.9$ nF

11.17

Use general first-order circuit:



–Zero at 1 kHz; Pole at 100 kHz

$-|T(O)| = 1;\ R_i(O) = 1\ \text{k}\Omega$

Thus: $R_i(\text{DC}) = R_1 = 1\ \text{k}\Omega$

$T(\text{DC}) = -R_2/R_1 = -1$

$R_2 = R_1 = 1\ \text{k}\Omega$

For a pole at 100 kHz

$$C_2R_2 = \frac{1}{W_O} \Rightarrow C_2 = \frac{1}{2\pi f_O R_2}$$

$$= 1.59\ \text{nF}$$

For the circuit $T(S) = \dfrac{a_1S + a_O}{S + W_O}$

Thus the Zero at $-a_O/a_1 = -2\pi\ 10^3$

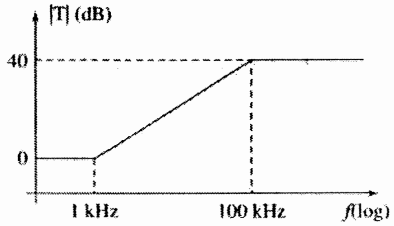
$C_1R_1 = a_1/a_O$

$$C_1 = \frac{1}{2\pi\ 10^3\ R_1}$$

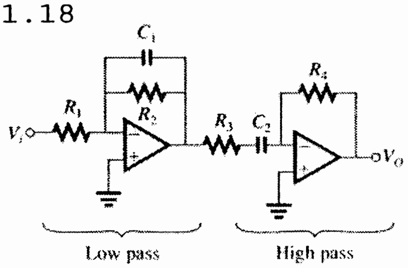
$$= 159\ \text{nF}$$

High freq gain $= \dfrac{-C_1}{C_2} = -100$

$= 40$ dB



11.18



gain $= 10^{12/20} = 3.98 \approx 4$

want $R_i = R_1$ large

$\therefore R_1 = 100\ \text{k}\Omega$

Total gain $= A_{LP}A_{HP} = 4$

$A_{LP} = -R_2/R_1 \Rightarrow R_2 = -A_{LP}R_1$ and

$R_2 \leq 100\ \text{k}\Omega$

$\therefore \text{make} A_{LP} = -1 A_{HP} = -4$

$R_2 = 100\text{k}\Omega$

$$R_2C_1 = \frac{1}{W_{O,LP}}$$

$$C_1 = \frac{1}{2\pi f_{O,LP}R_2} = \frac{1}{2\pi(10 \times 10^3)100 \times 10^3}$$

$$= 0.159\ \text{nF}$$

$$A_{HP} = \left.\begin{matrix} -R_4/R_3 = -4 \\ R_4 = 4R_3 \end{matrix}\right\}\text{make}\ \begin{matrix} R_4 = 100\ \text{k}\Omega \\ R_3 = 25\ \text{k}\Omega \end{matrix}$$

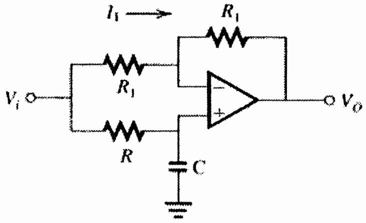
Now $R_3C_2 = 1/W_{O,HP}$

$$C_2 = \frac{1}{2\pi f_{O,HP}R_3}$$

$$= \frac{1}{2\pi(100 \times 10^3)25 \times 10^3}$$

$$= 63.7\ \text{nF}$$

11.19



At +ve terminal

$$V_+ = \frac{1/SC}{1/SC + R_1}V_i$$

$$= \frac{1}{1 + S\tau}V_i\ \ \tau = R_C$$

V– = V+ due to virtual short between terminals

$$\therefore I_1 = \left(V_i - \frac{1}{1 + S\tau}V_i\right)\frac{1}{R_1}$$

$$V_O = V_2 - I_1R_1$$

$$= \frac{V_i}{1 + S\tau} - \left(V_i - \frac{V_i}{1 + S\tau}\right)\frac{R_1}{R_1}$$

$$\frac{V_O}{V_i} = \frac{1 - (1 + S\tau) + 1}{1 + S\tau} = \frac{1 - S\tau}{1 + S\tau}$$

$$= \frac{\omega_O - S}{\omega_O + S} \quad \omega_O = \frac{1}{\tau}$$

$$= -\frac{S - \omega_O}{S + \omega_O} = T(S)$$

$$T(jW) = \frac{j\omega - \omega_O}{j\omega + \omega_O}$$

$$\phi(W) = 180° + \tan\left(\frac{\omega}{-\omega_O}\right) - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$$

$$= 360° - 2\tan^{-1}\left(\frac{\omega}{\omega_O}\right) \because \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$$

$$= -2\tan^{-1}(\omega/\omega_O) \quad = 180 - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$$

Now this equation can be rearranged:

$$\frac{W}{W_O} = \tan(-\phi/2) \Leftarrow \omega_O = \frac{1}{2} = \frac{1}{\mathrm{R_C}}$$

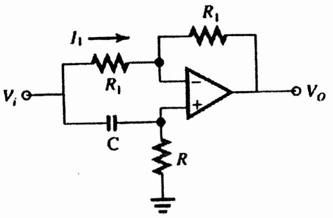
$$R_C W = \tan(-\phi/2)$$

$$\therefore R = \frac{\tan(-\phi/2)}{CW} = 10^4 \tan(-\phi/2)$$

$$\phi = -30°, -60°, -90°, -120°, -150°$$

$$R = 2.68\ \mathrm{k\Omega},\ 5.77\ \mathrm{k\Omega},\ 10\ \mathrm{k\Omega},\ 17.32\ \mathrm{k\Omega},\ 37.32\ \mathrm{k\Omega}$$

11.20



$$V+ = \frac{\mathrm{R}}{R + 1/SC} V_i = \frac{S}{S + \omega_O} V_i$$

$$\text{Where } \omega_O = \frac{1}{R_C}$$

$$I_1 = \frac{V_i - (S/S + \omega_O)V_i}{R_1}$$

$$V_O = \frac{S}{S + \omega_O} V_i - I_1 R_1$$

$$= \frac{S}{S + \omega_O} V_i - V_i\left(1 - \frac{S}{S + \omega_O}\right)$$

$$\frac{V_O}{V_i} = \frac{2S - S - \omega_O}{S + \omega_O} = \frac{S - \omega_O}{S + \omega_O}$$

Now :

$$\phi(W) = \tan^{-1}\left(\frac{\omega}{-\omega_O}\right) - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$$

$$\text{Note } \tan^{-1}\frac{\omega}{-\omega_O} = 180° - \tan^{-1}\frac{\omega}{-\omega_O}$$

$$= 180 - \tan^{-1}\left(\frac{\omega}{\omega_O}\right) - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$$

$$= 180 - 2\tan^{-1}\left(\frac{\omega}{\omega_O}\right)$$

Clearly $\phi(O) = 180°$ & $\phi(\omega \to \infty) = 0°$

11.21

Low pass $\omega_O = 10^3$ rad/s

$Q = 1$

DC gain = 1

$$T(S) = \frac{a_O}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$$

$$T(O) = a_O \omega_O^2 = 1$$

$$a_O = W_O^2 = 10^6$$

$$\therefore T(S) = \frac{10^6}{S^2 + 10^3 S + 10^6}$$

$$\omega_{\max} = \omega_O \sqrt{1 - 1/2Q^2}$$

$$= \frac{\omega_O}{\sqrt{2}}$$

$$= 0.707 \text{ rad/s}$$

$$|T_{\max}| = \frac{|a_O| Q}{\omega_O^2 \sqrt{1 - \frac{1}{4Q^2}}} \Leftarrow a_O = \omega_O^2$$

$$= \frac{|\omega_O^2|}{\omega_O^2 \sqrt{3/4}}$$

$$= 2/\sqrt{3}$$

$$= 1.15 \text{ V/V}$$

$$= 1.21 \text{ dB}$$

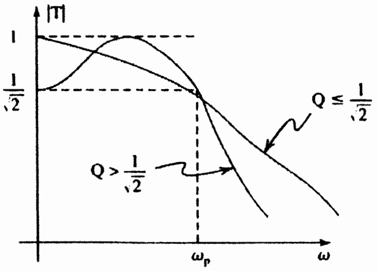
11.22

$W_P = 1$ rad/s

$A_{\max} = 3$ dB

$$10^{-3/20} = 0.708 \cong \frac{1}{\sqrt{2}}$$

There are many Q-values which may be used



$Q \leq 1/\sqrt{2}$ - no peaking

$Q > 1/\sqrt{2}$ - peaking

Solution 1 $Q \leq 1/\sqrt{2}$

For $Q = 1/\sqrt{2}$ the response is maximally flat.

Because this is desirable, use: $Q = \dfrac{1}{\sqrt{2}}$

$$T(S) = \frac{a_O}{S^2 + SW_O\sqrt{2} + W_O^2}$$

$$|T(O)| = \frac{a_O}{W_O^2} = 1$$

$$a_O = W_O^2$$

$$|T(j1)|^2 = \frac{W_O^2}{(W_O^2 - 1) + 2W_O^2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$W_O = 1$ rad/s

$$\therefore T_1(S) = \frac{1}{S^2 + \sqrt{2}S + 1}$$

Solution 2 $Q > 1/\sqrt{2}$

From the figure:

$$|T(O)| = 1/\sqrt{2} = \frac{a_O}{W_O^2}$$

$$\therefore a_O = \omega_O^2/\sqrt{2}$$

$$\text{Now}\,|T|_{\max} = \frac{|a_O|}{W_O^2\sqrt{1 - 1/4Q^2}} = 1$$

$$\frac{Q}{\sqrt{2}\sqrt{1 - 1/4Q^2}} = 1$$

$$Q = \sqrt{2}\sqrt{1 - 1/4Q^2}$$

$$\therefore Q^2 = 2\left(1 - \frac{1}{4Q^2}\right)$$

$$= 2 - \frac{1}{2Q^2}$$

$$Q^4 - 2Q^2 + \frac{1}{2} = 0$$

Solving for Q^2 gives:

$$Q^2 = 1 \pm \sqrt{2}$$

ASIDE:

$\because Q > 1/\sqrt{2}$

$Q^2 > 1/2$

$4Q^2 > 2$

$$\frac{1}{4Q^2} < \frac{1}{2}$$

$$\therefore 1 - \frac{1}{4Q^2} > 1/2$$

$$\therefore \left|1 - \frac{1}{4Q^2}\right| = 1 - \frac{1}{4Q^2}$$

$\Rightarrow Q = 0.5412$ or 1.3066

$\because Q > \dfrac{1}{\sqrt{2}}$ use $Q = 1.3066$

Now at the passband edge

$$|T(j1)| = 1/\sqrt{2}$$

$$|T(j1)|^2 = \frac{(W_O^2/\sqrt{2})^2}{(W_O^2 - 1)^2 + \dfrac{W_O^2}{Q^2}} = \frac{1}{2}$$

$$\frac{W_O^4}{2} = \frac{1}{2}\left[W_O^4 - 2W_O^2 + 1 + \frac{W_O^2}{Q^2}\right]$$

$$W_O^2(2 - 1/Q^2) = 1$$

$$W_O = 0.841$$

$$\therefore T_2(S) = \frac{W_O^2/\sqrt{2}}{S^2 + W_O/QS + W_O^2}$$

$$= \frac{0.5}{S^2 + 0.644s + 0.707}$$

If $W_S = 2$

$|T_1(j2)| = 0.242$ $|T_2(j2)| = 0.1414$

$\therefore A_{\min,1} = -12.3$ dB $A_{\min,2} = -17$ dB

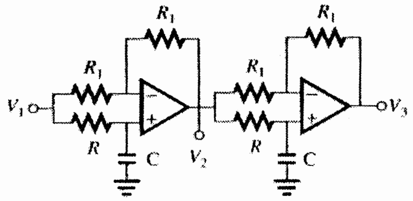
11.23

V_2 lags V_1 by 120°

V_3 lags V_2 by 120°

$w = 2\pi 60$ & $C = 1\ \mu$F

$$T(s) = \frac{s - w_O}{s + w_O} \qquad w_O = \frac{1}{RC}$$



$$\phi(\omega) = 180^\circ + \tan^{-1}\left(\frac{\omega}{-\omega_O}\right) - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$$

$$\text{sub: } \tan\left(\frac{\omega}{-\omega_O}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_O}\right)$$

$$\Rightarrow \phi(\omega) = -2\tan(\omega / \omega_O)$$

Now $\phi = -120^\circ$ at $\omega = 2\pi 60$

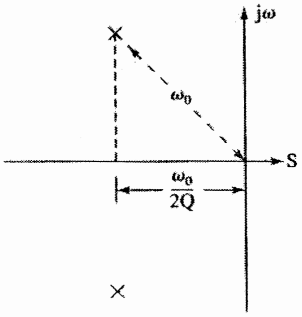
$$-120^\circ = -2\tan(WRC)$$

$$-60 = -\tan^{-1}(2\pi 60 \times R \times 10^{-6})$$

$$R = 4.59\text{ k}\Omega$$

R_1 can be arbitrarily chosen use $R_1 = 10\text{ k}\Omega$

11.24



Natural Modes:

$$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$w_O = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \quad 51.0$$

$$\frac{w_O}{2Q} = \frac{1}{2} \Rightarrow \frac{\omega_O}{Q} = 1$$

$$T(s) = \frac{a_2 s^2}{s^2 + s\dfrac{\omega_O}{Q} + \omega_O^2} = \frac{a_2 s^2}{s^2 + s + 1}$$

$$|T(j\infty)| = a_2 = 1$$

$$\therefore T(s) = \frac{s^2}{s^2 + s + 1}$$

11.25

For a 2nd-order bandpass

$$T(S) = \frac{a_1 S}{S^2 + S\dfrac{\omega_O}{Q} + \omega_O^2}$$

$$T(j\omega) = \frac{j\omega a_1}{(\omega_O^2 - w^2) + \dfrac{j\omega\omega_O}{Q}}$$

$$|T(j\omega)| = \frac{aW}{\left[(\omega_O^2 - \omega^2)^2 + \dfrac{\omega^2\omega_O^2}{Q^2}\right]^{1/2}}$$

Part (a):

$$|T(j\omega_1)| = |T(j\omega_2)|$$

$$\frac{aW_1}{\sqrt{(\omega_O^2 - \omega_1^2)^2 + \left(\dfrac{\omega_1\omega_O}{Q}\right)^2}}$$

$$= \frac{aW_2}{\sqrt{(\omega_O^2 - \omega_2^2)^2 + \left(\dfrac{\omega_2\omega_O}{Q}\right)^2}}$$

$$\omega_1^2\left[(\omega_O^2 - \omega_2^2)^2 + \cancel{\left(\frac{\omega_2\omega_O}{Q}\right)^2}\right]$$

$$= \omega_2^2\left[(\omega_O^2 - \omega_1^2)^2 + \cancel{\left(\frac{\omega_1\omega_O}{Q}\right)^2}\right]$$

$$\omega_1^2(\omega_O^4 - \cancel{2\omega_O^2\omega_2^2} + \omega_2^4) = \omega_2^2(\omega_O^4 - \cancel{2\omega_O^2\omega_1^2} + \omega_1^4)$$

$$\omega_1^2\omega_O^4 + \omega_1^2\omega_2^4 = \omega_2^2\omega_O^4 + \omega_2^2\omega_1^4$$

$$\omega_O^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^4 - \omega_1^2\omega_2^4$$

$$\omega_O^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^2(\omega_1^2 - \omega_2^2)$$

$$\omega_O^4 = \omega_1^2\omega_2^2$$

$$\omega_O^2 = \omega_1\omega_2 \quad \text{Q.E.D.}$$

(b) For Fig. 16.4:

$$\omega_{P1} = 8100\text{ rad/s}$$

$$\omega_{P2} = 10000\text{ rad/s}$$

$$A_{\max} = 1\text{dB}$$

$$\omega_O^2 = (8100)(10000)$$

$$\omega_O = 9000\text{ rad/s}$$

$$|T(jw_{P1})| = |T(j\omega_{P2})| = 10^{-1/20}$$

$$= 0.8913$$

$$|T(j\omega_O)| = \frac{\omega_O a_1}{\sqrt{(\omega_O^2 - \omega_O^2)^2 + \left(\dfrac{\omega_O^2}{Q}\right)^2}} = 1$$

$$\Rightarrow \frac{\omega_O a_1}{\omega_O^2/Q} = 1$$

$$\therefore \frac{Qa_1}{\omega_O} = 1 \Rightarrow a_1 = \frac{\omega_O}{Q}$$

$$|T(j\omega_{P1})|^2 = |T(j0.9\omega_O)|^2 = 0.8913^2$$

$$\frac{(\omega_O/Q)^2(0.9\omega_O)^2}{(\omega_O^2-(0.9\omega_O)^2)^2+\left(\frac{0.9\omega_O}{Q}\right)^2} = 0.8913^2$$

$$\left(\frac{\omega_O}{Q}(.9\omega_O)\right)^2$$

$$= 0.8913^2\left[(\omega_O^2-(0.9\omega_O)^2)^2+\left(\frac{0.9\omega_O}{Q}\right)^2\right]$$

$$\frac{0.81\omega_O^4}{Q^2} = 0.8913^2\left[W_O^4(1-0.81)^2+\frac{0.81\omega_O^4}{Q^2}\right]$$

$$\frac{0.81\omega_O^4}{Q^2}(1-0.8913^2) = 0.8913^2\omega_O^4\times(1-0.81)^2$$

SUBω_O = 9000 gives

$Q = 2.41$

Now $Q_1 = \frac{\omega_O}{Q} = 0.415\omega_O$

$$\therefore T(S) = \frac{0.415\omega_O S}{S^2+0.415\omega_O S+\omega_O^2}$$

IF WS_1 = 3000 rad/s

$$|T(j3000)| = \frac{0.415\omega_O(3000)}{\sqrt{(W_O^2-3000^2)^2+(\omega_O 3000\times .415)^2}}$$

$$= 0.1537$$

$$\therefore A_{\min} = -20\log(0.1537)$$

$$= 16.3\text{dB}$$

Now ω_{S1} and ω_{S2} are geometrically symmetrical about ω_O :

$$\omega_{S1}\omega_{S2} = \omega_O^2$$

$$\omega_{S2} = \frac{9000^2}{3000}$$

$$= 27000 \text{ rad/s}$$

11.26

$$Q = \frac{\omega_O}{BW\sqrt{10^{A/10}-1}}$$

$\omega_O = 2\pi(60)$ $BW = 2\pi 6$ $A = 20\text{dB}$

$= 1.005$

$$T(S) = a_2\frac{S^2+\omega_O^2}{S^2+S\frac{\omega_O}{Q}+\omega_O^2}$$

$$|T(0)| = \frac{a_2\omega_O^2}{\omega_O^2} = 1 \leftarrow \text{DC Gain}$$

$Q_2 = 1$

$$T(S) = \frac{S^2+(2\pi 60)^2}{S^2+S\frac{2\pi 60}{1.005}+(2\pi 60)^2}$$

$$T(S) = \frac{S+1.421\times 10^5}{S^2+375.1s+1.421\times 10^5}$$

11.27

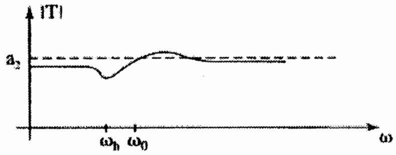
FOR ALL PASS:

$$T(S) = Q_2\frac{S^2-SW_O/Q+\omega_O^2}{S^2+SW_O/Q+\omega_O^2}$$

If Zero frequency < pole frequency

$$T(S) = Q_2\frac{S^2-SW_n/Q+\omega_n^2}{S^2+SW_O/Q+\omega_O^2}\quad \omega_n<\omega_O$$

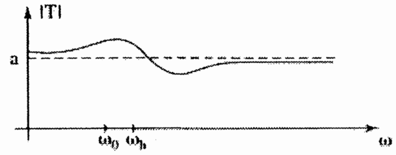
At DC: $|T| = a\frac{\omega_n^2}{\omega_O^2}$ where $\frac{\omega_n^2}{\omega_O^2}<1$



If Zero frequency > pole frequency

then $\omega_n > \omega_O$

At DC: $|T| = Q_2\frac{\omega_n^2}{\omega_O^2}$ where $\frac{\omega_n^2}{\omega_O^2}>1$



11.28

$$T(S) = \frac{S^2-SW_O/Q^1+\omega_O^2}{S^2+SW_O/Q_O+\omega_O^2}a$$

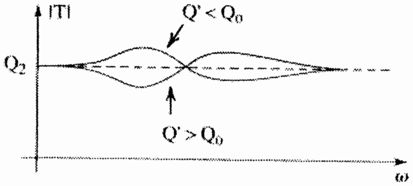
Zero Q < pole $Q \Rightarrow Q^1 < Q_O$

At W = W_O :

$$|T| = \frac{a_2\omega_O^2/Q'}{\omega_O^2/Q_O} = \frac{a_2 Q_O}{Q'} > a_2$$

If $Q^1 > Q_O$

$$|T(j\omega_O)| = \frac{Q_2 Q_O}{Q_1} < Q_2$$



11.29

$W_O = \frac{1}{\sqrt{LC}}$

If $L' = 1.01L$

$W_O' = (1.01LC)^{-1/2}$

$= 0.9950\frac{1}{\sqrt{LC}}$

$= 0.9950W_O$

$\therefore \Delta W_O = -0.5\%$

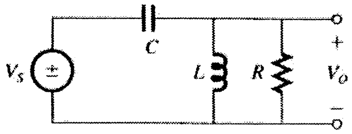
If $C^1 = 1.01C$

$W_O' = 0.9950W_O$

$\Delta W_O' = -0.5\%$

Changing R has no effect on W_O

11.30



Use voltage divider rule:

$$V_O = \frac{Z_{R\parallel L}}{Z_{R\parallel L} + Z_C}V_S$$

$$\frac{V_O}{V_S} = \frac{\left(\frac{1}{R} + \frac{1}{SL}\right)^{-1}}{\left(\frac{1}{R} + \frac{1}{SL}\right)^{-1} + \frac{1}{SC}}$$

$$= \frac{SC}{\left(\frac{1}{SL} + \frac{1}{R}\right) + SC}$$

$$\therefore T(S) = \frac{V_O(S)}{V_S(S)} = \frac{S^2}{S^2 + S/RC + \frac{1}{LC}}$$

11.31

Low Pass: $\omega_0 = 10^5$, $C = 0.1\mu F$
$Q = 1/\sqrt{2}$

$Q = \omega_0 CR$

$R = \frac{Q}{\omega_0 C}$

$= \frac{1}{\sqrt{2}\times 10^5 \times 0.1\times 10^{-6}}$

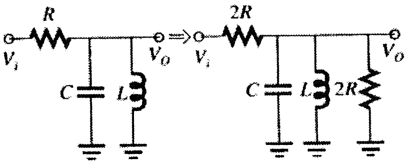
$= \underline{\underline{70.7\Omega}}$

$\omega_0 = 1/\sqrt{LC}$

$L = \frac{1}{\omega_0^2 C}$

$= \underline{\underline{1mH}}$

11.32



$\omega_O = \frac{1}{\sqrt{LC}}$

$Q = \omega w_O CR$

$A_{mid} = 1$

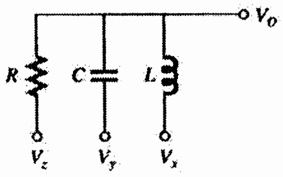
$\omega_O = \frac{1}{\sqrt{LC}}$

$Q = \omega_O C(2R \parallel 2R)$

$= \omega_O CR$

$A_{mid} = \frac{2R}{2R + 2R} = \frac{1}{2}$

11.33



$$V_O/V_z\big|_{V_y = V_x = 0} = T_{BP}(S)$$

$$V_O/V_y\big|_{V_x = V_z = 0} = T_{HP}(S)$$

$$\frac{V_O}{V_x}\bigg|_{V_y = V_z} = T_{LP}(S)$$

Using superposition

$$V_O = \frac{V_O}{V_x}V_x + \frac{V_O}{V_y}V_y + \frac{V_O}{V_z}V_z$$

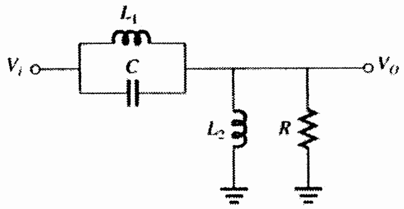
$$= T_{LP}V_x + T_{HP}V_y + T_{BP}V_z$$

$$\frac{\frac{1}{LC}V_x + S^2V_y + \frac{S}{RC}V_z}{S^2 + S/RC + \frac{1}{LC}}$$

$$\therefore V_O = V_x\frac{y_{LC}}{S^2 + S/R_C + 1/L_C}$$

$$V_x\frac{S^2}{S^2 + S/R_C + 1/L_C} + V_z\frac{S/R_C}{S^2 + S/R_C + 1/L_C}$$

11.34



From Eq 16.46

$$T(S) = \frac{S^2 + 1/L_1C}{S^2 + S(1/CR) + \frac{1}{(4 \| L_2)C}}$$

Required notch $\omega_n^2 = \frac{1}{L_1C} = (0.9\omega_O)^2$

but:

$$\omega_O^2 = \frac{1}{(L_1 \| L_2)C} \text{ where}$$

$$L_1 \| L_2 = \frac{1}{4^{-1} + L_2^{-1}} = \frac{L_1L_2}{L_1 + L_2}$$

$$= \frac{L_1 + L_2}{L_1L_2C}$$

$$= \frac{L_1 + L_2}{L_2}(.9\,\omega_O)^2$$

$$1 = \left(\frac{L_1}{L_2} + 1\right)0.9^2$$

$$\therefore L_1/L_2 = \frac{1}{0.9^2} - 1 = 0.2346$$

For $\omega << \omega_o$:-

$$|T| \cong \frac{1/L_1C}{1/(L_1 \| L_2)C} = \frac{L_2}{L_1 + L_2}$$

i.e. inductors dominate!

For $\omega >> \omega_o$, L_1 & L_2 are "open" C is shorted

$|T| \cong 1$

11.35

$$L = C_4 R_1 R_3 R_5/R_2$$

Choose $R_1 = R_2 = R_3 = R_5 = 10\text{ k}\Omega$

$\therefore L = C_4 \times 10^8$

For:

$L = 10\text{ H} = C_4 \times 10^8 \Rightarrow C_4 = 100\text{ nF}$

$L = 1\text{ H} \Rightarrow C_4 = 10\text{ nF}$

$L = 0.1\text{ H} \Rightarrow C_4 = 1\text{ nF}$

11.36

$A_{max} = 10\log(1 + \varepsilon^2) = 3\text{ dB}$

$\therefore \varepsilon = 0.998 \cong 1$

$\omega_o = \omega_p = 10^4$

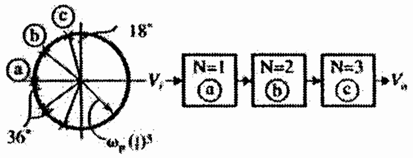
For circuit Q use fig 16.13 (a)

DC Grain

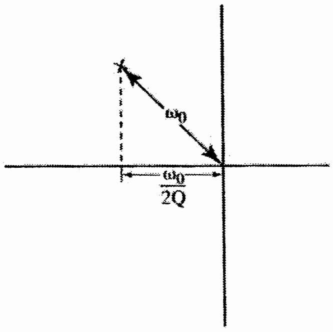
$= 1 = R_2/R_1 \Rightarrow R_1 = R_2 = 10\text{ k}\Omega$

$CR_2 = 1/\omega_o \Rightarrow C = 1/R_2\,\omega_o = \dfrac{1}{10^4\,10^4}$

$= 10\text{ nf}$



For circuit (b)



$\omega_o = 10^{4\text{ rad/s}}$

$\dfrac{\omega_o}{2Q} = \omega_o \cos 36^\circ$

$Q = \dfrac{1}{2\text{ Cos }36^\circ} = 0.618$

$$T(S) = \frac{\dfrac{k\,R_2}{C_4\,C_6\,R_1\,R_3\,R_5}}{S^2 + S/C_6\,R_6 + \dfrac{R_2}{C_4\,C_6\,R_1\,R_3\,R_5}}$$

$\omega_o^2 = \dfrac{R_2}{C_4\,C_6\,R_1\,R_3\,R_5}$

Let $= R_1 = R_3 = R_5 = R_2 = R$

$C_4 = C_6 = C$

$\omega_o^2 = \dfrac{1}{R^2\,C^2}$

USE $C_4 = C_6 = 100\text{ nF}$

$\therefore R = \dfrac{1}{\omega_o C} \Rightarrow R_1 = R_2 = R_3 = R_5 = 1\text{ k}\Omega$

Now using:

$\dfrac{\omega_o}{Q} = \dfrac{1}{C_6\,R_6}$ & $Q = 0.618$

$R_6 = \dfrac{Q}{C_6\,\omega_o} = 618\ \Omega$

For circuit (c) use

$\omega_o = 10^4$ which is the same as for circuit (b).

$\therefore C_4 = C_6 = 100\text{ nF}$

$R_1 = R_2 = R_3 = R_5 = 1\text{ k}\Omega$

Now: $Q = \dfrac{1}{2\cos 72^\circ} = 1.618$

$R_6 = Q/\omega_o C_6 = 1.618\text{ k}\Omega$

11.37

$f_O = 4\text{kHz}$ $\quad f_N = 5\text{kHz}$ $\quad Q = 10$

now $C_4 = 10\text{nF}$ and $k = 1 \equiv$ dc gain

$W_O = [C_4(C_{61} + C_{62})R_1R_3R_5/R_2]^{-1/2}$

$C_{61} + C_{62} = C_6$

Choose $C_6 = C_4 = 10\text{nF}$

$R_1 = R_3 = R_5 = R_2 = R$

$\therefore \omega_O = (C_4C_6R^2)^{-1/2}$

$R = \dfrac{1}{\omega_O C_4}$

$\Rightarrow R_1 = R_3 = R_5 = R_2 = 3.979\text{ k}\Omega$

$\omega_n = (C_4C_6R^2)^{-1/2}$

$C_{61} = \dfrac{1}{\omega_n^2 R^2 C_4} \Rightarrow C_{61} = 6.4\text{ nF}$

$C_{62} = 3.6\text{ nF}$

$Q = R_6\sqrt{\dfrac{C_{61} + C_{62}}{C_4} \cdot \dfrac{R_2}{R_1R_3R_5}}$

$= R_6\sqrt{\dfrac{1}{R_1^2}} = R_6/R_1 \Rightarrow R_6 = 39.79\text{ k}\Omega$

11.38

$\theta = 180°$ at f_o!

$\therefore$ Use $f_O = 1\text{kHz}$ $\quad Q = 1$

$W_O^2 = \dfrac{R_2}{C_4C_6R_1R_3R_5}$

Let $C = C_4 = C_6 = 1\text{nF}$

$R_1 = R_3R_5 = R_2 = R$

$= 1/C^2R^2$

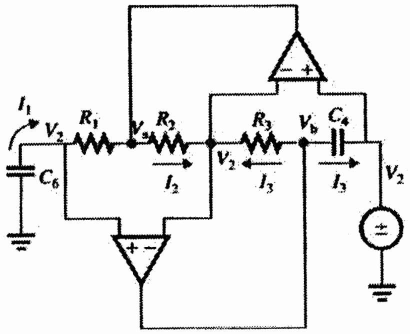
$R = \dfrac{1}{W_O C} = 159.16\text{ k}\Omega = R_1 = R_3 = R_5 = R_2$

$\dfrac{W_O}{Q} = \dfrac{1}{R_6C_6} \Rightarrow R_6 = \dfrac{Q}{C_6\omega_O}$

$= \dfrac{1}{10^{-9}2\pi10^3}$

$\therefore R_6 = 159.16\text{k}\Omega$

11.39



Because of virtual short at opamp input terminals all modes are at V_2!

$I_1 = -SC_6V_2$

Since no current goes into the opamp input terminals we have:

$$V_a = V_2 - I_1R_1$$
$$= V_2(1 + SC_6R_1)$$
$$I_2 = (V_a - V_2)\frac{1}{R_2}$$
$$= \frac{SC_6R_1}{R_2}$$
$$V_b = V_2 - I_2R_3$$
$$= V_2 - \frac{SC_6R_1R_3}{R_2}$$
$$I_3 = (V_b - V_2)SC_4$$
$$= -\frac{S^2C_4C_6R_1R_3}{R_2}V_2$$

Now the voltage source sees an input impedance given by:

$$Z_{in} = -\frac{V_2}{I_3} = \frac{R_2}{S^2C_4C_6R_1R_3}$$

As required

for $S = j\omega \Rightarrow S^2 = -\omega^2$

$$Z_{in}(j\omega) = \frac{-R_2}{C_4C_6R_1R_3}\cdot\frac{1}{\omega^2}$$
$$= -R(W)$$

ie. A PURE NEGATIVE RESISTANCE!

11.40

$$T(s) = k\,\frac{C_{61}}{C_{61}+C_{62}}\cdot\frac{S^2 + \dfrac{R_2}{C_4\,C_{61}\,R_1\,R_3\,R_5}}{S^2 + S/C_6\,R_6 + \dfrac{R_2}{C_4(C_{61}+C_{62})\,R_1\,R_3\,R_5}}$$

At DC $\Rightarrow s = 0$

$$T(o) = k\,\frac{C_{61}}{C_{61}+C_{62}}$$
$$\frac{R_2/C_4\,C_6\,R_1\,R_3\,R_5}{R_2/C_4(C_{61}+C_{62})\;R_1\,R_3\,R_5}$$
$$\Rightarrow T(o) = k \triangleq \text{DC Gain!}$$

Note that $C_{61} + C_{62}$ is the total capacitance across R_6

$$\omega_o^2 = R_2/C_4\,C_6\,R_1\,R_3\,R_5$$
$$\Rightarrow \omega_n = \sqrt{\frac{R_2}{C_4\,C_{61}\,R_1\,R_3\,R_5}}$$
$$\therefore C_6 = C_{61} + C_{62}$$
$$\frac{\omega_n^2}{\omega_o^2} = \frac{R_2/C_4\,C_{61}\,R_1\,R_3\,R_5}{R_2/C_4(C_{61}+C_{62})\;R_1\,R_3\,R_5}$$
$$= \frac{C_{61}}{C_{61}+C_{62}}$$
$$\frac{\omega_n^2}{\omega_o^2} = \frac{C_{61}}{C_6}$$
$$\therefore C_{61} = C_6\left(\frac{\omega_n}{\omega_o}\right)^2 = C\left(\frac{\omega_n}{\omega_o}\right)^2$$

Clearly from T(S) above:

$$\omega_n^2 = R_2/C_4\,C_6\,R_1\,R_3\,R_5$$
$$\Rightarrow \omega_o = \sqrt{\frac{R_2}{C_4\,(C_{61}+C_{62})\,R_1\,R_3\,R_5}}$$

11.41

$T(s)$

$$= k\,\frac{S^2 + (R_2/C_4\,C_6\,R_1\,R_3\,R_{51})}{S^2 + S/C_6\,R_6 + \left(\dfrac{R_2}{C_4\,C_6\,R_1\,R_3}\right)\left(\dfrac{1}{R_{51}} + \dfrac{1}{R_{52}}\right)}$$

clearly: $w_n = \sqrt{\dfrac{R_2}{C_4\,C_6\,R_1\,R_3\,R_{51}}}$

$$w_o = \sqrt{\frac{R_2}{C_4\,C_6\,R_1\,R_3}\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)}$$

At high frequencies $S \to \infty$

$T(\infty) = k \triangleq$ high freq gain

Observe that the equivalent resistance at the tue terminal of A_1 is:

$$\frac{1}{R_5} = \frac{1}{R_{51}} + \frac{1}{R_{52}} \quad \text{AND}$$

For the resonator (table 16.1)

$$R_5 = 1/\omega_o c \Rightarrow \frac{1}{R_5} = w_o c$$

$$\frac{\omega_o^2}{\omega_n^2} = \frac{R_2/C_4\,C_6\,R_1\,R_3\,R_5}{R_2/C_4\,C_6\,R_1\,R_3\,R_{51}} \Rightarrow R_{51} = R_5\,\frac{\omega_o^2}{\omega_n^2}$$

Now $\frac{1}{R_5} = \frac{1}{R_5 \frac{\omega_o^2}{\omega_n^2}} + \frac{1}{R_{52}}$

$$\frac{1}{R_{52}} = \frac{1}{R_5}\left[1 - \frac{\omega_n^2}{\omega_o^2}\right]$$

$$R_{52} = \frac{R_5}{1 - \omega_n^2 / \omega_o^2}$$

11.42

$T(s)$

$$= \frac{0.4508\,(S^2 + 1.6996)}{(S + 0.7294)\,(S^2 + 0.2786s + 1.0504)}$$

Part (a) Replace s with s/ω_p

$T(s)$

$$= \frac{0.4508\,(S^2/\omega_p^2 + 1.6996)}{\left(\frac{S}{\omega_p} + 0.7294\right)\left(\frac{S^2}{\omega_p^2} + \frac{0.27865}{\omega_p} + 1.0504\right)}$$

$T(s)$

$$= \frac{0.4508\,\omega_p\,(S^2 + 1.6996\,\omega_p^2)}{(S + 0.7294\,\omega_p)\,(S^2 + 0.2786\,\omega_p S + 1.0504\,\omega_p^2)}$$

Sub $\omega_p = 10^4$ rad / s

$T(s)$

$$= \frac{4508\,(S^2 + 1.6996 \times 10^8)}{(S + 7294)\,(S^2 + 2786s + 1.0504 \times 10^8)}$$

Part (b)

First decompose T(s) into 1st and 2nd- order sectionswith unity DC gain!

$T_1(s) = \frac{k_1}{S + 7294}$ $T_1(o) = \frac{k_1}{7294} = 1$

$\Rightarrow k_1 = 7294$

Now $k_1\,k_2 = 4508 \Rightarrow k_2 = 0.6180$

$$\therefore T_2(s) = \frac{0.6180\,(S^2 + 1.6996 \times 10^8)}{S^2 + 2786s + 1.0504 \times 10^8}$$

As a check:

$$T_2(o) = \frac{0.6180\,(1.6996 \times 10^8)}{1.0504 \times 10^8} = 1.000$$

AS EXPECTED!

$\therefore T(s) = T_1(s) \cdot T_2(s)$

$w_o = 7294$ rad / s DC Gain = 1

Let $C = 10$ nF

$R_1 = R_2 = \frac{1}{\omega_o C} \Rightarrow R_1 = R_2 = 13.71\ \text{k}\Omega$

11.43

$R_L = R_H = R_B / Q \Rightarrow R_B = QR_H$

$R_L = R_H$

$$\frac{V_O}{V_i} = -K\frac{\frac{R_F}{R_H}S^2 - S\left(\frac{R_F}{R_B}\right)\omega_o + \left(\frac{R_F}{R_L}\right)\omega_o^2}{S^2 + S\frac{\omega_o}{Q} + \omega_o^2}$$

$$= -K\,\frac{R_F}{R_H}\,\frac{S^2 - \frac{\omega_o}{Q}S + \omega_o^2}{S^2 + \frac{\omega_o}{Q}S + \omega_o^2}$$

Flat Gain = $-K\ R_F / R_H$

Part (b)

$-\ \omega_o = 10^4$ rad / s $Q = 2$ Flat Gain $= 10$

Choose C = 10 nF $\Rightarrow R = \frac{1}{\omega_o C} = 10\ \text{k}\Omega$

Choose $R_f = R_1 = 10\ \text{k}\Omega$

$\frac{R_3}{R_2} = 2Q - 1 = 3 \Rightarrow R_2 = 10\ \text{k}\Omega$

$R_3 = 30\ \text{k}\Omega$

Now $K = 2 - 1/Q = 1.5$

$\therefore$ Flat Gain $= 10 = (1.5)\,\frac{R_F}{R_H}$

$\therefore \frac{R_H}{R_F} = 0.15$

Choose $R_F = 100\ \text{k}\Omega$

$R_H = R_L = 15\ \text{k}\Omega$

$R_B = QR_H = 30\ \text{k}\Omega$

11.44

Note ω_n does not depend on R or C From

$$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_o}\right)^2$$

$\therefore \omega_n = \omega_o\sqrt{\frac{R_H}{R_L}}$ Nominally $R_H = R_L \pm 1\%$

Thus:

$\omega_n' = \omega_o\sqrt{\frac{1.01}{0.99}}$ $\omega_n'' = \omega_o\sqrt{\frac{0.99}{1.01}}$

$= 1.01\omega_o$ $= 0.99\ \omega_o$

$\therefore \omega_n$ can deviate from ω_o

by $\pm$ 1%

11.45

Use Tow Thomas to realize a LPN

$\omega_o = 10^4 \quad \omega_n = 1.2\omega_o \quad Q = 10$

DC Gain = 1

C = 10 nF r = 20 kΩ

$$R = \frac{1}{\omega_o C} = 10\ \text{k}\Omega$$

From 16.16 (e):

$$\text{DC Gain} = a_2 \frac{\omega_n^2}{\omega_o^2} = 1$$

$$a_2 \frac{1.2^2\,\omega_o^2}{\omega_o^2} = 1$$

$$a_2 \frac{1}{1.2^2} = \text{HF} \cdot \text{Gain}$$

$$C_1 = Ca_2 = \frac{10 \times 10^{-9}}{1.2^2} = 6.94\ \text{nF}$$

$$R_2 = \frac{R(\omega_o/\omega_n)^2}{\text{HF Gain}} = R\left(\frac{1}{1.2}\right)^2 \times (1.2)^2$$

$$= R = 10\ \text{k}\Omega$$

$$R_1 = R_3 = \infty$$

11.46

For all pass:

$T(S)$

$$-g^2\left(\frac{C_1}{C}\right) + S\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}$$

$$\omega_z^2 = \frac{1}{C^2RR_2} \cdot \frac{C}{C_1} \Rightarrow \omega_Z = \frac{1}{C\sqrt{RR_2}} \cdot \sqrt{\frac{C}{C_1}}$$

$$Q_Z = \frac{\omega_Z}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\frac{C}{C_1}}$$

$$Q_Z = \frac{\sqrt{\frac{1}{C^2RR_2}\frac{C}{C_1}}}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\left(\frac{C}{C_1}\right)}$$

$$\frac{1}{\sqrt{RR_2}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\sqrt{\frac{C}{C_1}}}$$

For All Pass $R_1 \Rightarrow \infty$

To adjust Q_3, trim r or R_2 (independent of W_z !)

Now $\omega_o = \frac{1}{CR}$ so do not trim R or C!

Note if we trim R_2 or C_1 to adjust W_z This will also affect Q_z. So the options are:

For W_z: (a) trim R_2 and (r or R_3) to maintain the value of Q_z

OR

(b) trim C_1, and r or R_3

Prefer not to trim a capacitor so use (*a*) !

11.47

$T(s)$

$$= \frac{0.4508\,(S^2 + 1.6996)}{(S + 0.7294)\,(S^2 + 0.27865 + 1.0504)}$$

Part (a) Replace s with s/ω_p

$\omega_p = 10^4\text{rad/s}.$

$T(s)$

$$= \frac{0.4508\,(S^2/\omega_p^2 + 1.6996)}{\left(\frac{S}{\omega_p} + 0.7294\right)\left(\frac{S^2}{\omega_p^2} + \frac{0.2786s}{\omega_p} + 1.0504\right)}$$

$T(s)$

$$= \frac{0.4508\,\omega_p\,(S^2 + 1.6996\,\omega_p^2)}{(S + 0.7294\,\omega_p)\,(S^2 + 0.27865\,\omega_p S + 1.0504\,\omega_p^2)}$$

$$= \frac{4508\,(S^2 + 1.6996 \times 10^8)}{(S + 7294)\,(S^2 + 27865 + 1.0504 \times 10^8)}$$

For FIRST ORDER SECTION

$\omega_o = 7294 \qquad \text{DC Gain} = 1$

Choose $C = 10$ nF

$$R_1 = R_2 = \frac{1}{\omega_o C} \Rightarrow R_1 = R_2 = 13.71\ \text{k}\Omega$$

For SECOND ORDER SECTION

$$\omega_n^2 = 1.6996 \times 10^8 \Rightarrow \omega_n = 13.037 \times 10^3$$

$$\omega_o^2 = 1.0504 \times 10^8 \Rightarrow \omega_o = 10.249 \times 10^3$$

$$\frac{\omega_o}{Q} = 2786 \Rightarrow Q = 3.6787$$

DC gain = 1

For Tow Thomas LPN

Choose C = 10 nF

$$R = \frac{1}{\omega_o C} = \frac{1}{10.249 \times 10^3 \times 10 \times 10^{-9}}$$

$$= 9.757\ \text{k}\Omega$$

Choose r = 20 kΩ

(e):

$$T(o) = a_Z\frac{\omega_n^2}{\omega_o^2} = 1 \Rightarrow a_Z\frac{\omega_o^2}{\omega_n^2} = 0.618$$

$$\therefore \text{HF gain} = a_Z = 0.618$$

$$C_1 = C \times \text{HF gain} \Rightarrow C_1 = 0.618\ \text{nF}$$

$$R_2 = R\,(\omega_o/\omega_n)^2$$

$$R_2 = 0.618\,R \Rightarrow R_2 = 6.03\ \text{k}\Omega$$

$$R_1 = R_3 = \infty \qquad QR = 35.89\ \text{k}\Omega$$

11.48

$$t(S) = \frac{S^2 + S\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}{S^2 + S\left(\frac{1}{C_1R_3} + \frac{1}{C_1R_4} + \frac{1}{C_2R_3}\right) + \frac{1}{C_1C_2R_3R_4}}$$

But $C_1 = C_2 = C$ & $R_3 = R_4 = R$, $RC = \tau$

$$\therefore t(S) = \frac{S^2 + S^2/RC + 1/R^2C^2}{S^2 + S\,3/RC + \frac{1}{R^2C^2}}$$

$$= \frac{S^2 + S^2/\tau + 1/\tau^2}{S^2 + S\frac{3}{\tau} + 1/\tau^2}$$

Zeros defined by $w_z = 1/\tau$

$$Q_z = \frac{1}{2}$$

$\Rightarrow$ Double Root at $S = -1/\tau$

Poles of t(S) are given by the quadratic formula:

$$S = \frac{-3}{2\tau} \pm \frac{\sqrt{5}}{2\tau} = \frac{-3 \pm \sqrt{5}}{2\tau}$$

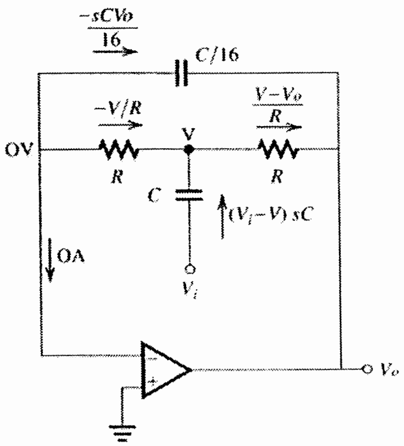
i.e. two roots on the negative real axis

If the network is placed in the negative feedback path of an ideal amplifier ($A = \infty$) then the poles are given by the zeros of $t(S)$:

Closed loop poles:

$S = -1/\tau$ (multiplicity = 2)

11.49



Note first $\frac{-sCV_O}{16} = \frac{-V}{R}$

$$V = -\frac{sCRV_O}{16}$$

ΣI at V

$$-\frac{V}{R} + sC(V_i - V) - \frac{V - V_O}{R} = 0$$

$$\frac{sCV_O\cancel{R}}{\cancel{R}\;16} + sCV_i + \frac{s^2C^2RV_O}{16} + \frac{sCV_O}{16}$$

$$+ \frac{V_O}{R} = 0$$

mult by: 16 R and let $RC = \tau$

$s\tau\, V_o + 16\tau\, V_i\, s + s^2\tau^2 V_o + s\tau V_o + 16V_o = 0$

$V_o\,[s^2\tau^2 + s \times 2^{\tau} + 16] = -16\, s^{\tau}\, V_i$

$$\therefore \frac{V_O}{V_i} = -\frac{16s\tau}{s^2\tau^2 + 2\tau\, s + 16}$$

$$\therefore T(s) = \frac{s\,16/RC}{s^2 + s^2/RC + 16/R^2C^2}$$

$$\text{Let } \omega_O^2 = \frac{16}{(RC)^2} \Rightarrow \omega_O = \frac{4}{RC}$$

$$\frac{\omega_O}{Q} = \frac{2}{RC} \Rightarrow Q = \frac{RC\omega_O}{2} = 2$$

$$\frac{\omega_O}{Q} = \frac{2}{RC} \Rightarrow Q = \frac{RC\omega_O}{2} = 2$$

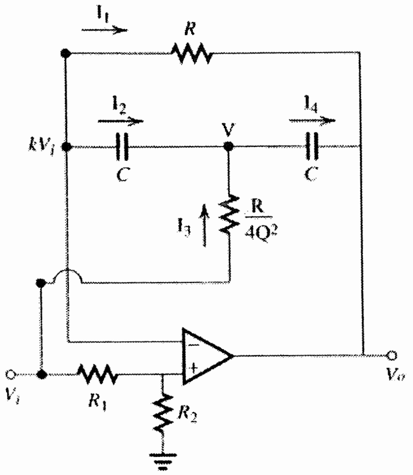
$$\therefore \frac{V_O}{V_i} = \frac{-4\omega_O s}{s^2 + s\frac{\omega_O}{Q} + \omega_O^2}$$

$$\left.\begin{aligned} |T|_{S=0} &= 0 \\ |T|_{S=\infty} &= 0 \end{aligned}\right\}\text{Bandpass}$$

$|T(j\omega_O)| = 4/1/2 = 8\frac{\text{V}}{\text{V}}$ CENTER FREQ

GAIN

11.50



$RC = 2Q/w_o$

$$k = \frac{R_2}{R_1 + R_2}$$

$V_+ = V_- = kV_i$ due to virtual short

$I_1 = -I_2$

$$\frac{kV_i - V_O}{R} = \frac{V - kV_i}{1}(SC)$$

$$V = \frac{1}{SCR}(kV_i - V_O + SCRKV_i)$$

$\Sigma\, I$ at $V = 0$

$$I_2 + I_3 - I_4 = 0$$

$$SC(kV_i - V) + \frac{4Q^2}{R}(V_i - V)$$

$$- SC(V - V_o) = 0$$

$$SC\left(kV_i - \frac{kV_i}{SCR}\,1\,\frac{V_O}{SCR} - kV_i\right)$$

$$+ \frac{4Q^2}{R}\left(V_i - \frac{kV_i}{SCR} + \frac{V_O}{SCR} - kV_i\right)$$

$$-SC\left(\frac{kV_i}{SCR} - \frac{V_O}{SCR} + kV_i - V_O\right) = O$$

$$\Rightarrow -\frac{kV_i}{\cancel{R}} + \frac{V_O}{\cancel{R}}$$

$$+ \frac{4Q^2}{\cancel{R}}\left(V_i - \frac{kV_i}{SCR} + \frac{V_O}{SCR} - kV_i\right)$$

$$-\frac{SC}{\cancel{R}}\left(\frac{kV_i}{SC} - \frac{V_O}{SC} + kRV_i - V_OR\right) = O$$

$$\Rightarrow \text{SUB CR} = \frac{2Q}{\omega_O} \;\;\&\;\; R = \frac{2Q}{C\omega_O}$$

$$- kV_i + V_o$$

$$+ 4Q^2V_i - \frac{{}^{2}\cancel{4}\,Q^2\,kV_i\omega_O}{S2\cancel{Q}} + \frac{V_O\omega_O\,\cancel{4}^{2}Q^2}{S2\cancel{Q}} - 4Q^2kV_i$$

$$- kV_i + V_O - Sk2Q/w_OV_i + SV_O\frac{2Q}{w_O} = O$$

$$V_O\left[1 + \frac{2Q\omega_O}{S} + 1 + \frac{2QS}{\omega_O}\right]$$

$$= V_i\left[k - 4Q^2 + \frac{2kQ\omega_O}{S} + 4Q^2k + k + \frac{2kQS}{w_O}\right]$$

$$\Rightarrow V_O\left[S^2\frac{2Q}{\omega_O} + 2S + 2Q\omega_O\right] = V_i\Big[S^2\frac{2kQ}{\omega_O} +$$

$$S(4Q^2k - 4Q^2 + 2\,K) + 2kQw_O\Big]$$

$$\Rightarrow \frac{V_O}{V_i} = \frac{S^2\frac{2kQ}{\omega_O} + S(4Q^2k - 4Q^2 + 2k) + 2kQ\omega_O}{S^2\frac{2Q}{\omega_O} + 2S + 2Q\omega_O}$$

$$= k\,\frac{S^2 + S\frac{\omega_O}{Q}\left(2Q^2 - \frac{2Q^2}{k} + 1\right) + \omega_O^2}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$$

Recall $k = \dfrac{R_2}{R_1 + R_2}$ and $\dfrac{1}{k} = 1 + \dfrac{R_1}{R_2}$

$$\Rightarrow \frac{V_O}{V_i} = \left(\frac{R_L}{R_1 + R_2}\right)\frac{S^2 + S\frac{\omega_O}{Q}\left(1 - \frac{R_L}{R_2}.2Q^2\right) + \omega_O^2}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$$

$$\therefore T(S) = \frac{R_2}{R_1 + R_2}$$

$$\frac{S^2 + S\frac{\omega_O}{Q}\left(1 - \frac{2Q^2R_1}{R_2}\right) + \omega_O^2}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$$

For All Pass

we want $T(S)\alpha \dfrac{S^2 + \frac{\omega_O}{Q}(-1)S + \omega_O^2}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2}$

$$\Rightarrow 1 - \frac{2Q^2R_1}{R_2} = -1$$

$$2Q^2\frac{R_1}{R_2} = 2$$

$$\frac{R_1}{R_2} = \frac{1}{Q^2}$$

$$\therefore \frac{R_2}{R_1} = Q^2 \;\&\; \frac{R_2}{R_1 + R_2} = \frac{R_2/R_1}{1 + R_2/R_1}$$

$$= \frac{Q^2}{1 + Q^2}$$

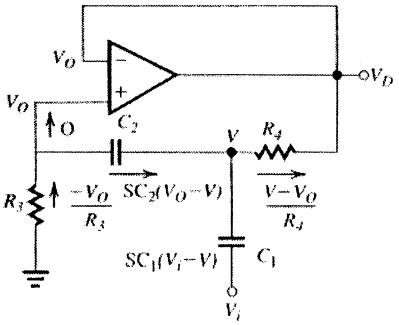
For Notch:

$$1 - 2Q^2\frac{R_1}{R_2} = O$$

$$\frac{R_1}{R_2} = \frac{1}{2Q^2}$$

$$\frac{R_2}{R_1} = 2Q^2 \;\&\; \frac{R_2}{R_1 + R_2} = \frac{2Q^2}{1 + 2Q^2}$$

11.51



$\therefore$ No current can't flow into the terminal

$$-\frac{V_O}{R_3} = SC_2(V_O - V)$$

$$V = V_O\left(1 + \frac{1}{SC_2R_3}\right)$$

ΣI @ $V = O$

$$-\frac{V_O}{R_3} + \frac{V_i - V}{1}SC_1 = \frac{V - V_O}{R_4}$$

$$V_O\left[-\frac{1}{R_3} + \frac{1}{R_4}\right] + V\left[-SC_1 - \frac{1}{R_4}\right] = -SC_1V_i$$

$$V_o\,[R_4 - R_3] + V\,[SC_1\,R_3R_4 + R_3] = V_iSC_1R_3R_4$$

$$V_O(R_4 - R_3) + V_O\left(1 + \frac{1}{SC_2R_3}\right)(SC_1R_3R_4 + R_3)$$

$$= SC_1R_3R_4\,V_i$$

$$V_O\left(R_4 - \cancel{R}_3 + SC_1R_3R_4 + \cancel{R}_3 + \frac{C_1}{C_2}R_4 + \frac{1}{SC_2}\right)$$

$$= SC_1R_3R_4\,V_i$$

$$V_o\,(S^2C_1C_2R_3R_4 + S\,C_1R_4 + SC_2\,R_4 + 1)$$

$$= S^2C_1R_3R_4C_2\,V_i$$

$$\therefore \frac{V_O}{V_i} = \frac{S^2C_1C_2R_3R_4}{S^2C_1C_2R_3R_4 + SC_1R_4 + SC_2R_4 + 1}$$

$$= \frac{S^2}{S^2 + S\left(\frac{1}{C_2R_3} + \frac{1}{C_1R_3}\right) + \frac{1}{C_1C_2R_3R_4}}$$

$$\text{Note}\quad \left.\begin{array}{l}|T(O)| = 0\\ |T(\infty)| = 1\end{array}\right\}\begin{array}{l}\therefore\ \text{High Pass}\\ \text{High Freq Gain} = 1\frac{\text{V}}{\text{V}}\end{array}$$

3 dB freq = 10^3rad/s, $Q = \frac{1}{\sqrt{2}}$ for max flat.

$$\therefore w_O = 10^3\,\frac{\text{rad}}{\text{s}}\quad C_1 = C_2 = 10\text{ nF}$$

clearly $w_O^2 = \frac{1}{C_1C_2R_3R_4}$ and

$$\frac{w_O}{Q} = \frac{1}{C_2R_3} + \frac{1}{C_1R_3} = \frac{C_1 + C_2}{C_1C_2R_3}$$

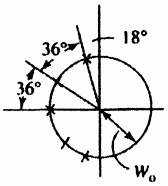
$$= \frac{2C}{C^2R_3} = \frac{2}{CR_3} = \sqrt{2} \times 10^3$$

$$R_3 = \frac{2}{10 \times 10^{-9} \times 10^3 \times \sqrt{2}}$$

$R_3 = 141.4\text{ k}\Omega$

$$R_4 = \frac{1}{w_O^2C_1C_2R_3} \Rightarrow R_4 = 70.7\text{ k}\Omega$$

11.52



$A_{max} = 3$ dB

$$\epsilon = (10^{3/10} - 1)^{-1/2} \cong 1$$

$$\omega_O = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N} = \omega_p = 2\pi 5000 = 10^4\pi$$

$$Q_1 = \frac{1}{2\text{Cos}36} = 0.618$$

$$Q_2 = \frac{1}{2\text{Cos}72} = 1.618$$

For first order section:

$\omega_o = 10^4\pi$ dc gain = 1

From 16.13 (a)

$R_1 = R_2 = 10\text{ k}\Omega$

$$C = \frac{1}{\omega_OR_2} = \frac{1}{10^4\pi 10^4} = 3.18\text{ nF}$$

Second order section $Q = 0.618$:

From 16.34 (c) $m = 4Q^2 = 1.528$

$$RC = \frac{2Q}{\omega_O}\ \text{let } R_1 = R_2 = 10\text{ k}\Omega$$

$$C = \frac{2Q}{\omega_OR} \Rightarrow C_4 = C = 3.93\text{ nF}$$

$$C_3 = \frac{C}{m} = 2.57\text{ nF}$$

Second Order Section $Q = 1.618$:

$$C = \frac{2Q}{\omega_OR}\quad m = 4Q^2 = 10.472$$

$$= 10.3\text{ nF} \Rightarrow R_1 = R_2 = 10\text{ k}\Omega$$

$C_4 = C = 10.3$ nF

$$C_3 = \frac{C}{m} = 0.984\text{ nF}$$

11.53

For a bandpass filter

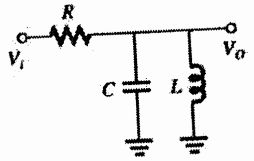
$$t(s) = \frac{\omega_O/Qs}{s^2 + s\omega_O/Q + \omega_O^2}$$

center freq. gain = 1

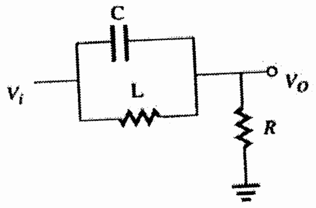
complementary transfer function :

$$t' = 1 - t$$

$$= \frac{s^2 + \omega_O^2}{s^2 + s\omega_O/Q + \omega_O^2} \equiv \text{NOTCH!}$$



⇒ INTERCHANGE V_i & gnd to get:



11.54

$$T(S) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

$$\omega_O = \frac{1}{\sqrt{LC}} \quad Q = R\sqrt{\frac{C}{L}}$$

For w_o

$$\frac{\partial\omega_O}{\partial L} = \frac{\partial(LC)^{-1/2}}{\partial L} = -\frac{1}{2}L^{-3/2}C^{-1/2} = \frac{\omega_O}{2L}$$

$$\frac{\partial\omega_O}{\partial C} = -\frac{\omega_O}{2C}$$

$$\frac{\partial\omega_O}{\partial R} = 0$$

$$\therefore S_L^{\omega_O} = \frac{\partial\omega_O}{\partial L}\frac{L}{\omega_O} = -1/2$$

$$S_C^{\omega_O} = \frac{\partial\omega_O}{\partial C}\times\frac{C}{\omega_O} = -1/2$$

$$S_R^{\omega_O} = \frac{\partial\omega_O}{\partial R}\frac{R}{\omega_O} = 0$$

For Q

$$\frac{\partial Q}{\partial L} = \frac{R\sqrt{C}}{L\sqrt{L}}\left(\frac{-1}{2}\right) = \frac{-Q}{2L}$$

$$\frac{\partial Q}{\partial C} = \frac{1}{2}\frac{R}{\sqrt{LC}} = \frac{1}{2}\frac{R\sqrt{C}}{C\sqrt{L}} = \frac{Q}{2C}$$

$$\frac{\partial Q}{\partial R} = \sqrt{C/L} = \frac{R}{R}\sqrt{C/L} = Q/R$$

$$S_L^Q = -\frac{Q}{2C}\times\frac{L}{Q} = -\frac{1}{2}$$

$$S_C^Q = \frac{Q}{2C}\times\frac{C}{Q} = \frac{1}{2}$$

$$S_R^Q = \frac{Q}{R}\cdot\frac{R}{Q} = 1$$

11.55

$$S^2 + S\frac{\omega_O}{Q}\left[1 + \frac{2Q^2}{A+1}\right] + \omega_O^2 = 0$$

Now the actual w_O and Q are given by:

$$\omega_{O,a} = \omega_O \text{ and } Q_a = \frac{Q}{1 + \frac{2Q^2}{(A+1)}}$$

$$S_A^{\omega_{O,a}} = 0$$

$$S_A^{Q_a} = \frac{A}{A+1}\,\frac{2Q^2(A+1)}{1 + 2Q^2/(A+1)}$$

$$\therefore S_A^{Q_a} \cong \frac{2Q^2}{A}$$

11.56

$R_1 = R_2$.

$$W_O = \frac{1}{\sqrt{C_3C_4R_1R_2}}$$

$$Q = \frac{1}{\sqrt{C_3C_4R_1R_2\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}}$$

$$\frac{\partial\omega_O}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2}}$$

$$\frac{\partial\omega_O}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2}} = \frac{-\omega_O}{2C_3}$$

$$S_{C_3}^{\omega_O} = \frac{\partial\omega_O}{\partial C_3}\frac{C_3}{\omega_O} = -\frac{1}{2}$$

clearly $S_{C_3}^{\omega_O} = S_{C_4}^{\omega_O} = S_{R_1}^{\omega_O} = S_{R_2}^{\omega_O} = -\frac{1}{2}$

$$\frac{\partial Q}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3C_4R_1R_2\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}} = \frac{-Q}{2C_3}$$

$$\therefore S_{C_3}^Q = -\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = \frac{Q}{2C_4} \Rightarrow S_{C_4}^Q = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial R_1} = \frac{1/\sqrt{R_1} - \sqrt{R_1/R_2}}{R_1\left(\frac{1}{\sqrt{R_1}} + \sqrt{\frac{R_1}{R_2}}\right)}\cdot\frac{Q}{2}$$

$$= \frac{\sqrt{R_2/R_1} - \sqrt{R_1/R_2}}{R_1\left(\sqrt{\frac{R_2}{R_1}} + \sqrt{\frac{R_1}{R_2}}\right)}\cdot\frac{Q}{2}$$

$$\therefore S_{R_1}^{Q} = \frac{\sqrt{R_2/R_1} - \sqrt{R_1/R_2}}{\sqrt{R_2/R_1} + \sqrt{R_1/R_2}}$$

If $R_1 = R_2 \Rightarrow S_{R_1}^{Q} = 0$

$S_{R_2}^{Q} = 0$

11.57

$$\omega_O = \frac{1}{\sqrt{C_4C_6R_1R_3R_4/R_2}}$$

$$Q = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}$$

$$\frac{\partial\omega_O}{\partial C_4} = \frac{-\omega_O}{2C_4}$$

$$\therefore S_{C_4}^{\omega_o} = \frac{-\omega_O}{2C_4} \times \frac{C_4}{\omega_O} = -\frac{1}{2}$$

Similarly $S_{C_6}^{\omega_o} = S_{R_1}^{\omega_o} = S_{R_3}^{\omega_o} = S_{R_5}^{\omega_o} = \frac{1}{2}$

$$\frac{\partial\omega}{\partial R_2} = \frac{\omega_O}{2R_2} \Rightarrow S_{R_2}^{\omega_o} = \frac{1}{2}$$

Now for Q:

$$\frac{\partial Q}{\partial R_6} = \frac{Q}{R_6} \Rightarrow S_{R_6}^{Q} = \frac{\partial Q}{\partial R_6}\frac{R_6}{Q} = +1$$

$$\frac{\partial Q}{\partial C_6} = \frac{Q}{2C_6} \Rightarrow S_{C_6}^{Q} = S_{R_2}^{Q} = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = -\frac{Q}{2C_4} \Rightarrow S_{C_4}^{Q} = S_{R_1,R_2,R_3}^{Q} = -\frac{1}{2}$$

11.58

change transferred $\Rightarrow Q = CV$

$= 10^{-12}(1)$

$= 1$ pC

For $f_O = 100$ kHz, average current is given by:

$$I_{AVE} = \frac{Q}{T} = 1\text{ pC} \times \frac{1}{100 \times 10^3}$$

$= 0.1$ μA

For each clock cycle, the output will change by the same amount as the change in voltage across C_2!

$$\therefore \Delta V = Q/C_2 = \frac{1\text{ pC}}{10\text{ pF}} = 0.1\text{ V}$$

For $\Delta V = 0.1$ V for each clock cycle, the amplifier will saturate in

$$\text{vcycles} = \frac{10\text{ V}}{0.1\text{ V}} = 100\text{ cycles}$$

$$\text{slope} = \frac{\Delta V}{\Delta t} = \frac{10\text{ V}}{(100\text{ cycles})(1/100 \times 10^3)}$$

$$= 10^4\frac{\text{V}}{\text{s}}$$

11.59

$f_C = 400$ kHz $f_O = 10$ kHz $Q = 20$

$C_1 = C_2 = 20\text{ pF} = C$

$C_3 = C_4 = \omega_O T_C C$

$$= 2\pi(10^4)\frac{1}{400 \times 10^3}20 \times 10^{-12}$$

$= 3.14$ pF

$$C_5 = \frac{\omega_O T_C C}{Q}$$

$$= \frac{C_3}{Q} = 0.157\text{ pF}$$

$$C_6 = \frac{\omega_O T_C C}{Q} \times \text{centre frequency gain}$$

$= 0.157$ pF

Note that the clock frequency has doubled. Hence the period, T_C, is halved. Therefore, for the same integratity capacitors, the resistors (switched capacitors) will change by the factor of z. so compensate for this by changing the switched caps by a factor of 1/2.

11.60

for $Q = 40$

$f_C = 200$ kHz $f_O = 10$ kHz

$C_1 = C_2 = 20\text{ pF} = C$

$C3 = C4 = \omega_O T_C C$

$$= 2\pi(10^4)\left(\frac{1}{200 \times 10^3}\right)20 \times 10^{-12}$$

$= 6.28$ pF

$$C_5 = \frac{\omega_O T_C C}{Q} = \frac{C_3}{Q} = 0.157\text{ pF}$$

$$C_6 = \frac{\omega_O T_C C}{Q} = C_5 = 0.157\text{ pF}$$

11.61

$\omega_O = 10^4, Q = 1/\sqrt{2}, f_C = 100$ kHz

$$\text{DC gain} \Rightarrow \frac{R_4}{R_6} \Rightarrow \frac{C_6}{C_4} = 1$$

$C_1 = C_2 = 10$ pF

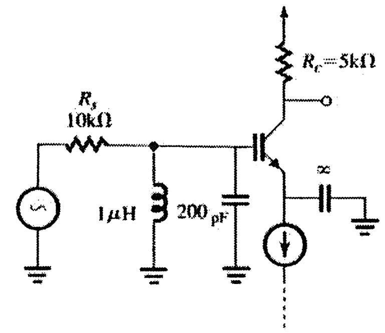
$C_3 = C_4 = C_6 = \omega_O T_C C$

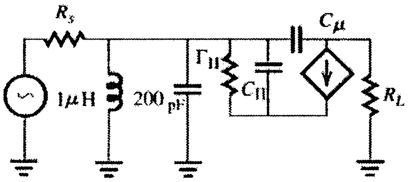
$$= 10^4\left(\frac{1}{100 \times 10^3}\right)10 \times 10^{-12}$$

$= 1\ \text{pF}$

$C_5 = C_4/Q = 1.41\ \text{pF}$

11.62





$r_e = 25\ \Omega,\ C_\mu = 1\ \text{pF},\ C_\pi = 10\ \text{pF},$

$\beta = 200$

$r_\pi = (\beta + 1)\sqrt{2} = 5.025\ \text{k}\Omega$

From base to collector

$$\frac{V_C}{V_b} = -\frac{\beta}{\beta + 1} \cdot \frac{R_2}{r_e} = -199 = \text{k}$$

Total capacitance at base

$C_T = C_\pi + 200\ \text{p} + C_\mu(1 - \text{k})$ Miller Effect

$= 10 + 200\ \text{p} + 1(1 + 199)$

$= 410\ \text{pF}$

$$\therefore \omega_O = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{10^{-6} \times 410 \times 10^{-12}}}$$

$= 49.4 \times 10^6\ \text{rad/s}$

centre frequency gain $= \dfrac{r_\pi}{R_S + r_\pi} \cdot \text{k}$

$$= \frac{5.025}{10 + 5.025} \times -199$$

$= -66.6\ \text{V/V}$

$$BW = \frac{1}{RC}$$

$$= \frac{1}{(R_S \| r_\pi)410\ \text{pF}}$$

$= 729 \times 10^3\ \text{rad/s}$

$$Q = \frac{\omega_O}{BW}$$

$= 49.4/0.7293$

$= 67.7$

11.63

$$Q_O = \frac{R_p}{\omega_O L} \Rightarrow R_p = Q_O \omega_O L$$

$= 200(2\pi 10^6)(10 \times 10^{-6})$

$= 12.57\ \text{k}\Omega$

$$\omega_O = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_O^2 L}$$

$$= \frac{1}{(2\pi 10^6)^2\ 10 \times 10^{-6}}$$

$= 2.533\ \text{nF}$

$$B = \frac{1}{RC}$$

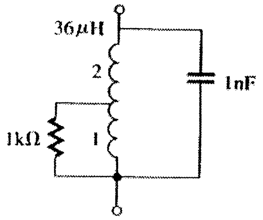
$$R_r = \frac{1}{(2\pi \times 10 \times 10^3)(2.533 \times 10^{-9})}$$

$= 6.283\ \text{k}\Omega$

$$\therefore \frac{1}{R_1 + R_p + R_r}$$

$\Rightarrow R_1 = 12.57\ \text{k}\Omega$ ie. $R_1 \| R_p = R_r$

11.64



$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

$= (2\pi(36 \times 10^{-6})10^{-9})^{-1}$

$= 838.8\ \text{kHz}$

$R_p = n^2 R$

$= 9\ (1\ \text{k}\Omega)$

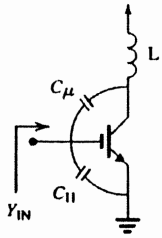
$= 9\ \text{k}\Omega$

$Q = R_p \omega_O L$

$$= \frac{9 \times 10^3}{2\pi\, 838.8 \times 10^3 \times 36 \times 10^{-6}}$$

$$= 47.4$$

11.65



for $\omega C_\mu << \frac{1}{WL}$

$\therefore \omega^2 << \frac{1}{LC_\mu}$

ie well below resonance

$\therefore$ gain $= -g_m(jwL)$

$$\therefore y_{in} = \frac{1}{\sqrt{r_\pi}} + j\omega C_\pi + j\omega C_\mu(1 + g_m j\omega L)$$

$$= \left(\frac{1}{r_\pi} - \omega^2 g_m C_\mu\right) + j\omega(C_\pi + C_\mu)$$

AS REQUIRED!

11.66

$$T(S) = \frac{a_1 s}{s^2 + s\frac{\omega_O}{Q} + \omega_O^2}$$

$$T(j\omega) = \frac{ja_1\omega}{\omega_O^2 - \omega^2 + \frac{j\omega\omega_O}{Q}}$$

$$T(j\omega_O) = \frac{ja_1\omega_O}{j\omega_O^2/Q} = \frac{a_1 Q}{\omega_O}$$

$$|T(j\omega)| = a_1\omega\left[(\omega_O^2 - \omega^2)^2 + \left(\frac{\omega\omega_O}{Q}\right)^2\right]^{-\frac{1}{2}}$$

$$= \frac{a_1\omega Q/\omega\omega_O}{\sqrt{1 + Q^2\left(\frac{\omega_O^2 - \omega^2}{\omega_O\omega}\right)^2}}$$

Now $\omega = \omega_o + \delta\omega$, $\frac{\delta\omega}{\omega_o} << 1$

and $\omega^2 \simeq \omega_O^2\left(1 + \frac{2\delta\omega}{\omega_O}\right)$

so $\omega_O^2 - \omega^2 = -2\delta\omega\omega_O$

$$\therefore |T(j\omega)| \cong \frac{a_1 Q/\omega_O}{\sqrt{1 + Q^2\left(\frac{2\delta\omega}{\omega}\right)^2}}$$

for $Q >> 1: Q^2\left(\frac{2\delta\omega}{\omega}\right)^2 \cong Q^2\left(\frac{2\delta\omega}{\omega_O}\right)$

$\because \omega \approx \omega_O$!

$$\Rightarrow |T(j\omega)| \cong \frac{|T(j\omega_O)|}{\sqrt{1 + Q^2\left(\frac{2\delta\omega}{\omega_O}\right)^2}}$$

$$= \frac{|T(j\omega_O)|}{\sqrt{1 + 4Q^2\left(\frac{\partial\omega}{\omega_O}\right)^2}}$$

For N bandpass sections, synchronously tuned in cascade, half power is given by:

$$\left(\frac{1}{\sqrt{1 + 4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2}}\right)^N = \frac{1}{\sqrt{2}}$$

$$\left(1 + 4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2\right)^N = 2$$

$$4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2 = 2^{1/N} - 1$$

$$\delta w = \frac{\omega_O}{2Q}\sqrt{2^{1/N} - 1}$$

$\therefore$ Bandwidth:

$$B = 2\delta\omega = \frac{\omega_O}{Q}\sqrt{2^{1/N} - 1}$$

11.67

For first order lowpass:

$$T(S) = \frac{\omega_O}{S + \omega_O} \quad |T(jw)| = \frac{\omega_O}{\sqrt{\omega^2 + \omega_O^2}}$$

for a bandpass response arount w_O with

$$\omega_O' = \frac{\omega_O}{2Q}:$$

$$|T(j\omega)| \cong \frac{\omega_O/2Q}{(\delta\omega)^2 + \left(\frac{\omega_O}{2Q}\right)^2}$$

$$= \frac{\omega_O/2Q}{\frac{\omega_O}{2Q}\sqrt{\left(\frac{2Q}{\omega_O}\right)^2(\partial W)^2 + 1}}$$

$$= \frac{1}{\sqrt{1 + 4Q^2(\delta\omega/w_O)^2}}$$

Now at $\omega = \omega_O$ or $\delta\omega = 0$

$|r(j\omega_o)| = 1$, then

$$T(j\omega) = \frac{|T(j\omega_O)|}{\sqrt{1+4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2}}$$

Part (b)

For N synchronously tuned sections in cascade; 3 dB bandwidth is given by:

$$(|T|/|T_D|)^N = \frac{1}{\sqrt{2}}$$

$$(|T|/|T_D|)^2 = \frac{1}{2^{1/N}} \text{ OR}$$

$$1+4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2 = 2^{1/N} \text{ OR}$$

$$2\delta\omega = \frac{\omega_O}{Q}\sqrt{2^{1/N}-1} \qquad (16.110)$$

Thus: $|T(j\omega)|_{\text{overall}} = |T(j\omega)|^N$

$$= \frac{|T(j\omega)|_{\text{overall}}}{\left[1+4Q^2\left(\frac{\delta\omega}{\omega_O}\right)^2\right]^{N/2}}$$

NOTE

$$Q = \frac{\omega_O}{Q}\sqrt{2^{1/N}-1}$$

$$= \frac{|T(j\omega_O)|_{\text{overall}}}{\left[1+4\frac{\omega_O^2}{B^2}(2^{1/N}-1)\left(\frac{\partial\omega}{\omega_O}\right)^2\right]^{\frac{N}{2}}}$$

$$= \frac{|T(j\omega_O)|_{\text{overall}}}{\left[1+4(2^{1/N}-1)\left(\frac{\delta\omega}{B}\right)^2\right]^{\frac{N}{2}}}$$

Part (c)(i)

for bandwidth = $2B$, ie $\delta w = \pm B$

$$Att = -20\log(1+(2^{1/N}-1)(1))^{-N/2}$$

$$= -10\text{N}\ \log(1+2^{2+1/N}-4)$$

$$= 10\text{N}\ \log(2^{2+1/N}-3)$$

N	1	2	3	4	2
Att(dB)	6.70	8.49	9.28	9.79	10.13

Part (ii)

3 dB bandwidth $\delta\omega = \pm B/2$

30 dB bandwidth $\frac{\delta\omega}{B} = x$

$$-30 = -20\frac{N}{2}\log(1+4(2^{1/N}-1)x^2)$$

$$3 = N\log(1+4(2^{1/N}-1)x^2)$$

$$x = \left[\frac{10^{3/N}-1}{4(2^{1/N}-1)}\right]^{1/2}$$

Ratio of 30 dB to 3 dB

$$BW = \frac{2Bx}{B} = 2x$$

N	1	2	3	4	5
Ratio	31.6	8.6	5.7		4.5

11.68

(a) For the narrowband approximation variation of Ω around O is equivalent to sw around w_O

Thus, a low-pass maximally flat filter of bandwidth $B/2$ and order N for which

$|\text{T}| = [(1+\Omega/B/2)^{2N}]^{\frac{-1}{2}}$ is transformed to a band-pass maximally flat filter of bandwidth $B/2$ and order 2N, and centre frequency ω_O for which:

$$|T| = \left(1+\left(\frac{\delta\omega}{B/2}\right)^{2N}\right)^{-1/2}$$

(b) For bandwidth $2B$, $\delta\omega = B$ &

$$|T| = \left(1+\left(\frac{B}{B/2}\right)^{2N}\right)^{-1/2}$$

$$= (1+2^{2N})^{-1/2} \text{ thus:}$$

N	1	2	3	4	5
$\|T\|$	0.447	0.242	0.124	0.062	0.031
$\|T\|_{dB}$	–6.99	–16.3	–18.1	–24.1	–30.1

For 30 dB bandwidth,

$$-30 = 20\log x \Rightarrow x = 10^{-3/2}$$

$$= \frac{1}{31.6}$$

$$\therefore 1+\left(\frac{\delta'\omega}{B/2}\right)^{2N} = (31.6)^2$$

$$\left(\frac{\delta'\omega}{B/2}\right)^{2N} = 999-1 = 998$$

Now the ratio of 30 dB to 3 dB bandwidth is

$$\text{ratio} = \frac{2\delta'\omega}{B} = \frac{\delta'\omega}{B/2} = 998^{\frac{1}{2N}}$$

N	1	2	3	4	5
ratio	31.6	5.62	3.16	2.37	1.99

11.69

$A_{\max} = 3\text{ dB} \Rightarrow \epsilon = \sqrt{10^{A_{\max}/10} - 1} \approx 1$

Poles of lowpass prototype are given by

Poles: $-\omega_p,\ \omega_p(-1/2 \pm j\sqrt{3}/2)$

Make $w_p = B/2$

$$\Rightarrow \text{poles:}\left\{\frac{-B}{2}, \frac{+B}{2}\left(-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right)\right\}$$

Using the low-pass to bandpass transformation:

Poles of the bandpass filter:

$$\frac{-B}{2} \pm j\omega_O$$

$$\frac{-B}{4} \pm j\left(\frac{\sqrt{3}}{4}B + \omega_O\right) \text{ and}$$

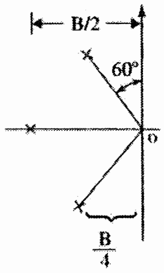
$$\frac{-B}{4} \pm j\left(\frac{\sqrt{3}}{4}B - \omega_O\right)$$

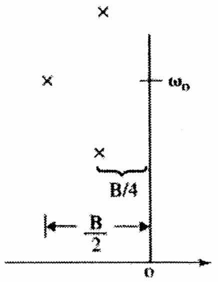
For the three circuits:

(1) $\omega_{O1} = \omega_O \quad B_1 = B \quad Q_1 = \omega_O/B$

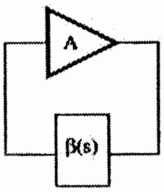
(2) $\omega_{O2} \cong \frac{\sqrt{3}}{4}B + \omega_O \quad B_2 = \frac{B}{2} \quad Q_2 \cong \frac{2\omega_O}{B}$

(3) $\omega_{O3} \cong \frac{\sqrt{3}}{4}B - \omega_O \quad B_3 = \frac{B}{2} \quad Q_3 \cong \frac{2\omega_O}{B}$





12.1



$A = A_O > 0$

$$\beta(s) = \frac{K\frac{\omega_O}{Q}s}{S^2 + s\frac{\omega_O}{Q} + \omega_O^2}$$

(a) for oscillations $1 - A\beta(s) = 0$

$$A_O K\frac{\omega_O}{Q}s = S^2 + s\frac{\omega_O}{Q} + \omega_O^2$$

$$\omega_O^2 - \omega^2 = j\omega\left(\frac{\omega_O}{Q}\right)(A_O K - 1)$$

at the freq. of oscillation, both Real & imaginary parts are O.

$\therefore \omega = \omega_O$ & $A_O K = 1$

(b)

$$L(j\omega) \overset{\Delta}{=} A\beta(j\omega) = \frac{AK\frac{\omega_O}{Q}j\omega}{(\omega_O^2 - \omega^2) + j\omega\left(\frac{\omega_O}{Q}\right)}$$

$$\therefore \phi(\omega) = 90° - \tan^{-1}\left(\frac{\omega\omega_O/Q}{\omega_O^2 - \omega^2}\right)$$

$$\text{Now } \frac{\partial}{\partial x}\tan^{-1}v = \frac{1}{1+v^2}\cdot\frac{\partial u}{\partial x}$$

$$\therefore \frac{\partial\phi}{\partial\omega} = \frac{1}{1 + \left(\frac{\omega\omega_O/Q}{\omega_O^2 - \omega^2}\right)^2}\cdot\frac{\partial}{\partial\omega}\left(\frac{\omega\omega_O/Q}{\omega_O^2 - \omega^2}\right)$$

$$= \frac{-(\omega_O^2 - \omega^2)^2}{(\omega_O^2 - \omega^2) + \left(\frac{\omega\omega_O}{Q}\right)^2}\cdot\left[\frac{\frac{\omega_O}{Q}(\omega_O^2 - \omega^2) - 2\omega\frac{\omega\omega_O}{Q}}{(\omega_O^2 - \omega^2)^2}\right]$$

$$\left.\frac{d\phi}{\partial\omega}\right|_{\omega = \omega_O} = \frac{-1}{\omega_O^4/Q^2}\cdot\frac{2\omega_O^3}{Q}$$

$$= -\frac{2Q}{\omega_O}$$

$$\text{(c) } \Delta\omega_O = \frac{\Delta\phi}{\partial\phi/\partial\omega} = \frac{\Delta\phi}{-2Q/\omega_O}$$

$$= \frac{-\Delta\phi\omega_O}{2Q}$$

$\therefore$ Per unit change in ω_O is given by

$$\frac{\Delta\omega_O}{\omega_O} = \frac{-\Delta\phi}{2Q}$$

12.2

For the circuit of problem .1, the poles, which are the zeros of the characteristic equation, are given by:

$1 - L(S) = 0$

$L(S) = 1$

$$\frac{AK\left(\frac{\omega_O}{Q}\right)S}{S^2 + S\frac{\omega_O}{Q} + \omega_O^2} = 1$$

$$S^2 + S\frac{\omega_O}{Q}(1 - AK) + \omega_O^2 = 0$$

$\therefore$ Poles are at :

$$S = \frac{-\frac{\omega_O}{Q}(1 - AK) \pm \sqrt{\left(\frac{\omega_O}{Q}\right)^2(1 - AK)^2 4\omega_O^2}}{2}$$

$$= -\omega_O\left[\frac{1 - AK}{2Q} \pm \sqrt{\left(\frac{1 - AK}{2Q}\right)^2 - 1}\right]$$

$$= -\omega_O\left(\frac{1 - AK}{2Q}\right)\left[1 \pm j\sqrt{\left(\frac{2Q}{1 - AK}\right)^2 - 1}\right]$$

Radial distance of $\omega_O \Rightarrow$

$$|S^2| = \omega_O^2\left(\frac{1 - AK}{2Q}\right)^2\left[1 + \left(\frac{2Q}{1 - AK}\right)^2 - 1\right]$$

$$= \omega_O^2$$

$\therefore |S| = \omega_O$ independent of A or K !

(a) For poles on jw- axis ⇒ real part = 0

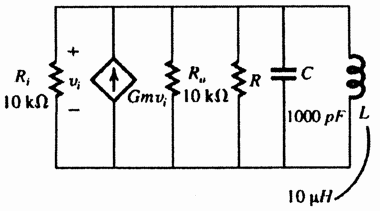
$\therefore -(1 - AK) = 0 \Rightarrow AK = 1$

(b) For poles in RHS ⇒ Real Part > 0

$-(1 - AK) > 0$

$AK > 1$

12.3



For resonator: $\omega_O = \frac{1}{\sqrt{LC}} = 10^7 \frac{\text{rad}}{S}$

$$\frac{\omega_O}{Q} = \frac{1}{RC}$$

$$R = \frac{Q}{\omega_O C} = \frac{100}{10^7 \times 1000 \times 10^{-12}} = 10\ \text{k}\Omega$$

Oscillation will occur at $\omega_O = 10^7 \frac{\text{rad}}{3}$

when $G_m = (R_i \parallel R_o \parallel R) = 1$ i.e. gain = 1

$$\therefore G_m = \frac{1}{10\text{ K} \parallel 10\text{ K} \parallel 10\text{ K}} = \frac{3}{10^4} = 300\ \frac{\mu\text{A}}{\text{V}}$$

12.4

At ω_O $A\beta = 1$

If $\beta(\omega_O)$ is -20 dB with a phase shift of 180° then clearly A should have a gain of 20 dB (i.e. $A(\omega_O) = 10$) with a phase shift of ±180°

i.e. $A = -10$

12.5

$$L_- = -V\frac{R_3}{R_2} - V_D\left(1 + \frac{R_3}{R_2}\right)$$

$$6 = 10\frac{R_3}{R_2} + 0.7\left(1 + \frac{R_3}{R_2}\right)$$

$$= 10.7\frac{R_3}{R_2} + 0.7$$

$\frac{R_3}{R_2} = 0.495$ By symmetry $\frac{R_4}{R_5} = 0.495$

Use $R_2 = R_5 = 10\ \text{k}\Omega$

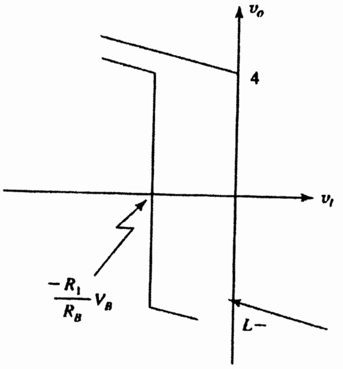
$\therefore R_3 = R_4 \cong 5\ \text{k}\Omega$

Slope of limiting characteristic

$$\frac{R_4}{R_5} = 0.1$$

$$\therefore R_1 = \frac{1}{0.1}R_4 = 50\ \text{k}\Omega$$

12.6



For V_B connected via R_B to the virtual ground, a current $= \frac{V_B}{R_B}$ flows into the node. To compensate, v_I must be moved by Δv_I, in a direction opposite to V_B to produce a current $\Rightarrow$

$$\frac{\Delta v_I}{R_1} = \frac{-v_B}{R_B}$$

$$\therefore \Delta v_I = -\frac{R_1}{R_B}V_B$$

$v_D = 0$ ~ assumed

$L_- = -5 = -15\ R_3/R_2$

$\frac{R_3}{R_2} = \frac{1}{3} = R_4/R_5$

Given $R_{in} = 100\ \text{k}\Omega \Rightarrow R_1 = 100\ \text{k}\Omega$

Slope $= R_4/R_1 \le 0.05$

$R_4 \le R_1 \times 0.05$

$R_4 \leq 5\ \text{k}\Omega \Rightarrow$ Let $R_4 = 4.3\ \text{k}\Omega$

$\because R_3 = R_4 \Rightarrow R_3 = 4.3\ \text{k}\Omega$

$R_2 = R_5 = 3R_4 = 12.9\ \text{k}\Omega$

For standard resistance values:

$R_2 = R_5 = 13\ \text{k}\Omega$

$$\therefore\ L- = -15\frac{R_3}{R_2} = -15 \times \frac{4.3}{12.9}$$

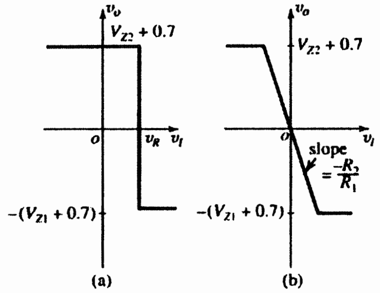
$$= -4.96\ \text{V} \cong -5\ \text{V}$$

Offset is $+5\ \text{V} \Rightarrow$ use $V_B = -15\ \text{V}$

and $5 = R_1 / R_B\ 15$

$\therefore R_B = 3R_1 = 300\ \text{k}\Omega$

12.7



12.8

$$\frac{V_a}{V_o} = \frac{1/sc \parallel R}{1/sc \parallel R + 1/sc + R}$$

$$= \frac{(R/sc)/\left(\frac{1}{sc} + R\right)}{\frac{(R/sc)}{\left(\frac{1}{sc} + R\right)} + \frac{1}{sc} + R}$$

$$= \frac{\frac{R}{sc}}{\frac{R}{sc} + \left(\frac{1}{sc} + R\right)^2} \times \frac{S^2C^2}{S^2C^2}$$

$$= \frac{SCR}{SCR + (1 + SCR)^2}$$

$$= \frac{SCR}{SCR + (1 + 2SCR) + S^2C^2R^2}$$

$$= \frac{\frac{1}{RC}S}{S^2 + S\frac{3}{RC} + \frac{1}{R^2C^2}}$$

Note $\frac{V_a}{V_o}$ has zeros at O and ∞

i.e. *A* Band pass!

$$\omega_O^2 = \frac{1}{R^2C^2} \Rightarrow \omega_O = \frac{1}{RC}$$

$$\frac{\omega_O}{Q} = \frac{3}{RC} \Rightarrow Q = \frac{1}{3}$$

For centre frequency gain:

$S = j\omega_O = j/RC$

$$\therefore \left.\frac{V_a}{V_o}\right|_{S = (j/RC)} = \frac{\frac{1}{RC}\ j/RC}{-\frac{1}{R^2C^2} + \frac{3}{RC}\left(\frac{j}{RC}\right) + \frac{1}{R^2C^2}}$$

$$= \frac{1}{3} = \text{centre freq. gain}$$

12.9

$$L(j\omega) = \frac{1 + R_2/R_1}{3 + j\left(WCR - \frac{1}{WCR}\right)} \qquad \text{Eq(17.11)}$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{WCR - \frac{1}{WCR}}{3}\right)$$

using $\frac{\partial \tan^{-1} v}{\partial x} = \frac{1}{1 + v^2}\frac{\partial v}{\partial x}$

$$\frac{\partial \phi}{\partial \omega} = \frac{-1}{1 + \left(\frac{WCR - \frac{1}{WCR}}{3}\right)^2} \cdot \frac{1}{3}\left(CR + \frac{1}{W^2CR}\right)$$

$$\left.\frac{\partial \phi}{\partial \omega}\right|_{\omega = \frac{1}{RC}} = \frac{-1}{3}(CR + CR) = \frac{-2}{3}CR$$

for $\Delta\phi = -0.1$ rad

$$\Delta\omega_O = \frac{\Delta\phi}{\partial\phi/\partial\omega} = \frac{-0.1}{-2/3\frac{1}{\omega_O}}$$

$$= 0.15\ \omega_O$$

$\therefore$ New frequency of oscillation

$$= 1.15\ \omega_O = \frac{1.15}{RC}$$

12.10

$$L(s) = \frac{1 + R_2/R_1}{3 + SCR + 1/SCR}$$

Poles of closed loop given by: $L(S) = 1$

$$1 + R_2/R_1 = 3 + SCR + \frac{1}{SCR}$$

$$O = S^2 + \frac{S}{RC}\left(2 - \frac{R_2}{R_1}\right) + \frac{1}{R^2C^2}$$

$$Q = \frac{1}{(2 - R_2/R_1)}$$

for $Q = \infty$ - poles on jω axis

$-R_2/R_1 = 2$

for poles in R.H.P $R_2/R_1 > 2$

12.11

, assuming resistance of limiting network is very low

At positive peak

$$v_O = \left(\frac{1 + 20.3\text{ K}}{10\text{ K}}\right) v_O = 3.03 v_I \qquad (1)$$

$$v_O - \left[\frac{R_5}{R_5 + R_6} \cdot (v_O - (-15))\right] - 0.7 = v_I \qquad (2)$$

Now for $10V_{p-p}$ out

$$\hat{v}_O = 5\text{ V}$$

$$\hat{v}_I = \frac{5}{3.03} = 1.65\text{ V}$$

using (2) $R_5 = 1\text{ k}\Omega$

$$5 - \left(\frac{1}{1 + R_6} \cdot (V_o + 15)\right) - 0.7 = 1.65$$

$$\frac{20}{1 + R_6} = 2.65$$

$$R_6 = \frac{20}{2.65} - 1$$

$$R_6 = 6.5\text{ k}\Omega = R3$$

If $R_3 = R_6 = \infty$ from (2)

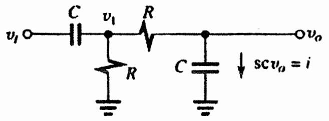
$$v_0 - \left(\frac{1}{1 + \infty}(V_0 + 15)\right) - 0.7 = \frac{v_0}{3.03}$$

$$v_0 - 0.7 = \frac{v_0}{3.3}$$

$$v_0 = 1.04\text{ V}$$

∴ oppoutput is 2 v_0 = 2.08 V_{p-p}.

12.12



$$\frac{v_1 - v_0}{R} = SCv_0 \Rightarrow v_1 - v_0(1 + SCR)$$

ΣI at v_1

$$\frac{v_1}{R} + SC(v_1 - v_I) + SCv_0 = 0$$

$$v_0(1 + SCR) + SCR(v_0 + v_0 SCR) - SCRv_I + SCRv_0 = 0$$

$$v_0(1 + SCR + SCR + S^2C^2R^2 + SCR) = SCRv_I$$

$$\beta(s) \triangleq \frac{v_0}{v_I} = \frac{SCR}{S^2C^2R^2 + 3SCR + 1}$$

$$= \frac{1}{3 + SCR + 1/(SCR)}$$

$$A = 1 + R_2/R_1$$

$$\beta(j\omega) = \frac{1}{3 + j\left(\omega CR - \frac{1}{\omega CR}\right)}$$

Zero phase when $\omega CR = \frac{1}{\omega CR}$

$$\omega = \frac{1}{CR}$$

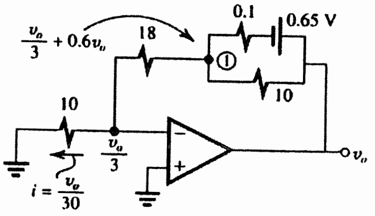
$$|\beta(W = 1/RC)| = \frac{1}{3}$$

for oscillations $1 + R_2/R_1 \geq 3 \Rightarrow \frac{R_2}{R_1} \geq 2$

$$L(s) = A\beta = \frac{1 + R_2/R_1}{3 + SCR + SCR}$$

$$L(jw) = \frac{1 + R_2/R_1}{3 + j\left(WCR - \frac{1}{WCR}\right)}$$

12.13



ΣI at node 1

$$\frac{v_O}{30} = \frac{v_O - \frac{v_O}{3} - 0.650}{10}$$

$$\frac{+ v_O - 0.65 - \frac{v_O}{3} - 0.65}{0.1}$$

$$0.00666 v_O + 0.666 v_O - 0.65$$

$$v_O = 10.156\text{ V}$$

∴ Max. ouput = 20.3 $Vp-p$

12.14

$$\omega_O = \frac{1}{RC} = 2\pi 10^4 \quad R = 10\text{ k}\Omega$$

$$C = \frac{1}{10^4 \times 2\pi \times 10^4} \Rightarrow C \cong 1.6\text{ nF}$$

$$\beta(j\omega) = \left[3 + j\left(\omega CR - \frac{1}{\omega CR}\right)\right]^{-1}$$

$$\therefore \phi(\omega) = -\tan^{-1}\left(\frac{WCR - \frac{1}{WCR}}{3}\right)$$

using $\frac{\partial \tan^{-1} v}{\partial x} = \frac{1}{1+v^2}\frac{\partial v}{\partial x}$ we get

$$\frac{\partial\phi(\omega)}{\partial\omega} = \frac{-1}{1+\left(\frac{WCR - \frac{1}{WCR}}{3}\right)^2}\left[\frac{RC + 1/W^2RC}{3}\right]$$

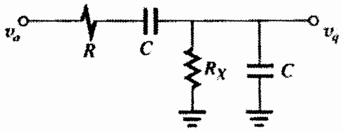
At $\omega = \omega_O = \frac{1}{RC}\ \frac{\partial\phi(\omega)}{\partial\omega} = \frac{-2}{3}RC$

Now 5.7° ≈ 0.1rad (lag = −0.1rad)

$\therefore \Delta\,\omega_o = \frac{-0.1}{-2/3\ \mathrm{RC}} = 0.15\ \omega_o = 1.5\ \mathrm{kHz}$

∴ New frequency of oscillation = 8.5 kHz

To restore Operation:



$$\beta(s) = \frac{Rx \parallel \frac{1}{SC}}{Rx \parallel \frac{1}{SC} + R + \frac{1}{SC}}$$

$$= \frac{\frac{Rx/SC}{Rx + 1/SC}}{\frac{Rx/SC}{Rx + \frac{1}{SC}} + R + \frac{1}{SC}}$$

$$= \frac{Rx/SC}{Rx/SC + RRx + \frac{R}{SC} + \frac{Rx}{SC} + \frac{1}{S^2C^2}}$$

$$\therefore \beta(s) = \frac{1}{2 + \frac{R}{Rx} + SCR + \frac{1}{SCRx}}$$

$$\phi = -\tan^{-1}\left(\frac{WCR - \frac{1}{WRxC}}{2 + R/Rx}\right)$$

Now it is required that $\phi = 5.7°$ at $\omega > \omega_O$!

where $\omega_O = 1/RC$

$$\therefore \omega_O RC - \frac{-1}{\omega_O RxC} = \left(2 + \frac{R}{Rx}\right)\tan^{(-5.7)}$$

$$1 - \frac{1}{\omega_O RxC} = (2 + R/Rx)(-0.1)$$

$$1 + 0.2 = \frac{1}{\omega_O RxC} - 0.1\frac{R}{Rx}$$

$$Rx = \frac{1/\omega_O C - 0.1\ \mathrm{R}}{1.2}$$

given:

$\omega_O = 2\pi 10^4$

$C = 1.6 \times 10^{-9}$

$R = 10^4$

$Rx = 7.5\ \mathrm{k\Omega}$

Now:

$$\beta(j\omega_O) = \frac{1}{2 + 10/7.5 + j(1 - 1/\omega_O CRx)}$$

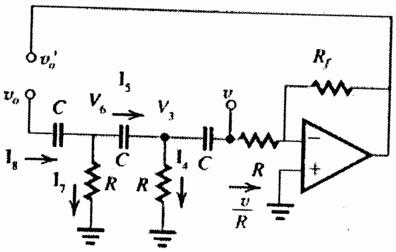
$$= (3.333 - j0.326)^{-1}$$

$$|\beta(j\omega_O)| = \frac{1}{3.35}$$

$\therefore 1 + R_2/R_1 = 3.35$ for oscillations

$\frac{R_2}{R_1} = 2.35$ (not 2 as before)

12.15



$$V_3 = v + \frac{1}{SCR}\frac{v}{} = v\left(1 + \frac{1}{SCR}\right)$$

$$I_4 = \frac{v\left(1 + \frac{1}{SCR}\right)}{R} = \frac{v}{R} + \frac{v}{SCR^2}$$

$$I_5 = I_4 + \frac{v}{R}\,i_s = \frac{2v}{R} + \frac{v}{SCR^2}$$

$$V_6 = V_3 + \frac{I_5}{sC}$$

$$= v + \frac{v}{sCR} + \frac{1}{sC}\left(2\frac{v}{R} + \frac{v}{sCR^2}\right)$$

$$V_6 = v + \frac{3v}{sCR} + \frac{v}{s^2C^2R^2}$$

$$I_7 = \frac{v}{R} + \frac{3v}{sCR^2} + \frac{v}{s^2C^2R^3}$$

12.16

$I_8 = I_5 + I_7$

$$I_8 = \frac{3v}{R} + \frac{4v}{sCR^2} + \frac{v}{s^2C^2R^3}$$

$$V_o = V_6 + \frac{I_8}{sC}$$

$$v_o = v + \frac{3v}{sCR} + \frac{v}{s^2C^2R^2} + \frac{3v}{sCR} + \frac{4v}{s^2C^2R^2} + \frac{v}{s^3C^3R^3}$$

$$= v + \frac{6v}{sCR} + \frac{5v}{s^2C^2R^2} + \frac{v}{s^3C^3R^3}$$

Now loop gain =

$$L(S) = \frac{-v_o^1}{v_o}$$

$$v_o^1 = \frac{R_f}{R} \cdot v$$

$$\therefore L(S) = \frac{\frac{R_f}{R} \cdot v}{v\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right)}$$

$$= \frac{s^3 R_f / R}{s^3 + \frac{6s^2}{RC} + \frac{5s}{C^2R^2} + \frac{1}{C^3R^3}}$$

$$L(j\omega) = \frac{-j\omega^3 R_f / R}{\frac{1}{C^3R^3} - \frac{6\omega^2}{RC} + j\left(\frac{5\omega}{C^2R^2} - \omega^3\right)}$$

$L(j\omega)$ is real if

$$\frac{6\omega_O^2}{RC} = \frac{1}{R^3C^3}$$

$$\omega_O = \frac{1}{\sqrt{6}RC}$$

$$L(j\omega_O) = \frac{\omega_0^2 R_f / R}{-\omega_0^2 + 5 / R^2C^2}$$

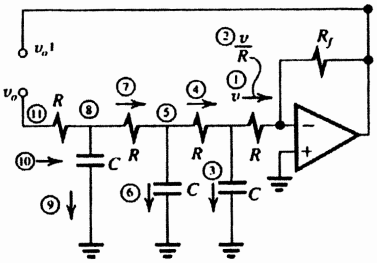
$$= \frac{R_f / R\omega_0^2}{-w_0^2 + 30\omega_0^2}$$

$$= \frac{R_f / R}{29}$$

Now loop Gain = 1 if $R_f = 29$ R

$\therefore$ Minimum value for $R_f = 29$ R

$$f_O = \frac{0.065}{RC}$$



(3) $i = scv$

(4) $scv = v/R$

(5) $v + \left(SCv + \frac{v}{R}\right)R = 2v + SCRv$

(6) $2SCv + S^2C^2Rv$

(7) = (6) + (4) = $3SCv + S^2C^2Rv + \frac{v}{R}$

(8) $2v + SCRv + v + 3SCRv + S^2C^2R^2v$

$= 3v + 4SCRv + S^2C^2R^2v$

(9) $3SCv + 4S^2C^2Rv + S^3C^3R^2v$

(10) = (7) + (9)

$= 6SCv + 5S^2C^2Rv + \frac{v}{R} + S^3C^3R^2v$

(11) = (8) + (10) $\times R$

$v_o = 4v + 10SCRv + 6S^2C^2R^2v + s^3c^3R^3v$

$$L(s) = \frac{v_o^1}{v_o} = \frac{vR_f / R}{v(S^3C^3R^3 + 6S^2C^2R^2 + 10SCR + 4)}$$

$$= \frac{R_f / R}{s^3C^3R^3 + 6S^2C^2R^2 + 10SCR + 4}$$

$$L(j\omega) = \frac{R_f / R}{(4 - 6\omega^2C^2R^2) + j(10\omega CR + (\omega^3C^3R^3)}$$

$L(j\omega)$ is purely real if

$$10\omega_O CR = \omega_O^3 C^3 R^3$$

$$\omega_O = \frac{1}{\sqrt{10}} \frac{1}{RC}$$

Given $R = 10$ kΩ, fo = 10 kHz

$$C = \frac{1}{\sqrt{10} \times 10^4 \times 2\pi 10^4}$$

$= 0.503$ nF

Now,

$$|L(j\omega_O)| = \frac{R_f / R}{4 - 6\omega_O^2 R^2 C^2} \quad \text{sub for } \omega_o$$

$$= \frac{R_f / R}{4 - 6\frac{1}{10R^2C^2}R^2C^2}$$

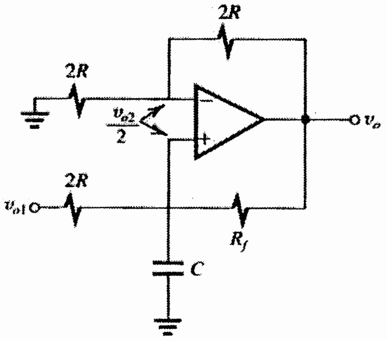
$$= \frac{R_f / R}{4 - 6/10} \geq 1$$

$\therefore R_f / R \geq 3.4$

$R_f \geq 34\ \text{k}\Omega$

12.17

for 2nd indicator



From the voltage divider around the upper branch:

$$v+ = v- = \frac{1}{2}v_{02}$$

$\Sigma I = 0$ at the input

$$\frac{\frac{1}{2}v_{02} - v_{01}}{2R} + SC\frac{v_{02}}{2} + \frac{\frac{v_{02}}{2} - v_{02}}{R_f} = 0$$

$$\frac{v_{02} - 2v_{01}}{2R} + SCv_{02} - \frac{v_{02}}{R_f} = 0 \quad R_f = \frac{2R}{H\Delta}$$

$$v_{02}\left(\frac{1}{2} + SC - \frac{H\Delta}{2R}\right) = \frac{v_{01}}{R}$$

$$v_{02}\left(SCR - \frac{\Delta}{2}\right) = v_{01}$$

$$\therefore \frac{v_{02}}{v_{01}} = \frac{1}{SCR - \Delta/2}$$

Now: $\frac{v_{01}}{v_x} = \frac{-1}{SCR}$

$$\therefore L(S) = \frac{-1/SCR}{SCR - \Delta/2}$$

Characteristic equation $L(s) = 1$

$$\therefore S^2C^2R^2 - \frac{SCR\Delta}{2} + 1 = 0$$

$\therefore$ Poles are

$$S_p = \frac{\frac{RC\Delta}{2} \pm \sqrt{\frac{R^2C^2\Delta^2}{4} - 4C^2R^2}}{2R^2C^2}$$

$$= \frac{\Delta/2 \pm 2j\sqrt{1 - (\Delta/4)^2}}{2RC}$$

for $\Delta \ll 1$ $\left(1 - \left(\frac{\Delta}{4}\right)^2\right)^{1/2} \approx \left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)$

$$\therefore S_p \cong \left[\Delta/2 \pm j2\left(1 - \frac{1}{2}\left(\frac{\Delta}{4}\right)^2\right)\right]\frac{1}{2RC}$$

$$= \frac{\frac{\Delta}{2} \pm j\left(2 - \left(\frac{\Delta}{4}\right)^2\right)}{2RC}$$

Now:

$R_e[S_p] > 0 \Rightarrow$ Poles in R.H.P.!

for $\Delta \ll 1$

$$S_p \cong \frac{\Delta/2 \pm j2}{2RC} = \frac{1}{RC}\left(\frac{\Delta}{4} \pm j\right) \text{ Q.E.D}$$

12.18

The transmission of the filter normalized to the centre frequency, w_o is:

$$|T(jw)| = \frac{\omega\omega_0 / Q}{(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}$$

$$= \frac{(1/Q)\left(\frac{\omega_0}{\omega}\right)}{\left(\left(\frac{\omega_0}{\omega}\right)^2 - 1\right)^2 + \frac{1}{Q^2}\left(\frac{\omega_0}{\omega}\right)^2}$$

Relative to the amplitude of the fundamental

(a) The second harmonic = 0

(b) The third harmonic

$$= \frac{1}{3}\frac{\frac{1}{20} \times \frac{1}{3}}{\left(\frac{1}{9} - 1\right)^2 + \left(\frac{1}{20}\right)^2\left(\frac{1}{9}\right)} = 6.25 \times 10^{-3}$$

(c) The fifth harmonic

$$= \frac{1}{5}\frac{\frac{1}{20} \times \frac{1}{5}}{\left(\frac{1}{25} - 1\right)^2 + \left(\frac{1}{20}\right)^2\left(\frac{1}{25}\right)} = 2.08 \times 10^{-3}$$

(d) The 4th harmonic = 6th = 10th = 0

7th harmonic = 1.04×10^{-3}

9th harmonic = 0.625×10^{-3}

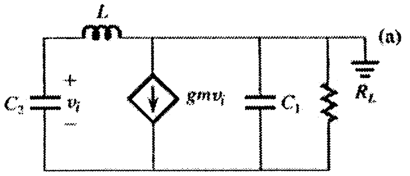
$$\therefore \frac{\text{RMS of } 2^{nd} \text{ to } 10^{th} \text{ harmonic}}{\text{RMS of fundamental}} \text{is}$$

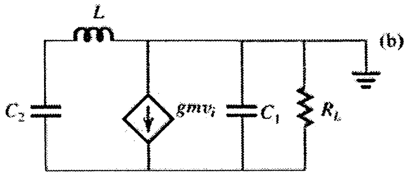
$$[6.25^2 + 2.08^2 + 1.04^2 + 0.625^2]^{1/2} \times 10^{-3}$$

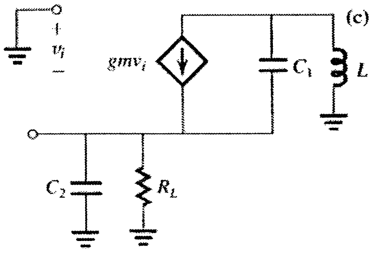
$= 6.7 \times 10^{-3}$ OR 0.7%

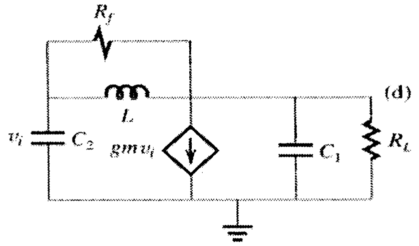
12.19

Consider the small signal models for each circuit.
Assume r_π very large:

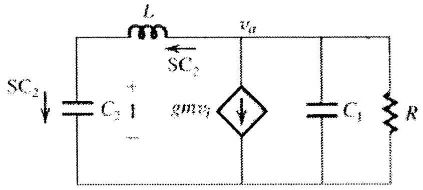

L
(a)
C_2
+
v_i
−
gmv_i
C_1
R_L


L
(b)
C_2
gmv_i
C_1
R_L


(c)
+
v_i
−
gmv_i
C_1
L
C_2
R_L


R_f
(d)
L
v_i
C_2
gmv_i
C_1
R_L

Given $R_f >> w_0 L$, circuits (a), (b) and (d) are the same except for the reference (ground) node.
For circuit (a), (b) & (d)


L
v_a
SC_2
SC_2
+
C_2 1
−
gmv_i
C_1
R

-Break the loop at v_i and assume unit return.

$$v_a = 1 + SC_2 sL$$

$$= 1 + S^2 C_2 sL$$

$\Sigma I = 0$ at v_a

$$g_m + SC_2 + SC_1(1 + S^2 C_2 sL) + \frac{(1 + S^2 C_2 sL)}{R} = 0$$

$$\therefore g_m + 1/R + S(C_1 + C_2) + \frac{S^2 C_2 L}{R} + S^3 C_2 sL = 0$$

This is the characteristic equation.
For $s = j\omega$:

$$g_m + \frac{1}{R} - \frac{\omega^2 C_2 L}{R} + j((C_1 + C_2)\omega - \omega^3 (C_1 C_2 L)) = 0$$

IMAGINARY PART = 0:

$$C_1 + C_2 = \omega^2 C_1 + C_2 L$$

$$\omega = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}} \equiv \text{Frequency of Oscillation}$$

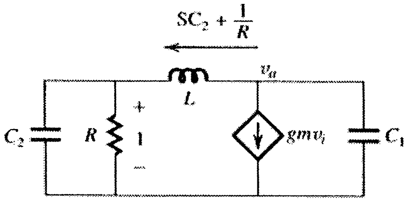
REAL PART = 0

$$g_m + \frac{1}{R} = \frac{\omega^2 C_2 L}{R}$$

$$g_m R = \left(\frac{C_1 + C_2}{C_1 C_2 L}\right) C_2 L - 1$$

$$g_m R = \frac{C_2}{C_1} \equiv \text{LIMIT ON GAIN}$$

For Circuit (c)


$SC_2 + \frac{1}{R}$
L
v_a
+
C_2
R
1
−
gmv_i
C_1

$$v_a = \left(SC_2 + \frac{1}{R}\right) SL + 1$$

$\Sigma I = 0$ at v_a, $v_i = 1$

$$g_m + SC_2 + \frac{1}{R} + SC_1\left[SL\left(SC_2 + \frac{1}{R}\right) + 1\right] = 0$$

$$g_m + \frac{1}{R} + SC_2 + S^3 C_1 C_2 L + \frac{S^2 C_1 L}{R} + SC_1 = 0$$

THE CHARACTRISTIC EQUATION ≡

$$g_m + \frac{1}{R} + S(C_1 + C_2) + \frac{S^2 C_2 L}{R} + S^3 C_1 C_2 L = 0$$

Note this is the same as above, with $C_1 \leftrightarrow C_2$

$$\therefore \omega_O = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}} \text{ and } g_m R = \frac{C_1}{C_2}$$

12.20

(a) frequency of oscillation $\omega_0 = \frac{1}{\sqrt{LC}}$

gain >> 1 gain $= \frac{RC}{2r_e} = \frac{RC}{2v_T/I/2}$

$= \frac{IRC}{4v_T}$

for $v_T = 0.025$ V then

$IRC \geq 4v_T$

$RC \geq 0.1/I$ for oscillations to start.

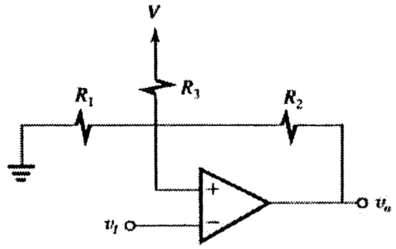
(b) For $RC = \frac{1}{I}$ (kΩ) we have

$$\text{gain} = \frac{1/I}{2\left(\frac{2v_T}{I}\right)} = \frac{1}{4 \times 0.025} = 10$$

Oscillations will start (10>1) and grow until Q1, Q2 go into cutoff. Output will go from V_{CC} to $V_{CC} - IRC = V_{CC} - 1$.

Therefore, output will be $1\ V_{P-P}$. Fundamental has a P-P amplitude of $\frac{4}{\pi} = 1.27\ V_{P-P}$

12.21



(a) ΣI at $+ v_L$ node:

$$\frac{V_{TH}}{R_1} = \frac{V - V_{TH}}{R_3} + \frac{L_T - V_{TH}}{R_2}$$

$$V_{TH}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V}{R_3} + \frac{L_T}{R_2}$$

$$V_{TH} = (V/R_3 + L_T/R_2)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$= \left(\frac{V}{R_3} + \frac{L_T}{R_2}\right)R_1 \parallel R_2 \parallel R_3$$

Similarly

$$V_{TL} = \left(\frac{V}{R_3} + \frac{L-}{R_2}\right)(R_1 \parallel R_2 \parallel R_3)$$

(b) Now

$$V_{TH} = 5.1 = \left(\frac{15}{R_3} + \frac{13}{R_2}\right)\left(\frac{1}{10} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$\frac{5.1}{10} + \frac{5.1}{R_2} + \frac{5.1}{R_3} = \frac{15}{R_3} + \frac{13}{R_L}$$

$$0.51 = \frac{7.9}{R_2} + \frac{9.9}{R_3} \qquad (1)$$

AND

$$V_{TL} > 4.9 = \left(\frac{15}{R_3} - \frac{13}{R_2}\right)\left(\frac{1}{10} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$0.49 = \frac{-17.9}{R_2} + \frac{10.1}{R_3} \qquad (2)$$

$$(1) \times \frac{10.1}{9.9} \Rightarrow 0.52 = \frac{8.06}{R_2} + \frac{10.1}{R_3}$$

$$(2) \Rightarrow 0.49 = \frac{-17.9}{R_2} + \frac{10.1}{R_3}$$

SUBTRACT TO GET $\Rightarrow$

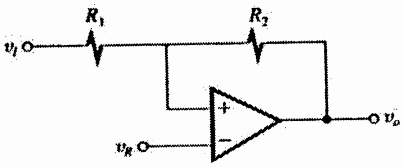
$$0.52 - 0.49 = \frac{8.06 + 17.9}{R_2}$$

$$R_2 = \frac{25.96}{0.0303} = 856.8\ \text{k}\Omega$$

$$\frac{10.1}{R_3} = 0.49 + \frac{17.9}{856.8}$$

$R_3 \cong 19.8$ kΩ

12.22



(a) for $v_I = v_{TL}$ and $v_O = L_+$ initially

$$\frac{L_+ - v_R}{R_2} = \frac{v_R - v_{TL}}{R_1}$$

$$v_{TL} = v_R - \frac{R_1}{R_2} v_R - \frac{R_1}{R_2} L_+$$

$$\therefore v_{TL} = v_R\left(1 - \frac{R_1}{R_2}\right) - \frac{R_1}{R_2} L_+$$

Similarly

$$\frac{L_- v_R}{R_2} = \frac{v_R - v_{TH}}{R_1}$$

$$v_{TH} = v_R(1 + R_2 / R_1) - \frac{R_1}{R_2} L_-$$

(b) Given

$L_+ = -L_- = V$

$R_1 = 10\ \text{k}\Omega$

$V_{TL} = 0$

$V_{TH} = V/10$

Substituting these values we get:

$$O = V_R(1 + 10 / R_2) - 10 / R_2 V \qquad (1)$$

$$\frac{V}{10} = V_R(1 + 10 / R_2) + 10 / R_2 V \qquad (2)$$

$$(1) - (2) - \frac{V}{10} = -\frac{20}{R_2} v$$

$$R_2 = 200\ \text{k}\Omega$$

$$O = V_R(1 + 10 / 200) - \frac{10}{200}\ \text{V}$$

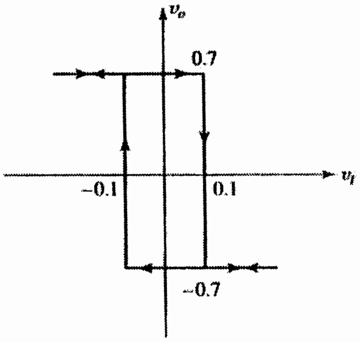
$$V_R = \frac{10 / 200\ \text{V}}{1 + 10 / 200} = 47.62\ \text{mV}$$

12.23

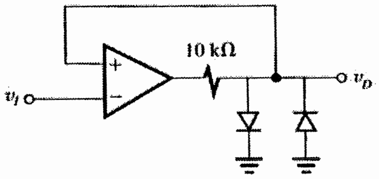
Output levels = ±0.7 V

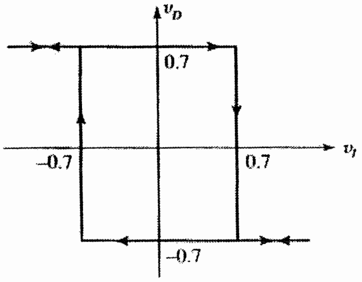
$$\text{Threshold levels} = \pm\frac{10}{10 + 60} \times 0.7 = 0.1\ \text{V}$$

$$i_{D,\max} = \frac{12 - 0.7}{10} - \frac{0.7}{10 + 60} = 1.12\ \text{mA}$$



12.24





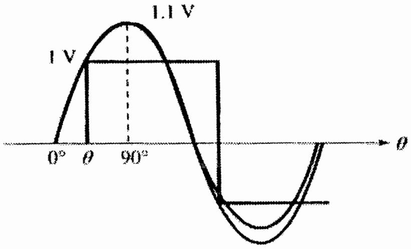
12.25

(a) A 0.5 V peak sine wave, is not large enough to change the state of the circuit. Hence, the output will be either +12 V or −12 V at DC.

(b) The 1.1 V peak will change the state when

$1.1 \sin\theta = 1$

$\theta = 65.40$



∴ The output is a symmetric square wave at frequency *f*, and lags the sine wave by an angle of 65.4°. The square wave has a swing of ± 12 V. Since $v_{TH} - v_{TL} = 1$ V, if the average shifts by an amount so either the +ve or -ve swing is <1 V, then no change of state will occur. Clearly, if the shift is 0.1 V, the output will be a DC voltage.

12.26

For $L+ = -L- = 7.5$ V
$V_Z = 6.8$ V with $V_D = 0.7$ V.

For $V_{TH} = -V_{TL} = 7.5\text{ V} \Rightarrow R_1 = R_2$

For $v_i = 0 \quad I_{R_2} = 0.1\text{mA} = \dfrac{7.5}{R_1 + R_2}$

$\Rightarrow R_1 = R_2 = 37.5\text{ k}\Omega$

$$I_D = 1\text{mA} = \frac{12 - 7.5}{R} - \frac{7.5}{2R_1}$$

$$1 = \frac{4.5}{R} + 0.1$$

$R = 4.1\text{ k}\Omega$

12.27

$$T = 2\tau \ln \frac{1 + \beta}{1 - \beta} \qquad \beta = \frac{R_1}{R_1 + R_2} = \frac{10}{26}$$

$$T = 2(10 \times 10^{-9})(62 \times 10^3) l_n\left(\frac{1 + 10/26}{1 - 10/26}\right)$$

$T = 1.006\text{ms} \Rightarrow \text{f} = 994.5\text{ Hz}$

12.28

for $\pm 5V_{\text{outputs}}$

$V_z = 5 - 2V_{\text{DIODE}} = 5 - 1.4 = 3.6$ V

For $\pm 5V_{\text{out}}$:

$R_1 = R_2 \; L_+ = -L_- = 5$ V

$V_{TH} = -V_{TL} = 5$ V

Max current in feedback network = 0.2 mA

$$\therefore 0.2 = \frac{5}{R_1 + R_2} \Rightarrow R_1 = R_2 = 25\text{ k}\Omega$$

Max diode current = 1 mA

$$\therefore \frac{13 - 5}{R_z} = (0.2 + 1)\text{mA}$$

$$R_z = \frac{8}{1.2} = 6.67\text{ k}\Omega$$

Now from Fig 17.25(c)

$$\text{slope} = \frac{-L_-}{RC} = \frac{V_{TH} - V_{TL}}{T/2}$$

for $f = 1$kHz
$T = 10^{-3}$ sec.
$C = 0.01\ \mu$F

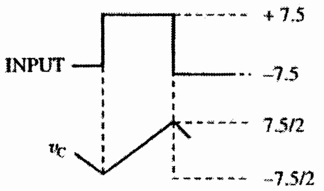
$$\frac{5}{RC} = \frac{10}{10^{-3}/2} \Rightarrow R = 25\text{ k}\Omega$$

12.29

For 15 V_{PP} output $v_z = 15/2 - 0.7$

$= 6.8$ V

For the integrator:



i.e. V_C should ramp between V_{TH} & V_{TL}!

$$v_C(t_1) = \frac{1}{RC}\int_{t_0}^{t_1} v\text{dt} + v_c(t_0)$$

$-v$ is a square wave

$$\frac{7.5}{2} = \frac{1}{RC}(t_1 - t_0)(7.5 - (-7.5)) - \frac{7.5}{2}$$

$$(t_1 - t_0) = \frac{T}{2}$$

$$7.5 = \frac{1}{RC}\frac{T}{2}(15)$$

$$1 = \frac{T}{RC} \Rightarrow R = \frac{T}{RC} = \frac{1}{fC}$$

$$= \frac{1}{10^4(0.5 \times 10^{-9})}$$

$\therefore R = R_{1-6} = 200\text{ k}\Omega$

Minimum level current = 1 mA

$$\frac{13 - 7.5}{R_z} = 1 + \frac{7.5}{R_1 + R_2} + \frac{7.5 - V_C}{R_5}$$

Maximum current into the integrator when

$$V_C = \frac{-7.5}{2}$$

$$\therefore \frac{5.5}{7.5} = 1 + \frac{7.5}{400} + \frac{11.25}{200}$$

$\therefore R_7 = 5.12\text{k}\Omega \overset{\text{use}}{\Rightarrow}$

$R_7 = 5.1\ \text{k}\Omega$

Integrator output is triangular, with period $= 100\mu\text{s}$ and ± 7.5 V peaks. (i.e. $2\times$ voltage at capacitor)

12.30

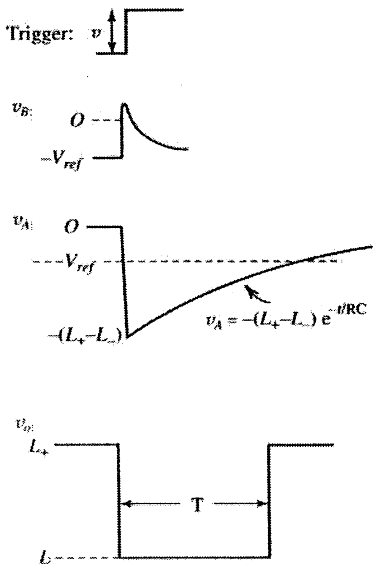
See sketches that follow:

$V_A(t = T) = V_{ref} = -(L_+ - L_-)e^{-T/RC}$

$\frac{V_{ref}}{L_+ - L_-} = e^{-T/RC}$

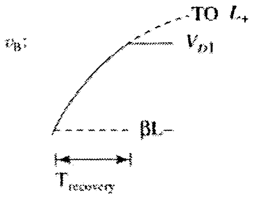
$T = -RC\ln\left(\frac{V_{ref}}{L_+ - L_-}\right) = RC\ln\left(\frac{L_+ - L_-}{V_{ref}}\right)$

Q.E.D.



12.31

For recovery, v_B goes from $\beta L-$ to L_+ until D_1 conducts at $V_{D1} = 0.7$ V



For recovery

$v_B = -0.1(12) + (12 + 1.2)(1 - e^{-t/\tau})$

$= 12 - 13.2e^{-t/\tau}$

At T recovery:

$V_{D1} = 12 - 13.2e^{-Tr/\tau}$

$\tau = R_3C_1$

$Tr = -R_3C_1\ln\left(\frac{V_{D1} - 12}{13.2}\right)$

$= -(6171)(0.1 \times 10^{-6})\ln\left(\frac{11.3}{13.2}\right)$

$= 96\ \mu\text{s}$

12.32

Choose $C_1 = 1\text{nF}$ $\qquad C_2 = 0.1\text{nF}$

$R_1 = R_2 = 100\ \text{k}\Omega \Rightarrow \beta \cong \frac{1}{2}$

$T \cong C_1R_3\ln\left(\frac{0.7 + 13}{-13(0.5 - 1)}\right)$

$10^{-4} = 10^{-9}R_3\ln\left(\frac{13.7}{13(0.5)}\right)$

$R_3 = 134.1\ \text{k}\Omega$

Need $R_4 >> R_1 \Rightarrow$ choose $R_4 = 470\ \text{k}\Omega$

Min trigger voltage $= (\beta L_+ - V_{D2} + V_{D1})$

$= 6.5$ V

For recovery

$v_B = 13 - (13 - \beta L_-)e^{-t/\tau}$

$= 13 - 19.5e^{-t/\tau} = 0.7$

$\therefore t_{recovery} = -\tau\ln\left(\frac{12.3}{19.5}\right)$

$= -(134.1 \times 10^3)(10^{-9})(-0.4608)$

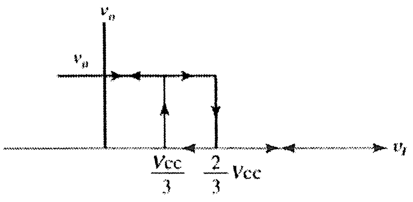
$= 61.8\ \mu\text{s}$

12.33

For $v_I > 2/3V_{CC}$ comp − 1 = "1" and comp −2 = "0" and flip flop is reset. I.E. $v_O = 0$V.

Now v_O will not change until $v_I = 1/3V_{CC}$, when comp - 2 = "1" and comp -1 = "0" and FF is set: I.E. $V_O = V_{CC}$

For $\frac{1}{3}V_{CC} < v_I < \frac{2}{3}V_{CC}$, comp − 1 = comp - 2 = "0" and no change of state will occur.



i.e. an inverting bistable circuit.

12.34

(a) C = 1nF

$v_C = V_{CC}(1 - e^{-t/\tau})$

where $\tau = RC$

Pulse width of 10 µs when $v_C = V_{TH}$

$= \frac{2}{3}V_{CC}$

$\therefore \frac{2}{3} = 1 - e^{-t/RC}$

$t = T = 10\ \mu s$

$-\frac{T}{RC} = \ln\left(\frac{1}{3}\right) \Rightarrow R = \frac{-T}{C\ln(1/3)}$

$= 9.1\ k\Omega$

(b) for $T = 20\mu s$ $R = 9.1\ k\Omega$, C = 1nF

$\therefore V_{TH} = 15(1 - e^{-t/RC})$

$$= 15\left(1 - e^{\frac{-20 \times 10^{-6}}{9.1 \times 10^{3} \times 10^{-9}}}\right)$$

= 13.3 V

12.35

$C = 680\ pF$ $f = 50\ kHz$

$T = 20\ \mu s = T_H + T_L$

For 75% Duty $T_H = 15\ \mu s$

$T_L = 5\ \mu s$

From Eq (17.43) we have:

$T_L = CR_B \ln 2$

$\therefore R_B = \frac{5 \times 10^{-6}}{680 \times 10^{-12} \ln 2} = 10.6\ k\Omega$

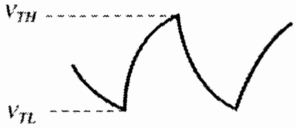
From Eq (17.41)

$T_H = C(R_A + R_B)\ln(2)$

$R_A = \frac{15 \times 10^{-6}}{680 \times 10^{-12}\ln(2)} - 10.6 \times 10^3$

$= 21.2\ k\Omega$

12.36



For the rise:

$V_C = V_{CC} - (V_{CC} - V_{TL})e^{-t/C(R_A + R_B)}$

$V_{TH} = V_{CC} - (V_{CC} - V_{TL})e^{-T_H/C(R_A + R_B)}$

$\frac{V_{CC} - V_{TH}}{V_{CC} - V_{TL}} = e^{-T_H/C(R_A + R_B)}$

$T_H = C(R_A + R_B)\ln\left(\frac{V_{CC} - V_{TL}}{V_{CC} - V_{TH}}\right)$

For exponential fall:

$V_C = V_{TH}e^{-t/CR_B}$

$\therefore V_{TL} = V_{TH}e^{-T_L/CR_B}$

$T_L = CR_B\ln\left(\frac{V_{TH}}{V_{TL}}\right)$

for $V_{TH} = 2\ V_{TL} \Rightarrow T_L = C\ R_B\ln(2)$

(b) $C = 1nF$, $R_A = 7.2\ k\Omega$, $R_B = 3.6\ k\Omega$

$V_{CC} = 6\ V$ $V_{TH\ ext} = 0$

$\therefore T_H + T_L = T = \ln 2(R_A + 2R_B)C$

$T = 9.98\ \mu s \rightarrow f = 100\ kHz$

Duty cycle $= \frac{T_H}{T_H + T_L} = \frac{R_A + R_B}{R_A + 2R_B} = 0.75$

$\Rightarrow 75\%$

(c) $V_{CC} = 5V$,

$V_{TH} = \frac{2}{3} \times 5 = \frac{10}{3} = 3.33\ V$

for IV input $V^{1}_{TH} = 4.33\ V$

$V^{1}_{TL} = \frac{1}{2}V^{1}_{TH} = 2.17\ V$

$T^{1}_{H} = 10^{-9}(3.6 + 7.2) \times 10^3 \ln\left(\frac{5 - 2.17}{5 - 4.33}\right)$

$= 15.6\ \mu s$

$T^{1}_{L} = 10^{-9} \times 3.6 \times 10^3 \ln 2 = 2.5\ \mu s$

$\therefore f = \frac{1}{(15.6 + 2.5)10^{-6}} = 55.2\ kHz$

duty cycle $= \frac{15.6}{2.5 + 15.6} = 86.2\%$

for IV input $V^{11}_{TH} = 2.33$

$V^{11}_{TL} = 1.17$

$\therefore T^{11}_{H} = 10^{-9}(3.6\ 7.2)10^3\ln\left(\frac{5 - 1.17}{5 - 2.33}\right)$

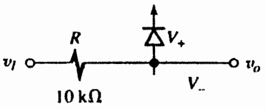
$= 3.92\ \mu s$

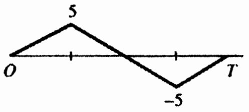
$T^{11}_{L} + T^{1}_{L} = 2.5\ \mu s$

$\therefore f = \frac{10^6}{(3.92 + 2.5)} = 156\ kHz$

duty cycle $= \frac{3.92}{2.5 + 3.92} = 61\%$

12.37





$$v_O = A\sin\frac{2\pi}{T}t$$

Slope of v_O at $t = 0$:

$$\frac{\partial v_O}{\partial t} = A\frac{2\pi}{T}\cos\left(\frac{2\pi}{T}\right)t|_t = 0$$

$$= \frac{A2\pi}{T} = \text{SLOPE AT ZERO CROSSING}$$

Slope of Δ^- wave $= \frac{5}{T/4} = \frac{20}{T}$

$$\therefore \frac{20}{T} = \frac{A2\pi}{T}$$

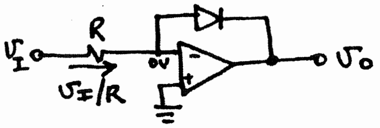
$A = 3.18$ V

∴ Clamp voltage:

$V_T = -V_- = 3.18 - 0.7$

$= 2.48 = 2.5$ V

12.38



$\rightarrow v_I > 0$

$\rightarrow$ voltage across diode is $-v_O$

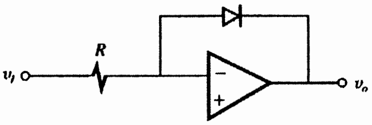
$$i_D = \frac{v_I}{R} = I_s e^{-v_O/nV_T}$$

$$-\frac{v_O}{nV_T} = \ln\left(\frac{v_I}{RI_s}\right)$$

$$v_O = -nV_T\ln\left(\frac{V_I}{RI_s}\right), \quad v_I > 0$$

Q.ED.

12.39



$$= -nV_T\ln\left(\frac{v_1}{RI_S}\right)$$

Now.

$V_A = -nV_T\ln\frac{V_1}{RI_S}$ $\quad R = 1$ kΩ

$V_B = -nV_T\ln\frac{V_2}{RI_S}$ $\quad V_1, V_2 > 0$

$V_C = +nV_T\ln\frac{1}{RI_S}$

$V_D = -(V_A + V_B + V_C)$

$$= nV_T\left(\ln\left[\frac{V_1}{RI_S}\times\frac{V_2}{RI_S}\times\frac{RI_S}{1}\right]\right)$$

$$= nV_T\ln\left(\frac{V_1V_2}{RI_S}\right)$$

$$i_{D4} = I_s e^{V_D/nV_T}$$

$$= I_s \times \frac{V_1V_2}{RI_S} = \frac{V_1V_2}{R}$$

$$v_O = -i_{D4}R = -\frac{V_1V_2}{R}\times R$$

$\therefore v_O = -v_1v_2$ ANALOG MULTIPLIER

To check $v_1 = 0.5$, $v_2 = 2$

$$I_{D1} = 0.5\text{ mA} \rightarrow V_A = -0.7 + nV_T\ln\left(\frac{0.5}{1}\right)$$

$$= 0.7 + 2(0.025)\ln\left(\frac{1}{2}\right)$$

$$= -0.6653\text{ V}$$

$I_{D2} = 2\text{ mA} \rightarrow V_B = (0.7 + 0.05\ln(2))(-1)$

$= -0.7347\text{ V}$

$I_{D3} = 1\text{ mA} \rightarrow V_C = 0.700\text{ V}$

$V_D = -(-0.6653 - 0.7347 + 0.7) = 0.7\text{ V}$

$V_D = V_{D4} = 0.7\text{ V} \Rightarrow I_{D4} = 1\text{ mA}$

$\therefore v_O = -1\text{ V}$ i.e. $2 \times 0.5 = 1$

For $v_1 = 3$, $v_2 = 2$:

$I_{D1} = 3\text{mA} \rightarrow V_A = -(0.7 + 0.05\ln 3)$

$= -0.7549\text{ V}$

$I_{D2} = 2\text{mA} \rightarrow V_A = -(0.7 + 0.05\ln 2)$

$= -0.7347\text{ V}$

$I_{D3} = 1\text{mA} \rightarrow V_C = 0.7\text{ V}$

$\therefore V_D = V_{D4} = -(V_A + V_B + V_C) = +0.7896\text{ V}$

$$\therefore \frac{I_{D4}}{1\text{ mA}} = \frac{I_S e^{V_D/0.05}}{I_S e^{0.7/0.05}}$$

$$I_{D4} = e^{\frac{0.7986 - 0.7}{0.05}} = 6\text{ mA}$$

$\therefore v_O = -6\text{ V}$ i.e. $2 \times 3 = 6$.

For squarer: $v_1 = 2$ through $\frac{1}{2}\text{ k}\Omega$ resistor

$I_{D1} = 4\text{mA} \rightarrow V_A = -(0.7 + 0.05\ln 4)$

$= -0.7693$

$V_D = -(-0.7693) = 0.7693\text{ V}$

$$I_{D4} = e^{\frac{0.7693 - 0.7}{0.05}} = 3.999\text{ mA}$$

$\therefore V_D = -3.999\text{ V}$ i.e. $2^2 = 4$.

12.40

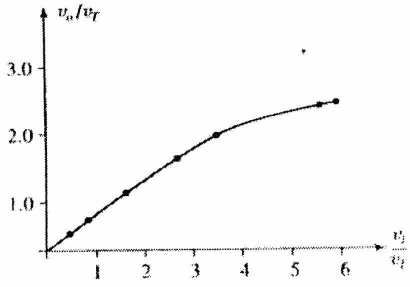
Say $V_{BE} = \tilde{V}_D @ I\ n = 1$

for $v_O = 0.25 v_T$:

$$I_R = \frac{0.25V_T}{R} = \frac{0.25V_T}{\frac{2.5V_T}{I}} = \frac{I}{10}$$

$$V_{BE1} = \tilde{V}_D + nV_T\ln\left(\frac{I + I/10}{I}\right)$$

$\cong \tilde{V}_D + V_T\ln(1.1)$



$$V_{BE2} = \tilde{V}_D + nV_T\ln\left(\frac{I - I/10}{I}\right)$$

$\cong \tilde{V}_D + V_T\ln(0.9)$

$V_I = -V_{BE2} + V_D + V_{BE1}$

$= V_T[\ln(1.1) + 0.25 - \ln(0.9)]$

$= 0.451 V_T$

For $v_O = 0.5V_T$

$$I_R = \frac{0.5\text{ I}}{2.5} = 0.2\text{ I}$$

$V_I = V_T[\ln(1.2) + 0.5 - \ln(0.8)]$

$= 0.905 V_T$

$V_O = V_T\ I_R = 0.4\text{ I}$

$V_I = V_T[\ln 1.4 + 1 - \ln 0.6]$

$= 1.847 V_T$

$V_O = 1.5V_T\ I_R = 0.6\text{ I}$

$V_I = V_T(\ln 1.6 + 1.5 - \ln 0.4)$

$= 2.886 V_T$

$V_O = 2V_T\ I_R = 0.8\text{ I}$

$V_I = V_T(\ln 1.8 + 2 - \ln 0.2) = 4.197 V_T$

$V_O = 2.4V_T\ I_R = 0.96\text{ I}$

$V_I = V_T(\ln 1.96 + 2.4 - \ln 0.04) = 6.292 V_T$

$V_O = 2.42V_T\ I_R = 0.968\text{ I}$

$V_I = V_T(\ln 1.968 + 2.42 - \ln 0.032)$

$= 6.519 V_T$

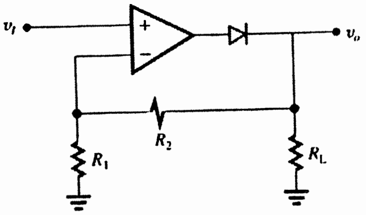
Ideal curve given by

$$v_O = 2.42V_T\sin\left(\frac{v_I}{6.6V_T} \times 90^\circ\right)$$

$$\frac{v_I}{v_T} = \frac{6.6}{90}\sin^{-1}\left(\frac{v_O}{2.42V_T}\right)$$

v_O / v_T	0.25	0.50	1.00	1.50	2.00	2.40	2.42
v_I / v_T	0.451	0.905	1.85	2.89	4.20	6.29	6.52
v_I / v_T (ideal)	0.435	0.874	1.79	2.81	4.09	6.06	6.60

12.41

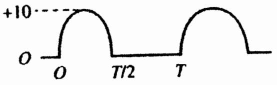


for $v_I \geq 0$

$$v_O = \left(1 + \frac{R_2}{R_1}\right)v_I$$

for a gain of 2 $R_1 = R_2 = 10\text{ k}\Omega$

for $v_I = 10V_{pp}$ sine wave $v_O \Rightarrow$



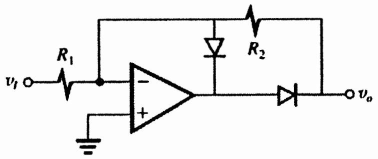
$$\text{Avg} = \frac{1}{T}\int_0^{T/2} 10\sin\frac{2\pi}{T}t\ dt$$

$$= \frac{1}{T}\frac{T}{2\pi}\cos\frac{2\pi}{T}t \times (-10)\Big|_{t=0}^{T/2}$$

$$= \frac{-10}{2\pi}(\cos\pi - \cos 0)$$

$$= 10/\pi = 3.18\text{ V}$$

12.42



for $v_I < 0 \Rightarrow v_O = -R_2/R_1$

$R_{in} = R_1 = 100\text{ k}\Omega$ $\therefore R_2 = 200\text{ k}\Omega$

12.43

for high R_{in}, use $R_1 = 1\text{ M}\Omega$

Ac gain is given by R_2/R_1

$\Rightarrow R_2 = 1\text{ M}\Omega$

Now for 1 *VrMs* sine, peak is 1.414 V. The value of V_1 is then $\frac{1.414}{\pi} = 0.450\text{ V}$

For 10 V out at second stage gain (dc)

$$= \frac{10}{0.450} = 22.2$$

$\therefore R_4/R_3 = 22.2$

& Choose $\frac{1}{2\pi R_4 C} = 10\text{ Hz}$ (i.e. corner frequency)

To make C small, make $R_4 = 1\text{ M}\Omega$

$\therefore C = 15.9\text{ nF}$

$$R_3 = \frac{1\text{ M}\Omega}{22.2} = 45\text{ k}\Omega$$

12.44

At the +ve terminal $V_+ = -5\text{V}$

for $v_I > -5$ D_1 is "ON" and faces virtual short.

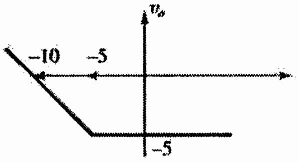
$\therefore V_- = -5$, and no current will flow in feedback R.

$\therefore v_O = -5\text{ V}$

for $v_I < -5$ D_1 is "off" and

$$\frac{v_O}{v_I} = \frac{-5 - v_I}{R} = \frac{v_O + 5}{R}$$

$$\Rightarrow v_O = -v_I$$



12.45

for $v_I < 0$

D_2 "off"

$$\frac{v_{01}}{v_I} = -1$$

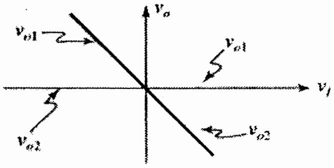
$$\frac{v_{02}}{v_I} = 0$$

for $v_I > 0$

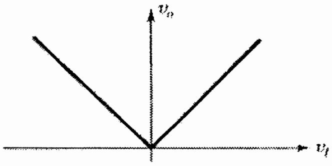
D_1 "off"

$$\frac{v_{01}}{v_I} = 0$$

$$\frac{v_{02}}{v_I} = -1$$



12.46



For $v_I < 0$ – Diode is on, and cathode is forced to $\approx$ 0 V.

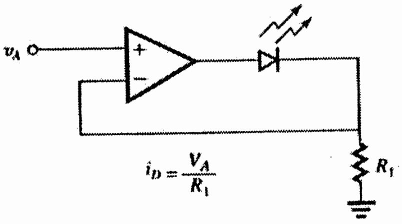
$\therefore (v_O / v_I) = -1$

For $v_I > 0$ – Diode is off, and the cathode now follows v_I since no current flows in resistor. So v_O must follow v_I so that no current flows in feedback resistor.

$$\therefore \frac{v_O}{v_I} = +1$$

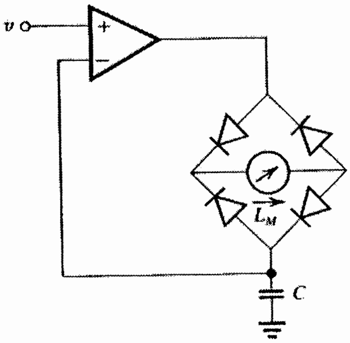
12.47

Simply place the LED in the feedback path.



$$i_D = \frac{V_A}{R_1}$$

12.48



$i_m = |i_c| = c\frac{dv}{dt}$ using $R = 1\text{ k}\Omega$

$i_m = |i_R| = \frac{|v|}{R} = \frac{|v|}{1\text{ k}\Omega} \Rightarrow i_m = |v|$ mA

Now $v = V \sin 2\pi 60t$

$\Rightarrow i_m = C \times 2\pi 60 |\cos(2\pi 60t)|$

for equivalence:

$$\frac{V}{10^3}|\sin 2\pi 60t| = 2\pi 60 VC|\cos 2\pi 60t|$$

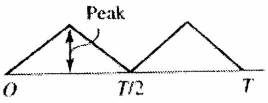
$$\therefore C = \frac{1}{2\pi 60 10^3} = 2.65\ \mu\text{F}$$

At 120 Hz: $i_m = 2\pi 120 VC|\cos 2\pi 60t|$

$$i_{m120} = 2i_{m60}$$

At 180 Hz: $i_{m180} = 3i_{m60}$

For Δ -wave



with R.

$i_m = 1$ mA, $R = 1$ kΩ

∴ Full wave rectified wave has average voltage = N.

$\therefore V_{\text{peak}} = 2$ V

with C:

$$\text{slope} = \frac{V_{\text{peak}}}{T/4} = 4V_{\text{peak}}f$$

$= 4 \times 2 \times 60 = 480$

Now: current through the capacitor will be a square wave (50% duty cycle)

Peak current $= 2.65 \times 10^{-6} \times 480$

$= 1.27$ mA .

$\therefore i_m = i_{\text{avg}} = 1.27$ mA

12.49

10 V pulses of 10 μs , and large C_{LARGE}, will cause the op amp to current limit.

Charge transfered in one pulse:

$Q = (10\text{ mA})(10\ \mu\text{S})$

$= 10^{-7}$ C

Voltage change per pulse:

$$\Delta V = Q/C = \frac{10^{-7}}{10 \times 10^{-6}} = 10\text{ mV}$$

after: 1 pulse	V_c = 10 mV
2 pulses	20 mV
10 pulses	100 mV
to reach 0.5 V	require 50 pulses
1.0 V	100 pulses
2.0 V	200 pulses

12.50

For $1V_{pp}$, peak detector output $V_O = 0.5$ V .

Ripple voltage = (1%) 0.5 = 5 mV

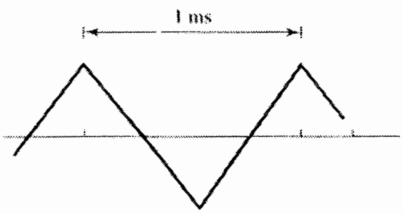
Total leakage = 10 + 1 = 11 nA

∴ total charge lost:

$\Delta Q = 11\text{ nA} \times 1\text{ ms} = 11$ PC

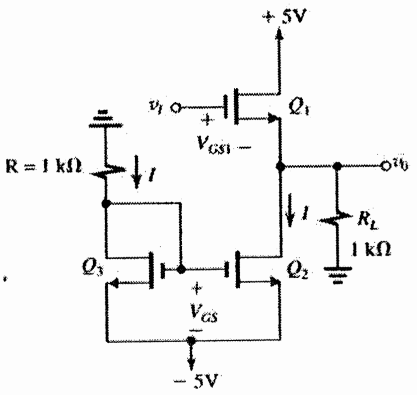
∴ Required capacitance:

$$C = \frac{Q}{\Delta V} = \frac{11 \times 10^{-12}}{5 \times 10^{-3}} = 2.2\text{ nF}$$



13.1

First we determine the bias current I as follows:



$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_t)^2$$

But $V_{GS} = 5 - IR$

$= 5 - I$

Thus

$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(5 - I - V_t)^2$$

$I = 10(5 - I - 1)^2$

$\Rightarrow I^2 - 8.1I + 16 = 0$

$I = 3.416\text{ mA}$ and $V_{GS} = 1.584\text{ V}$

The upper limit on v_O is determined by Q_1 leaving the saturation region (and entering the triode region). This occurs when v_I exceeds V_{DI} by V_t volts,

$v_{I\max} = 5 + 1 = +6\text{ V}$

To obtain the corresponding value of v_o we must find the corresponding value of V_{GS1}, as follows:

$v_o = v_I - V_{VSI}$

$$i_L = \frac{v_O}{R_L} = \frac{v_I - V_{GSI}}{1}$$

$= v_I - V_{GS1} = 6 - V_{GS1}$

$i_1 = I + i_L$

$= 3.416 + 6 - V_{GS1}$

$= 9.416 - V_{GS1}$

But $i_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS1} - V_t)^2$

Thus, $9.416 - V_{GS1} = 10(V_{GS1} - 1)^2$

$\Rightarrow V_{GS1}^2 - 1.9\,V_{GS1} + 0.0584 = 0$

$V_{GS1} = 1.869\text{ V}$

$v_{O\max} = 6 - 1.869$

$= +4.131\text{ V}$

The lower limit of v_o is determined either by Q_1 cutting off,

$v_o = -IR_L = -3.416 \times 1 = -3.416\text{ V}$

or by Q_2 leaving sturation,

$v_o = V_{GS} - V_t$

$= -5 + 1.584 - 1 = -4.416\text{ V}$

Thus, $v_{O\min} = -3.416\text{ V}$

The corresponding value of v_I is determined by moving that since Q_1 is on the verge of cut-off,

$V_{GS1} = V_t = 1\text{ V}$ and

$v_I = -3.416 + 1 = -2.416\text{ V}$

13.2

For a load resistance of 100 Ω and v_o ranging between −5 V and +5 V, the maximum current through Q_1 is

$I + \frac{5}{0.1} = I + 50$, mA and the minimum current is $I - \frac{5}{0.1} = I - 50$, mA.

For a current ratio of 10,

$$\frac{I + 50}{I - 50} = 10$$

$\Rightarrow I = 61.1\text{ mA}$

$$R = \frac{9.3\text{ V}}{61.1\text{ mA}} = 152\ \Omega$$

The incremental voltage gain is $A_v = \frac{R_L}{R_L + r_{e1}}$

For $R_L = 100\ \Omega$;

At $v_o = +5\ \text{V}$, $I_{E1} = 61.1 + 50 = 111.1\ \text{mA}$

$$r_{e1} = \frac{25}{111.1} = 0.225\ \Omega$$

$$A_v = \frac{100}{100 + 0.225} = 0.998\ \text{V/V}$$

At $v_o = 0\ \text{V}$, $I_{E1} = 61.1\ \text{mA}$

$$r_{e1} = \frac{25}{61.1} = 0.409\ \Omega$$

$$A_v = \frac{100}{100.409} = 0.996\ \text{V/V}$$

At $v_o = -5\ \text{V}$, $I_{E1} = 61.1 - 50 = 11.1\ \text{mA}$

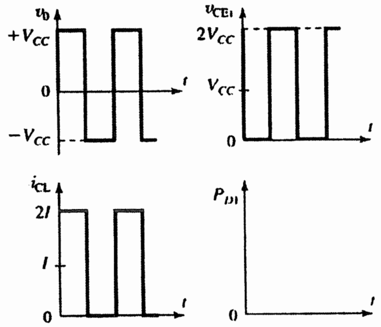
$$r_{e1} = \frac{25}{11.1} = 2.25\ \Omega$$

$$A_v = \frac{100}{102.25} = 0.978\ \text{V/V}$$

Thus the incremental gain changes by $0.998 - 0.978 = 0.02$ or about 2% over the range of v_o.

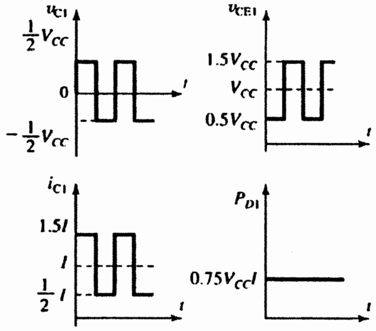
13.3

For v_o being a square wave of $\pm V_{CC}$ levels:



$P_{D1}|_{\text{average}} = 0$ For the corresponding sine wave curve $P_{D1}|_{\text{avg}} = \frac{1}{2} V_{CC} I$

For v_o a square wave of $\pm V_{CC}/Z$ levels :



$$P_{D1}|_{\text{average}} = 0.75 V_{CC} I$$

For a sine-wave output of $V_{CC}/2$ peak amplitude,

$$v_{C1} = \frac{1}{2} V_{CC} \sin\theta$$

$$i_{C1} = I + \frac{\frac{1}{2}V_{CC}}{R_L}\sin\theta = I + \frac{1}{2} I \sin\theta$$

$$v_{CE1} = V_{CC} - \frac{1}{2} V_{CC} \sin\theta$$

$$P_{D1} = \left(V_{CC} - \frac{1}{2}V_{CC}\sin\theta\right)\left(I + \frac{1}{2}I\sin\theta\right)$$

$$= V_{CC}I - \frac{1}{4}V_{CC}I\sin^2\theta$$

$$= V_{CC} - \frac{1}{4}V_{CC}I \times \frac{1}{2}(1 - \cos 2\theta)$$

$$= \frac{7}{8} V_{CC}I + \frac{1}{8} V_{CC}I \cos 2\theta$$

$$P_{D1}|_{\text{average}} = \frac{7}{8} V_{CC} I$$

13.4

In all cases, the average voltage across Q_2 is equal to V_{CC}. Thus, since Q_2 conducts a constant current I, its angle power dissipation is $V_{CC} \cdot I$.

13.5

$V_{CC} = 16, 12, 10$ and $8\ \text{V}$
$I = 100\ \text{mA}$ $R_L = 100\ \Omega$

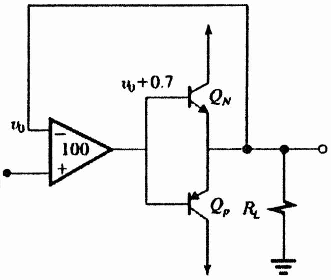
$\hat{V}_o = 8\ \text{V}$

$$\eta = \frac{1}{4}\left(\frac{\hat{V}_o}{IR_L}\right)\left(\frac{\hat{V}_o}{V_{CC}}\right)$$

$$= \frac{1}{4}\left(\frac{8}{10}\right)\left(\frac{8}{V_{CC}}\right) = \frac{1.6}{V_{CC}}$$

V_{CC}	16	12	10	8
η	10%	13.3%	16%	20%

13.6



With v_I sufficiently positive so that Q_N is conducting the situation shown obtains. Then,

$$(v_I - v_o) \times 100 = v_o + 0.7$$

$$\Rightarrow v_o = \frac{1}{1.01}(v_I - 0.007)$$

This relationship applies for $v_I \geq 0.007$. Similarly, for v_I sufficiently negative so that Q_P conducts, the voltage at the output of the amplifier becomes $v_o - 0.7$,

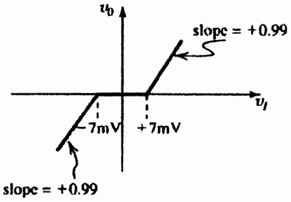
thus

$$(v_I - v_o) \times 100 = v_o - 0.7$$

$$\Rightarrow v_o = \frac{1}{1.01}(v_I + 0.007)$$

This relationship applies for $v_I \geq -0.007$.

The result is the transfer characteristic



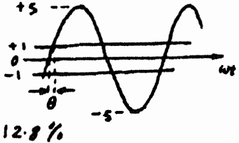
Without the feedback arrangement, the deadband becomes ±700 mV and the slope change a little (to nearly +1 V/V).

13.7

With $R_L = \infty$ and $v_I = +5$ V, v_O will be $v_I - V_{GSt} = v_I - V_t = 4$ V (since the current is virtually zero and thus $V_{GS} \simeq V_t$). Thus the resulting peak output voltage will be $\underline{\underline{4\text{ V}}}$.

$\sin\theta = \frac{1}{5}$

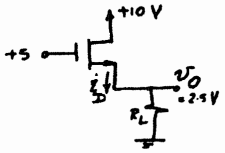
$\Rightarrow \theta = 11.54°$



Cross-over interval $= 4\theta$

Fraction of cycle $= \frac{4\theta}{360°} = \underline{12.8\%}$

For $v_I = +5$ V and $v_O = +2.5$ V, $V_{GS} = 2.5$ V, then



$i_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(v_{GS} - V_t)^2$

$= 0.1\,(2.5-1)^2 = 0.225$ mA

Thus, $R_L = \frac{2.5}{0.225} = \underline{11.1\text{ k}\Omega}$

13.8

For $V_{CC} = 10$ V and $R_L = 100\ \Omega$, the maximum sine-wave output power occurs when $\hat{V}_o = V_{CC}$ and is $P_{L\max} = \frac{1}{2}\frac{V_{CC}^2}{R_L}$

$$= \frac{1}{2} \times \frac{100}{100} = 0.5 \text{ W}$$

Correspondingly,

$$P_{S+} = P_{S-} = \frac{1}{\pi}\frac{\hat{V}_o}{R_L}V_{CC}$$

$$= \frac{1}{\pi} \times \frac{10}{100} \times 10 = 0.318 \text{ W}$$

For a total supply power of

$P_s = 2 \times 0.318 = 0.637$W

The power conversion efficency η is

$$\eta = \frac{P_L}{P_S} \times 100 = \frac{0.5}{0.637} \times 100 = 78.5\%$$

For $\hat{V}_o = 5$ V,

$$P_L = \frac{1}{2}\frac{\hat{V}_o^2}{R_L} = \frac{1}{2} \times \frac{25}{100} = \frac{1}{8} \text{ W}$$

$$P_{S+} = P_{S-} = \frac{1}{\pi}\frac{\hat{V}_o}{R_L}V_{CC}$$

$$= \frac{1}{\pi} \times \frac{5}{100} \times 10 = \frac{1}{2\pi}$$

$$P_S = \frac{1}{\pi} \text{ W} = 0.318 \text{ W}$$

$$\eta = \frac{1/8}{1/\pi} \times 100 = \frac{\pi}{8} \times 100 = 39.3\%$$

13.9

$V_{CC} = 5\text{ V}$

For maximum η,

$\hat{V}_o = V_{CC} = 5\text{ V}$

The output voltage that results in maximum device dissipation is given by Eq. (12.20),

$$\hat{V}_o = \frac{2}{\pi}V_{CC}$$

$$= \frac{2}{\pi} \times 5 = 3.18\text{ V}$$

If operation is always at full output voltage, $\eta = 78.5\%$ and thus

$$P_{\text{dissipation}} = (1-\eta)P_s$$

$$= (1-\eta)\frac{P_L}{\eta} = \frac{1-0.785}{0.785}P_L = 0.274P_L$$

$$P_{\text{dissipation/device}} = \frac{1}{2} \times 0.274P_L = 0.137P_L$$

For a rated device dissipation of 1 W, and using a factor of 2 safety margin,

$$P_{\text{dissipation/device}} = 0.5\text{ W}$$

$$= 0.137P_L$$

$$\Rightarrow P_L = 3.65\text{ W}$$

$$3.65 = \frac{1}{2} \times \frac{25}{R_L}$$

$$\Rightarrow R_L = 3.425\ \Omega \text{ (i.e. } R_L \geq 3.425\ \Omega\text{)}$$

The corresponding output power (i.e., greatest possible output power) is 3.65W.

If operation is allowed at $\hat{V}_o = \frac{1}{2}V_{CC} = 2.5\text{ V}$,

$$\eta = \frac{\pi}{4}\frac{\hat{V}_o}{V_{CC}} \text{ (Eq. 12.15)}$$

$$= \frac{\pi}{4} \times \frac{1}{2} = 0.393$$

$$P_{\text{dissipation/device}} = \frac{1}{2}\,\frac{1-\eta}{\eta}P_L = 0.772P_L$$

$$0.5 = 0.772P_L$$

$$\Rightarrow P_L = 0.647\text{ W}$$

$$= \frac{1}{2}\,\frac{2.5^2}{R_L}$$

$$\Rightarrow R_L = 4.83\ \Omega \text{ (i.e., } \geq 4.83\ \Omega\text{)}$$

13.10

$$P_L = \frac{1}{2}\,\frac{\hat{V}_o^2}{R_L}$$

$$100 = \frac{1}{2}\,\frac{\hat{V}_o^2}{16}$$

$$\hat{V}_o = 56.6\text{ V}$$

$$V_{CC} = 56.6 + 4 = 60.6 \rightarrow 61\text{ V}$$

Peak current from each supply $= \frac{\hat{V}_o}{R_L} = \frac{56.6}{16}$

$$= 3.54\text{ A}$$

$$P_{s+} = P_{s-} = \frac{1}{\pi} \times 3.54 \times 61$$

Thus, $P_s = \frac{2}{\pi} \times 3.54 \times 61$

$$= 137.4\text{ W}$$

$$\eta = \frac{P_L}{P_S} = \frac{100}{137.4} = 73\%$$

Using Eq. (12.22),

$$P_{DN\max} = P_{DP\max} = \frac{V_{CC}^2}{\pi^2 R_L} = \frac{61^2}{\pi^2 \times 16}$$

$$= 23.6\text{W}$$

13.11

$$P_L = \frac{\hat{V}_o^2}{R_L}$$

$$P_{S+} = P_{S-} = \frac{1}{2}\left(\frac{\hat{V}_o}{R_L}\right)V_{SS}$$

$$P_S = \frac{\hat{V}_o}{R_L}V_{SS}$$

$$\eta = \frac{P_L}{P_S} = \frac{\hat{V}_o^2/R_L}{\hat{V}_o V_{SS} R_L} = \frac{\hat{V}_o}{V_{SS}}$$

$\eta_{\max} = 1(100\%)$, obtained for $\hat{V}_o = V_{SS}$

$$P_{L\max} = \frac{V_{SS}^2}{R_L}$$

$$P_{\text{dissipation}} = P_S - P_L$$

$$= \frac{\hat{V}_o}{R_L}V_{SS} - \frac{\hat{V}_o^2}{R_L}$$

$$\frac{\partial P_{\text{dissipation}}}{\partial \hat{V}_o} = \frac{V_{SS}}{R_L} - \frac{2\hat{V}_o}{R_L}$$

$$= 0 \text{ for } \hat{V}_o = \frac{V_{SS}}{2}$$

Correspondingly, $\eta = \frac{V_{SS}/2}{V_{SS}} = \frac{1}{2}$ or 50%

13.12

$$A_v = \frac{R_L}{R_L + R_{out}}$$

where, $R_{out} = \frac{r_e}{2} = \frac{V_T}{2 I_Q}$.

For $A_v \geqslant 0.99$ with $R_L \geqslant 100\ \Omega$,

$$0.99 = \frac{100}{100 + R_{out}} \Rightarrow R_{out} = 1\ \Omega$$

$$\frac{V_T}{2 I_Q} = 1 \Rightarrow I_Q = \underline{\underline{12.5\ mA}}$$

$$V_{BB} = 2 V_{BE}$$

$$= 2 \left[0.7 + V_T \ln \frac{12.5}{100} \right]$$

$$= \underline{\underline{1.296\ V}}$$

This table is for 13.13

v_o(V)	i_L (mA)	i_N (mA)	i_P (mA)	V_{BEN} (V)	V_{EBP} (V)	V_I (V)	V/V	R_{out} (Ω)	V/V	i_I	R_{in} (Ω)
+10.0	100	100.04	0.04	0.691	0.495	10.1	0.99	0.25	1.00	2	5050
+5.0	50	50.08	0.08	0.673	0.513	5.08	0.98	0.50	1.00	1	5080
+1.0	10	10.39	0.39	0.634	0.552	1.041	0.96	2.32	0.98	0.2	5205
+0.5	5	5.70	0.70	0.619	0.567	0.526	0.95	4.03	0.96	0.1	5260
+0.2	2	3.24	1.24	0.605	0.581	0.212	0.94	5.58	0.95	0.04	5300
+0.1	1	2.56	1.56	0.599	0.587	0.106	0.94	6.07	0.94	0.02	5300
0	0	2	2	0.593	0.593	0	–	6.25	0.94		
−0.1	−1	1.56	2.56	0.587	0.599	−0.106	0.94	6.07	0.94	−0.02	5300
−0.2	−2	1.24	3.24	0.581	0.605	−0.212	0.94	5.58	0.95	−0.04	5300
−0.5	−5	0.70	5.70	0.567	0.619	−0.526	0.95	4.03	0.96	−0.1	5260
−1.0	−10	0.39	10.39	0.552	0.634	−1.041	0.96	2.32	0.98	−0.2	5205
−5.0	−50	0.08	50.08	0.513	0.673	−5.08	0.98	0.50	1.00	−1	5080
−10.0	−100	0.04	100.04	0.495	0.691	−10.1	0.99	0.25	1.00	−2	5050

$$I_Q = \frac{V_T}{4} = \frac{25 \times 10^{-3}}{4} = 6.25 \text{ mA}$$

$$V_{BB} = 2V_{BE} = 2\left[0.7 + V_T \ln\left(\frac{6.25}{100}\right)\right]$$

$$= 1.26 \text{ V}$$

13.13

The current i_I can be obtained as

$$i_I = \frac{i_N}{\beta_N + 1} - \frac{i_P}{\beta_P + 1} = \frac{i_L}{\beta + 1}$$

$\therefore \beta_N = \beta_P = \beta = 49$

Using values of v_I from the table one can evaluate R_{in}

$$\therefore R_{in} = \frac{v_I}{i_I}$$

Using resistance reflection rule

$R_{in} \cong \beta R_L = 49 \times 100$

$= 4900\ \Omega$

For large input signal the two values of R_{in} are somewhat same. For the small values of v_I, the calculated value in the table is larger.

13.14

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_{out}} \text{ and}$$

$$R_{out} = \frac{V_T}{i_P + i_N} = \frac{V_T}{I_Q + I_Q} \text{ at } v_o = 0$$

$$\text{a. } \epsilon = 1 - \left.\frac{v_o}{v_I}\right|_{v_o = 0}$$

$$= 1 - \frac{R_L}{R_L + R_{out}} = 1 - \frac{R_L}{R_L + \dfrac{V_T}{2I_Q}} = \frac{V_T/2I_Q}{R_L + (V_T/2I_Q)}$$

$$\epsilon = \frac{V_T/2I_Q}{R_L + (V_T/2I_Q)} = \frac{V_T}{2R_LI_Q + V_T}$$

If $2I_QR_L >> V_T$

$$\epsilon = \frac{V_T}{2I_QR_L}$$

b. Quiescent Power Dissipation = $2V_{CC}I_Q = P_D$

c. ϵ × Quiescent Power Dissipation =

$$\frac{V_T}{2I_QR_L} \times 2V_{CC}I_Q = V_T \times \left(\frac{V_{CC}}{R_L}\right)$$

$$\therefore \epsilon P_D = V_T\left(\frac{V_{CC}}{R_L}\right)$$

$$\text{d. } \epsilon P_D = V_T\frac{V_{CC}}{R_L} = 25 \times 10^{-3} \times \frac{15}{100}$$

$= 3.75$ mW

$$P_D = \frac{3.75 \times 10^{-3}}{\epsilon}$$

ϵ	P_D in mW
0.05	75
0.02	187.5
0.01	375

13.15

$I_Q \simeq I_{bias} = 0.5$ mA, neglecting the base current of Q_N. More precisely,

$$I_Q = I_{\text{bias}} - \frac{I_Q}{\beta + 1}$$

$$\Rightarrow I_Q = \frac{I_{\text{bias}}}{1 + \frac{1}{\beta + 1}} \simeq 0.98 \times 0.5 = 0.49 \text{ mA}$$

The largest positive output is obtained when all of I_{bais} flows into the base of Q_N, resulting in

$$v_o = (\beta_N + 1) I_{\text{bias}} R_L$$

$$= 51 \times 0.5 \times 100\ \Omega = 2.55 \text{ V}$$

The largest possible negative output voltage is limited by the saturation of

Q_P to $-10 + V_{\text{ECSat}} = -10$ V

To achieve a maximum positive output of 10 V without changing I_{bias}, β_N must be

$$10 = (\beta_N + 1) \times 0.5 \times 100\ \Omega$$

$$\Rightarrow \beta_N = 199$$

Alternatively, if β_N is held at 50, I_{bias} must be increased so that

$$10 = 51 \times I_{\text{bias}} \times 100\Omega$$

$$\Rightarrow I_{\text{bias}} = 1.96 \text{mA}$$

for which,

$$I_Q = \frac{I_{\text{bias}}}{1 + \frac{1}{\beta + 1}} = 1.92 \text{ mA}$$

13.16

At 20°C, $I_Q = 1\text{mA} = I_S\, e^{(0.6/0.025)}$

$$\Rightarrow I_S \text{ (at 20°C)} = 3.78 \times 10^{-11} \text{ mA}$$

At 70°C, $I_S = 3.78 \times 10^{-11} (1.14)^{50}$

$$= 2.64 \times 10^{-8} \text{ mA}$$

At 70°C, $V_T = 25 \frac{273 + 70}{273 + 20} = 29.3 \text{ mV}$

Thus, I_Q (at 70°C) $= 2.64 \times 10^{-8}\, e^{0.6/0.0293}$

$$= 20.7 \text{ mA}$$

Additional current $= 20.7 - 1 = 19.7$ mA

Additional power $= 2 \times 20 \times 19.7 = 788$mW

Additional temperature rise $= 10 \times 0.788 = 7.9$°C,

At 77.9°C:

$$V_T = \frac{25}{293}(273 + 77.9) = 29.9 \text{ mV}$$

$$I_Q = 3.78 \times 10^{-11} \times (1.14)^{57.9}\, e^{(0.6/0.0299)}$$

$$= 37.6 \text{ mA}$$

etc., etc.

13.17

Since the peak positive output current is 200 mA, the base current of Q_N can be as high as

$\frac{200}{\beta_N + 1} = \frac{200}{51} \simeq 4$ mA. We select

$I_{bias} = 5$ mA, thus providing the multiplier with a minimum current of 1 mA.

Under quiescent conditions ($v_o = 0$ and $i_L = 0$) the base current of Q_N can be neglected.

Selecting $I_R = 0.5$ mA leaves $I_{C1} = 4.5$ mA. To obtain a quiescent current of 2 mA in the output transistors, V_{BB} should be

$$V_{BB} = 2V_T \ln \frac{2 \times 10^{-3}}{10^{-15}} = 1.19 \text{ V}$$

Thus

$$R_1 + R_2 = \frac{V_{BB}}{I_R} = \frac{1.19}{0.5} = 2.38 \text{ k}\Omega$$

At a collector current of 4.5 mA, Q_1 has

$$V_{BE1} = V_T \ln \frac{4.5 \times 10^{-3}}{10^{-14}} = 0.671 \text{ V}$$

The value of R_1 can now be determined as

$$R_1 = \frac{0.671}{0.5} = 1.34 \text{ k}\Omega \text{ and}$$

$$R_2 = 2.58 - 1.34 = 1.04 \text{ k}\Omega$$

13.18

(a) $V_{BE} = 0.7$ V at 1 mA

At 0.5 mA,

$$V_{BE} = 0.7 + 0.025 \ln \frac{0.5}{1} = 0.683 \text{ V}$$

Thus $R_1 = \frac{0.683}{0.5} = 1.365\ \text{k}\Omega$

and $R_2 = 1.365\ \text{k}\Omega$

(b) For $I_{\text{bias}} = 2$ mA, I_C increases to nearly 1.5 mA for which

$V_{BE} = 0.7 + 0.025\ \ln\frac{1.3}{1} = 0.710$ V

Note that $I_R = \frac{0.710}{1.365} = 0.52$ mA is very nearly equal to the assumed value of 0.50 mA. Thus no further iterations are required.

$V_{BB} = 2V_{BE} = 1.420$ V

(c) For $I_{\text{bias}} = 10$ mA, assume that I_R remains constant at 0.5 mA, thus $I_{C1} = 9.5$ mA

and $V_{BE} = 0.7 + 0.025\ \ln\frac{9.5}{1} = 0.756$ V

at which

$I_R = \frac{0.755}{1.365} = 0.554$ mA

Thus,

$I_{C1} = 10 - 0.554 = 9.45$ mA

and $V_{BE} = 0.7 + 0.025\ \ln\frac{9.45}{1} = 0.756$ V

Thus $V_{BB} = 2 \times 0.756 = 1.512$ V

(d) Now for $\beta = 100$,

$I_{R1} = \frac{0.756}{1.365} = 0.554$ mA

$I_{R2} = 0.554 + \frac{9.45}{101} = 0.648$ mA

$I_C = 10 - 0.648 = 9.352$ mA

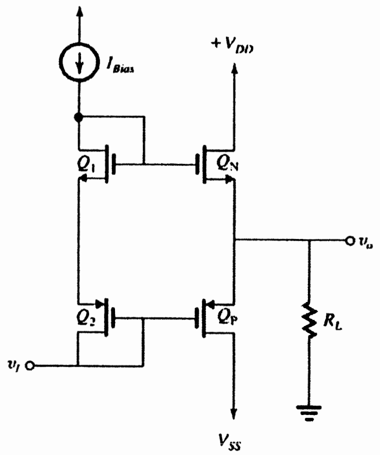
Thus, $V_{BE} = 0.7 + 0.025\ \ln\frac{9.352}{1} = 0.756$ V

$V_{BB} = 0.756 + I_{R2} R_2$

$= 0.756 + 0.648 \times 1.365$

$= 1.641$ V

13.19



a. under quiescent condition

Voltage gain $= \frac{v_O}{v_i} = \frac{R_L}{R_L + R_{\text{out}}}$

As shown in problem 11.24, for matched transistors

$R_{\text{out}} = \frac{1}{2g_m}$

Substitute for R_{out} above for $\frac{v_O}{v_i}$

$$\frac{v_O}{v_i} = \frac{R_L}{R_L + \frac{1}{2g_m}}$$

b. Voltage gain $= 0.98 = \frac{R_L}{R_L + \frac{1}{2g_m}}$

$$0.98 = \frac{1000}{1000 + \frac{1}{2g_m}}$$

$\Rightarrow g_m = 24.5$ mA/V

For Q_1, $I_{\text{Bias}} = I_D$

$\therefore 0.1 = \frac{1}{2}k_1 V_{OV}^2$

$$0.1 = \frac{1}{2} \times 20 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.1 \text{ V}$$

For Q_N

$$g_m = k_n V_{OV}$$

$$24.5 = k_n \times 0.1$$

$$k_n = 245 \text{ mA/V}^2$$

$$n = \frac{k_n}{k_1} = \frac{245}{20}$$

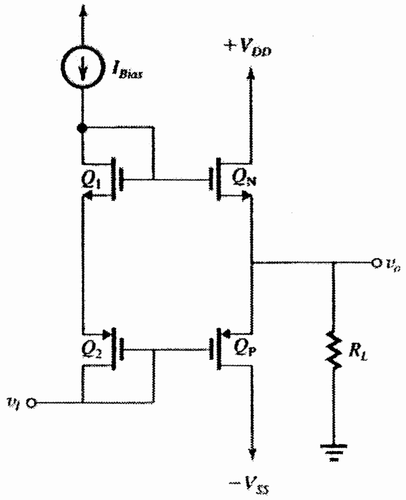
$$= 12.25$$

and $I_Q = nI_{\text{bias}}$

$$= 12.25 \times 0.1$$

$$= 1.225 \text{ mA}$$

13.20



$$I_Q = I_{\text{Bias}} \frac{(W/L)_n}{(W/L)_1}$$

$$1 = 0.1 \frac{(W/L)_n}{(W/L)_1}$$

$$\frac{(W/L)_n}{(W/L)_1} = 10$$

$$Q_1\text{: } I_{\text{Bias}} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 V_{ov}^2$$

$$0.1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_1 \times (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 20$$

$$Q_2\text{: } 0.1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_2 \times (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 50$$

$$Q_N\text{: } 1 = \frac{1}{2} \times 0.250 \times \left(\frac{W}{L}\right)_N \times (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_N = 200$$

$$Q_P\text{: } 1 = \frac{1}{2} \times 0.100 \times \left(\frac{W}{L}\right)_P \times (0.2)^2$$

$$\left(\frac{W}{L}\right)_P = 500$$

b. From the circuit we get $-v_I + V_{GSP} + v_o = 0$

Since $v_o = 0$

$v_I = V_{GSP}$

$$V_{SGP} = |V_{ov}| + |V_t|$$

$$= 0.2 + 0.45$$

$$= -0.65 \text{ V}$$

$$\therefore v_I = V_{GSP} = -0.65 \text{ V}$$

c. Using equation 11.4

$$v_{o\max} = V_{DD} - V_{ov}|_{\text{Bias}} - V_{\text{GSN}}$$

To find V_{GSN},

$$i_{\text{DNmax}} = \frac{1}{2} k_n' \frac{W}{L} (V_{\text{GSN}} - V_t)^2$$

$$10 = \frac{1}{2} \times 0.250 \times 200\ (V_{\text{GSN}} - V_t)^2$$

$$\Rightarrow V_{\text{GSN}} - V_t = 0.63 \text{ V}$$

$$V_{\text{GSN}} = V_t + 0.63 = 0.45 + 0.63 \simeq 1.1 \text{ V}$$

$$\therefore v_{o\max} = 2.5 - 0.2 - 1.1 = 1.2 \text{ V}$$

13.21

$I_Q = 3$ mA, $|V_{ov}| = 0.15$ V

$$g_{mn} = g_{mp} = \frac{2I_D}{V_{ov}} = \frac{2 \times 3}{0.15} = 0.04 \text{ A/V}$$

$$= 40 \text{ mA/V}$$

Using equation 11.57

$$R_{\text{out}} = \frac{1}{\mu(g_{mp} + g_{mn})} = \frac{1}{5(0.04 + 0.04)}$$

$$= 2.5\ \Omega$$

13.22

$$|\text{Gain Error}| = \frac{1}{2\mu g_m R_L}$$

From equation 11.57

$$R_{out} = \frac{1}{\mu(g_{mp} + g_{mp})}$$

$$= \frac{1}{2\mu g_m} \text{ when } g_{mp} = g_{mn} = g_m$$

$$\therefore |\text{Gain Error}| = \frac{1}{2\mu g_m} \times \frac{1}{R_L}$$

$$= \frac{R_{\text{out}}}{R_L}$$

$$|\text{Gain Error}| = \frac{1}{2\mu g_m R_L}$$

$$0.05 = \frac{1}{2 \times 10 \times g_m \times 100}$$

$g_m = 0.01$ A/V $= 10$ mA/V

$$g_m = \frac{2I_Q}{V_{ov}}$$

$$V_{ov} = \frac{2I_Q}{g_m} = \frac{2 \times 1}{10}$$

$V_{ov} = 0.2$ V

13.23

a. $I_Q = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2$

$$1.5 \times 10^{-3} = \frac{1}{2} \times 100 \times 10^{-6} \left(\frac{W}{L}\right)_p (0.15)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_p \simeq 1333.3$$

$$\left(\frac{W}{L}\right)_N = \frac{(W/L)_P}{(k_n'/k_{Q_p}')} = \frac{1333.3}{(250/100)}$$

$= 533.3$

b. $g_m = \frac{2I_Q}{V_{ov}} = \frac{2 \times 1.5}{0.15} = 20$ mA/V

$= 0.02$ A/V

$$R_{\text{out}} = \frac{1}{2\mu g_m} \quad g_{mp} = g_{mn} = g_m$$

$$2.5 = \frac{1}{2\mu \times 0.02}$$

$$\mu = \frac{1}{2.5 \times 2 \times 0.02}$$

$\mu = 10$

c. Gain Error $= -\frac{V_{ov}}{4\mu I_Q R_L}$

$$= -\frac{0.15}{4 \times 10 \times 1.5 \times 10^{-3} \times 50}$$

$= -0.05$

Gain Error $= 5\%$

d. In the quiescent state $v_o = 0$

The voltage at the output of each amplifier will be $= \mu(v_o - v_I) = -\mu v_I$

e. Q_N turn off when the voltage at its gate drops from quiescent value of -1.85 V to -2V, at which point $V_{GSN} = V_{tN}$ and an equal change of -0.15 V appear at the output of the top amplifier.

$$i_p = \frac{1}{2} k_p \left(\frac{W}{L}\right)_p (0.3)^2$$

$$= \frac{1}{2} \times 0.100 \times 1333.3 \times 0.3^2$$

$i_p = 6$ mA

$v_o = 6 \times 10^{-3} \times 50\,\Omega = 0.3$ V

So for $v_o > 0.3$ V, Q_P conducts all the current.

f. the situation at $v_o = v_{omax}$ will occur when Q_P will go from saturation to triode region and it will be approximatly 2 V.

Linear range of v_o from 2 to -2 V

13.24

$$\theta_{JA} = \frac{150 - 25}{0.2} = 625^\circ\text{C/W} = 0.625^\circ\text{C/mW}$$

At 70°C, Power rating

$$= \frac{150 - 70}{0.625} = 128 \text{ mW}$$

$$T_J = 50 + 0.625 \times 100 = 112.5^\circ\text{C}$$

13.25

(a) $\theta_{JA} = \frac{T_{Jmax} - T_{AO}}{P_{DO}}$

$$= \frac{100 - 25}{2} = 37.5^\circ \text{ C/W}$$

(b) At $T_A = 50^\circ$C

$$P_{Dmax} = \frac{T_{Jmax} - T_A}{\theta_{JA}}$$

$$= \frac{100 - 50}{37.5} = 1.33 \text{ W}$$

(c) $T_J = 25^\circ + 37.5 \times 1 = 62.5^\circ$C

13.26

$$T_C - T_A = \theta_{CA} P_D$$

$$= (\theta_{CS} + \theta_{SA}) P_D$$

$$\Rightarrow P_D = \frac{T_C - T_A}{\theta_{CS} + \theta_{SA}} = \frac{90 - 30}{0.5 + 0.1} = 100 \text{ W}$$

$$T_J - T_C = \theta_{JC} P_D$$

$$130 - 90 = \theta_{JC} \times 100$$

$$\Rightarrow \theta_{JC} = 0.4^\circ \text{ C/W}$$

13.27

$$\theta_{JC} = \frac{T_J - T_C}{P_D} = \frac{180^\circ - 50^\circ}{50} = 2.6^\circ \text{ C/W}$$

$$T_J - T_S = \theta_{JS} P_D$$

$$180^\circ - T_S = (\theta_{JC} + \theta_{CS}) P_D$$

$$\Rightarrow T_S = 180 - (2.6 + 0.6) \times 30 = 84^\circ$$

$$T_S - T_A = \theta_{SA} P_D$$

$$84 - 39 = \theta_{SA} \times 30$$

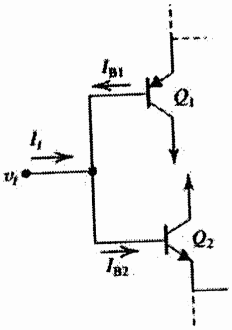
$$\Rightarrow \theta_{SA} = 1.5^\circ\text{C/W}$$

Required heat-sink length $= \frac{4.5^\circ\text{C/W/Cm}}{1.5^\circ\text{C/W}}$

$= 3$ cm

13.28

(a) For $R_L = \infty$:



At $v_I = 0$ V,

$$I_{B1} = I_{B2} = \frac{2.87}{200}$$

$$I_I = I_{B2} - I_{B1} = 0$$

At $v_I = +10$ V,

$$I_{B1} = \frac{0.88}{200}\ \text{mA} = 4.4\ \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200}\ \text{mA} = 24.4\ \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = 20\ \mu\text{A}$$

At $v_I = -10$ V,

$$I_{B1} = \frac{4.87}{200}\ \text{mA} = 24.4\ \mu\text{A}$$

$$I_{B2} = \frac{0.88}{200}\ \text{mA} = 4.4\ \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = -20\ \mu\text{A}$$

(b) For $R_L = 100\ \Omega$:

At $v_I = 0$ V, $I_I = 0$

At $v_I = +10$ V,

$$I_{B1} = \frac{0.38}{200} = 1.9\ \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200} = 24.4\ \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = 22.5\ \mu\text{A}$$

At $v_I = -10$ V, $I_I = -22.5\ \mu$A

13.29

Circuit operating near $v_I = 0$ and is fed with a signal source having zero resistance.

The resistance looking as shown by the arrow X is

$= R_1 \parallel r_{e1}$

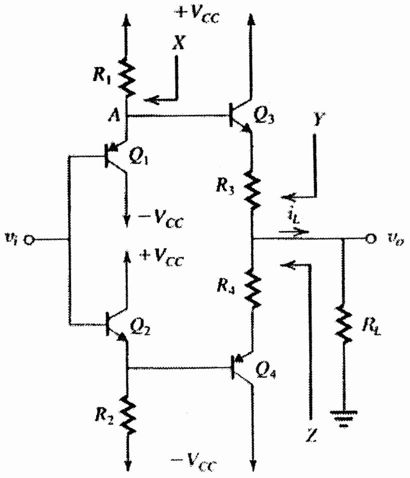
This resistance is reflected from base to the emitter of Q$_3$ is $= (\beta_3 + 1)/(R_1 \parallel r_{e1})$

This resistance seen as shown by arrow Y$_1$, from the upper half of the circuit =

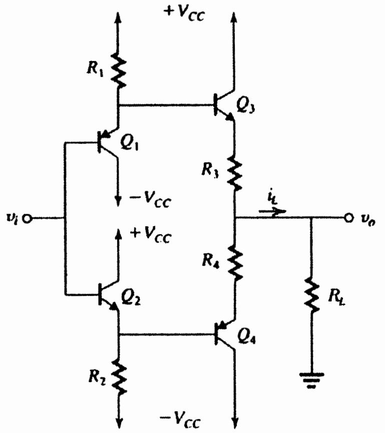
$R_3 + r_{e3} + (\beta_3 + 1)/(R_1 \parallel r_{e1})$

A similar resistance is as shown by the arrow Z and both of these resistances (seen arrow Y and arrow Z) are parallal, therefore

$$R_{\text{out}} = \frac{1}{2}[R_3 + r_{e3} + (R_1 \parallel r_{e1})/(\beta_3 + 1)]$$



13.30



At $v_I = 5$V, the voltage V_{R1} across the resistor R_1 is

$v_{R1} = V_{CC} - 0.7 - 5 = 4.3$ V

$i_{R1} = 2 \times 10$ mA

The current i_{R1} should be enough to allow for i_{B1} as much as 10 mA and only a 2 to 1 variation in i_{E1}

$$\therefore R_1 = \frac{V_{R1}}{i_{R1}} = \frac{4.3}{20\text{ m}} = 215\ \Omega = 0.215\text{ k}\Omega$$

Similarly $R_2 = 215\ \Omega = 0.215$ kΩ

Now solve for R_3 and R_4

For $v_I = 0$ and $V_{EB1} \simeq 0.7$ V

$$i_{R1} = \frac{10 - 0.7 - 0}{215} = 43.3\text{ mA}$$

$$v_{EB1} = 0.7 + 25 \times 10^{-3}\ln\left(\frac{43.3}{10}\right) = 0.7366\text{ V}$$

$\simeq 0.74$ V

In Q_3, $I_Q = 40$ mA and $I_{S3} = 3I_{S1} = 30$ mA

$$v_{BE3} = 0.7 + 25 \times 10^{-3}\ln\left(\frac{40}{30}\right) = 0.7072\text{ V}$$

$$R_3 = \frac{V_{EB1} - V_{BE3}}{I_{Q3}} = \frac{0.7366 - 0.7072}{40 \times 10^{-3}} \cong 0.74\ \Omega$$

Similarly $R_4 = 0.74\ \Omega$

Output Resistance at $v_I = 0$

$$R_{out} = \frac{1}{2}\left(R_3 + r_{e3} + \frac{r_{e1} \parallel R_1}{\beta_3 + 1}\right)$$

$$\simeq \frac{1}{2}\left(R_3 + r_{e3} + \frac{r_{e1}}{\beta_3 + 1}\right) \text{ Since } r_{e1} \parallel R_1 \simeq r_{e1}$$

This $\frac{1}{2}$ is there because of two paths to output.

$$r_{e1} \simeq r_{e3} = \frac{25\text{ mV}}{40\text{ mA}} = 0.625\ \Omega$$

$$R_{out} = \frac{1}{2}\left(0.735 + 0.625 + \frac{0.625}{50 + 1}\right)$$

$\simeq 0.69\ \Omega$

Output voltage for $v_I = 1$ V and $R_L = 2\ \Omega$

Let $v_O \simeq 1$ V

$$i_L = \frac{1\text{ V}}{2\ \Omega} = 500\text{ mA}$$

$$i_{B3} = \frac{500}{50} = 10\text{ mA}$$

$$I_{E1} \simeq \frac{10 - 0.7 - 1}{0.215\text{ k}\Omega} - 10 = 28.6\text{ mA}$$

So

$$V_{EB1} = 0.7 + 25 \times 10^{-3}\ln\left(\frac{28.6}{10}\right) = 0.726\text{ V}$$

$V_{B3} = v_I + V_{EB1} = 1 + 0.726 = 1.726$ V

Assuming $i_{E4} \simeq 0$

$$V_{BE3} = 0.7 + 0.025\ln\left(\frac{500}{30}\right)$$

$\therefore i_{E3} = i_L = 500$ mA

$= 0.770$ V

$$\therefore i_L = \frac{1.726 - 0.770}{0.74 + 2} = 0.349\text{ A} \simeq 0.35\text{ A}$$

This value of i_L gives

$v_O = 2\ \Omega \times 0.349\text{ A} = 0.698$ V

The voltage drop across the series combination of R_4 and the emitter base junction of Q_4 can be determined as follows

$V_{B4} = V_{E2} = V_I - V_{BE2} = 1 - 0.74$

$= 0.26$ V

$V_{B4} = 0.26$ V, leaves a drop across V_{BE4} and R_4 of $v_O - V_{B4}$, that is $0.698 - 0.26 = 0.438$ and this will give $i_{E4} \simeq 0$ as assumed earlier.

Do one more iteration

$i_L \simeq 0.35$ A

$$i_{B3} \cong \frac{0.35}{51} \simeq 7\text{ mA}$$

$$i_{E1} = \frac{10 - 1 - 0.73}{0.215\text{ k}\Omega} - 7 = 31.5\text{ mA}$$

$$V_{EB1} = 0.7 + 0.025\ln\left(\frac{31.5}{10}\right) = 0.729\text{ V}$$

$$V_{B3} = 1 + 0.729 = 1.729\text{ V}$$

$$V_{EB3} = 0.7 + 0.025\ln\left(\frac{31.5}{10}\right) = 0.729\text{ V}$$

$= 0.761$ V

Here $i_{E3} = i_L = 0.35\text{ A} = 350$ mA

$$i_L = \frac{1.729 - 0.761}{2 + 0.74} = 0.353\text{ A}$$

$v_O = 2\ \Omega \times 0.353$ A

$= 0.706$ V

13.31

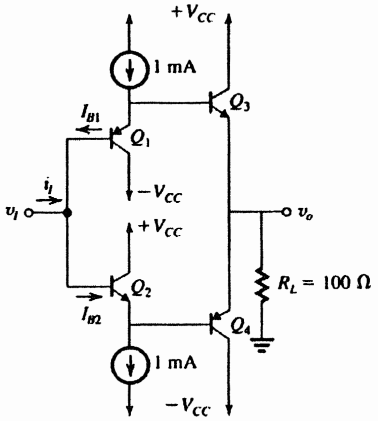
a. $v_I = 0$ and transistors have $\beta = 100$

$I_Q \cong I_{E3} = I_{E4} = I_{E1} = I_{E2} \simeq 1\ \text{mA}$

More precisely $I_Q = \frac{\beta}{\beta+1} \times 1 = 0.99\ \text{mA}$

Input bias current in zero because $I_{B1} = I_{B2}$

output voltage = 0 V



$+V_{CC}$
1 mA
Q_3
I_{B1}
Q_1
i_I
v_I
$-V_{CC}$
$+V_{CC}$
v_o
Q_2
$R_L = 100\ \Omega$
I_{B2}
Q_4
1 mA
$-V_{CC}$

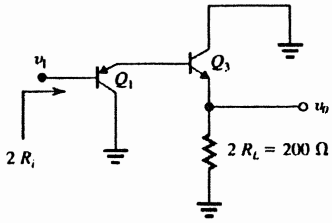
b. From the equivalent half circuit

$$2R_i = (\beta_1 + 1)[r_{e3} + (\beta_3 + 1)(r_{e3} + 2R_2)]$$

$$r_{el} = r_{e3} = \frac{V_I}{I_E} = \frac{25}{1} = 25\ \Omega$$

$$2R_i = (100 + 1)[25 + (100 + 1)(25 + 2 \times 100)]$$

$$\Rightarrow R_i = 1.15\ \text{M}\Omega$$



v_I
Q_1
Q_3
v_o
$2\ R_i$
$2\ R_L = 200\ \Omega$

$$A_v = \frac{v_o}{v_i} = \frac{2R_L}{2R_L + r_{e3} + \frac{r_{el}}{\beta_3 + 1}}$$

$$= \frac{200}{200 + 25 + \frac{25}{101}}$$

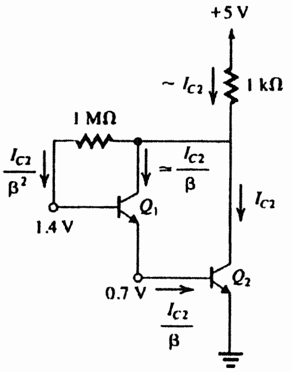
$\simeq 0.89$ V/V

$$2R_{out} = r_{e3} + \frac{r_{el}}{\beta + 1}$$

$$= 25 + \frac{25}{101}$$

$$R_{out} = 12.6\ \Omega$$

13.32



+5 V
$\sim I_{C2}$
1 kΩ
1 MΩ
$\frac{I_{C2}}{\beta^2}$
$\simeq \frac{I_{C2}}{\beta}$
Q_1
I_{C2}
1.4 V
0.7 V
$\frac{I_{C2}}{\beta}$
Q_2

a. D C Analysis

Current through 1 kΩ $\simeq I_{C2}$

$$5 = 1\text{k} \times I_{C2} + 1\text{M} \times \frac{I_{C2}}{\beta^2} + 1.4$$

$$I_{C2} = \frac{3.6}{1 + \frac{1000}{\beta^2}} \text{ in mA}$$

= 3.3 mA

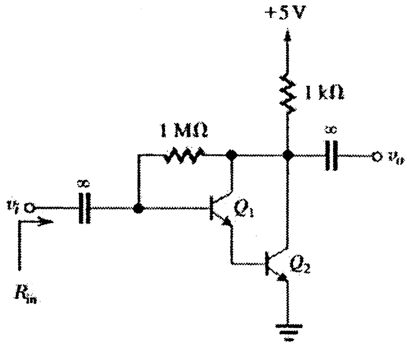
$$I_{C1} = \frac{I_{C2}}{\beta} = \frac{3.3}{100} = 0.033\ \text{mA}$$

b. $i_C = g_{m2}\, v_{be2}$

$$= g_{m2} \frac{(\beta_2 + 1)r_{e2}}{r_{e1} + (\beta_2 + 1)r_{e2}} \times v_i$$

$$\text{But } r_{e1} = \frac{V_T}{I_{E1}} = \frac{V_T}{\frac{I_{C2}}{\beta_2 + 1}} \cong r_{e2}(\beta_2 + 1)$$

$$i_C = g_{m2} \frac{r_{e2}}{\frac{r_{e1}}{\beta_2 + 1} + r_{e2}} \cdot v_i$$



$$i_C = \frac{g_{m2} r_{e2}}{r_{e2} + r_{e2}} \cdot v_i$$

But $g_{m2}\, r_{e2} \simeq 1$

$$i_C \simeq \frac{v_i}{2 r_{e2}}$$

For this circuit g_m equi. is

$$g_m \text{ equi} \simeq \frac{i_C}{v_i} = \frac{1}{2 r_{e2}} = \frac{1}{2 \times \dfrac{V_T}{I_{C2}}} = 66 \text{ mA/V}$$

Now $v_O \simeq -i_C \times 1 \text{ k}$

$$= -g_m\text{equi} \times v_i \times 1 \text{ k}$$

$$\frac{v_O}{v_i} = -66 \frac{\text{mA}}{\text{V}} \times 1 \text{ k} = -66 \text{ V/V}$$

c. $i_i = i_{b1} + i_{1\text{M}\Omega \text{ resistor}}$

$$\simeq \frac{i_C}{\beta^2} + \frac{v_i - v_o}{1 \text{ M}\Omega}$$

$$= \frac{1}{\beta^2} \times \frac{v_i}{2 r_{e2}} + \frac{v_i - (-66\, v_i)}{1 \text{ M}\Omega}$$

$$i_i = \frac{v_i}{2\beta^2 r_{e2}} + \frac{67 v_i}{1 \text{ M}\Omega}$$

$$= v_i \left[\frac{1}{2 \times 100^2 \times \dfrac{25}{3.3}} + \frac{67}{1\text{M}} \right]$$

$$= v_i\,(6.6 + 67) \times 10^{-6}$$

$$R_{in} = \frac{v_i}{i_i} = 13.6 \text{ k}\Omega$$

13.33

The quiescent current through Q_2 and Q_4 is to be 2 mA. Then

$$V_{BE2} = V_{BE4} = 0.7 + 0.025 \ln\left(\frac{2}{10}\right) = 0.660 \text{ V}$$

For Q_1 and Q_3, $I_C \simeq \frac{2}{100} = 0.02$ mA, then

$$V_{BE1} = V_{EB3} = 0.7 + 0.025 \ln \frac{0.02}{1} = 0.602 \text{ V}$$

$$I_{B1} = \frac{20\ \mu\text{A}}{100} = 0.2\ \mu\text{A}$$

$$I_{bias} = 100 \times 0.2 = 20\ \mu\text{A}$$

$$I_{R_1, R_2} = \frac{1}{10} \times 20\ \mu\text{A} = 2\ \mu\text{A}$$

$$I_{C5} = 20 - 2 = 18\ \mu\text{A}$$

$$V_{BE5} = 0.7 + 0.025 \ln \frac{0.018}{1} = 0.600 \text{ V}$$

$$V_{BB} = V_{BE1} + V_{BE2} + V_{EB3} = \underline{\underline{1.864 \text{ V}}}$$

$$R_1 + R_2 = \frac{1.864}{2\ \mu\text{A}} = 932 \text{ k}\Omega$$

$$R_1 = \frac{0.600}{2\ \mu\text{A}} = \underline{\underline{300 \text{ k}\Omega}}$$

$$R_2 = 932 - 300 = \underline{\underline{632 \text{ k}\Omega}}$$

For $v_O = -10$ V and $R_L = 1$ kΩ:

$$i_L = \frac{-10}{1} = -10 \text{ mA}$$

Assume that the current through Q_2 becomes almost zero, thus

$$I_{C4} = 10 \text{ mA}$$

i.e. the current through Q_4 increases by a factor of 5. It follows that the current through Q_3 must increase by the same factor, thus V_{EB3} becomes

$$V_{EB3} = 0.602 + 0.025 \ln 5$$

$$= 0.642 \text{ V} \quad (\text{an increase of } 0.04\text{V})$$

Let us check the current through Q_2. Since we assumed Q_1 and Q_2 to be almost cut off, all of I_{bias} now flows through the V_{BE} multiplier, an increase of 0.2 μA. Assuming that most of this increase occurs in I_{C5}, V_{BE5} becomes

$$V_{BE5} = 0.7 + 0.025 \ln \frac{0.0182}{1} = 0.600 \text{ V}$$

Thus the voltage across the V_{BE}-multiplier remains approximately constant and the voltage $(V_{BE1} + V_{BE2})$ decreases by the same value that V_{EB3} increases by. That is

$$V_{BE1} + V_{BE2} = 0.660 + 0.602 - 0.04$$

Since the current through each of Q_1 and Q_2 decreases by the same factor (call it m),

$$0.025 \ln m + 0.025 \ln m = -0.04 \text{ V}$$

$$\Rightarrow m = 0.45$$

Thus $I_{C2} = 0.45 \times 2 = 0.9$ mA

New iteration: $I_{C4} = 10.9$ mA (an increase by a factor $\simeq 5.5$).

$$V_{EB3} = 0.602 + 0.025 \ln 5.5$$
$$= 0.645 \text{ V}$$
$$v_I \simeq \underline{\underline{-10.645 \text{ V}}}$$

For $v_O = +10$ V and $R_L = 1$ kΩ:

Assume that Q_4 is now conducting a negligible current. Thus, $I_{C2} \simeq I_L = 10$ mA. i.e. the current through each of Q_1 and Q_2 increases by a factor of 5. Thus

$$V_{BE2} = 0.66 + 0.025 \ln 5$$
$$= 0.700 \text{ V}$$
$$V_{BE1} = 0.602 + 0.025 \ln 5 = 0.642 \text{ V}$$
$$I_{B1} = 5 \times 0.2 = 1\ \mu\text{A}$$

Thus the current through the multiplier becomes 19 μA, and assuming that most of the decrease occurs in I_{C5},

$$V_{BE5} = 0.7 + 0.025 \ln \frac{0.017}{1}$$
$$= 0.598 \text{ V}$$

Thus the voltage across the multiplier becomes

$$V_{BB} = 0.598 \times \frac{932}{300} = 1.858 \text{ V}$$

It follows that V_{EB3} becomes

$$V_{EB3} = 1.858 - 0.700 - 0.642 = 0.516 \text{ V}$$

i.e. V_{EB3} decreases by $0.600 - 0.516 = 0.084$ V and correspondingly I_{C3} decreases by a factor of $e^{\frac{-0.084}{0.025}} = 0.035$. ~~becoming~~ Thus I_{C4} becomes $0.035 \times 2 = 0.07$ mA, close to the zero value assumed. Thus no further iteration are required and

$$v_I \simeq 10 + 0.7 + 0.642 - 1.858$$
$$= \underline{\underline{+9.484 \text{ V}}}$$

13.34

Now Q_5 has $I_S = 10^{-13}$A. Thus,

$$2 \times 10^{-3} = 10^{-13} e^{V_{BE}/V_T}$$

$$V_{BE} = 0.025 \ln \frac{2 \times 10^{-3}}{10^{-13}}$$

$$= 0.593 \text{ V}$$

$$R_{E1} = \frac{0.593}{150 \text{ mA}} \simeq 4\Omega$$

For a normal peak current of 100 mA, the voltage drop across R_{E1} is 400 mV and its collector current is $10^{-13} e^{400/25} = 0.89\ \mu\text{A}$

13.35

$2 \times 10^{-3} = 10^{-14} e^{V_{BE}/V_T}$

$\Rightarrow V_{BE} = 0.650 \text{ V}$

$R_{E1} = \dfrac{0.650 \text{ V}}{50 \text{ mA}} = 13 \ \Omega$

For a peak output current of 33.3 mA,

$V_{BE} = 13 \times 33.3 = 433 \text{ mV}$

$I_{CS} = 10^{-14} e^{433/25} = 0.33 \ \mu\text{A}$

13.36

$2 \times 10^{-3} = 10^{-14} e^{V_{EB5}/V_T}$

$V_{EB5} = 0.025 \ln(2 \times 10^{-11})$

$= 0.650 \text{ V}$

$R = \dfrac{0.650 \text{ V}}{150 \text{ mA}} = 4.3 \ \Omega$

For a peak output current of 100 mA,

$V_{EB5} = 430 \text{ mV}$

$I_{C5} = 10^{-14} e^{430/25} = 0.3 \ \mu\text{A}$

13.37

At 125°C,

$V_Z = 6.8 + (125 - 25) \times 2 = 7.0 \text{ V}$

$V_{E1} = 7.0 - (0.7 - 100 \times 0.002)$

$= 6.5 \text{ V}$

$V_{BE2} = 0.5 \text{ V}$

$R_2 = \dfrac{0.5 \text{ V}}{100 \ \mu\text{A}} = 5 \text{ k}\Omega$

$R_1 = \dfrac{6.5 - 0.5}{100 \ \mu\text{A}} = 60 \text{ k}\Omega$

At 25°C, $V_Z = 6.8 \text{ V}$,

$V_{E1} = 6.8 - 0.7 = 6.1 \text{ V}$

$V_{B2} = 6.1 \times \dfrac{5}{60 + 5} = 0.469 \text{ V}$

$I_{C2} = 100 \, e^{(469 - 700)/25} = 0.01 \ \mu\text{A}$

13.38

$V_{B1} \simeq 0$

$V_{E1} \simeq +0.7 \text{ V}$

$V_{E3} \simeq +1.4 \text{ V}$

$V_{C10} = 20 - 0.7 = 19.3 \text{ V}$

$I_{E3} = \dfrac{19.3 - 1.4}{50} = 0.358 \text{ mA}$

$I_{B5} = I_{E1} = \dfrac{0.358}{21} = 17.05 \ \mu\text{A}$

$I_{B1} = \dfrac{17.05}{21} = 0.81 \ \mu\text{A}$

$V_{B1} = 0.81 \ \mu\text{A} \times 150 \text{ k}\Omega = 0.122 \text{ V} \simeq 0$

i.e. $I_{E1} = I_{E2} \simeq 17 \ \mu\text{A}$

$I_{E3} = I_{E4} \simeq 358 \ \mu\text{A}$

$I_{E5} \simeq I_{E6} = \dfrac{20}{21} \times 358 = 341 \ \mu\text{A}$

$I_{R1} = I_{R2} = 358 \ \mu\text{A}$

$V_o = 0.12 + 1.4 + 25 \text{ k}\Omega \times 0.358 \text{ mA}$

$= 10.5 \text{ V}$

13.39

for 8Ω load, we see that $V_S = 16\text{V}$ allows more than 1.5 W power dissipation for some input signals. Thus we use

$V_S = 14 \text{ V}$

For THD = 3%, $P_{L\max} = 1.9 \text{ W}$

$1.9 = V_o^2 / R_L = V_o^2 / 8$

$V_o = \sqrt{8 \times 1.9}$

Peak-to-Peak output sinusoid

$= 2\sqrt{2} \sqrt{8 \times 1.9} = 11 \text{ V}$

13.40

For $i_L = 1\text{A}$, $i_{C5} \simeq 1\text{A}$ and $i_{B5} = \frac{1\text{A}}{50} = 20 \text{ mA}$

$i_{E3} = 0.9 \times 20 = 18 \text{ mA}$,

$\uparrow$ 10%

For $i_L = 20 \text{ mA}$,

$i_{C5} = 2 \text{ mA} \quad I_{B5} = \frac{2}{50} = 0.04 \text{ mA}$

$i_{C3} = \frac{50}{51} \times 18 = 17.65 \text{ mA}$

$i_{R3} = i_{C3} - i_{B5} = 17.61 \text{ mA}$

Thus, $R_3 = \frac{0.7}{17.61} = 39.8 \simeq 40\ \Omega$

Similarly, $R_4 = 40\ \Omega$

Since $i_{B5} \le 20 \text{ mA}$, $i_{C3} \le \frac{0.7 \text{ V}}{40\ \Omega} + 20 \text{ mA}$

i.e. $i_{C3} \le 37.5 \text{ mA}$

$i_{B3} \le \frac{37.5}{50} = 0.75 \text{ mA}$

Allowing for a factor of safety of 2, we select R_1 so that the current through it is 1.5 mA. Now, for $V_o = 11 \text{ V}$, $V_{E1} = 11.7 \text{ V}$,

$R_1 = \frac{15 - 11.7}{1.5} = 2.2 \text{ k}\Omega$

Similarly,
$R_2 = 2.2 \text{ k}\Omega$

13.41

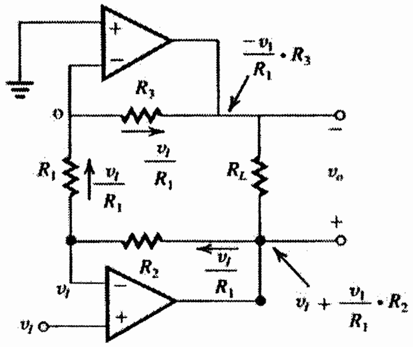
$$\frac{v_o}{v_I} = 2K = 2\left(1 + \frac{R_2}{R_1}\right) = 10$$

$$\Rightarrow \frac{R_2}{R_1} = 4 \qquad R_2 = 40 \text{ k}\Omega$$

Also, $K = \frac{R_4}{R_3} = 5$

$$\Rightarrow R_4 = 50 \text{ k}\Omega$$

13.42



As shown on the diagram

$$v_O = \left(v_I + \frac{V_I}{R_1} \cdot R_2\right) - \left(\frac{v_I}{R_1} \cdot R_3\right)$$

$$= v_I\left(1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}\right) = v_I\left(1 + \frac{R_2 + R_3}{R_1}\right)$$

The largest sine wave output is obtained when the output voltage of one op amp is +13 V and the output voltage of the other op amp is −13 V, which results in A 26 V peak output

For $\frac{v_O}{v_I} = 10 = 1 + \frac{R_2 + R_3}{R_1}$ choose

$R_I = 1 \text{ k}\Omega$ and $(R_2 + R_3) = 9 \text{ k}\Omega$

To keep the output complementary

$\frac{R_3}{R_1} = 1 + \frac{R_2}{R_1}$ here $R_1 = 1 \text{ k}\Omega$

$$\Rightarrow R_3 = 1 + R_2$$

So $R_2 = 4 \text{ k}\Omega$, $R_3 = 5 \text{ k}\Omega$

13.43

For

$$I_{QN} = I_{QP} = 10 \text{ mA} = \mu C_{OX}\frac{W}{L}(V_{GS} - V_t)^2$$

$$10 = 100(V_{GS} - 2)^2$$

$$\Rightarrow V_{GS} = 2.32 \text{ V}$$

$$V_R = 2|V_{GS}| = 4.63 \text{ V}$$

For $I_R = 10$ mA,

$$R = \frac{4.63 \text{ V}}{10 \text{ mA}} = 463 \ \Omega$$

$$V_{BB} = 4.63 + 4 \times 0.7 = 7.43 \text{ V}$$

$$I_{R2} = I_{R4} = \frac{100}{2} = 50 \ \mu\text{A}$$

$$R_2 = R_4 = \frac{700 \text{ mV}}{50 \ \mu\text{A}} = 14 \text{ k}\Omega$$

Now, since V_{GS} changes by $2 \times -3 \text{ mV}/°\text{C} = -6 \text{ mV}/°\text{C}$ while V_{BE1}, V_{BE2}, V_{BE3} and V_{BE4} remain constant, V_{BB} changes by -6 mV/°C. But the voltage across the Q_5 multiplier remains constant. Thus the voltage across the Q_6 multiplier should by made to change by -6 mV/°C which can be achieved by making

$$1 + \frac{R_3}{R_4} = 3$$

$$\Rightarrow R_3 = 2R_4 = 28 \text{ k}\Omega$$

The voltage across the Q_5 multiplier is $V_{BB} - 3V_{BE6} = 7.43 - 2.1 = 5.33$ V

$$\text{Thus, } 5.33 = \left(1 + \frac{R_1}{R_2}\right) \times 0.7$$

$$\Rightarrow \frac{R_1}{R_2} = 6.61$$

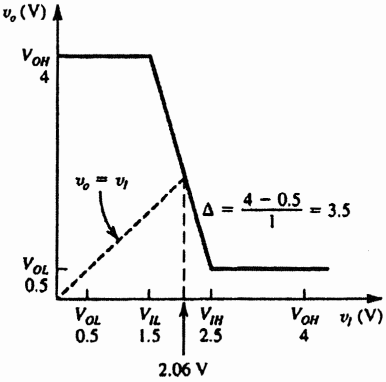
But $R_2 = 14$ kΩ, thus $R_1 = 92.6$ kΩ

14.1

$NM_H = V_{OH} - V_{IH} = 3.3 - 1.7 = 1.6$ V

$NM_L = V_{IL} - V_{OL} = 1.3 - 0 = 1.3$ V

14.2



(a) $NM_H = V_{OH} - V_{IH} = 4 - 2.5 = 1.5$ V

$NM_L = V_{IL} - V_{OL} = 1.5 - 0.5 = 1$ V

(b) In the transition region

$$V_O = 4 - 3.5(V_I - 1.5)$$
$$= 9.25 - 3.5V_I$$

If

$$V_O = V_I \Rightarrow 4.5V_O = 9.25$$
$$V_O = V_I = 2.06 \text{ V}$$

(c) Slope = −3.5 V/V

14.3

$NM_H = V_{OH} - V_{IH} = 0.8V_{DD} - 0.6V_{DD} = 0.2V_{DD}$

$NM_L = V_{IL} - V_{OL} = (0.4 - 0.1)V_{DD} = 0.3V_{DD}$

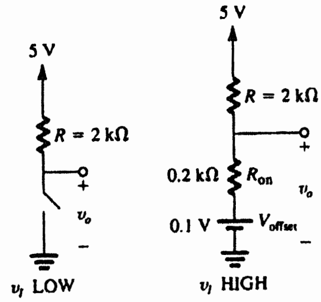
width of transition region

$= V_{IH} - V_{IL} = 0.2V_{DD}$ for a minimum NM of

1 V $\Rightarrow 0.2V_{DD} = 1$

$V_{DD} = 5$ V

14.4



$v_{IL} = 1$ V $v_{IH} = 2$ V

(a) $V_{OL} = \frac{5 - 0.1}{2.2} = 0.2 + 0.1 = 0.545$ V

$V_{OH} = 5$ V

$NM_H = V_{OH} - V_{IH} = 3$ V

$NM_L = V_{IL} - V_{OL} = 0.455$ V

(b) $V_{OH} = 5 - N(0.2 \times 10^{-3})R = 5 - 0.4N$

$NM_H = 5 - 0.4N - 2 = 3 - 0.4N = 0.455 \quad \therefore \; N = 6$

(c) (i) $P_{D_{v_o, LOW}} = (5 - 0.1)^2/2.2 \text{ k}\Omega = 10.9$ mW

(ii) $P_{D_{v_o, HIGH}} = 5 \times (0.2 \times 6) = 6$ mW

14.5

Ideal 3V logic implies :

$V_{OH} = V_{DD} = \underline{\underline{3.0V}}$; $V_{OL} = \underline{\underline{0.0V}}$;

$V_{th} = V_{DD}/2 = 3.0/2 = \underline{\underline{1.5V}}$;

$V_{IL} = V_{DD}/2 = \underline{\underline{1.5V}}$; $V_{IH} = V_{DD}/2 = \underline{\underline{1.5V}}$

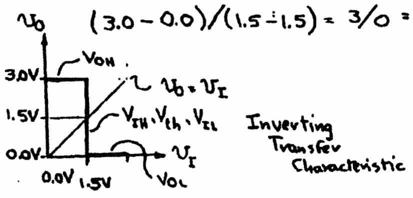
$NM_H = V_{OH} - V_{IH} = 3.0 - 1.5 = \underline{\underline{1.5V}}$

$NM_L = V_{IL} - V_{OL} = 1.5 - 0.0 = \underline{\underline{1.5V}}$

The gain in the transition region is :

$(V_{OH} - V_{OL})/(V_{IH} - V_{IL}) =$

$(3.0 - 0.0)/(1.5 - 1.5) = 3/0 = \underline{\underline{\infty V/V}}$



14.6

Nearly ideal 3.3V logic, assumed ideal:

→ $V_{OH} = \underline{\underline{3.3V}}$, $V_{OL} = \underline{\underline{0.0V}}$, $V_{th} = 0.4(3.3) = \underline{\underline{1.32V}}$

Now, at V_{th}, $v_O = v_I$, so to reach $v_O = 1.32V$ the required input is $1.32/(-50) = -26.4mV$

Thus, $V_{IL} = 1.32 - 26.4 \times 10^{-3} = \underline{\underline{1.294\ V}}$

Likewise, $V_{IH} = 1.32 + (3.3 - 1.32)/50 = \underline{\underline{1.360V}}$

Best possible noise margins are:

$NM_H = V_{OH} - V_{IH} = 3.30 - 1.360 = \underline{\underline{1.940\ V}}$

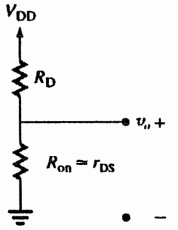
$NM_L = V_{IL} - V_{OL} = 1.294 - 0.0 = \underline{\underline{1.294V}}$

For noise margins only 7/10 of these, and V_{OH}, V_{OL} still ideal:

$V_{IH} = 3.3 - 0.7(1.940) = \underline{\underline{1.942\ V}}$, and
$V_{IL} = 0.0 + 0.7(1.294) = \underline{\underline{0.906V}}$

Correspondingly, the large-signal voltage gain is:
$G = (3.3 - 0.0)/(0.906 - 1.942) = \underline{\underline{-3.18\ V/V}}$

14.7



Equivalent circuit for output-low state

The output high level for the simple inverter circuit shown in Fig 13.2 of the Text is

$V_{OH} = V_{DD} \Rightarrow V_{DD} = 2\ V.$

When the output is low, the current drawn from the supply can be calculated as:

$$I = \frac{V_{DD}}{R_D + R_{on}} = 20\ \mu A$$

Therefore: $R_D + r_{DS} = \dfrac{2}{20 \times 10^{-6}} = 100\ k\Omega$

Also:

$$V_{OL} = 0.1\ V = \frac{r_{DS}}{R_D + r_{DS}} \times V_{DD}$$

$$\Rightarrow r_{DS} = 100\ k\Omega \times \frac{0.1}{2} = 5\ k\Omega$$

Hence: $R_D = 100\ K - 5\ K = 95\ k\Omega$

$$r_{DS} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_t)} = \frac{1}{100 \times 10^{-6} \times \frac{W}{L}(2 - 0.5)}$$

$= 5\ k\Omega$

$$\frac{W}{L} = \frac{10}{1.5 \times 5} = 1.3$$

when the output is low:

$P_D = V_{DD}I_{DD} = 2 \times 20\ \mu A = 40\ \mu W$

when the output is high, the transistor is off:

$P_D = 0\ W$

14.8

$V_{OH} = V_{DD} = 2.5\ V$

The power drawn from the supply during the low-output state is:

$P_{DD} = V_{DD} \Rightarrow 125\ \mu\omega = 2.5 \times I_{DD}$

$\Rightarrow I_{DD} = 50\ \mu A$

In this stage:

$$I_{DD} = \frac{V_{DD} - V_{OL}}{R_D} \Rightarrow 50\ \mu A = \frac{2.5 - 0.1}{R_D}$$

$\Rightarrow R_D = 48\ k\Omega$

In order to determine $\frac{W}{L}$, we note that

$$k_n R_D = 1 / V_X \text{ or } k'_n \frac{W}{L} R_D = \frac{1}{V_X}$$

Therefore, we need to first calculac V_X.

$$V_{OL} - \frac{V_{DD}}{1 + \frac{V_{DD} - V_t}{V_X}} \text{ or equivalently:}$$

$$0.1\ V = \frac{2.5}{1 + \frac{2.5 - 0.5}{V_X}} \Rightarrow V_X = \frac{2}{24} = 0.083\ V$$

Hence, $k'_n \frac{W}{L} R_D = \frac{1}{V_x}$ gives:

$$100 \times 10^{-6} \times \frac{W}{L} \times 48 \times 10^3 = \frac{1}{0.083} \Rightarrow \frac{W}{L} = 2.5$$

$V_{IL} = V_t + V_X = 0.5 + 0.083 = 0.583V$

$V_M = V_t + \sqrt{2(V_{DD} - V_t)V_x + V^2_x} - V_x$

$= 0.5 + \sqrt{2(2.5 - 0.5)0.083 + 0.083^2} - 0.083$

$V_M = 1V$

$V_{IH} = V_t + 1.63\sqrt{V_{DD}V_x} - V_x$

$= 0.5 + 1.63\sqrt{2.5 \times 0.083} - 0.083 = 1.16V$

$NM_H = V_{OH} - V_{IH} = 2.5 - 1.16 = 1.34V$

$NM_L = V_{IL} - V_{OL} = 0.583 - 0.1 = 0.483V$

14.9

$V_{t2} = V_{t0} + r[\sqrt{V_{OH} + 2\varphi_F} - \sqrt{2\varphi_F}]$,

$V_{OH} - V_{DD} - V_{t2}$

Interation 1: $V_{t2} = 0.5V$

$V_{OH} = 1.8 - 0.5 = 1.3V$

Interation 2:

$V_{t2} = 0.5 + 0.3[\sqrt{1.3 + 0.8} - \sqrt{0.8}] = 0.67V$

$V_{OH} = 1.8 - 0.67 = 1.13V$

Interation 3:

$V_{t2} = 0.5 + 0.3[\sqrt{1.13 + 0.8} - \sqrt{0.8}] = 0.65V$

$V_{OH} = 1.8 - 0.65 = 1.15V$

Interation 4:

$V_{t2} = 0.5 + 0.3[\sqrt{1.15 + 0.8} - \sqrt{0.8}] = 0.65V$

$V_{OH} = 1.8 - 0.65 = 1.15V$

$\therefore V_{t2} = 0.65V$ and $V_{OH} = 1.15V$

$\Delta V_{OH} = 1.3 - 1.15 = 0.15V$ V_{OH} is reduced by 0.15V due to the body effect on Q_2

14.10

$V_{IH} \simeq V_M + \frac{V_M}{K_r} = 0.63 + \frac{0.63}{5} = 0.756V$

The value calculated the long way in Example 13.2 is: $V_{IH} = 0.75V$ and is very close to the above approximation.

14.11

Given: $V_{OL} \simeq 0.05V$

$V_{OH} = V_{DD} - V_t = 2.5 - 0.5 = 2V$

$V_{IL} = V_t = 0.5V$

$$V_{OL} \simeq \frac{(V_{DD} - V_t)^2}{2k^2_r(V_{DD} - 2V_t)} = \frac{(2.5 - 0.5)^2}{2k^2_r(2.5 - 2 \times 0.5)}$$

$$= \frac{4}{3k^2_r} \simeq 0.05V \Rightarrow K_r = 5.2$$

V_M

$$= \frac{V_{DD} + (K_r - 1)V_t}{(K_r + 1)} = \frac{2.5 + (5.2 - 1) \times 0.5}{5.2 + 1}$$

$= 0.74V$

$$V_{IH} \cong V_M + \frac{V_M}{K_r} = 0.74V + \frac{0.74V}{5.2} = 0.88V$$

$NM_H = V_{OH} - V_{IH} = 2 - 0.88 = 1.12V$

$NM_L = V_{IL} - V_{OL} = 0.5 - 0.05 = 0.45V$

To obtain $\frac{W}{L}$.

$$K_r = \sqrt{\frac{W/L_1}{W/L_2}} \Rightarrow K_r = \sqrt{\frac{W/L_1}{\frac{1}{W/L_1}}} = \left(\frac{W}{L}\right).$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 5.2\left(\frac{W}{L}\right)_2 = \frac{1}{5.2} = 0.19$$

$$I_{DD} = i_{D2} = \frac{1}{2}K_{n1}(V_{DD} - V_{OL} - V_t)^2$$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times 0.19 \times (2.5 - 0.05 - 0.5)^2$$

$= 36.1\ \mu A$

$P_D = V_{DD}I_{DD} = 2.5 \times 36.1\mu = 90.2\ \mu W$

14.12

$E_{dissipated/cycle} = CV_{DD}^2 = 10 \times 10^{-15} \times 2.5^2$

$E_{dissipated/cycle} = 62.5FJ$

$P_{dyn} = fcV^2_{DD} = 1 \times 10^9 \times 10 \times 10^{-15} \times 2.5^2$

$= 62.5\mu W$

This is the power consumption for one inverter. For a chip with 1 million inverters, the power consumption is:

$P_{dyn(chip)} = 62.5 \times 10^{-6} \times 10^6 = 62.5W$

To determine the average current drawn from the supply, we note that

$$P_{dyn} = I_{DDavg}V_{DD} \Rightarrow I_{DD-avg} = \frac{62.5}{2.5} = 25A$$

14.13

$P_{dynamic} = fCV_{DD}^2 = 100 \times 10^6 \times 10 \times 10^{-12} \times 25 =$ 25 mW

$P = V_{DD}\, I_{avg} = 5I_{avg} = 25$ mW

$I_{avg} = 5$ mA

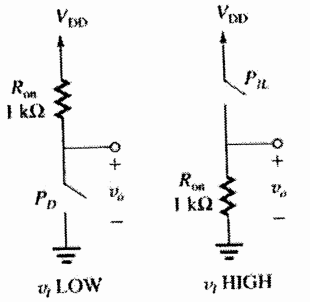
14.14

(a) $V_{OL} = 0$

$V_{OH} = 5$

$NM_L = V_{IL} - V_{OL} = 2.5 - 0 = 2.5V$

$NM_H = V_{OH} - V_{IH} = 5 - 2.5 = 2.5V$



(b)

$V_o(t) = 0 - (0 - 5)e^{-t/R_{on}C} = 5e^{-t/R_{on}C}$

For $t_{PHL} \Rightarrow V_o(t) = 5e^{-t/R_{on}C} = \frac{1}{2}(5) = 2.5$

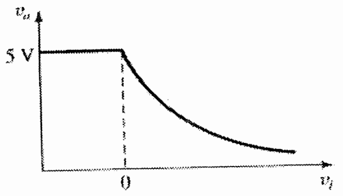
$t_{PHL} = -(10^3)(10^{-12})\ln\frac{2.5}{5} = 0.69ns$

For $t_{PHL} V_o(t) = 5e^{-t_H/R_{on}C} = 4.5V$

$t_1 = 0.1$ ns $V_o(t) = 5e^{-t_L/R_{on}C} = 0.5V$

$t_2 = 2.3$ ns

$\therefore t_{THL} = t_2 - t_1 = 2.2$ ns



(c)

$V_o(t) = 5 - (5 - 0)e^{-t/R_{on}C} = 5 - 5e^{-t/R_{on}C}$

$V_O = 5 - 5e^{-t_{PLH}/R_{on}C} = 2.5$

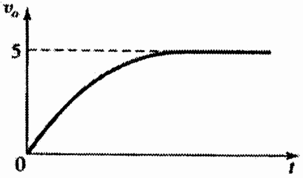
$t_{PLH} = 0.69$ ns

For t_{PLH},

$V_o(t) = 5 - 5e^{-t_1/R_{on}C} = 0.5 \Rightarrow t_1 = 0.10$ ns

$V_o(t) = 5 - 5e^{-t_2/R_{on}C} = 4.5 \Rightarrow t_2 = 2.3$ ns

$t_{PLH} = 2.3 - 0.1 = 2.2ns$



14.15

(a) Generally, $t_p = (t_{PHL} + t_{PLH})/2$, but due to current ratio, $t_{PHL} = 0.5t_{PLH}$.

Thus $1.5t_{PHL} = 2(1.2ns)$, whence

$t_{PLH} = 2.4/1.5 = 1.6$ ns, and $t_{PHL} = 0.8$ ns

Check:

$t_p = (1.6 + 0.8)/2 = 1.2ns$

(b) Generally, $t_p = CV/I = kC$

Originally, $1.2n = kC$ (1)

Then, $1.7(1.2)n = k(C + 1p)$ (2)

Dividing $\left(\frac{2}{1}\right)$: $1.7 = (c + 1p)/C$

Thus, $1.7C = C + 1$, $0.7C = 1$,

$C = 1.43pF$

(the combined load and output capacitances)

(c) With the load inverter removed:

$0.6(1.2n) = k(1.43 - C_{in})$ (3)

Dividing $\left(\frac{3}{1}\right)$ $0.6 = (1.43p - C_{in})/1.43$

Thus,

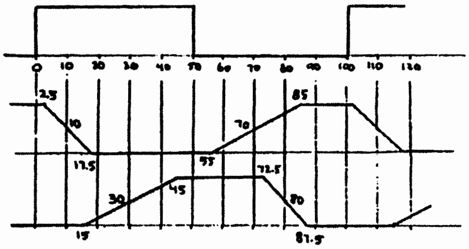
$C_{in} = 1.43(1 - 0.6) = 0.57pF$;

$C_{out} = 1.43 - 0.57 = 0.86pF$

14.16

The results depend on whether the gates are inverting or non-inverting.

For inverting gates, the timing diagram is:



Note: For simplicity, 0% to 100% (rather than 10% to 90%) both in the diagram above and calculation to follow:

For inverting gates (as shown, above):

a) For a rising input, time to 90% change of output of second gate is $10 + 20 + 30/2 = 45\text{ns}$

b) For a falling input, time to 90% change of output of 2nd gate is $20 + 10 + 15/2 = 37.5\text{ns}$

For non-inverting gates:

a) Time to 90% rise is $10 + 10 + 15/2 = 27.5\text{ns}$

b) Time to 90% fall is $20 + 20 + 30/2 = 55\text{ns}$

The propagation delay for these gates is

$t_P = (t_{PHL} + t_{PLH})/2 = (10 + 20)/2 = 15\text{ns}$

14.17

Note that this question ignores the possibility of dynamic power dissipation: Average propagation delay is $t_p = (50 + 70)/2 = 60$ ns

Average power loss at 50% duty cycle

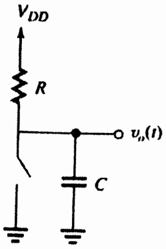
$= (1 + 0.5)/2 = 0.75$ mW

Delay-Power product is

$DP = 60 \times 10^{-9} \times 0.75 \times 10^{-3}$ or

$DP = 45 \times 10^{-12}\ \text{J} = 45$ pJ

14.18



$v_O(t)$ begins at V_{OL} and rises toward V_{OH} (in this case $V_{OH} = V_{DD}$) according to

$v_o(t) = v_\infty - (v_\infty - v_{oi})e^{-t/CR}$

$= V_{OH} - (V_{OH} - V_{OL})e^{-t/CR}$

$= V_{OH} - (V_{OH} - V_{OL})e^{-t/\tau_1}$, $\tau_1 = CR$

Q.E.D.

$v_O(t)$ reaches $\frac{1}{2}(V_{OH} + V_{OL})$ at $t = t_{PLH}$,

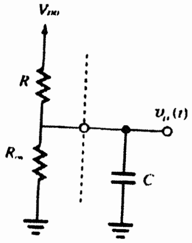
$\frac{1}{2}(V_{OH} + V_{OL}) = V_{OH} - (V_{OH} - V_{OL})e^{-t_{PLH}/\tau_1}$

$\Rightarrow t_{PLH} = \tau_1 \ln 2 = 0.69CR$ Q.E.D.

(b)

$v_O(t) = v_\infty - (v_\infty - v_{oi})e^{-t/\tau_2}$,

$\tau_2 = C(R//R_{on})$



14.19

$$V_M = \frac{r(V_{DD} - |v_{tp}|)}{1 + r} \text{ where}$$

$$r = \sqrt{\frac{kp}{k_n}} = \sqrt{\frac{\mu_p w_p}{\mu_n w_n}}$$

a) For $w_p = 3.5w_n$ or matched case:

$\frac{w_p}{w_n} = \frac{\mu_n}{\mu_p}$ we have $r = 1$ and:

$$V_M = \frac{1(2.5 - 0.5) + 0.5}{1 + 1} = \frac{V_{DD}}{2} = 1.25\text{V},$$

$$A = (w_p + w_n)L = 4.5w_nL$$

b) For $w_p = w_n$: $r = \sqrt{\frac{1}{3.5}} \times 1 = 0.53$

$$V_M = \frac{0.53(2.5 - 0.5) + 0.5}{1 + 0.53} = 1.02\text{V}$$

The shift in NM_L is approximately equal to the shift in V_M, that is:

$\Delta V_M = 1.25 = 1.02 = 0.23\text{V}$, hence NM_L is reduced by 0.23V.

$A = (w_p + w_n)L = (w_n + w_n)L = 2w_nL$, therefore the area is reduced by

$(4.5 - 2)w_nL = 2.5w_nL = 2.5 \times 1.5 \times 0.25 \times 0.25$

$= 0.23\mu\text{m}^2$ or by $\frac{25}{4.5} = 0.56$, 56%

c) For $w_p = 2w_n$: $r = \sqrt{\frac{1 \times 2}{3.5}} = 0.76$,

$$V_M = \frac{0.76(2.5 - 0.5) + 0.5}{1 + 0.76} = 1.15\text{V}$$

The shift in V_M is $1.25 - 1.15 = 0.1\text{V}$, hence, the NM_L is approximately reduced by 0.1V or comparing to NM_L in P13.26 above, it is reduced by 9.4%.

$A = (w_p + w_n)L = (2w_n + w_n)L = 3w_nL$,

therefore the area is reduced by $1.5w_nL$ or

$1.5 \times 1.5 \times 0.25 \times 0.25 = 0.14\mu\text{m}^2$ or by

$\frac{1.5}{4.5} = 0.33$ or 33%.

14.20

In the low-output state, V_{in} is high and V_{out} is low and therefore NMos operatres in triode region:

$$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)_n \left((V_{GS} - V_{tn})V_{DS} - \frac{1}{2}V_{DS}^2\right)$$

$$= k_n'\left(\frac{W}{L}\right)_n \left((V_{DD} - 0.2V_{DD}) \times 0.1V_{DD} - \frac{1}{2}(0.1V_{DD})^2\right)$$

$$I_D = k_n'\left(\frac{W}{L}\right)_n (0.08V^2{}_{DD} - 0.005V^2{}_{DD})$$

$$= 0.075k_n'\left(\frac{W}{L}\right)_n V^2{}_{DD}$$

For $I = 0.5\text{mA}$,

$V_{DD} = 2.5\text{V}$, $k_n' = 115\mu\text{A}/\text{V}^2$, we'll have:

$$0.5 \times 10^{-3} = 0.075 \times 115 \times 10^{-6}\left(\frac{W}{L}\right)_n \times 2.5^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 9.3$$

14.21

For $v_I = 1.5\text{V}$, the NMOS operates in triode mode while the PMOS is cut off.

$r_{DSn} = [k_n(v_I - V_t)]^{-1} = [100 \times 10^{-6}(1.5 - 0.5)]^{-1}$

$= 10\text{k}\Omega$

Thus,

$v_a = 100 \times 10^{-3} \times 10^4/(10^4 + 10^5) = 9.09\text{mV}$

For $v_I = -1.5\text{V}$, the PMOS operates with

$r_{DSp} = [k_p(|v_I|) - V_t] = [(100 \times 10^{-6})(1.5 - 0.5)]^{-1}$

$= 10^5\Omega$

Thus

$v_a = 100 \times 10^{-3} \times 10^5/(10^5 + 10^5) = 50\text{mV}$

14.22

Since at M, both Q_N and Q_P operate in saturation, their currents are given

Substituting $V_I = V_O = V_M$ and equating the two currents results in:

$$i_{DN} = i_{DP} \Rightarrow \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_I - V_{tn})^2$$

$$= \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p (V_{DD} - V_M - |V_{tp}|)^2$$

$$\frac{K_p}{k_n} = \frac{(V_M - V_{tn})^2}{(V_{DD} - V_M - |V_{tp}|)^2}. \text{ Considering}$$

$$r = \sqrt{\frac{K_p}{R_n}}, \text{ we have: } r = \frac{V_M - V_{tn}}{V_{DD} - |V_{tp}| - V_M}$$

Now, for

$$V_M = 0.6V_{DD} = 0.6 \times 1.8 = 1.08\text{V}$$

$$r = \frac{1.08 - 0.5}{1.8 - 0.5 - 1.08} = 2.64$$

$$r = \sqrt{\frac{\mu_p w_p}{\mu_n w_n}} = r\frac{w_p}{w_n} = \frac{(2.64)^2}{V_4} = 27.9$$

14.23

The peak current happens when $V_I = V_M$ and since Q_p and Q_N are matched

$$V_M = \frac{V_{DD}}{2} = \frac{1.8}{2} = 0.9V$$

Noting that both transistors are in saturation region, Find the current.

$$I = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_M - V_{tn})^2$$

$$= \frac{1}{2}K_n'\left(\frac{W}{L}\right)_n\left(\frac{V_{DD}}{2} - V_{tn}\right)^2$$

For $k_n' = 300\mu A/V^2$,

$\left(\frac{W}{L}\right)_n = 1.5\ V_{DD} = 1.8V\ V_{tn} = 0.5V$:

$$I_{peak} = \frac{1}{2} \times 300 \times 10^{-6} \times 1.5 \times \left(\frac{1.8}{2} - 0.5\right)^2$$

$$= 36\mu A$$

14.24

since $V_{tn} = |V_{tp}|$, then $\alpha_n = \alpha_p$, then $\alpha_n = \alpha_p$. From above, we have $\alpha_n = \alpha_p = 2.32$.

$$t_p = \frac{1}{2}(t_{PLH} + t_{PHL})$$

$$= \frac{1}{2}\left(\frac{\alpha_n C}{k'_n\left(\frac{W}{L}\right)_n V_{DD}} + \frac{\alpha_p C}{k'_p\left(\frac{W}{L}\right)_n V_{DD}}\right)$$

Since QN and QP are matched, then $k_p = k_n$ and

$$t_p = \frac{\alpha_n C}{k_n V_{DD}} = \frac{2.32 \times 20 \times 10^{-15}}{K_n \times 1.2V}$$

$$t_p \le 20ps \Rightarrow \frac{38.7 \times 10^{-15}}{k_n} \le 2.0 \times 10^{-12} \Rightarrow k_n \approx 1.9m$$

Now if we substitude in

$$k_n = (k_n')\left(\frac{W}{L}\right) = 430 \times 10^{-6} \times \left(\frac{W}{L}\right)_n = 1.9m$$

$$\Rightarrow \left(\frac{W}{L}\right)_n = 4.4$$

Since Q_P and Q_N are matched and $k_n = k_p$, then

$$\left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_n \times \frac{k_n}{k_p} 4.4 \times 4 = 17.6$$

14.25

Using the equivalent resistance approach, we first find R_N.

$$R_N = \frac{12.5}{\left(\frac{W}{L}\right)_n} = \frac{12.5}{1.5} = 8.33k\Omega$$

to determine t_{PHL}.

$$t_{PHL} = 0.69 R_N C = 0.69 \times 8.33 \times 10^3 \times 10 \times 10^{-15}$$

$= 57.5$ ps.

to determine R_p:

$$R_p = \frac{30}{\left(\frac{W}{L}\right)_p} = \frac{30}{3} = 10k\Omega$$

to determine t_{PLH}:

$$t_{PLH} = 0.69 R_p C = 0.69 \times 10 \times 10^3 \times 10 \times 10^{-15}$$

$= 69$ ps

$$t_p = \frac{1}{2}(57.5 + 69) = 63.2 \text{ ps}$$

Note that while the value obtained for t_{PHL} is higher than that found using the average currents method, the value for t_{PLH} is about the same.

14.26

$t_{PHL} = 0.69 R_N C$, $t_{PLH} = 0.69 R_p C$

Since $t_{PHL} = t_{PLH}$, then $R_N = R_p = R$

For $t_p \le 40ps$, we have to have:

$$\frac{1}{2}(t_{PHL} + t_{PLH}) \le 40 \text{ ps or}$$

$$\frac{1}{2}(0.69 \times 2R \times C) \le 40ps$$

$$\therefore R \le \frac{40 \times 10^{-12} \times 2}{0.69 \times 2 \times 10^{-15} \times 10} \Rightarrow R \le 5.8k\Omega$$

To determine the transistor widths in 0.18 μm technology,

$L_n = L_p = .18\ \mu m$

$$R_N = \frac{12.5}{\left(\frac{W}{L}\right)_n} k\Omega \text{ or}$$

$$\frac{12.5}{\left(\frac{W}{L}\right)_n} K \le 5.8k\Omega \Rightarrow \left(\frac{W}{L}\right)_n \ge 2.2$$

$$\Rightarrow w_n \ge 0.4\mu m$$

$$R_p = \frac{30}{\left(\frac{W}{L}\right)_p} k\Omega \text{ or}$$

$$\frac{30}{\left(\frac{W}{L}\right)_p} K \le 5.8k\Omega \Rightarrow \left(\frac{W}{L}\right)_p \ge 5.2$$

$$\Rightarrow w_p \ge .94\mu m$$

14.27

$$\alpha_n = 2/\left[\frac{7}{4} - \frac{3V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2\right]$$

$$= 2/\left[\frac{7}{4} - \frac{3 \times 0.7}{3.3} + \left(\frac{0.7}{3.3}\right)^2\right] = 1.73$$

$$t_{PHL} = \frac{\alpha_n C}{k_n'\left(\frac{W}{L}\right)_n V_{DD}} = \frac{1.73 \times (2\text{fF} \times 0.75 + 1\text{fF})}{180 \times 10^{-6} \times \frac{0.75}{0.5} \times 3.3}$$

$= 4.85$ ps

Since, $V_{tn} = |V_{tp}|$, then $\alpha_n = \alpha_p = 1.73$. We also have $\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p$, hence:

$$t_{PLH} = t_{PHL} \times \frac{k_n'}{k_p'} = 4.85 \times 3 = 14.55\text{ps}$$

$$t_p = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(4.85 + 14.55) = 9.7 \text{ ps}$$

If both devices are matched, then $k_p' = k_n'$.

$t_{PLH} = t_{PHL}$ and

$$t_p = \frac{1}{2}(t_{PLH} + t_{PHL}) = t_{PHL} = 4.85 \text{ ps}$$

14.28

In order to determine the propagation delay, we first need to calculate the total value for C, using

$$C = 2C_{gd1} + 2C_{gd2} + C_{db1} + C_{db2} + C_{g3} + C_{g4} + C_w$$

where $C_{gd1} = 0.4w_n = 0.4 \times 0.75 = 0.3\text{fF}$

Since transistors are matched

$$k_p'\left(\frac{W}{L}\right)_p = k_n'\left(\frac{W}{L}\right)_n \Rightarrow w_p = \frac{180}{45} \times 0.75 = 3\mu\text{m}$$

$C_{gd2} = 0.4 \times w_p = 0.4 \times 3 = 1.2\text{fF}$

$C_{db1} = 1.0 \times w_n = 1 \times 0.75 = 0.75\text{fF}$

$C_{db2} = 1.0 \times w_p = 1.0 \times 3 = 3.0\text{fF}$

$C_{g3} = (WL)_s C_{ox} + C_{gdov3} + C_{gsov3}$

$= (0.75 \times 0.5) \times 3.7 + 0.4 \times 0.75 + 0.4 \times 0.75$

$= 1.99\text{fF}$

$C_{g4} = 3 \times 0.5 \times 3.7 + 2 \times 0.4 \times 3 = 7.95\text{fF}$

$C = 2 \times 0.3 + 2 \times 1.2 + 0.75 + 3 + 1.99 + 7.95 + 2$

$= 18.7\text{fF}$

to determine t_{PHL}:

$$\alpha_n = 2/\left[\frac{7}{4} - \frac{3 \times 0.7}{3.3} + \left(\frac{0.7}{3.3}\right)^2\right] = 1.74$$

then, t_{PHL}

$$= \frac{1.74 \times 18.7 \times 10^{-15}}{180 \times 10^{-6} \times \frac{0.75}{0.5} \times 3.3} = 36.5\text{ps}$$

Since $|V_{tp}| = V_{tn}$ and transistors are matched,

$t_{PHL} = t_{PLH} = t_p \Rightarrow t_p = 36.5$ ps.

Considering that t_{PHL} and t_{PLH} both are proportional to C, then for an increase of 50% in t_p, C also has to be increased by 50%. Hence,

$\Delta C = 18.7 \times 0.5 = 9.4$ fF

14.29

$$\frac{t_{pnew}}{t_{pold}} = \frac{C_{int} + C_{ext}/s}{C_{int} + C_{ext}} \Rightarrow \frac{30}{60} = \frac{10 + 20/s}{10 + 20}$$

$$\Rightarrow 15 = 10 + \frac{20}{5} \Rightarrow s = 4$$

Note that $S = \frac{R_{eq}}{R_{eq}}$ and hence R_{eq} has to be reduced by a factor of 4 or equivalently $\left(\frac{W}{L}\right)_n$ and $\left(\frac{W}{L}\right)_p$ have to be increased by a factor of 4.

14.30

Dynamic power is $P_D = fCV^2_{DD}$; Static Power is P_s.

Now, $9.0 = P_s + 120 \times 10^6 C^2 5$ and

$4.7 = P_s + 50 \times 10^6 C^2 5$

Subtracting, $4.3 = 70 \times 10^6 C(25)$

Whence

$C = 4.3/(25 \times 70 \times 10^6) = 2457\text{pF}$

and $P_s = 9.0 - 120 \times 10^6 (25) 2457 \times 10^{-12}$

$= 9.0 - 7.37 = 1.63\text{W}$

For 70% of the gates active, total gates

$= 0.7 \times 10^6$

Capacitance per gate is

$2457 \times 10^{-12}/(0.7 \times 10^6) = 3.5\text{fF}$

14.31

$$C = 2C_{gd1} + C_{gd2} + C_{db1} + C_{db2} + C_{g3} + C_{g4} + C_n$$

$$W = W_n = W_p = 0.75\mu\text{m}$$

$$C_{gd1} = \frac{.4\text{f}}{\mu\text{m}} \cdot W_n = \frac{.4\text{f}}{\mu\text{m}} \times .75\mu\text{m} = 0.3\text{fF}$$

$$C_{gd2} = \frac{0.4\text{f}}{\mu\text{m}} \cdot W_p = \frac{.4\text{f}}{\mu\text{m}} \times .75\ \mu\text{m} = 0.3\text{fF}$$

$$C_{db1} = C_{db2} = \frac{1\text{f}}{\mu\text{m}} \cdot W = \frac{1\text{f}}{\mu\text{m}} \times 0.75\mu\text{m}$$

$$= 0.75\text{fF}$$

$$C_{g3} = C_{g4} = (WL)C_{ox} + C_{gdov} + C_{gsov}$$

$$= (0.75\ \mu\text{m} \times 0.5\ \mu\text{m})\frac{3.7\text{f}}{\mu\text{m}^2} + 2 \times \frac{0.4\text{f}}{\mu\text{m}} \times 0.75\ \mu\text{m}$$

$$C_{g3} = C_{g4} = 1.99\text{fF}$$

$$C = 2 \times 0.3\text{f} + 2 \times 0.3\text{f} + 2 \times 0.75\text{f}$$

$$+ 2 \times 1.99f + 2f = 8.7\ \text{fF}$$

$$\alpha_n = 2 / \left[\frac{7}{4} - \frac{3 \times 0.7}{3.3} + \left(\frac{0.7}{3.3}\right)^2\right] = 1.74$$

$$t_{PHL} = \frac{1.74 \times 8.7 \times 10^{-15}}{180 \times 10^{-6} \times \frac{0.75}{0.5} \times 3.3} = 17\ \text{ps}$$

$$\alpha_p = 2\left[\frac{7}{4} - \frac{3 \times 0.7}{3.3} + \left(\frac{0.7}{3.3}\right)^2\right] = 1.74$$

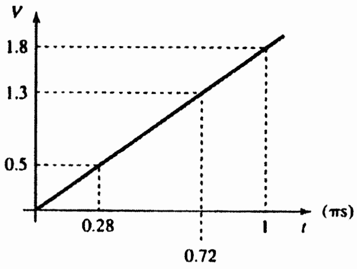
$$t_{PLH} = \frac{1.74 \times 8.7 \times 10^{-15}}{45 \times 10^{-6} \times \frac{0.75}{0.5} \times 3.3} = 68\text{ps}$$

$$t_P = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(17\text{p} + 68\text{p}) = 42.5\text{ps}$$

$$P_D = fCV_{DD}^2 = 250 \times 10^6 \times 8.7 \times 10^{-15} \times (3.3)^2$$

$$= 23.7\mu\text{W}$$

14.32



$$I_{\text{peak}} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n\left(\frac{V_{DD}}{2} - V_{tn}\right)^2$$

$$I_{\text{peak}} = \frac{1}{2} \times 450\frac{\mu\text{A}}{\text{V}^2}\left(\frac{1.8}{2} - 0.5\right)^2 = 36\ \mu\text{A}$$

The time when the input reaches V_t is:

$$\frac{0.5}{1.8} \times 1^{\text{ns}} = 0.28\text{ns}$$

The time when the input reaches $V_{DD} - V_t$ is

$$\frac{1.8 - 0.5}{1.8} \times 1^{\text{ns}} = 0.72^{\text{ns}}$$

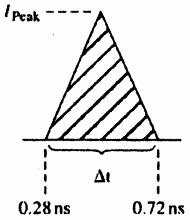
So the base of the triangle is

$\Delta t = 0.72 - 0.28 = 0.44\text{ns}$ wide

$$E = \frac{1}{2}I_{\text{peak}} \times V_{DD} \times \Delta t = \frac{1}{2} \times 36\ \mu\text{A} \times 1.8 \times 0.44\ \text{ns}$$

$$= 14.3\text{fJ}$$

$$P = f \times E = 100 \times 10^6 \times 14.3 \times 10^{-15} = 1.43\ \mu\text{W}$$



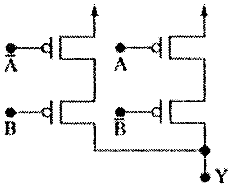
14.33

$Y = A\bar{B} + \bar{A}B \rightarrow \bar{Y} = \overline{A\bar{B} + \bar{A}B} = \overline{A\bar{B}} \cdot \overline{\bar{A}B}$

or $Y = (\bar{A} + B)(A + \bar{B}) = AB + \bar{A}\bar{B}$

PUN for $Y = A\bar{B} + \bar{A}B$:

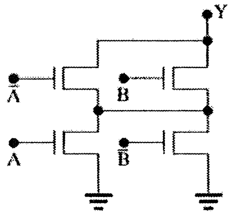
u1



$\bar{A}$
A
B
$\bar{B}$
Y

PDN dual to u1

D1

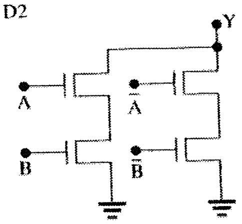


Y
$\bar{A}$
B
A
$\bar{B}$

*Note, however that D1 can be redrawn as shown, then its colums (series links) converted to rows (parallal links of a PUN:

PDN for $Y = AB + \bar{A}B$:

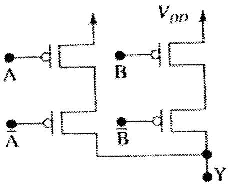
D2



Y
A
$\bar{A}$
B
$\bar{B}$

PUN dual to D2:

u2

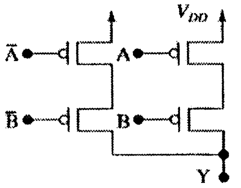


V_{DD}
A
B
$\bar{A}$
$\bar{B}$
Y

The two circuits required are U_1 with D1 and U_2 with D_2.

14.34

$Y = AB + \bar{A}\bar{B}$. Directly, the PUN is as follows: u1



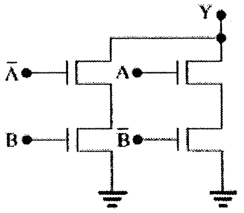
V_{DD}
$\bar{A}$
A
$\bar{B}$
B
Y

Now,

$\bar{Y} = \overline{AB + \bar{A}\bar{B}} = \overline{AB} \,.\, \overline{\bar{A}\bar{B}} = (\bar{A} + \bar{B})/(A + B)$

or $\bar{Y} = \bar{A}B + A\bar{B}$

Directly, the PDN is:

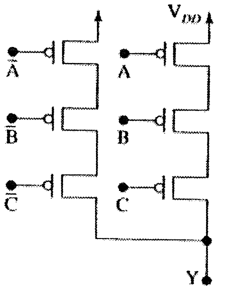


Y
$\bar{A}$
A
B
$\bar{B}$

14.35

$Y = ABC + \bar{A}\bar{B}\bar{C}$. Directly, the PUN is as shown below:

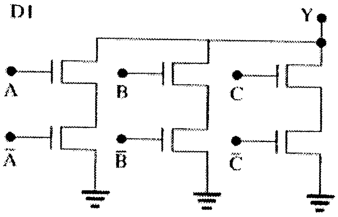
u1



V_{DD}
$\bar{A}$
A
$\bar{B}$
B
$\bar{C}$
C
Y

The corresponding dual PDN is shown above below.

D1



Y
A
B
C
$\bar{A}$
$\bar{B}$
$\bar{C}$

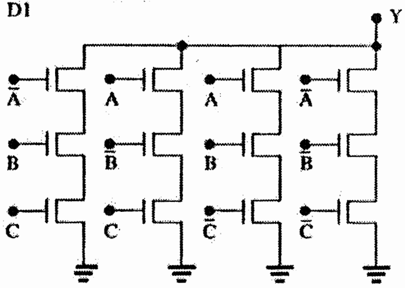
14.36

a) Even-parity circuit:

$\bar{Y} = \bar{A}BC + A\bar{B}C + AB\bar{C} + \bar{A}\bar{B}\bar{C}$,

b) PDN directly is:

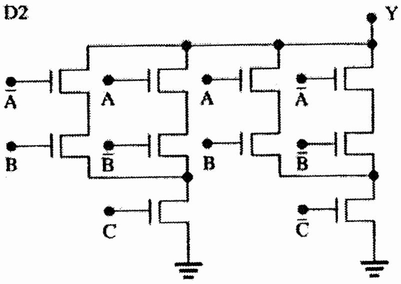
D1


Y
$\bar{A}$
A
A
$\bar{A}$
B
$\bar{B}$
B
$\bar{B}$
C
C
$\bar{C}$
$\bar{C}$

It uses 12 transistors.

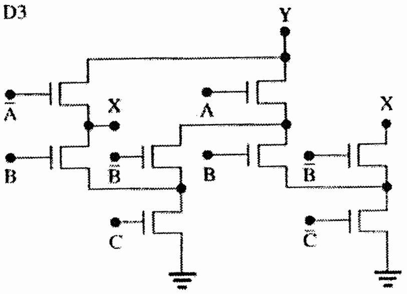
(c) PDN reduced to 10 transistors:

D2


Y
$\bar{A}$
A
A
$\bar{A}$
B
$\bar{B}$
B
$\bar{B}$
C
$\bar{C}$

PDN reduced to 8 transistors: (X and X are joined)

D3


Y
$\bar{A}$
X
A
X
B
$\bar{B}$
B
$\bar{B}$
C
$\bar{C}$

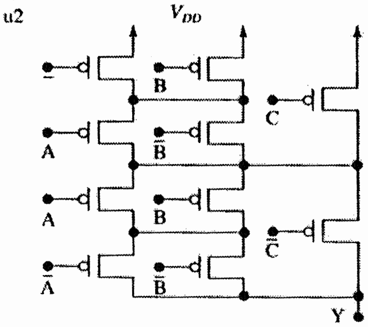
[Think circuit is not "planar", but has one "cross-over" (*x*-*x*): it has no convenient dual]

PUN as the dual of D_2:

[Think of the structure of the dual of D_1 when constructing this]

The complete circuit, using u_2 and D_2 has 20 transistors.

u2


V_{DD}
B
C
A
$\bar{B}$
A
B
$\bar{C}$
$\bar{A}$
$\bar{B}$
Y

14.37

Sum, $S = A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC$

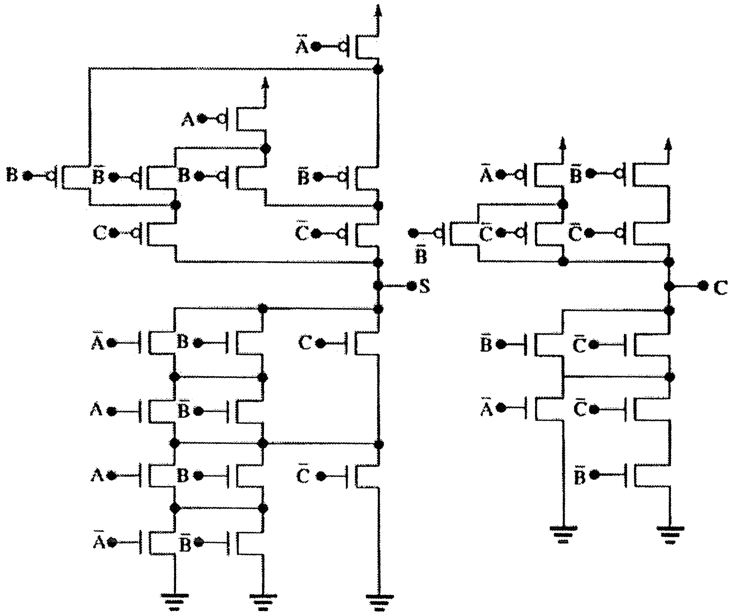
Carry $C_o = AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC$

$= AB + AC + BC = A(B + C) + BC$

Create the PUN, directly, simplifying that for S as in P13.50 above as

$S = \bar{A}(B\bar{C} + \bar{B}C) + A(\bar{B}\bar{C} + BC)$

This figure is for 14.37



14.38

For matched-inverter equivalence of the circuit

$p_A = p : p_B = p_C = p_D = 2p$

and

$n_A = n_B = 2n : n_C = n_D = 2(2n) = 4n.$

14.39 blank

14.40

Ignore the capacitances of the transistors themselves; For the matched NAND.

$t_{PLH} = t_{PHL} = t_p$. For the "uncompensated NAND, $t_{PLH} = t_p$, $t_{PHL} = t_p / 4$. Thus, t_{PLH} are the same, but t_{PHL} is 4 times greater with no matching.

14.41

For design a), there are $2(6)+2 = \underline{\underline{14}}$ transistors:

All 7 NMOS use $(W/L)_n = n$

1 PMOS uses $(W/L)_p = p$

6 PMOS use $(W/L)_p = 6p$

Total Area $= 7(1.2)0.8 + 1(3.6)0.8 + 6(6)(3.6)0.8 = \underline{\underline{113.3 \mu m^2}}$

For design b), there are $2(3)2 + 1(2)2 = \underline{\underline{16}}$ transistors:

6 NMOS use $(W/L)_n = n$

6 PMOS use $(W/L)_p = 3p$

2 PMOS use $(W/L)_p = p$

2 NMOS use $(W/L)_n = 2n$

Total equivalent devices is $6n + 18p + 2p + 2n = 10n + 20p$

Total equivalent area is $[10 + 3(20)]n = 70n$, and

Total Area $= 70(1.2)0.8 = \underline{\underline{67.2 \mu m^2}}$, or $\underline{59\%}$ of a)

14.42

Corresponding to a matched inverter characterized by n and p where $k_p = k_n = k$, the two-input NOR uses transistors n and 2p where $k_p = 2k_n$

a) For A grounded, V_{thB} occurs near $V_{DD}/2$, with Q_{PB} and Q_{NB} in saturation and Q_{PA} in trade. Let $V_{th} = v$ and the voltage across Q_{PA} be x.

Thus $i_D = k_p[(5-1)x - x^2/2]$

and $i_D = \frac{1}{2}k_p(5 - x - v - 1)^2$

and $i_D = \frac{1}{2}k_n(v-1)^2$

For $k_p = 2k_n$

$i_p = 2k_n(4x - x^2/2) = k_n(8x - x^2)$ (1)

and $i_D = k_n(4x - x - v)V^2$ (2)

and $i_D = \frac{1}{2}k_n(v-1)^2$ (3)

From 2) 3): $\pm(v-1)(0.707) = 4 - x - v$

Thus,

$1.707v = 4.707 - x$ or
$x = 4.707v - 1.707v$

$0.293v = 3.293 - x$
$x = 3.293 - 0.293v$

Now $x \approx 0$, in which case

$v = 4.707/1.707 = 2.38$ or

$v = 3.293/0.293 = 11.2$ (clearly too large)

Thus

$x = 4.707 - 1.707v$ (4)

Now, from 1), 3): $(v-1)^2 = 2(8x - x^2)$
with 4)

$v^2 - 2v + 1 = 16(4.707 - 1.707v) - 2$

$(4.707 - 1.707v)^2$ or

$v^2 - 2v + 1 = 75.32 - 27.32v - 44.31 + 32.13v$

$-5.83v^2$

or

$6.83v^2 + v(-2 + 27.32 - 32.13)$

$+ (1 - 75.32 + 44.31) = 0$

or

$6.833v^2 - 6.81v - 30.01 = 0$

whence

$v = (-6.81 \pm (6.81^2 - 4(6.83)30.01)^{\frac{1}{2}}/2(6.83)$

$=(6.81 \pm 29.43)/13.66 = 2.65\text{V}$

Check: [>2.5V probably OK since one PMOS is full on]

Thus $V_{th} = 2.65\text{V}$

b) For A and B joined, the PMOS can be approximated as a single device with twice the length for which the width is twice that in a matched inverter. Thus, for the equivalent PMOS device $(W/L)_{peq} = P$ and $k_p = k$. For each of the two NMOS $(W/L)_n = n$ and $k_n = k$.

Thus at $v_{th} = v$ with all devices in saturation:

$i_D = 2k/2(v-1)^2 = (k/2)(5 - v - 1)^2$

$2(v-1)^2 = (4-v)^2$, and

$\pm\sqrt{2}(v-1) = (4-v)$

Thus, $1.414v - 1.414 = 4 - v$,

$2.414v = 5.414$,

whence $V_{th} = v = 2.24\text{ V}$

See this is reduced from the single-input value (of 2.65V)!

Note that this fact can be used to control the relative threshold of multiple gates connected to a single fanout node in order to guarantee operation sequence for slowly changing signals.

14.43

a) $t_p \alpha \frac{\alpha C}{k' V_{DD}}$, and k' is scaled by S, and C and V_{DD} are scaled by $\frac{1}{S}$ thus t_p is scaled by

$$\frac{\frac{1}{S}}{S \times \frac{1}{S}} = \frac{1}{S}$$

$S = 4 \Rightarrow t_p$ is scaled by $\frac{1}{4}$ (t_p decreases)

The maximum operating speed is $\frac{1}{2t_p}$ and therefore is scaled by 4.

$P_{dyn} = f_{max} C V^2_{DD}$ and thus is scaled by

$S \times \frac{1}{S} \times \frac{1}{S^2} = \frac{1}{S^2} = \frac{1}{16}$ (P_{dyn} decreases) power density $= \frac{P_{\text{dyn}}}{\text{area}}$ and thus is scaled by $\frac{\frac{1}{S^2}}{\frac{1}{S^2}} = 1$

i.e., remains unchanged.

PDP is scaled by $\frac{1}{S^3}$ (power is scaled by $\frac{1}{S^2}$ and delay by $\frac{1}{S}$ and thus it is scaled by $\frac{1}{64}$ (PDP decreases)

b) If V_{DD} and V_{tn} only scaled by $\frac{1}{2}$ while $S = 4$ we have:

$t_p = \frac{\alpha C}{k' V_{DD}}$ and $\alpha = \frac{2}{\frac{7}{4} - \frac{3V_{tn}}{V_{DD}} + \left(\frac{V_{tn}}{V_{DD}}\right)^2}$ so

α remains unchanged and t_p is scaled by

$$\frac{\frac{1}{S}}{S \times \frac{1}{2}} = \frac{\frac{1}{4}}{4 \times \frac{1}{2}} = \frac{1}{8}$$

The maximum operating speed is $\frac{1}{2t_p}$ and therefore is scaled by 8.

$P_{\text{dyn}} = f_{\max} C V^2_{DD}$ and thus is scaled by

$$8 \times \frac{1}{5} \times \frac{1}{2^2} = 8 \times \frac{1}{4} \times \frac{1}{2} = 1$$

Power density $= \frac{P_{\text{dyn}}}{\text{area}}$ is thus scaled by

$$\frac{1}{\frac{1}{S^2}} = \frac{1}{\frac{1}{16}} = 16$$

PDP is scaled by $1 \times \frac{1}{8} = \frac{1}{8}$

14.44

$V_{DSsat} = \left(\frac{L}{\mu_n}\right) v_{sat} \Rightarrow \mu_n = \frac{L}{V_{DDsat}} v_{sat}$

$v_{sat} \simeq 10^7 \text{cm/s} = 10^5 \text{m/s}$, $L = 0.18\ \mu\text{m}$ and $V_{DSsat} = 0.6$ V

$$\mu_n = \frac{0.18 \times 10^{-6}\text{m}}{0.6\text{V}} \times 10^5 \text{m/s} = 0.03 \frac{\text{m}^2}{\text{V} \cdot \text{S}}$$

$$= 300 \frac{\text{cm}^2}{\text{V} \cdot \text{S}}$$

For PMOS we have $V_{DSsat} = 1$ V, thus

$$\mu_p = \frac{0.18 \times 10^{-6}\ \text{m}}{1\ \text{V}} \times 10^5 \text{m/s} = 0.018 \frac{\text{m}^2}{\text{V} \cdot \text{S}}$$

$$= 180 \frac{\text{cm}^2}{\text{V} \cdot \text{S}}$$

From equation (13.92) of the Text, we have:

$$E_{cr} = \frac{V_{DSsat}}{L}$$

For NMOS: $E_{cr} = \frac{0.6\ \text{V}}{0.18\ \mu\text{m}} = 3.33 \times 10^6 \frac{\text{V}}{\text{m}}$;

$= 3.33 \times 10^{-4} \frac{\text{V}}{\text{cm}}$

For PMOS: $E_{cr} = \frac{1\ \text{V}}{0.18\ \mu\text{m}} = 5.56 \times 10^6 \frac{\text{V}}{\text{m}}$

$= 5.56 \times 10^4 \frac{\text{V}}{\text{cm}}$

14.45

assuming $g_{\text{sat}} = 10^7$cm/s, then:

$$V_{Dsat_n} = \frac{L}{\mu_n} \times g_{\text{sat}} = \frac{0.13 \times 10^{-6}}{325 \times 10^{-4}} \times 10^7 \times 10^{-2}$$

$= 0.4$ V

$$V_{Dsat_p} = \frac{L}{\mu_p} \times g_{\text{sat}} = \frac{0.13 \times 10^{-6}}{200 \times 10^{-4}} \times 10^7 \times 10^{-2}$$

$= 0.65$ V

14.46

$$t_{PHL} = \frac{CV_{DD}}{2I_{av}}$$

Since based on the assumption in this problem, Q_N turns on immediately (V_I rises instantaneously to V_{DD}) and it operates in the velocity-saturation region then $I_{av} = I_{D\text{sat}}$ thus, $t_{PHL} = \frac{CV_{DD}}{2I_{D\text{sat}}}$

b) From equations (13.68) and (13.70) of the Text we have:

$t_{PHL} = 0.69 R_N C$ and

$$R_N = \frac{12.5\ \text{k}\Omega}{(W/L)_n} \Rightarrow t_{PHL} = 0.69\text{C}\frac{12.5 \times 10^3}{(W/L)_n}$$

c) If the formula in (a) and (b) yield the same result we have:

$\frac{CV_{DD}}{2I_{D\text{sat}}} = 0.69\text{C}\frac{12.5 \times 10^3}{(W/L)_n}$ and from equation (13.94) of the Text we have:

$$I_{D\text{sat}} = \mu_n C_{ox} \left(\frac{W}{L}\right)_n V_{DSsat} \left(V_{GS} - V_t - \frac{1}{2} V_{DSsat}\right)$$

where in this case $V_{GS} = V_{DD}$ thus:

$$\frac{V_{DD}}{2 \times 0.69 \times 12.5 \times 10^3}$$

$$= \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DSsat}\left(V_{DD} - V_t - \frac{1}{2}V_{DSsat}\right)}{\left(\frac{W}{L}\right)_n}$$

$$\Rightarrow \frac{1.2}{17250} = 325 \times 10^{-6}\left(1.2 - 0.4 - \frac{V_{DSsat}}{2}\right)$$

$$\frac{1.2}{17250} = 325 \times 10^{-6} V_{DSsat}\left(1.2 - 0.4 - \frac{V_{DSsat}}{2}\right)$$

$$1.2 = 5.61 V_{DSsat}\left(0.8 - \frac{V_{DSsat}}{2}\right)$$

$$\Rightarrow 2.805 V^2{}_{DSsat} - 4.488 V_{DSsat} + 1.2 = 0$$

$V_{DSsat} = 1.261$ V or $V_{DSsat} = 0.339$ V

The answer $V_{DSsat} = 1.261$ V is not acceptable as it is above V_{DD}. Thus, $V_{DSsat} = 0.339$ V

14.47

$$I_{DSsatn} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DSsatn}\left(V_{GS} - V_{tn} - \frac{1}{2}V_{DSsatn}\right)$$

and

$$I_{DSsatp} = \mu_n C_{ox}\left(\frac{W}{L}\right)_p |V_{DSsatp}| |V_{GS}| - |V_{tp}| - \left(-\frac{1}{2}|V_{DSsatp}|\right)$$

Since $|V_{GS}| = V_{DD}$(i.e., for NMOS $V_{GS} = V_{DD}$ and for PMOS $|V_{GS}| = V_{DD}$ and $I_{DSsatn} = I_{DSsatp}$ we have:

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DSsatn}\left(V_{DD} - V_{tn} - \frac{1}{2}V_{DSsatn}\right)$$

$$= \mu_p C_{ox}\left(\frac{W}{L}\right)_p |V_{DSsatp}|\left(V_{DD} - |V_{tp}| - \frac{1}{2}|V_{DSsatp}|\right)$$

$L_n = L_p \Rightarrow$ Thus,

$$\frac{w_p}{w_n} = \frac{\mu_n}{\mu_p}\frac{V_{DSsatn}}{|V_{DSsatp}|}\frac{V_{DD} - V_{tn} - \frac{1}{2}V_{DSsatn}}{V_{DD} - |V_{tp}|\left|\frac{1}{2}V_{DSsatp}\right|}$$

b) $$\frac{w_p}{w_n} = \frac{\mu_n}{\mu_p}\frac{V_{DSsatn}}{|V_{DSsatp}|}\frac{V_{DD} - V_{tn} - \frac{1}{2}V_{DSsatn}}{V_{DD} - |V_{tp}|\frac{1}{2}V_{DSsatp}}$$

$$= 4 \times \frac{0.34}{0.6}\;\frac{1.2 - 0.4 - \frac{0.34}{2}}{1.2 - 0.4 - \frac{0.6}{2}} = 2.86$$

14.48

a) $R = 27\ \text{m}\Omega/\square \times \frac{10\ \text{mm}}{0.5\ \mu\text{m}} = 540\ \Omega$

b) $C = 0.1\ \text{fF}/\mu\text{m} \times 10\text{mm}$

$= 0.1\ \text{fF}/\mu\text{m} \times 10000\ \mu\text{m} = 1000\ \text{fF} = 1\ \text{pF}$

c) $t_{\text{delay}} = 0.69RC = 372.6$ ps

15.1

Here $V_{DD}/4 = 5/4 = 1.25V$

Now, for v_O rising, the NMOS is cutoff, and the PMOS is in triode mode with:

$i_{Dp} = k_p[(v_{SG} - V_t)v_{SD} - v_{SD}^2/2]$, and here

$i_D = k_p[(5-0.8)(5-1.25) - (5-1.25)^2/2]$

$= k_p(15.75 - 7.03) = \underline{8.72 k_p}$

Now, for v_O falling, the net current extracted from the load is $i_{Dn} - i_{Dp}$ which should be i_{Dp}

Thus $i_{Dn} = 2 i_{Dp} = 2(8.72)k_p$, for triode operation where $i_{Dn} = k_n[(5-0.8)1.25 - 1.25^2/2]$

Overall, $i_{Dn} = 2(8.72)k_p = k_n(5.25 - 0.78) = 4.47 k_n$

Thus $k_p = (4.47/(2(8.72)))k_n = \underline{0.256} k_n$

Check using Eq 10.39, where $r = k_n/k_p = 3.91$:

$V_{OL} = (V_{DD} - V_t)[1 - (1 - 1/r)^{1/2}]$

$= (5 - 0.8)[1 - (1 - 1/3.91)^{1/2}] = \underline{0.577V}$ ✓

From Eq 13.35, $V_{IL} = V_t + (V_{DD} - V_t)/(r(r+1))^{1/2}$

$= 0.8 + 4.2/(3.91 \times 4.91)^{1/2} = \underline{\underline{1.76V}}$

From Eq 13.38, $V_{IH} = V_t + (2/\sqrt{3r})(V_{DD} - V_t)$

$= 0.8 + (2/\sqrt{3(3.91)})4.2 = \underline{\underline{3.25V}}$

From Eq 13.36, $V_M = V_t + (V_{DD} - V_t)/(r+1)^{1/2}$

$= 0.8 + 4.2/(4.91)^{1/2} = \underline{\underline{2.70V}}$

Now, $NM_H = V_{OH} - V_{IH} = 5.00 - 3.25 = \underline{\underline{1.75V}}$

and $NM_L = V_{IL} - V_{OL} = 1.76 - 0.58 = \underline{\underline{1.18V}}$

15.2

$$\alpha_p = 2\Big/\left[\frac{7}{4} - 3\left(\frac{0.4}{1.2}\right) + \left(\frac{0.4}{1.2}\right)^2\right] = 2.3$$

$$t_{PLH} = \frac{\alpha_p C}{k_p V_{DD}} = \frac{2.3(10f)}{\left(\frac{430\ \mu}{4}\right)\cdot\left(\frac{W}{L}\right)_p \cdot 1.2}$$

$$r = \frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}{\mu_p C_{ox}\left(\frac{W}{L}\right)_p} = 4$$

$$\frac{430\ \mu(1)}{\frac{430\ \mu}{4}\left(\frac{W}{L}\right)_p} = 4 \Rightarrow \left(\frac{W}{L}\right)_p = \frac{4}{4} = 1$$

t_{PLH} = 0.18 nsec or 180 psec

using eq. (14.17) and (14.18)

$$\alpha_n = 2\Big/\left[1 + \frac{3}{4}\left(1 - \frac{1}{r}\right) - \left(3 - \frac{1}{r}\right)\left(\frac{V_t}{V_{DD}}\right)^2\right] = 2.6$$

$$t_{PHL} \cong \frac{\alpha_n C}{k_n V_{DD}} = \frac{2.6(10f)}{(430\ \mu)(1)(1.2)} = 50 \text{ psec}$$

$$t_P = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(180p + 50p) = 115 \text{ psec}$$

15.3

$NM_L = V_t - (V_{DD} - V_t)[1 - (1 - \frac{1}{r})^{1/2} - (r(r+1))^{-1/2}]$

Now, $\partial NM_L/\partial r = -(V_{DD} - V_t)[-\frac{1}{2}(1 - \frac{1}{r})^{-1/2}(-\frac{1}{r^2}) - (-\frac{1}{2}(r(r+1))^{-3/2}(2r+1))]$

Maximum occurs where:

$-\frac{1}{2}(1 - 1/r)^{-1/2}(1/r^2) = -\frac{1}{2}(r(r+1))^{-3/2}(2r+1)$

Square both sides: $(1 - 1/r)^{-1}/r^4 = r^{-3}(r+1)^{-3}(2r+1)^2$

or $\frac{1}{1 - 1/r} = \frac{r}{(r+1)^3}(2r+1)^2 = \frac{r}{r-1}$

15.4 blank

15.5

$NM_H = (V_{DD} - V_t)(1 - 2/\sqrt{3r})$

This is zero, when $1 - 2/\sqrt{3r} = 0$, or $\sqrt{3r} = 2$, or $3r = 4$, or $r = \underline{\underline{1.33}}$

For $r = 1$, $NM_H = 4.2(1 - 2/\sqrt{3}) = \underline{\underline{-0.65}}V$

For $r = 2$, $NM_H = 4.2(1 - 2/\sqrt{6}) = \underline{\underline{0.77V}}$

For $r = 4$, $NM_H = 4.2(1 - 2/\sqrt{12} = \underline{\underline{1.78V}}$

For $r = 8$, $NM_H = 4.2(1 - 2/\sqrt{24} = \underline{\underline{2.48V}}$

For $r = 16$, $NM_H = 4.2(1 - 2/\sqrt{48} = \underline{\underline{2.99V}}$

But, what about NM_L? (For $r = 16$, it is 0.92V)

15.6

noise margins are equal when

$$V_t - (V_{DD}-V_t)[1-(1-\tfrac{1}{r})^{1/2} - 1/(r(r+1))^{1/2}] = (V_{DD}-V_t)[1-2/(3r)^{1/2}]$$

or $V_t/(V_{DD}-V_t) = 2 - 2/(3r)^{1/2} - (1-1/r)^{1/2} - 1/(r(r+1))^{1/2}$ ---(1)

Here $V_t/(V_{DD}-V_t) = 0.8/(5.0-0.8) = \underline{0.1904}$

Try various values of r to solve (1):

For $r=2$, $f(r) = 2 - 2/6^{1/2} - (1-1/2)^{1/2} - 1/(2(3))^{1/2}$
$= 2 - 0.816 - 0.707 - 0.408 = \underline{0.069}$

For $r=3$, $f(r) = 2 - 2/9^{1/2} - (1-1/3)^{1/2} - 1/(3(4))^{1/2}$
$= 2 - 0.667 - 0.816 - 0.289 = \underline{0.228}$

Try $r=2.8$, $f(r) = 2 - 2/(3(2.8))^{1/2} - (1-1/2.8)^{1/2} - 1/(2.8(3.8))^{1/2}$
$= 2 - 0.690 - 0.802 - 0.307 = \underline{0.201}$

Try $r=2.7$, $f(r) = 2 - 2/(3(2.7))^{1/2} - (1-1/2.7)^{1/2} - (1/(2.7(3.7)))^{1/2}$
$= 2 - 0.703 - 0.793 - 0.316 = \underline{0.188}$

Conclude $r \simeq \underline{\underline{2.72}}$, for which the margins are:

$NM = NM_H = NM_L = (V_{DD}-V_t)(1-2/(3r)^{1/2})$
$= 4.2(1-2/(3(2.72))^{1/2}) = \underline{\underline{1.26V}}$

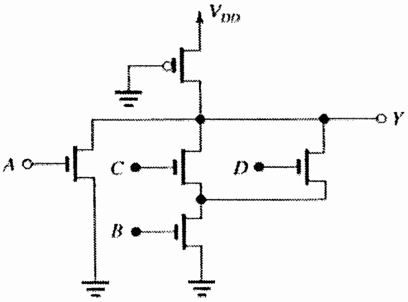
15.7 blank

15.8

$Y = \overline{A + B(C+D)}$,

whence $\overline{Y} = A + B(C+D)$

Thue the PDN can be formed directly as shown :



Now, $t_{PLH}/t_{PHL} = (k_n/k_p)(1-0.46/r)$

$= r(1-0.46/r) = 2.72 - 0.46 = 2.26$

Now,

$t_{PLH} = 1.7(1 \times 10^{-12})/(25 \times 10^{-6}(1.33/0.8)(5))$

$= 8.24\text{ns}$

and $t_{PHL} = 8.24/2.26 = 3.65\text{ns}$

and $t_p = (8.24 + 3.65)/2 = 5.95\text{ns}$

Now, dynamic power is approximately $fC\,V^2_{DD}$ since the output swing is not quite V_{DD}.

For equal static and dynamic power

$f \times 1 \times 10^{-12} \times 5^2 = 1.82 \times 10^{-3}$

whence

$f = 1.82 \times 10^{-3}/(25 \times 10^{-12}) = 72.8\text{MHz}$

for which the period is

$1/(72.8 \times 10^6) = 13.7$ ns

Now, for transition times in the same proportion as propagation delays

$t_{TLH}/t_{TTHL} = 8.24/3.65 = 2.26$

Now, for full output swing, there must be time for 2 full transitions in each cycle :

Thus

$t_{THL} \simeq 13.7/(1 + 2.26) = 4.19\text{ns}$ and

$t_{TLH} \simeq 4.19(2.26) = 9.47\text{ns}$

Since these values are of the same order as the propagation delays. Full swing operation is likely not possible at 72.8MHz.

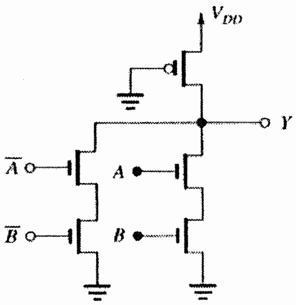
15.9

For an Exclusiue OR, $Y = A\overline{B} + \overline{A}B$, and

$\overline{Y} = \overline{A\overline{B} + \overline{A}B} = \overline{A\overline{B}} \cdot \overline{\overline{A}B} = (\overline{A} + B)(A + \overline{B})$

or $\overline{Y} = \overline{A}\overline{B} + AB$

The PDN results directly :

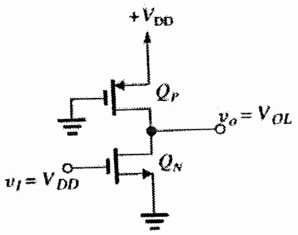


15.10

For a pseudo-NMOS NOR gate, independant of the number of inputs, the worst-case value of V_{OL} occurs for one input high (and a single NMOS conducting

From Eq. 10.39, $V_{OL} = (V_{DD} - V_t)[1 - (1 - 1/r)^{1/2}]$,
for which $0.2 = (5.0 - 0.8)[1 - (1 - 1/r)^{1/2}]$,
and $(1 - 1/r)^{1/2} = 1 - 0.2/4.2 = 0.952$
Thus $1 - 1/r = 0.907$
$1/r = 0.093$, and $r = \underline{10.76}$
Thus $k_n/k_p = 10.76 = 75(1.8/1.2)/(25(W/L)_p)$
Thus $(W/L)_p = (75/25)(1.8/1.2)/10.76 = \underline{0.418}$
Thus for $W_p = 1.8\mu m$, $L_p = 1.8/0.418 = \underline{4.31\mu m}$
and $(W/L)_p = \underline{(1.8/4.31)}$

15.11



$V_{Dsat} = 0.6V$

$V_{DD} = 1.2V$

$V_t = 0.4V$

$\mu_n C_{ox} = 4\mu_p C_{ox} = 430\mu A / V^2$

$L = 0.13\mu m$

In the case of Q_p,

$V_{SG} - V_t = 1.2V - 0.4V = 0.8V$, which is $> V_{Dsat}$

For reliable logic levels and noise margins, $V_{SD} > V_{Dsat}$ so that Q_p is operating in the velocity saturation region.

Ignoring channel-length modulation,

$$I_{Dsat} = \mu_p C_{ox}\left(\frac{W}{L}\right)_p |V_{Dsat}| \cdot \left[V_{SG} - |V_t| - \frac{1}{2}|V_{Dsat}|\right]$$

$$= \frac{1}{4}(430\mu A / V^2)\left(\frac{W}{L}\right)_p (0.6V) \bullet$$

$$\left[1.2V - 0.4V - \frac{1}{2}(0.6V)\right]$$

$$I_{Dsat} = 32.25\left(\frac{W}{L}\right)_p \mu A$$

For Q_N,

$V_{GS} - V_t = 1.2V - 0.4V = 0.8V$

and $V_{DSat} = 0.6V$

$V_{GS} - V_t > V_{DSat}$, but $V_{DS} < V_{DSat}$

This defines the triode region.

$$i_{DN} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{DS}\left[(V_{GS} - V_t) - \frac{1}{2}V_{DS}\right]$$

$$i_{DN} = (430\mu A / V^2)\left(\frac{W}{L}\right)_n \left[(0.8V)V_{DS} - \frac{1}{2}V^2{}_{DS}\right]$$

The unknowns in this problem are $\left(\frac{W}{L}\right)_n$, $\left(\frac{W}{L}\right)_p$ and V_{DS}. One possibility would be to match the source and sink currents for charging the output capacitance.

Without further information, let us assume as in Exercise 13.26

that $\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p = 1.5$

In this case,

$I_{DSat} = 32.25(1.5)\mu A = 48.4\mu A$

In the static state,

$i_{DN} = i_{DP}$ so that

$$(430\mu A / V^2)(1.5)\left[(0.8V)v_{DS} - \frac{1}{2}v^2{}_{DS}\right]$$

$$= 48.4\mu A$$

$$0.8v_{DS} - \frac{1}{2}v^2{}_{DS} = \frac{48.4\mu A}{430\mu A(1.5)} = 0.075$$

$$v_{DS}^2 - 1.6v_{DS} + 0.15 = 0$$

$$v_{DS} = \frac{1.6 \pm \sqrt{(1.6)^2 - 4(1)(0.15)}}{2} = 0.8 \pm 0.7\ V$$

$V_{OL} = v_{DS} = 0.1V$

15.12

(a) $V_{OH} = V_{DD} - V_t$

and $V_t = V_{to} + \gamma(\sqrt{V_{OH} + 2\phi_f} - \sqrt{2\phi_f})$

so, $V_t = V_{to} + \gamma(\sqrt{V_{DD} - V_t + 2\phi_f} - \sqrt{2\phi_f})$

Substituting values, we have

$V_t = 0.5V + 0.3V^{1/2} \times$

$(\sqrt{1.8V - V_t + 0.85V} - \sqrt{0.85V})$

$V_t - 0.22V = (0.3V^{1/2})(\sqrt{2.65V - V_t})$

Squaring both sides,

$V^2_t - 0.44V_t + 0.048 = 0.09(2.65 - V_t)$

or $V_t^2 - 0.35V_t - 0.191 = 0$

Solving this qundratic, we obtain

$V_t = 0.646\text{V}$

So that,

$V_{OH} = V_{DD} - V_t = 1.8\text{V} - 0.646\text{V} = 1.15\text{V}$

(b) $i_{DP} = \frac{1}{2}\mu p C_{ox}\left(\frac{W}{L}\right)_p (V_{DD} - V_{OH} - V_{to})^2$

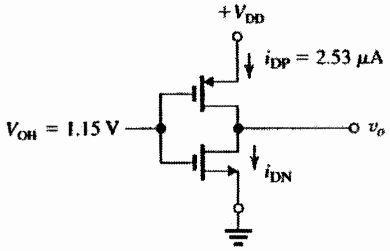
$= \frac{1}{2}75\mu(\text{A}/\text{V}^2)\left(\frac{0.54}{0.18}\right)(1.8\text{V} - 1.15\text{V} - 0.5\text{V})^2$

$i_{DP} = 2.53\mu\text{A}$

$P_D = V_{DD}i_{DP} = 1.8\text{V}(2.53\mu\text{A}) = 4.6\mu W$

To find the inverter's output voltage, we note that

$i_{DN} = i_{DP} = 2.53\mu\text{A}$



Since $V_{DS} < V_{GS} - V_t$ (triode region), we can to find v_o:

$i_{DN} = k_n\left[(v_I - V_t)v_o - \frac{1}{2}v_o^2\right]$

where $V_t = V_{to}$

$2.53\mu\text{A} = 300\mu\text{A}/\text{V}^2\left(\frac{0.54}{0.18}\right) \times$

$\left[(1.15\text{V} - 0.5\text{V})v_o - \frac{1}{2}v_o^2\right]$

$\frac{2.53\mu\text{A}}{300\mu\text{A}/\text{V}^2(1.5)} = 0.65\text{ V } v_o - \frac{1}{2}v_o^2$

or,

$v_o^2 - 1.3v_o + 0.0112 = 0$

solving for v_o, we get

$v_o = 0.01\text{V}$

(c) To find t_{PLH}, we can follow the procedure of

$i_{D(0)} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{to})^2$

$= \frac{1}{2}300\mu(\text{A}/\text{V}^2)1.5(1.8 - 0.5)^2\text{V}^2 = 380.3\mu\text{A}$

$V_t(at\ v_o = 0.9\text{V}) = V_{to} + \gamma(\sqrt{v_O + 2\phi_f} - \sqrt{2\phi_f})$

$= 0.5\text{ V} + 0.3\text{V}^{1/2}(\sqrt{0.9\text{V} + 0.85\text{V}} - \sqrt{0.85\text{V}})$

$V_t = 0.62\text{ V}$

$i_D(t_{PLH}) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_o - V_t)^2$

$= \frac{1}{2}(300\mu\text{A}/\text{V}^2)(1.5)(1.8\text{V} - 0.9\text{V} - 0.62\text{V})^2$

$= 17.6\mu\text{A}$

$i_D|_{av} = \frac{380.3\mu\text{A} + 17.6\mu\text{A}}{2} = 199\mu\text{A}$

$t_{PLH} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_{D|av}} = \frac{10(10^{-15})F\left(\frac{1.8\text{ V}}{2}\right)}{199(10^{-6})\text{A}}$

$= 0.045\text{ ns}$

(d) For V_I going LOW

$i_{D(o)} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{to})^2$

$= \frac{1}{2}(300\mu\text{A}/\text{V}^2)(1.5)(1.8\text{V} - 0.5\text{V})^2 = 380.3\mu\text{A}$

At $t = t_{PHL}$,

$i_D(t_{PHL}) = k_n\left[(V_{DD} - V_{to})v_o - \frac{1}{2}v^2_o\right]$

$= 300\mu\text{A}/\text{V}^2\left(\frac{1}{2}\right) \times$

$\left[(1.8\text{V} - 0.5\text{V})(0.9\text{V}) - \frac{1}{2}(0.9\text{V})^2\right]$

$i_D(t_{PHL}) = 114.8\mu\text{A}$

$i_D|_{av} = \frac{1}{2}(380.3\mu\text{A} + 114.8\ \mu\text{A}) = 247.6\mu\text{A}$

so that,

$t_{PHL} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{10(10^{-15})F\left(\frac{1.8\text{V}}{2}\right)}{247.6(10^{-6})\text{A}}$

$= 0.036\text{ns}$

(e) t_p

$= \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2}(0.045\text{ ns} + 0.036\text{ ns})$

$= 0.04\text{ ns}$

15.13

For a) see directly that $X = 1.\overline{A} = \overline{A}$

and $Y = X \cdot \overline{B} = \overline{A} \cdot \overline{B}$

For b) see directly that $Y = \overline{A} \cdot \overline{B}$

For each circuit node Y nominally satisfies both conditions. However in a) with A high and B low, Y is not pulled down completely to ground, but remains at V_{tp}, due to the PMOS threshold. Circuit b) does not have this problem, but node X is floating for A,B both high. However, X is not an output node. The body effect makes this worse! Notice that b) is exactly a complemantary CMOS NOR gate for which $Y = \overline{A} \cdot \overline{B} = \overline{A + B}$

For V_{DD} replaced by an inverter driven by C,

$Y = \overline{C}(\overline{A} \cdot \overline{B}) = \overline{A} \cdot \overline{B} \cdot \overline{C} = \overline{A + B + C}$,

a 3-input NOR for both a) and b).

Practically speaking, however, there is a problem because as noted above, the series PMOS do not operate well with a low input. In fact Y is pulled down only to one thueshold drop below ground, when C is high.

15.14

For the switch gate and input both at $V_{DD} = 3.3\text{V}$, the switch output is

$V_{OH} = V_{DD} - V_t$

where $V_t = V_{to} + \gamma[\sqrt{V_{OH} + 2\phi_F} - \sqrt{2\phi_F}]$

Substituting for V_{OH}, we get :

$V_t = V_{to} + \gamma[\sqrt{V_{DD} - V_t + 2\phi_F} - \sqrt{2\phi_F}]$

$= 0.8\text{V} + 0.5\text{V}^{1/2}[\sqrt{3.3\text{V} - V_t + 0.6\text{V}} - \sqrt{0.6\text{V}}]$

So that,

$V_t = 0.413\text{ V} + 0.5\text{ V}^{1/2}\sqrt{3.9\text{ V} - V_t}$

$V_t - 0.413\text{ V} = 0.5\text{V}^{1/2}\sqrt{3.9\text{ V} - V_t}$

Squaring both sides, we get

$V_t^2 - 0.826V_t + 0.171 = 0.975 - 0.25V_t$

or, $V_t^2 - 0.576V_t - 0.804 = 0$

Solving this quadratic, we find that

$V_t = 1.23\text{V}$

$V_{OH} = V_{DD} - V_t = 3.3\text{V} - 1.23\text{V} = 2.07\text{V}$

with the input Low and the gate switch HIGH,

$V_{OL} \to 0\text{V}$

If $V_{OH} = 2.07\text{V}$, the PMOS transistor of the inverter is in the saturation region. Since the inverter transistors are matched,

$\left(\frac{W}{L}\right)_p = \frac{k_n'}{k_p'}\left(\frac{W}{L}\right)_n$ so that

$$i_{DP} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_{DD} - V_{OH} - V_{to})^2$$

$$i_{DP} = \frac{1}{2}(25\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(3) \times$$

$$(3.3\text{V} - 2.07\text{V} - 0.8\text{V})^2 = 10.4\mu\text{A}$$

For t_{PLH}, at $t = 0$,

$$i_D(0) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{to})^2$$

$$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(3.3\text{V} - 0.8\text{V})^2 = 352\mu\text{A}$$

At $v_O = \frac{V_{DD}}{2}$,

$$V_t = V_{to} + \gamma\left[\sqrt{\frac{V_{DD}}{2} + 2\phi_F} - \sqrt{2\phi_F}\right]$$

$$= 0.8\text{ V} + 0.5\text{ V}^{1/2}\left[\sqrt{\frac{3.3\text{ V}}{2} + 0.6\text{ V}} - \sqrt{0.6\text{ V}}\right]$$

$$= 1.16\text{ V}$$

$$i_D(t_{PLH}) = \frac{1}{2}\mu_n C_{OX}\left(\frac{W}{L}\right)_n (V_{DD} - v_o - v_t)^2$$

$$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)\left(3.3\text{V} - \frac{3.3\text{V}}{2} - 1.16\text{V}\right)^2$$

$$= 13.5\mu\text{A}$$

$$i_D\big|_{av} = \frac{(352\mu\text{A} + 13.5\mu\text{A})}{2} = 183\mu\text{A}$$

$$t_{PLH} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D\big|_{av}} = \frac{100(10^{-15})_F(1.65V)}{183\mu\text{A}}$$

$$= 0.9\text{ ns}$$

For t_{PHL}, $V_t = V_{to}$ and

$$i_{D(o)} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{DD} - V_{to})^2$$

$$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(3.3\text{V} - 0.8\text{V})^2 = 352\mu\text{A}$$

$$i_D(t_{PHL}) = \mu_n C_{ox}\left(\frac{W}{L}\right)_n\left[(V_{DD} - V_{to})v_o - \frac{1}{2}v_o^2\right]$$

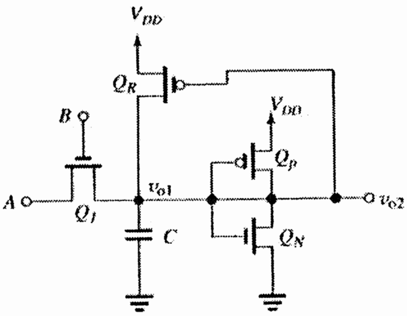
$$= 75\mu\text{A}/\text{V}^2\left(\frac{1.2}{0.8}\right)$$

$$\left[(3.3\text{ V} - 0.8\text{ V})\left(\frac{3.3\text{ V}}{2}\right) - \frac{1}{2}\left(\frac{3.3\text{ V}}{2}\right)^2\right] = 311\mu\text{A}$$

$$i_D\big|_{av} = \frac{1}{2}(352\mu\text{A} + 311\mu\text{A}) = 332\mu\text{A}$$

$$t_{PHL} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D\big|_{av}} = \frac{100(10^{-15})F(1.65\text{ V})}{332\mu\text{A}} = 0.5\text{ ns}$$

15.15



For the inverter, with

$v_{o2} = V_{DD} - |V_{to}| = 3.3\text{ V} - 0.8\text{ V} = 2.5\text{ V}$

Q_N is in the saturation region, so that

$$i_{DN} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (v_{o1} - v_{to})^2$$

$$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(v_{o1} - 0.8\text{ V})^2$$

$$= 56.25(v_{o1} - 0.8\text{ V})^2\mu\text{A}$$

Q_p is operating in the triode region so

$$i_{DP} = k_P'\left(\frac{W}{L}\right)_p [(V_{DD} - v_{o1} - V_{to})(v_{DS})$$

$$-\frac{1}{2}(v_{DS})^2]$$

$$= (25\mu\text{A}/\text{V}^2)\left(\frac{3.6}{0.8}\right)\times$$

$$\left[(3.3\text{V} - v_{o1} - 0.8\text{ V})(0.8\text{ V}) - \frac{1}{2}(0.8\text{ V})^2\right]$$

$$= 112.5[1.68 - 0.8v_{o1}]\mu\text{A}$$

Since $i_{DP} = i_{DN}$, we set these equal :

$$56.25(v_{o1} - 0.8\text{V})^2 = 189 - 90v_{o1}$$

$$56.25(v^2_{o1} - 1.6v_{o1} + 0.64) = 189 - 90v_{O1}$$

Simplifying, we get

$$v_{o1}^2 - 2.72 = 0$$

$$v_{o1} = \sqrt{2.72} = 1.65\text{ V}$$

for Q_1,

$$v_t = v_{to} + \gamma[\sqrt{v_{o1} + 2\phi_f} - \sqrt{2\phi_f}]$$

$$V_t = 0.8\text{V} + 0.5\text{V}^{1/2}[\sqrt{1.65\text{V} + 0.6\text{V}} - \sqrt{0.6\text{V}}]$$

$$= 1.16\text{ V}$$

Capacitor charging current before Q_R turns on is due to the current supplied by Q_r.

$$\text{At } v_{o1} : i_D = \frac{1}{2}k_n\left(\frac{W}{L}\right)_n (V_{DD} - v_{O1} - v_t)^2$$

$$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(3.3\text{V} - 1.65\text{V} - 1.16\text{V})^2$$

$$= 13.5\mu\text{A}$$

At $v_{O1} = 0\text{V}$,

$$i_D = \frac{1}{2}k_n\left(\frac{W}{L}\right)_n (V_{DD} - v_{to})^2$$

$$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(3.3\text{V} - 0.8\text{V})^2 = 351.6\mu\text{A}$$

$$i_D|_{av} = \frac{1}{2}(13.5\mu\text{A} + 351.6\mu\text{A}) = 182.6\mu\text{A}$$

$$t_{PLH} = \frac{Cv_{O1}}{i_D|_{av}} = \frac{20(10^{-15})\text{F}(1.65\text{V})}{182.6(10^{-6})\text{A}} = 0.18\text{ ns}$$

(b) For the inventer,

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t)$$

$$= \frac{1}{8}[5(3.3\text{ V}) - 2(0.8\text{ V})] = 1.86\text{ V}$$

For this value,

$$i_{D1} = k_n'\left(\frac{W}{L}\right)_n\left[(V_{DD} - v_{to})(v_{DS1}) - \frac{1}{2}(v_{DS1})^2\right]$$

$$i_{D1} = (75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)\times$$

$$\left[(3.3\text{ V} - 0.8\text{ V})(1.86\text{ V}) - \frac{1}{2}(1.86\text{ V})^2\right]$$

$$i_{D1} = 328.5\mu\text{A}$$

The current in

$$Q_R = k_P'\left(\frac{W}{L}\right)_R (V_{DD} - v_{to})^2 = \frac{i_{D1}}{2}$$

So,

$$(25\mu\text{A}/\text{V}^2)\left(\frac{W}{L}\right)_R (3.3\text{ V} - 0.8\text{ V})^2 = \frac{328.5\mu\text{A}}{2}$$

$$\left(\frac{W}{L}\right)_R = \frac{328.5\mu\text{A}}{2} = \frac{1}{(25\mu\text{A}/\text{V}^2)(2.5\text{ V})^2} = 1.05$$

OR,

$$\left(\frac{W}{L}\right)_R = \frac{W}{0.8\mu\text{m}} \Rightarrow W = 0.84\mu\text{m, and}$$

$$\left(\frac{W}{L}\right)_R = \frac{0.84\mu\text{m}}{0.8\mu\text{m}}$$

Initially, at $v_{O1} = V_{DD}, i_{DR} = 0$, since

$$V_{DSR} = o\text{, and } i_{D1} = \frac{1}{2}k_n\left(\frac{W}{L}\right)_n (V_{DD} - v_{to})^2$$

$$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)\left(\frac{1.2}{0.8}\right)(3.3\text{V} - 0.8\text{V})^2 = 352\mu\text{A}$$

At $V_{O1} = V_{IH} = 1.86\text{ V}$,

$$i_{DR} = k_P\left(\frac{W}{L}\right)_R [(V_{SG} - v_{to})\times$$

$$(V_{DD} - V_{IH}) - \frac{1}{2}(V_{DD} - V_{IH})^2]$$

$= (25\mu A / V^2)(1.05)[(3.3V - 0.8V)(3.3V - 1.86V)$

$-\frac{1}{2}(3.3V - 1.86V)^2]$

$= 67.3\mu A$

$i_{D1} = k_n'\left(\frac{W}{L}\right)_n[(V_{GS} - v_{to})(V_{DS}) - \frac{1}{2}(V_{DS})^2]$

$= (75\mu A / V^2)\left(\frac{1.2}{0.8}\right)[(3.3V - 0.8V)(1.86V)$

$-\frac{1}{2}(1.86V)^2] = 328.5\mu A$

$i_C|_{av} = \frac{1}{2}(328.5 + 352 - 67.3 - 0)\mu A = 306.6\mu A$

$t_{PHL} \simeq$

$\frac{C\Delta v_{OI}}{i_C|_{av}} = \frac{20(10^{-15})F(3.3V - 1.86V)}{306.6\mu A} = 94 \text{ ps}$

15.16

(a) When the input goes HIGH, Q is ON and V_{OH} will approach$+V_{DD}$

(b) when the input goes Low and Q is ON,

$V_{OL} \to |V_{tp}|$

(c) $i_D(o) = \frac{1}{2}k_P(V_{DD} - |V_{to}|)^2$

$i_D(o) = \frac{1}{2}(225\mu A / V^2)(1.8 \text{ V} - 0.5 \text{ V})^2 = 190\mu A$

when $v_o = \frac{V_{DD}}{2} = 0.9 \text{ V}$,

$i_D(t_{PLH}) = k_P[(V_{DD} - |V_{to}|)\left(\frac{V_{DD}}{2}\right)$

$-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2]$

$i_D(t_{PLH}) = 225\mu A / V^2[(1.8 \text{ V} - 0.5 \text{ V})(0.9 \text{ V})$

$-\frac{1}{2}(0.9 \text{ V})^2]$

$= 172\mu A$

$i_D|_{av} = \frac{1}{2}(190\mu A + 172\mu A) = 181\mu A$

$t_{PLH} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{C(0.9V)}{181\mu A} = 5000C$

15.17

$V_{OH} = V_{DD} = 1.8 \text{ V}$

$V_{OL} = 0 \text{ V}$

(b) $i_{DN}(o) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{to})^2$

$= \frac{1}{2}(300\mu A / V^2)(1.5)(1.8V - 0.5V)^2 = 380.3\mu A$

$i_{DP}(o) = \frac{1}{2}k_P'\left(\frac{W}{L}\right)_p(V_{DD} - V_{to})^2$

$= \frac{1}{2}(75\mu A / V^2)(1.5)(1.8 - 0.5)^2 V^2$

$= 95.1\mu A$

$i_{DN}(t_{PLH})$ can be found by finding V_t

when $v_O = \frac{V_{DD}}{2}$:

$V_t = V_{to} + \gamma(\sqrt{v_O + 2\phi_f} - \sqrt{2\phi_f})$

$V_t = 0.5 \text{ V} + 0.3 \text{ V}^{1/2}(\sqrt{0.9 \text{ V} + 0.85 \text{ V}} - \sqrt{0.85 \text{ V}})$

$V_t = 0.62\ \text{V}$

$i_{DN}(t_{PLH}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n\left(V_{DD} - \frac{V_{DD}}{2} - V_t\right)^2$

$= \frac{1}{2}(300\mu\text{A}/\text{V}^2)(1.5)(1.8 - 0.9 - 0.62)^2$

$= 17.64\mu\text{A}$

$i_{DP}(t_{PLH}) = k_p'\left(\frac{W}{L}\right)_p [(V_{DD} - V_{to})\left(\frac{V_{DD}}{2}\right)$

$-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2]$

$= (75\mu\text{A}/\text{V}^2)(1.5)[(1.8\ \text{V} - 0.5\ \text{V})(0.9\ \text{V})$

$-\frac{1}{2}(0.9\ \text{V})^2]$

$= 86.1\mu\text{A}$

$i_D|_{av} \simeq$

$\frac{1}{2}[i_{DN}(0) + i_{DP}(0) + i_{DN}(t_{PLH}) + i_{DP}(t_{PLH})]$

$i_D|_{av} \simeq$

$\frac{1}{2}[380.3 + 95.1 + 17.6 + 86.1]\mu\text{A} = 290\mu\text{A}$

$$t_{PLH} = \frac{C\frac{V_{DD}}{2}}{i_D|_{av}} = \frac{15(10^{-15})F\left(\frac{1.8\text{V}}{2}\right)}{290\mu\text{A}} = 0.047\ \text{ns}$$

(c) For the situation in Fig. 14.12(b),

$i_{DN}(o) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{to})^2$

$= \frac{1}{2}(300\mu\text{A}/\text{V}^2)(1.5)(1.8\ \text{V} - 0.5\ \text{V})^2 = 380.3\mu\text{A}$

$i_{DP}(o) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(V_{DD} - |V_{to}|)^2$

$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)(1.5)(1.8\text{V} - 0.5\text{V})^2 = 95.1\mu\text{A}$

At $V_o = \frac{V_{DD}}{2} = 0.9\ \text{V}$,

$i_{DN}(t_{PHL}) = k_n'\left(\frac{W}{L}\right)_n\left[(V_{DD} - V_{to})v_o - \frac{1}{2}v_o^2\right]$

$= (300\mu\text{A}/\text{V}^2)(1.5)[(1.8\text{V} - 0.5\text{V})(0.9\text{V})$

$-\frac{1}{2}(0.9\ \text{V})^2]$

$= 344.3\mu\text{A}$

To estimate $i_{DP}(t_{PHL})$, we find $|V_{tp}|$ at

$v_o = \frac{V_{DD}}{2}$:

$|V_{tp}|(at\ V_o = 0.9\ \text{V}) = |V_{to}| + \gamma \times$

$[\sqrt{V_{DD} - v_o + 2\phi_f} - \sqrt{2\phi_f}]$

$= 0.5\ \text{V} + 0.3\ \text{V}^{1/2}[\sqrt{1.8\ \text{V} - 0.9\ \text{V} + 0.85\ \text{V}}$

$- \sqrt{0.85\ \text{V}}]$

$= 0.62\ \text{V}$

$i_{DP}(t_{PHL}) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(V_{DD} - v_o - V_t)^2$

$= \frac{1}{2}(75\mu\text{A}/\text{V}^2)(1.5)(1.8\ \text{V} - 0.9\ \text{V} - 0.62\ \text{V})^2$

$= 4.41\mu\text{A}$

$i_D|_{av} \simeq$

$\frac{1}{2}[i_{DN}(0) + i_{DP}(0) + i_{DN}(t_{PHL}) + i_{DP}(t_{PHL})]$

$= \frac{1}{2}[380.3 + 95.1 + 344.3 + 4.41]\mu\text{A} = 412\mu\text{A}$

$$t_{PHL} = \frac{C\left(\frac{V_{DD}}{2}\right)}{i_D|_{av}} = \frac{15(10^{-15})F(0.9\ \text{V})}{412\mu\text{A}}$$

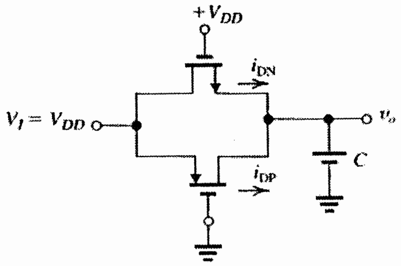
$= 0.033\ \text{ns}$

Q_P will turn off when $v_o = |V_{tp}| = 0.5\ \text{V}$

(d) $t_p \simeq \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2}(0.047 + 0.033)\ \text{ns}$

$= 0.04\ \text{ns}$

15.18



With V_I going to V_{DD} and $v_o(\text{o}) = 0\ \text{V}$,

$$R_{Neq} = \frac{V_{DD} - v_o}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{to} - v_o)^2}$$

$$= \frac{1.8\ \text{V} - 0\ \text{V}}{\frac{1}{2}(300\mu\text{A}/\text{V}^2)(1.5)(1.8\ \text{V} - 0.5\ \text{V} - 0v)^2}$$

$= 4.7\ \text{k}\Omega$

$$R_{Peq} = \frac{V_{DD} - 0}{\frac{1}{2}k_p'\left(\frac{W}{L}\right)_p(V_{DD} - |V_{tp}|)^2}$$

$$= \frac{1.8\ \text{V} - 0}{\frac{1}{2}(75\mu\text{A}/\text{V}^2)(1.5)(1.8\ \text{V} - 0.5)^2}$$

$R_{Peq} = 18.9\ \text{k}\Omega$

$R_{TG}(V_o = 0) = R_{Neq} \| R_{Peq} = 4.7\ \text{k}\Omega \| 18.9\ \text{k}\Omega$

$= 3.76\ \text{k}\Omega$

when $V_o = 0.9\ \text{V}$ Q_N is still considered in the saturation region.

So, R_{Neq} $(V_o = 0.9\text{ V})$

$$= \frac{V_{DD} - v_O}{\frac{1}{2}k_n'(W/L)(V_{DD} - V_{tn} - v_o)^2}$$

Where $V_{tn} = V_{to} + \gamma(\sqrt{V_o + 2\phi_f} - \sqrt{2\phi_f})$

$= 0.5 + 0.3\text{ V}^{1/2}(\sqrt{0.9\text{ V} + 0.85\text{ V}} - \sqrt{0.85\text{ V}})$

$= 0.62\text{ V}$

$$R_{Neq} = \frac{1.8\text{ V} - 0.9\text{ V}}{\frac{1}{2}(300\mu\text{A}/\text{V}^2)(1.5)(1.8\text{ V} - 0.62\text{ V} - 0.9\text{ V})^2}$$

$= 51\text{ k}\Omega$

$$R_{Peq} = \frac{1}{k_p'\left(\frac{W}{L}\right)_p\left[V_{DD} - |V_{tp}| - \frac{1}{2}(V_{DD} - V_O)\right]}$$

R_{Peq}

$$= \frac{1}{(75\mu\text{A}/\text{V}^2)(1.5)\left[1.8\text{ V} - 0.5\text{ V} - \frac{1}{2}(1.8\text{ V} - 0.9\text{ V})\right]}$$

$= 10\text{ k}\Omega$

$R_{TG}(V_o = 0.9\text{V}) = R_{Neq} \| R_{Peq} = 51\text{ k}\Omega \| 10\text{ k}\Omega$

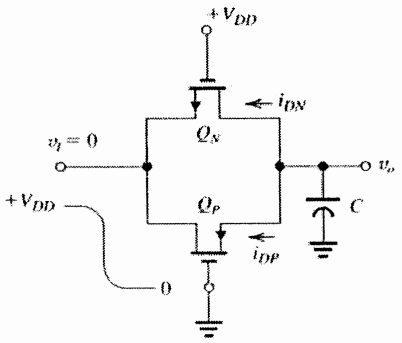
$= 8.36\text{ k}\Omega$

$R_{TG}|_{av} = \frac{1}{2}(3.76\text{ k}\Omega + 8.36\text{ k}\Omega) = 6.06\text{ k}\Omega$

$t_{PLH} \approx 0.69R_{TG}C = 0.69(6.06\text{ k}\Omega)(1.5)(10^{-15})F$

$= 62.7\text{ ps}$

15.19



C is charged so that $V_O = V_{DD}$

When v_I goes Low to $0V$, Q_N is initially in the saturation region with

$i_{DN} = \frac{1}{2}k_n(V_{DD} - V_{to})^2$

until $V_{DSN} = V_{GS} - V_{tn} = V_{DD} - V_{tn}$

$$R_{Neq} = \frac{v_o - 0}{\frac{1}{2}k_n(V_{DD} - V_{to})^2} = \frac{2v_o}{k_n(V_{DD} - V_{to})^2}$$

for $v_o \geq V_{DD} - V_{tn}$

When Q_N enters the triode region,

$$i_{DN} = k_n\left[(V_{DD} - V_{to})v_o - \frac{1}{2}v_o^2\right]$$

for $v_O \leq V_{DD} - V_{to}$

$$\text{Then, } R_{Neq} = \frac{v_o}{k_n\left[(V_{DD} - V_{to})v_o - \frac{1}{2}v_o^2\right]}$$

$$= \frac{1}{k_n\left[(V_{DD} - V_{to}) - \frac{1}{2}v_o\right]}$$

For Q_P initially,

$i_{DP} = \frac{1}{2}k_P(v_o - |V_{tp}|)^2$ so that

$$R_{Peq} = \frac{2v_o}{k_P(v_o - |V_{tp}|)^2}$$

where

$|V_{tp}| = |V_{to}| + \gamma(\sqrt{V_{DD} - v_o + 2\phi_f} - \sqrt{2\phi_f})$

until $v_o = |V_{tp}|$

For $v_o = V_{DD}$

$$R_{Neq}(v_o = V_{DD}) = \frac{2V_{DD}}{k_n'\left(\frac{W}{L}\right)_n(V_{DD} - V_{to})^2}$$

$$= \frac{2(1.8\text{V})}{(300\mu\text{A}/\text{V}^2)(1.5)(1.8\text{ V} - 0.5\text{ V})^2} = 4.7\text{ k}\Omega$$

$$R_{Neq}(v_o = V_{DD}) = \frac{2V_{DD}}{k_p'(W/L)_p(V_{DD} - |V_{to}|)^2}$$

$$= \frac{2(1.8\text{V})}{(75\mu\text{A}/\text{V}^2)(1.5)(1.8\text{ V} - 0.5\text{ V})^2} = 18.9\text{ k}\Omega$$

$R_{TG}(v_o = V_{DD}) = R_{Neq} \| R_{Peq} = 4.7\text{ k}\Omega \| 18.9\text{ k}\Omega$

$= 3.76\text{ k}\Omega$

At $v_o = \frac{V_{DD}}{2} = 0.9\text{ V}$

$V_{DD} - V_{tn} = 1.8\text{ V} - 0.5 = 1.3\text{ V}$

since $V_{DS} = v_o - o = 0.90$

Q_N is in the triode region.

$$R_{Neq} = \frac{1}{k_n'\left(\frac{W}{L}\right)_n\left[(V_{DD} - V_{to}) - \frac{1}{2}v_o\right]}$$

$$= \frac{1}{(300\mu\text{A}/\text{V}^2)(1.5)\left(1.8\text{V} - 0.5\text{ V} - \frac{0.9\text{ V}}{2}\right)}$$

$R_{Neq}(0.9\text{ V}) = 2.6\text{ k}\Omega$

At $v_o = 0.9\text{ V}$,

$|V_{tp}| = |V_{to}| + \gamma(\sqrt{V_{DD} - v_o + 2\phi_f} - \sqrt{2\phi_f})$

$= 0.5\text{ V} + 0.3\text{ V}^{1/2}\sqrt{1.8\text{ V} - 0.9\text{ V} + 0.85\text{ V}}$

$- \sqrt{0.85\text{ V}})$

$= 0.62\text{ V}$

$$R_{Peq} = \frac{2v_o}{k_p'\left(\frac{W}{L}\right)_p(v_o - |V_{tp}|)^2}$$

$$= \frac{2(0.9\ \text{V})}{(75\mu\text{A}/\text{V}^2)(1.5)(0.9\text{V} - 0.62\text{V})^2}$$

$= 204\ \text{k}\Omega$

$R_{TG}(v_o = 0.9\text{V}) = 2.6\ \text{k}\Omega \parallel 204\ \text{k}\Omega = 2.57\ \text{k}\Omega$

$R_{TG}|_{av} = \frac{1}{2}(3.76\ \text{k}\Omega + 2.57\ \text{k}\Omega) = 3.17\ \text{k}\Omega$

$t_{PHL} = 0.69R_{TG}C = 0.69(3.17\ \text{k}\Omega)(15)(10^{-15})F$

$= 32.8\ \text{ps}$

(This is close to the answer of Problem 14.19)

15.20

$$R_{TG} = \frac{12.5}{\left(\frac{W}{L}\right)_n}\ \text{k}\Omega = \frac{12.5}{1.5}\ \text{k}\Omega = 8.3\ \text{k}\Omega$$

$t_P \simeq t_{PLH} \simeq t_{PLH} \simeq 0.69R_{TG}C = 0.69(8.3\ \text{k}\Omega) \times (10)(10^{-15})F = 57.3\text{ps}$

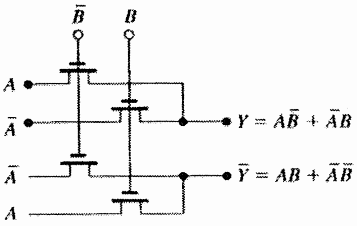
15.21

$$t_p = 0.69CR_{TG} \cdot \frac{n(n+1)}{2}$$

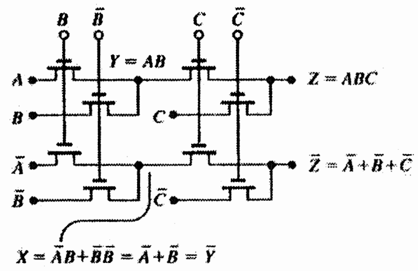
$$= 0.69(10)(10^{-15})F(10\ \text{k}\Omega)\frac{16(16+1)}{2} = 9.38\ \text{ns}$$

15.22

Need a CPL circuit for $Y = A\bar{B} + \bar{A}B$ and $\bar{Y} = AB + \bar{A}\bar{B}$ (See Exercise 14.8b


$\bar{B}$
B
A
$\bar{A}$
$\bar{A}$
A
$Y = A\bar{B} + \bar{A}B$
$\bar{Y} = AB + \bar{A}\bar{B}$

15.23

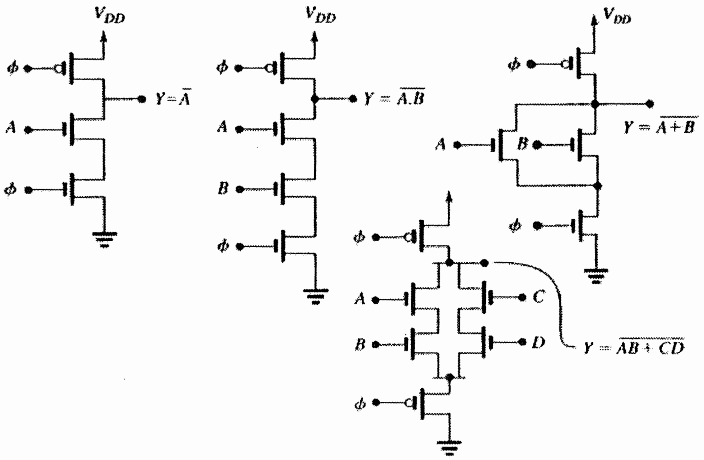

B
$\bar{B}$
C
$\bar{C}$
$Y = AB$
A
B
C
$Z = ABC$
$\bar{A}$
$\bar{B}$
$\bar{C}$
$\bar{Z} = \bar{A}+\bar{B}+\bar{C}$
$X = \bar{A}B+\bar{B}\bar{B} = \bar{A}+\bar{B} = \bar{Y}$

Require a CPL for Z = ABC and

$\bar{Z} = \overline{ABC} = \bar{A} + \bar{B} + \bar{C}$

Extend Fig 14.18 to 3 variables by dealing in pairs, creating $Y = AB$, then $Z = YC$ with $\bar{Y} = \bar{A} + \bar{B}$, then $\bar{Z} = \bar{Y} + \bar{C}$

15.24


V_{DD}
ϕ
$Y = \bar{A}$
A
ϕ
V_{DD}
ϕ
$Y = \overline{A.B}$
A
B
ϕ
V_{DD}
ϕ
$Y = \overline{A+B}$
A
B
ϕ
ϕ
A
C
B
D
ϕ
$Y = \overline{AB + CD}$

15.25

At $v_Y = 0.3\text{V}$, $i_{DP} = \frac{1}{2}(75/3)(2.4/0.8)(3.0-0.8)^2 = \underline{\underline{181.5\mu A}}$

At $v_Y = 2.7\text{V}$, $i_{DP} = (75/3)(2.4/0.8)[(3.0-0.8)0.3 - 0.3^2/2]$
$= \underline{\underline{46.1\mu A}}$

Thus $i_{D_{av}} = (181.5 + 46.1)/2 = \underline{114\mu A}$

and $t_{PLH} = t_r = 15\times10^{-15}(2.7-0.3)/(114\times10^{-6}) = \underline{\underline{316\text{ ps}}}$

15.26

For a 0.5V change, $t = C\Delta v/I_\ell = 30(10^{-15})0.5/10^{-12} = \underline{\underline{15\text{ms}}}$

Since the precharge interval is much shorter than the evaluate, the period of the minimum clocking frequency can be as great as 15ms, for which $f_{min} = 1/(15\times10^{-3}) = \underline{\underline{67\text{Hz}}}$

15.27

a) $C_1 = 5\text{fF}$:

Now, for v_{C1} rising to $V_{DD} - V_t = 5-1 = 4\text{V}$, and assuming Q_1 continues to conduct, v_Y will fall by an amount $(C_1/C_L)(\Delta v_{C1}) = 5/30(4) = \underline{0.67\text{V}}$ to $5.0 - 0.67 = 4.33\text{V}$. Since this exceeds 4.0, the assumption that Q_1 continues to conduct is verified. Thus v_Y drops by $\underline{\underline{0.67\text{V}}}$

Note that if the body effect is included, it will likely to be impossible to raise v_{C1} to 4V. Thus 0.67V is the largest possible change.

b) $C_1 = 10\text{fF}$:

In view of the previous analysis, assume that ultimately $v_Y = v_{C1} = v$. Now, the charge change in each capacitor is the same:

$Q = CV \longrightarrow 10(v-0) = 30(5-v)$

and $10v = 150 - 30v$, $40v = 150$, and $v = \underline{3.75\text{V}}$

Thus v_Y drops by $5-3.75 = \underline{\underline{1.25\text{V}}}$ to 3.75V

15.28

(a) Since Q_1 and Q_{e1} are in series, W remains the same, but the effective length doubles. So,

$$\left(\frac{W}{L}\right)_{eq1} = \left(\frac{W}{2L}\right) = \frac{1}{2}\left(\frac{W}{L}\right)_n$$

Similarly,

$$\left(\frac{W}{L}\right)_{eq1} = \left(\frac{W}{2L}\right) = \frac{1}{2}\left(\frac{W}{L}\right)_n$$

(b)

$$i_{D1}\,(v_{y1} = V_{DD}) = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq1}(V_{DD} - 0.2V_{DD})^2$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq1}(0.64\ V_{DD}^2)$$

$$= 0.32k_n'\left(\frac{W}{L}\right)_{eq1}V_{DD}^2$$

$$= 0.16k_n'\left(\frac{W}{L}\right)_n V_{DD}^2 = 0.16k_nV_{DD}^2$$

At $v_{y1} = V_t$:

$$i_{D1}\,(v_{y1} = V_t) = k_n'\left(\frac{W}{L}\right)_{eq1} \times$$

$$\left[(V_{DD} - 0.2V_{DD})(0.2V_{DD}) - \frac{1}{2}(0.2V_{DD})^2\right]$$

$$= k_n'\left(\frac{W}{L}\right)_{eq1}[0.16V_{DD}^2 - 0.02V_{DD}^2]$$

$$= k_n'\left(\frac{W}{L}\right)_{eq1} V_{DD}^2 = 0.07k_n'\left(\frac{W}{L}\right)_n V_{DD}^2$$

$$= 0.07k_n V_{DD}^2$$

$i_{D1}|_{av}$

$$= \frac{1}{2}\left[0.16k_n'\left(\frac{W}{L}\right)_n V_{DD}^2 + 0.07k_n'\left(\frac{W}{L}\right)_n V_{DD}^2\right]$$

$$= 0.115k_n'\left(\frac{W}{L}\right)_n V_{DD}^2 = 0.115k_n V_{DD}^2$$

(c) $\Delta t = \frac{C_{L1}(V_{DD} - V_t)}{i_{D1}|_{av}} = \frac{C_{L1}(0.8V_{DD})}{0.115k_n V_{DD}^2}$

$$= \frac{6.96C_{L1}}{k_n V_{DD}}$$

(d) Q_{eq2} will conduct during the time that v_{y1} drops from V_{DD} to V_t The transition half point is

when $v_{y1}|_{av} = \frac{V_{DD} - 0.2V_{DD}}{2} + 0.2V_{DD}$

$v_{y1}|_{av} = 0.6V_{DD}$

$i_{D2}|_{av} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_{eq2}(0.6V_{DD} - 0.2V_{DD})^2$

$$= 0.08k_n'\left(\frac{W}{L}\right)_{eq2} V_{DD}^2 = 0.04k_n V_{DD}^2$$

(e) $\Delta v_{y2} = -\frac{i_{D2}|_{av}\Delta t}{C_{L2}}$

Since $C_{L1} = C_{L2}$

$$\Delta v_{y2} = -\frac{0.04k_n V_{DD}^2(6.96C_{L1})}{C_{L1}k_n V_{DD}} = -0.278V_{DD}$$

So that v_{y2} is $V_{DD} - 0.278\ V_{DD} = 0.72\ V_{DD}$

15.29

The precharge time can be approximated as the rise time of the output voltage. In Example 14.3, $t_r \simeq 0.19$ ns. Assuming that the evaluation time is relatively short, the total cycle time can be estimated as being slighty longer than $t_r + t_{PHL}$. With $t_{PHL} \simeq 0.25$ ns, the maximum clocking frequency is $f < \frac{1}{T} = \frac{1}{(t_r + t_{PHL})}$

$$= \frac{1}{(0.19 + 0.25)(10^{-9})\text{s}} = 2.27\ \text{GHz}$$

15.30

(a) $V_{OH} = 0 - 0.75 = -0.75$ V

$V_{OL} = 0 - 0.75 - IR = -(0.75 + IR)$

(b)

$V_{th} = -(IR/2 + 0.75) = -(0.75 + IR/2)$

(c) For $i = 0.99$ I,

$v_{BE} = 750 + 25\ \ln(0.99) = 750$ mV

$i = 0.01$ I,

$v_{BE} = 750 + 25\ \ln(0.01) = 635$ mV

For

$0.99I$ in Q_R,

$$v_I = -\left(0.75 + \frac{IR}{2}\right) - (0.750 - 0.635)$$

$$= -(0.875 + IR/2)$$

(d) For $0.01I$ in Q_R,

$v_I = -(0.75 + IR/2) + 0.115$

$= -(0.635 + IR/2)$

(e) $V_{IH} = -(0.635 + IR/2)$

$V_{IL} = -(0.875 + IR/2)$

(f) $NM_H = -0.75 - [-(0.635 + IR/2)]$

$= IR/2 - 0.115$

$NM_L = -(0.875 + IR/2) - [-(0.75 + IR)]$

$= IR/2 - 0.115$

(g) $V_{IH} - V_{IL}$

$= -(0.635 + IR/2) - [-(0.875 + IR/2)]$

That is; $IR/2 - 0.115 = 0.230$

and $IR = 2(0.345) = 0.690$ V

(h) $V_{OH} = -0.75$ V;

$V_{OL} = -0.75 - 0.69 = -1.44$ V;

$V_{IL} = -(0.875 + 0.345) = -1.22$ V;

$V_{IH} = -(0.635 + 0.345) = -0.98$ V;

$V_R = -(0.750 + 0.345) = -1.095$ V.

15.31

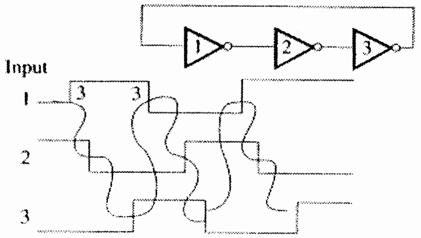
See that once started the process continues; that is we have,an oscillation. In each cycle, each gate output rises and falls.

Thus the period is 3(3+7) = 30 ns

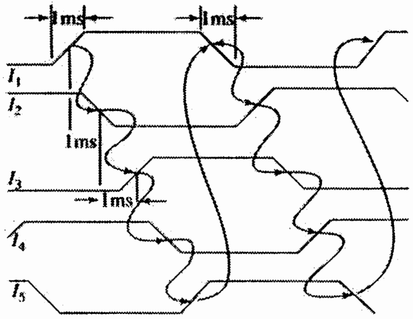
Frequency is 1/30 = 33.3 MHz.

Any output is high for 3+7+3 = 13 ns and low for 7+3+7 = 17 ns

Check: 30 ns.



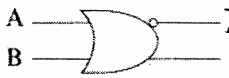
15.32


1ms
1ms
1ms
1ms
I_1
I_2
I_3
I_4
I_5

10 transitions per cycle, each of 1ns duration:
Period = 10 ns
Frequency = 100 MHz

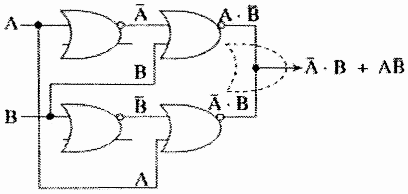
15.33

Tying two outputs together as yields a WIRED- OR operation. The most direct implementation is ORing the outputs of two AND gates. The AND function can be obtained using Demorgan's theorem:



$$\overline{A+B} = \bar{A}\cdot\bar{B}$$

Using NOR gates as inverters, $\bar{A}B$ and $A\bar{B}$ are obtained:


A
B
$\bar{A}$
$A\cdot\bar{B}$
B
$\bar{B}$
$\bar{A}\cdot B$
A
$\bar{A}\cdot B + A\bar{B}$

when $v_I = V_{IL} = -1.435$ V,
$I_E = 3.97$ mA, $v_{OR} = V_{OL} = -1.77$ V
when $v_I = V_R = -1.32$ V, $I_E = 4.00$ mA,
$v_{OR} = -1.31$ V
when $v_I = V_{IH} = -1.205$ V,
$I_E = 4.12$ mA, $v_{OR} = V_{OH} = -0.88$ V

At point x Transistor A's emitter current is 1% of I_E or

$$I_{E_A} = (0.01)(3.97\text{ mA}) = 39.7\ \mu\text{A}$$

$$\text{So that } r_{eA} = \frac{V_T}{I_{E_A}} = \frac{25\text{ mV}}{39.7\ \mu\text{A}} = 630\ \Omega$$

Transistor B's emitter current is 99% of I_E or

$$I_{EB} = (0.99)(3.97\text{ mA}) = 3.93\text{ mA}$$

$$\text{So that } r_{eB} = \frac{V_T}{I_{EB}} = \frac{25\text{ mV}}{3.93\text{ mA}} = 6.4\ \Omega$$

$$I_{E2} = \frac{v_{OR} - V_{EE}}{R_T} = \frac{-1.77\text{ V} - (-2\text{ V})}{50\ \Omega}$$

$$= 4.6\text{ mA}$$

$$\text{So that } r_{e2} = \frac{V_T}{I_{E2}} = \frac{25\text{ mV}}{4.6\text{ mA}} = 5.4\ \Omega$$

$$R_{in2} = (\beta + 1)(r_{e2} + R_T)$$

$$= (101)(5.4\ \Omega + 50\ \Omega) = 5.6\text{ k}\Omega$$

Solving for the incremental gain,

$$v_{OR} = \frac{V_{be2} R_T}{r_{e2} + R_T}$$

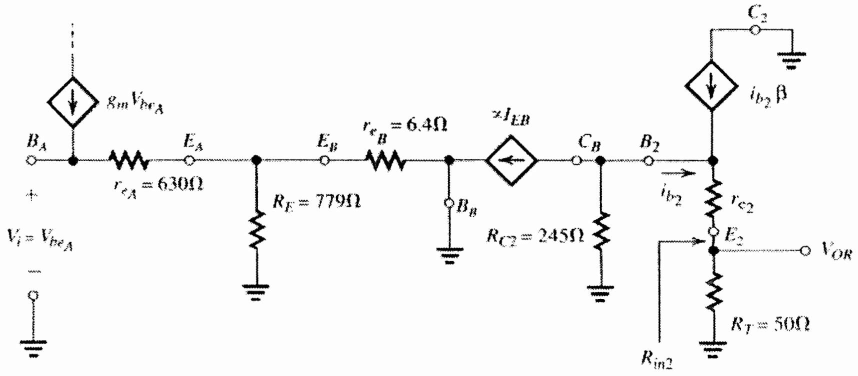
$$V_{be2} = -\alpha i_{eB}(R_{c2} \| R_{in2})$$

Since $R_E >> r_{eB}$,

$$i_{eB} \approx \frac{-v_i}{r_{eA} + r_{eB}} \text{ so that}$$

$$\frac{v_{OR}}{v_i} = \frac{R_T}{r_{e2} + R_T} \cdot \frac{\alpha(R_{C2} \| R_{in2})}{r_{eA} + r_{eB}}$$

15.34


$g_m V_{be_A}$
B_A
+
$V_i = V_{be_A}$
−
E_A
$r_{e_A} = 630\Omega$
$R_E = 779\Omega$
E_B
$r_{e_B} = 6.4\Omega$
B_B
αI_{EB}
C_B
$R_{C2} = 245\Omega$
B_2
i_{b_2}
C_2
$i_{b_2}\beta$
r_{e2}
E_2
V_{OR}
$R_T = 50\Omega$
R_{in2}

$$\frac{v_{OR}}{v_i} = \frac{50\ \Omega}{5.4\ \Omega + 50\ \Omega} \cdot \left(\frac{100}{101}\right) \cdot \frac{(245\ \Omega \parallel 5.6\ \text{k}\Omega)}{630\ \Omega + 6.4\ \Omega}$$

$= 0.33$ V/V

At point m,

$$I_{EA} = I_{EB} = \frac{I_E}{2} = \frac{4\ \text{mA}}{2} = 2\ \text{mA}$$

$$r_{eA} = r_{eB} = \frac{V_T}{I_{EA}} = \frac{25\text{mV}}{2\text{mA}} = 12.5\ \Omega$$

$$I_{E2} = \frac{v_{OR} - V_{EE}}{R_T} = \frac{-1.31 - (-2\text{V})}{50\ \Omega} = 13.8\ \text{mA}$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25\ \text{mV}}{13.8\ \text{mA}} = 1.81\ \Omega$$

$$R_{in2} = (\beta + 1)(R_T + r_{e2}) = (101)(50 + 1.81)$$

$$= 5.23\ \text{k}\Omega$$

$$\text{Gain} = \frac{V_{OR}}{V_i} = \frac{50\ \Omega}{50\ \Omega + 1.81\ \Omega} \cdot \left(\frac{100}{101}\right)$$

$$\cdot \frac{(245\Omega \parallel 5.23\text{k}\Omega)}{12.5\Omega + 12.5\Omega} = 8.95\ \text{V/V}$$

At point Y,

$$I_{EA} = 0.99(4.12\ \text{mA}) = 4.08\ \text{mA}$$

$$r_{eA} = \frac{V_T}{I_{EA}} = \frac{25\ \text{mV}}{4.08\ \text{mA}} = 6.13\ \Omega$$

$$I_{EB} = (0.01)(4.12\ \text{mA}) = 41.2\ \mu\text{A}$$

$$r_{eB} = \frac{V_T}{I_{EB}} = \frac{25\ \text{mV}}{41.2\ \mu\text{A}} = 607\ \Omega$$

$$I_{E2} = \frac{v_{OR} - V_{EE}}{R_T} = \frac{-0.88\ \text{V} - (-2\ \text{V})}{50\ \Omega}$$

$= 22.4$ mA

$$r_{e2} = \frac{25\ \text{mA}}{22.4\ \text{mA}} = 1.1\ \Omega$$

$$R_{in2} = (101)(50\Omega + 1.1\Omega) = 5.16\ \text{k}\Omega$$

$$\text{Gain} = \frac{v_{OR}}{v_i} = \frac{50\ \Omega}{50\ \Omega + 1.1\ \Omega} \cdot \left(\frac{100}{101}\right)$$

$$\cdot \frac{(245\ \Omega \parallel 5.16\ \text{k}\Omega)}{607\ \Omega + 6.13\ \Omega} = 0.37\ \text{V/V}$$

15.35

Assume I_E is constant at 4 mA.

(a) Currents are: 3.6 mA and 0.4 mA

$\therefore$ Emitter-Base voltage difference $= V_T \ln \dfrac{3.6}{0.4}$

or 25 ln 9 = 54.9 mV.

Thus $V_{IL} = -1.32 - .055 = -1.375$ V

$V_{IH} = -1.32 + .055 = -1.265$ V

(b) Currents are: $4(0.999) = 3.996$ mA and

$.001 \times 4 = 0.004$ mA

$\therefore$ Emitter-Base voltage difference =

$$V_T \ln\left(\frac{3.996}{.004}\right)$$

15.36

$NM_H = 0.325$ V, of which 50% is 162 mV, for $\beta = 100$, and $V_{BE2} = 0.83$ V, $I_{E2} = 22.4$ mA.

Approximately:

$$-2 + \frac{50}{50 + \dfrac{245}{\beta + 1}} \cdot (2 - 0.83) = -0.88 - 0.162$$

$$\text{or}\ \frac{50(1.17)}{50 + \dfrac{245}{\beta + 1}} = 0.958$$

$$50 + \frac{245}{\beta + 1} = 61.06$$

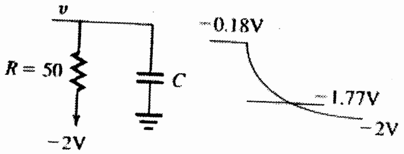
$$\text{Hence}\ \beta = \frac{245}{11.06} - 1 = 21.2$$

Check: For $V_o = -.88 - .162 = -1.042$ V

$$I_{E2} = \frac{2 - 1.042}{50} = 19.2\ \text{mA}$$

and $V_T \ln\left(\dfrac{22.4}{19.2}\right) = 3.85$ mV – OK, Since small, can ignore.

15.37



$$v = -0.88 + (.88 - 2)(1 - e^{-t/RC})$$

or $v = -2 + 1.12\, e^{-t/50\,C}$

After 1ns, $v = -1.77$ V

i.e., $-1.77 = -2 + 1.12\, e^{-1/50\,C}$

or $e^{-1/50\,C} = \dfrac{2-1.77}{1.12}$ and

$-1/50\,C = -1.583$

Thus

$$C = \frac{10^{-9}}{50(1.583)} = 12.6 \times 10^{-12}\ \text{F} = 12.6\ \text{pF}$$

15.38

$v = 2/3 \times 30\ \text{cm/ns} \times 20\ \text{cm/ns}$

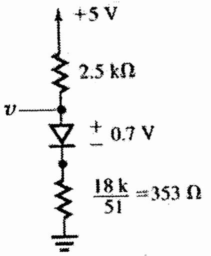
$$\text{Rate} = \frac{\text{Rise Time}}{\text{Return Time}} = 5/1 = \frac{3.5}{2L/20}$$

$$L = \frac{3.5\ \text{ns} \times 20\ \text{cm/ns}}{5 \times 2} = 7\ \text{cm}$$

15.39

$V_{OL} = 0.7$ V; $V_{OH} = +5.0$ V

More precisely, for $V_{OL} = v$



$$v = \frac{0.353}{2.5 + .353}(5 - 0.7) + 0.7 = 1.23\ \text{V}$$

i.e., $V_{OL} = 1.23$ V

Logically: A is high if one of A *or* B *and* one of C *or* D are high.

That is A = (A + B) · (C + D)

15.40

For $V_1 = V_o = V_{DD/2} = 5/2 = 2.5$ V.

$i_{DN} = (1/2)\, k_n' \,(W/L)_n\,(V_{GS} - V_t)^2$

$= (1/2)(100)10^{-6}(2/1)(2.5 - 0.7 - 1)^2$

$= 64\ \mu\text{A}$

Now, the collector current of

$Q_2 = \beta i_B = \beta i_{DN}$

$= 100(64 \times 10^{-6}) = 6.4$ mA

Corresponding, the totem-pole current is

$i_{EQ2} = (6400 + 64)10^{-6} = 6.46$ mA

Now, for $i_{EQ1} = i_{EQ2}$, $i_{DP} = i_{DN} = 64\ \mu\text{A}$

Thus

$64 = 1/2(100/2.5)(W/L)_p(5 - 2.5 - 0.7 - 1)^2$

where $(W/L)_p = 2.5(2/1) = (5\mu\text{m}/1\mu\text{m})$

15.41

At the threshold V_{th}, $v_o = v_I = V_{th} = v$, and the two MOS operate in saturation with equal currents. Thus

$1/2\,(100)\,(2/1)\,(5 - y - 0.7 - 1)^2$

Thus, $(3.3 - v)^2 = 2.5(v - 1.7)^2$

and $(3.3 - v) = \pm\sqrt{2.5}\,(v - 1.7)$.

Usefully, $(3.3 - v) = (1.58v - 2.69)$,

hence $2.58\,v = 5.99$, and $v = V_{th} = 2.32$ V

For this value,

$i_{DN} = 1/2\,(100)\,(2/1)\,(2.32 - 0.7 - 1)$

$= 38.4\ \mu\text{A}$

and the totem-pole current is $(\beta + 1)i_{DN}$

or $101(38.4)10^{-6} = 3.88$ mA

15.42

The problem as stated is very general, and correspondingly, its solution can be long and complex.

the specifications of matched MOS having

$(W/L)_p = 2.5\,(W/L)_n$.

For R_2: With $v_{DS} = V_{t/3} = 1/3 = 0.333$ V

$i_{DN} = 100(10^{-6})(2/1) \times$

$[(5 - 0.7 - 1)0.33 - 0.33^2/2] = 209\ \mu\text{A}$

Now, if 50% of this is lost in R_2,

$R_2 = 0.7/(0.50 \times 209) = 6.70\ \text{k}\Omega$

Now if 20% is lost in R_2,

$R_2 = 0.7/(0.20 \times 209) = 16.7\ \text{k}\Omega$

For R_1: $i_{Dp} = (100/2.5)10^{-6}(2.5(2/1)) \times$

$[(5 - 0 - 1)0.33 - 0.33^2/2] = 256\ \mu\text{A}$

Now, if 50% if this is lost in R_1,

$R_1 = (5 - 0.333)/(0.5 \times 256) = 36.5\ \text{k}\Omega$

Now, if 20% is lost in R_1

$R_1 = 2.5(36.5) = 91.1\ \text{k}\Omega$

In comparison:

For the 50% case,

$R_1/R_2 = 36.5/6.70 = 5.45$

For the 20% case,

$R_1/R_2 = 91.1/16.7 = 5.45$

(why should their equality be obvious?)

Thus, in general $R_1/R_2 = 5.45$

15.43

For t_{PLH}

At $V_O = 0$ V,

$i_{Dp} = \frac{1}{2}(100/2.5)(2/1)(5.0 - 1)^2 = 640\mu A$

At

$V_O = 2.5$ V,

$i_{DP} = (100/2.5)(2/1)[(5 - 1)2.5 - 2.5^2/2]$

$= 550\mu A$

Thus $i_{DPav} = (640 + 550)/2 = 595\mu A$

and $i_{Bav} = (100 + 1)595 = 60.1$ mA

Thus

$t_{PLH} = CV/I = 2 \times 10^{-12} \times 2.5/(60.1 \times 10^{-3})$

$= 83.2$ ps

For t_{PHL}:

At

$v_O = 5.0$ V,

$i_{DN} = \frac{1}{2}(100)(2/1)(5 - 0.7 - 1)^2 = 1.09$ mA

At

$v_O = 2.5$ V, $i_{DN} = 100(2/1)$

$[(5 - 0.7 - 1)(2.5 - 0.7) - (2.5 - 0.7)^2/2]$

$= 864\ \mu A$

Thus

$i_{DNav} = (1089 + 864)/2 = 977\mu A$

and $i_{Dav} = 101(977 \times 10^{-6}) = 98.6$ mA

Thus

$t_{PHL} = CV/I = 2 \times 10^{-12}(2.5)/(98.6 \times 10^{-3})$

$= 50.7$ ps

Thus $t_p = (83.2 + 50.7)/2 = 67.0$ ps

Note that this solution embodies two assumptions

1) Internal capacitances can be neglected.

2) Transitions are from ideal 0 V and 5 V output-signal level.

If outputs of $(5 - 0.7) = 4.3$ V and $(0 + 0.7) = 0.7$V apply, t_p becomes about $67 \times (2.5 - 0.7)/2.5 = 48$ ps

15.44

$R_1 = R_2 = 5\ k\Omega$ robs the base of some of its drive current, namely $0.7/5 \times 10^3 = 140\mu A$. Using results from the solution of P 14.46 above:

For t_{PLH}

: $i_{Bav} = 595 - 140 = 455\mu A$ and

$i_{Bav} = 101(455 \times 10^{-6}) = 46.0$ mA

Thus

$t_{PHL} = 2 \times 10^{-12} \times 2.5/4.6 \times 10^{-3} = 108.7$ ps

For t_{PLH}:

$i_{Dav} = 977 - 140 = 837\mu A$

and $i_{Bav} = 101(837 \times 10^{-6}) = 84.5$ mA

Thus

$t_{PHL} = 2 \times 10^{-12} \times 2.5/84.5 \times 10^{-3} = 59.2$ ps

Thus $t_p = (59.2 + 108.7)/2 = 84$ ps

15.45

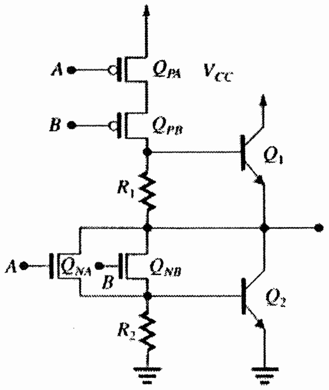
For the BiCMOS NAND of Fig 14.39 to have a dynamic response some what line that of the inverter of Fig. 14.37e:

$(W/L)_{PA} = (W/L)_{PB} = (W/L)_P$

and $(W/L)_{NA} = (W/L)_{NB} = 2(W/L)_N$

15.46

A BiCMOS 2-input NOR is as shown:



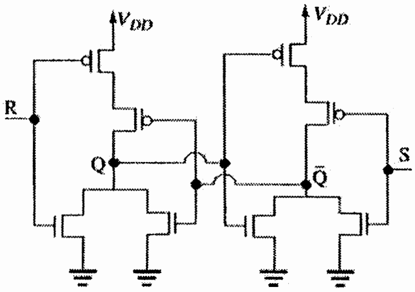
In terms of the basic matched inverter:

$(W/L)_{PA} = (W/L)_{PB} = 2(W/L)_P$

$(W/L)_{NA} = (W/L)_{NB} = (W/L)_N$

where $(W/L)_P$ and $(W/L)_N$ characterize the inverter.

16.1



16.2

$$\mu_n \frac{1}{2}\left(\frac{W}{L}\right)_5\left[(V_{DD}-V_{tp})\frac{V_{DD}}{2}-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$=\mu_p\left(\frac{\mu_n}{\mu_p}\right)\left(\frac{W}{L}\right)_n\left[(V_{DD}-V_{tp})\left(\frac{V_{DD}}{2}\right)-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

Assuming $V_{tn} = V_{pt}$ we have:

$$\mu_n\frac{1}{2}\left(\frac{W}{L}\right)_5 = \mu_p\left(\frac{\mu_n}{\mu_p}\right)\left(\frac{W}{L}\right)_n \Rightarrow \frac{1}{2}\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_n$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 2\left(\frac{W}{L}\right)_n$$

If the flip-flop is fabricated in a 0.13-μm process, we have:

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_3 = 1 \Rightarrow W_1 = W_3 = 1 \times L_{min}$$

$$= 0.13\ \mu\text{m}$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_4 = \left(\frac{\mu_n}{\mu_p}\right)\left(\frac{W}{L}\right)_n = 4 \times 1 = 4$$

$$\Rightarrow W_2 = W_4 = 4 \times 0.13 = 052\ \mu\text{m}$$

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = 2\left(\frac{W}{L}\right)_n$$

$$= 2 \Rightarrow W_5 = W_6 = W_7 = W_8 = 2\ \mu\text{m}$$

16.3

output $v_Q = \frac{V_{DD}}{2}$, and assuming a single equivalent transistor for Q_5 and Q_6 where $\left(\frac{W}{L}\right)_{eq} = \frac{1}{2}\left(\frac{W}{L}\right)_5 = \frac{1}{2}\left(\frac{W}{L}\right)_6$

Use eq. 13.100.

For equivalent *n* transistor

$$\Rightarrow V_{GS} - V_t = 1.8 - 0.5 = 1.3 > V_{DS_{satn}}$$

$$= 0.6\text{V}$$

For $P_{\text{transistor}}$

$$\Rightarrow |V_{GS}| - |V_t| = 1.8 - 0.5 = 1.3 > V_{DS_{satp}}$$

$$= 1\text{V}$$

Both operating in velocity saturation :

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_{eq} V_{DS_{satn}}\left(V_{GS} - V_{tn} - \frac{1}{2}V_{DS_{satn}}\right)$$

$$\times (1 + \lambda_n V_{DS}) = \mu_p C_{ox}\left(\frac{W}{L}\right)_2 |V_{DS_{satp}}|$$

$$\times \left(|V_{GS}| - |V_{tp}| - \frac{1}{2}|V_{DS_{satp}}|\right)(1 + |\lambda_p| V_{DS})$$

$$300 \times 10^{-6} \times \left(\frac{W}{L}\right)_{eq} \times 0.6\left(1.8 - 0.5 - \frac{1}{2} \times 0.6\right)$$

$$\times \left(1 + .1 \times \frac{1.8}{2}\right) = 75 \times 10^{-6}\left(\frac{1.08}{0.18}\right)$$

$$\times 1\left(1.8 - 0.5 - \frac{1}{2} \times 1\right)\left(1 + .1 \times \left(\frac{1.8}{2}\right)\right)$$

$$\left(\frac{W}{L}\right)_{eq} = 2$$

$$\left(\frac{W}{L}\right)_{eq} = \frac{1}{2}\left(\frac{W}{L}\right)_5 = 2$$

$$\therefore \left(\frac{W}{L}\right)_5 = 4,$$

$$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_5 = 4 = \frac{0.72\ \mu\text{m}}{0.18\ \mu\text{m}}$$

This value is greater
thus requiring 33% more width area of both *n* transistors as a minimum.

16.4

$$V_m = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{r + 1} \text{ where}$$

$$r = \sqrt{\frac{\mu_p W_P}{\mu_n W_n}}$$

$$W_p = W_n = 0.27\ \mu\text{m and } \mu_n = 4\mu_p$$

$$\therefore r = \sqrt{\frac{1}{4}} = 0.5$$

$$V_m = \frac{0.5(1.8 - 0.5) + 1}{0.5 + 1} = 1.1\text{V}$$

(threshold voltage)
Assuming

$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8$ and Q_5, Q_6 have an equivalent single transistor

$\left(\frac{W}{L}\right)_{eq} = \frac{1}{2}\left(\frac{W}{L}\right)_5 = \frac{1}{2}\left(\frac{W}{L}\right)_6$, the equivalent *n* transistor and Q_2 are in triode region with the same current flowing through them.

$$300\mu_n \times \left(\frac{W}{L}\right)_5\left[(1.8-0.5)\frac{1.8}{2}-\frac{1}{2}\left(\frac{1.8}{2}\right)^2\right]$$

$$= 75\times10^{-6}\times\left(\frac{0.27}{0.18}\right)\left[(1.8-0.5)\frac{1.8}{2}-\frac{1}{2}\left(\frac{1.8}{2}\right)^2\right]$$

$$\left(\frac{W}{L}\right)_5 = 0.375 \Rightarrow \left(\frac{W}{L}\right)_5 = 1$$

(cannot have less than minimum)

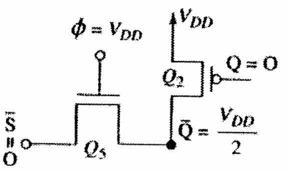
$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = \frac{0.18\ \mu\text{m}}{0.18\ \mu\text{m}}$$

16.5

Q_2 is conducting and Q_5 is conducting and operating in triode region:

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_5\left[(V_{DD}-V_{tn})\left(\frac{V_{DD}}{2}\right)-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$

$$= \mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[(V_{DD}-|V_{tp}|)\frac{V_{DD}}{2}-\frac{1}{2}\left(\frac{V_{DD}}{2}\right)^2\right]$$



$$\mu_n\left(\frac{W}{L}\right)_5 = \mu_p\left(\frac{W}{L}\right)_2$$

$$\therefore \left(\frac{W}{L}\right)_5 = \frac{\mu_p}{\mu_n}\left(\frac{W}{L}\right)_2$$

16.6

Note that the devices are matched, with

a) $K_n = K_p = 20(12/6) = 40\ \mu\text{A/V}^2$, and $|V_t| = 1$ V.

For $V_I = 2.5$ V

$V_o = 2.5$ V

For $v_I = 0$ V, 5 V : one device is on, one off;

$v_o = 5$ V, 0 V

For $v_I = 1$ V, 4 V : one on, one off;

$v_o = 5$ V, 0 V

For $v_I = 1.5$ V, 3.5 V : one in saturation, one in triode mode.

$$i_D = \frac{1}{2}(40)(1.5-1)^2 = 40[(5-1.5-1)v_o - v_o^2/2]$$

Thus $0.125 = 2.5v_o - v_o^2/2$

or $v_o^2 - 5v_o + 0.25 = 0$

and $V_o[- -5 \pm \sqrt{5^2 - 4(0.25)}]/2$

$= (5 \pm 4.8484)/2 = 0.05\text{V}$

Thus $v_o = 0.05\text{V}$ or 4.95V

For $v_I = 2.0$ V, 3.0 V :

$1/2(2-1)^2 = (5-2-1)v_o - v_o^2/2$

or $(2-1)^2 = 2\times2v_o - v_o^2$

and $v_o^2 - 4v_o + 1 = 0$

Whence $v_o = (- -4 \pm \sqrt{4^2 - 41.1})/2$

$= (4 \pm 3.464)/2 = 0.27$ V

Thus $v_o = 0.27$ V or 4.73 V

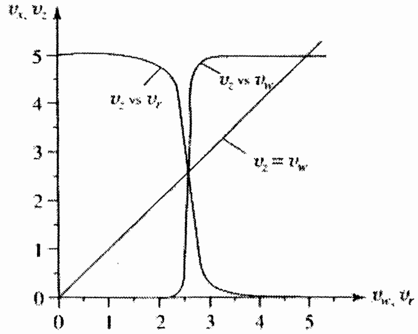
For $v_I = 2.25$ V or 2.75V :

$(2.25-1)^2 = 2(5-2.25-1)v_o - v_o^2$

$1.5625 = 3.5 - v_o - v_o^2$

$v_o^2 - 3.5v_o + 1.5625 = 0$

b)



Whence

$$v_o = (- -3.5 \pm \sqrt{3.5^2 - 4(1.5625)})/2$$

$$= (3.5 \pm 2.45)/2 = 0.525$$

Thus $v_o = 0.525$ V or 4.475 V

For $v_I = 2.5$ V, $v_o = 3.5$ V by symmetry

Now, having plotted v_Z versus v_Y (or v_x versus v_w)

use the graph to find v_z versus v_w.

Work backwards : first v_z, then $v_y = v_x$ than v_w.

For $v_z = 2.5$ V, $v_y = v_x = v_w = 2.5$ V

For $v_z = 4.4$ V, $v_y = 2.25$ V;

For $v_z = 4.5$ V $v_y = 1.50$ V; for

$v_x = 1.50$ V $v_w = 2.65$ V

(c) $v_z = v_w$ line at:

point A: (0, 0)

point B: (2.5, 2.5)

point C: (5, 5)

At point B, the current flow in each inverter is:

$$i_D = \frac{1}{2}(40)(2.5 - 1)^3 = 45 \ \mu\text{A/V}$$

where for each transistor, $r_o = 100/(45 \times 10^{-6})$ $= 2.32$ MΩ

and $g_m = 2\left(\frac{1}{2}\right)40(2.5 - 1) = 60 \ \mu\text{A/V}$

Thus for each inverter operating at (2.5, 2.5), the voltage gain is $-(g_m + g_m)(r_o \| r_o)$

$= -g_m V_o = -60 \times 10^{-6} \times 2.22 \times 10^6 = 133$ V/V

Thus an estimate of the slope of the v_z versus v_w curve at B is (13 s) = 17.7 × 10³ V/V

Correspondingly a lower bound on the width of the transition region is (5 − 0)/(17.7 × 10³), or 0.28 mV, that is ±0.14 mV around 2.5 V.

16.7

The approximate transfer characteristic of each Inverter passes through points : (0,5),(2.0,4.6), (2.42,0.4), (5,0).

For the linear centre segment between (2.0,2.46),(2.42,0.4) an equation is $v_O = a - bv_I$

Here : $4.60 = a - 2.00b$, and

$0.40 = a - 2.42b$

Subtract : $4.20 = 0.42b \rightarrow b = \underline{10}$

Now, $4.60 = a - 2(10) \rightarrow a = 4.6 + 20 = \underline{24.6}$

Check: $0.4 = 24.6 - 2.42(10)$ ✓

Thus the middle part of the characteristic is

$v_O = 24.6 - 10v_I$

For each device, $v_O = v_I = v$, when

$v = 24.6 - 10v$, or $11v = 24.6$, or $v = \underline{2.236\text{V}}$

where the gain is $\Delta v_O/\Delta v_I = -b = \underline{-10\text{V/V}}$

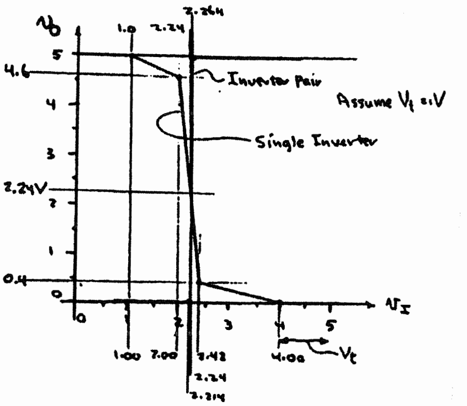
Thus point B on the open-loop characteristic is

$v_W = v_Z = \underline{\underline{2.236\text{V}}}$, where the loop gain can be approximated to be at least $(-10)^2 = \underline{\underline{100\text{V/V}}}$

The open-loop characteristic reaches $v_O = 5$V, where $v_I = 2.236 + (5 - 2.236)/100 = \underline{2.264\text{V}}$

and it reaches 0V where

$v_I = 2.236 - 2.236/100 = \underline{2.214\text{V}}$



16.8

(a) Q_5, Q_6 are conducting for $D = 1$ or 0

$\bar{Q} = \bar{D}$ and $Q = D$

If $D = 1$ then $\bar{Q} = 0$ and $Q = 1$

Q_1 conducts

If $D = 0$ then $\bar{Q} = 1$ and $Q = 0$

Q_4 conducts

(b) If $D = 1$ then $\bar{Q} = 0$ and $Q = 1$

when ϕ goes low, (Q_1, Q_2) conduct (Q_3 also conducts)

The value at the gate of Gz stays high (through Q_1, Q_2) $\bar{Q} = 0$ and $Q = 1$ (value is "latched")

(c) If $D = 0$ then $\bar{Q} = 1$ and $Q = 0$

when ϕ goes low, (Q_3, Q_4) conduct so gate value at G_2 is low (through Q_3, Q_4) to keep $\bar{Q} = 1$ and $Q = 0$

(d) No. The operation connects either V_{DD} or ground directly to gate of G_z which maintains values at $\bar{Q}$ and Q.

16.9

A 1Mb array requires n address bits where $2^n = 10^6$, or $n \log_{10} 2 = 6$, $n = 6/\log_{10} 2 = 19.93$

Thus 20 bits are needed to address every cell.

For 16-bit words, $2^4 = 16$ and 4 bits are not needed. Thus $20 - 4 =$ 16 bits of address are sufficient.

Check: $m = \log_{10}(10^6/16)/\log_{10}(2) = 4.796/0.301 = 15.93$

Use 16 ✓

Note: A "1Mb array" actually holds $2^{20} = 1024^2 = 1048{,}576$ cells.

16.10

The cell area is $10^6 \times 0.38 \times 10^{-6} \times 0.76 \times 10^{-6}$

$= 0.289 \times 10^{-3}\text{m}^2$

The chip area is $19 \times 10^{-3} \times 38 \times 10^{-3}$

$= 0.722 \times 10^{-3}\text{m}^2$

Thus the peripheral circuits and interconnect occupy $(0.722 - 0.289)10^{-3} = 0.433$ mm^2

or $\left(\frac{433}{722}\right) \times 100 = 60\%$ of the clip area.

16.11

$$\left(\frac{(W/L)_5}{(W/L)_1}\right) \le \frac{1}{\left(1 - \frac{0.5}{1.8 - 0.5}\right)^2} - 1 = 1.64$$

minimum area when $\left(\frac{W}{L}\right)_5 = 1$ so

$W_5 = 0.18$ µm Q_5 is saturated and Q_1 is in triode. Currents are equal:

$$\frac{1}{2}\mu_n C_{ox}(1)(1.8 - 0.5 - 0.2)^2 = \mu_n C_{ox}\left(\frac{W_1}{0.18\mu}\right)$$

$$\left[(1.8 - 0.5)0.2 - \frac{1}{2}(0.2)^2\right]$$

Solving for $W_1 = 0.45$ µm

Check condition above :

$$\left(\frac{1}{\frac{0.45\ \mu}{0.18\ \mu}}\right) = 0.4 < 1.64$$

16.12

$$\frac{\left(\frac{W}{L}\right)_a}{\left(\frac{W}{L}\right)_n} \le \frac{1}{\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2} - 1$$

$$= \frac{1}{\left(1 - \frac{0.5}{2.5 - 0.5}\right)^2} - 1 = 0.78$$

$\left(\frac{W}{L}\right)_a \le 0.78 \times 1.5$ or $\left(\frac{W}{L}\right)_a \le 1.17$

16.13

(a) 0.25 µm: $V_{DD} = 2.5$ V and $V_t = 0.5$ V

$$A{:}\left(\frac{V_{\bar{Q}} = V_t}{V_{DD} - V_{tn}}\right) = \frac{0.5}{2.5 - 0.5} = \frac{0.5}{2} = 0.25$$

$$\Rightarrow \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \approx 0.8$$

$$\Rightarrow \frac{(W/L)_5}{(W/L)_1} \approx 0.8$$

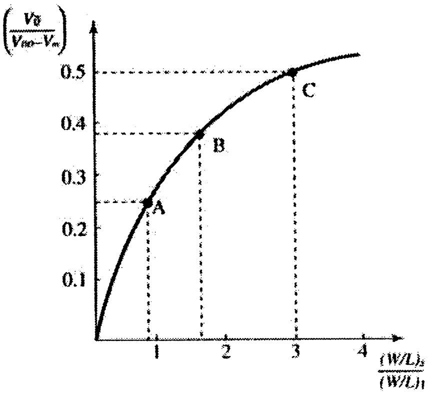
(b) 0.18 µm: $V_{DD} = 1.8$ V and $V_t = 0.5$ V

$$\text{B:}\left(\frac{0.5}{1.8 - 0.5}\right) = 0.385 \Rightarrow \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \approx 1.7$$

$$\Rightarrow \frac{(W/L)_5}{(W/L)_1} = 1.64$$

(c) 0.13 μm: $V_{DD} = 1.2$ V and $V_t = 0.4$ V

C: $\left(\frac{0.4}{1.2 - 0.4}\right) = 0.5$



$$\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} = 3$$

$$\left(\frac{W}{L}\right)_5 / \left(\frac{W}{L}\right)_1 = 3$$

16.14

When $V_{\bar{Q}} \leq V_t = 0.5$ V :

Q_5: $V_{GS} - V_t = 1.8 - 0.5 = 1.3 > 0.6$ V and

$V_{DS} = 1.8 - 0.5 > V_{DS_{sat}} = 0.6$

Q_1: $V_{GS} - V_t = 1.8 - 0.5 = 1.3 > 0.6$ V and

$V_{DS} = 0.5 - 0 < V_{DS_{sat}} = 0.6$

(not in velocity saturation)
Only Q_5 is in velocity saturation.
using Eq. 13.100:

$$i_D = \mu_n C_{ox}\left(\frac{W}{L}\right)_5 V_{DS_{sat}}\left(V_{GS} - V_t - \frac{1}{2}V_{DS_{sat}}\right)$$

$$\times (1 + \lambda V_{DS})$$

Neglecting $\lambda(\lambda = 0)$

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_5 0.6\left(1.8 - 0.5 - 0.5 - \frac{1}{2}(0.6)\right)$$

$$= \mu_n C_{ox}\left(\frac{W}{L}\right)_1 0.6\left[(1.8 - 0.5)0.5 - \frac{1}{2}(0.5)^2\right]$$

$$\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} = 1.75$$

without velocity saturation: (Eq. 15.4)

$$\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1} \leq \frac{1}{\left(1 - \frac{0.5}{1.8 - 0.5}\right)^2} - 1 = 1.64$$

16.15

With body effect considerations:

$$V_t = V_{t0} + r[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}]$$

$$V_t = 0.5 + 0.3[\sqrt{0.8 + 0.5} - \sqrt{0.8}] = 0.574 \text{ V}$$

$$\left(\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1}\right) \leq \left[\frac{1}{\left(1 - \frac{0.574}{1.8 - 0.574}\right)^2} - 1\right] = 2.54$$

without body effect: $V_{tn} = V_{t0}$

$$\left(\frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_1}\right) \leq \left[\frac{1}{\left(1 - \frac{0.5}{1.8 - 0.5}\right)^2} - 1\right] = 1.64$$

16.16

$\left(\frac{W}{L}\right)_5$ for body effect can have a large maximum ratio.

$$\left(\frac{\left(\frac{W}{L}\right)_5}{1}\right) \leq \left[\frac{1}{\left(1 - \frac{0.4}{1.2 - 0.4}\right)^2} - 1\right] = 3$$

For V_Q kept below V_{tn}, Eq. 15.10 becomes

$$\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2 = 1 - \left(\frac{\mu_p}{\mu_n}\right)\frac{\left(\frac{W}{L}\right)_4}{\left(\frac{W}{L}\right)_6}$$

$$\left(\frac{W}{L}\right)_4 = \frac{\left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2}{(\mu_p / \mu_n)}$$

Assuming $\frac{\mu_n}{\mu_p} = 4$

$$\left(\frac{W}{L}\right)_4 = 3$$

$\Rightarrow \Delta t = \frac{C_{\bar{B}}\,\Delta V}{I_5}$ where I_5 is obtained from

$$I_5 = \frac{1}{2} \times 430 \times 10^{-6} \times 1 \times (1.2 - 0.4 - 0.4)^2$$

$$= 34.4\ \mu\text{A}$$

$$\Delta t = \frac{C_{\bar{B}}\,\Delta V}{I_5} = \frac{2 \times 10^{-12} \times 0.2}{34.4 \times 10^{-6}} = 11.6 \text{ ns.}$$

(c) $\left(\frac{W}{L}\right) = 3$

(Eq. 15.1):

$$I_5 = \frac{1}{2} \times 430 \times 10^{-6} \times 3(1.2 - 0.4 - 0.4)^2$$

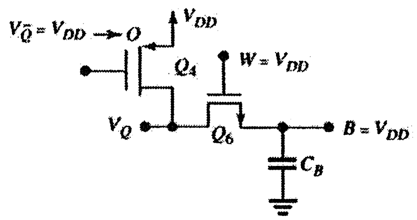
$$= 103\ \mu\text{A}$$

$$\Delta t = \frac{C_{\bar{B}}\,\Delta V}{I_5} = \frac{(2 \times 10^{-12} \times 0.2)}{103 \times 10^{-6}} = 3.9 \text{ ns.}$$

16.17

Storing a 0: $V_Q = 0$, $V_{\bar{Q}} = V_{DD}$

To write a 1 → B line raised to V_{DD}, $\bar{B}$ line lowered, and word line raised to V_{DD}. V_Q changes to V_{DD} and $V_{\bar{Q}} = 0$. Relevant transistors:



Q_4 in saturation and Q_6 in triode, which is the same as the text for writing a 0.

16.18

$$\left(\frac{W}{L}\right)_p \le \left(\frac{W}{L}\right)_n \times 4 \times \left[1 - \left(1 - \frac{0.5}{2.5 - 0.5}\right)^2\right]$$

$$\left(\frac{W}{L}\right)_p \le 1.75\left(\frac{W}{L}\right)_n$$

16.19

For a 1Mb square array there are 1024 rows and 1024 columns.

Thus the bit-line capacitance is $10^{-15}(1024 \times 1 + 12)$ or 1.036 pF

When storing a '1', the voltage on C_S is $(V_{DD} - V_t)$ or $(5 - 1.5) = 3.5V$. With precharge to $V_{DD}/2 = 2.5V$, the change in voltage on $C_S = 3.5 - 2.5 = 1.0V$,

For $C_S = 25fF$, the bit-line voltage resulting is $25/(25 + 1036) \times 1 = 23.6 mV$

When storing a '0', the voltage on C_S is $0V$ and the change is $2.5 - 0 = 2.5V$ with a resulting bit-line signal of $25/(25 + 1036) \times 2.5 = 58.9 mV$

16.20

Let $\left(\frac{W}{L}\right)_5 = \frac{0.13 \mu m}{0.13 \mu m}$

Let $V_{\bar{Q}} = V_{tn} = 0.4V$.

$I_5 = I_1$

$$\frac{1}{2}\mu_n C_{ox} \times 1 \times (1.2 - 0.4 - 0.4)^2$$

$$= \mu_n C_{ox} \times \left(\frac{W}{L}\right)_1 \times \left[(1.2 - 0.4)0.4 - \frac{1}{2}(0.4)^2\right]$$

Solving for $\left(\frac{W}{L}\right)_1 = .33$ so choose

$$\left(\frac{W}{L}\right)_1 = \frac{0.13 \mu m}{0.13 \mu m} = 1$$

Checking

$$1 \le \left[\frac{1}{\left(1 - \frac{0.4}{1.2 - .4}\right)^2} - 1\right] = 3$$

Let $\left(\frac{W}{L}\right)_4 = \frac{0.13 \mu m}{0.13 \mu m}$

Let $V_Q = V_{tn} = 0.4V$.

$\therefore I_4 = I_6$

$$\frac{1}{2}\mu_p C_{ox} \times 1 \times (1.2 - 0.4)^2 = \mu_p C_{ox} \times \left(\frac{W}{L}\right)_6$$

$$\times \left[(1.2 - 0.4)0.4 - \frac{1}{2}(0.4)^2\right]$$

$$\left(\frac{W}{L}\right)_6 = 1.33 \therefore \left(\frac{W}{L}\right)_6 = 2 = \frac{0.26 \mu m}{0.13 \mu m},$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \frac{0.13 \mu m}{0.13 \mu m}$$

Checking

$$\frac{\left(\frac{W}{L}\right)_p}{\left(\frac{W}{L}\right)_n} \le \left(\frac{\mu_n}{\mu_p}\right)\left[1 - \left(1 - \frac{V_{tn}}{V_{DD} - V_{tn}}\right)^2\right]$$

(Assume $\left(\frac{\mu_n}{\mu_p} = 4\right)$

$$\frac{1}{2} \le 4\left[1 - \left(1 - \frac{0.4}{1.2 - 0.4}\right)^2\right] = 3$$

16.21

If the memory array has n columns, it has 2n rows and $2n^2$ cells
Refresh time is

$2n\,(30)\,10^{-9}\text{s} = (1.00 - 0.98)\ \text{s} \times 10^{-3}\text{s}$

Whence

$n = 0.02 \times 8 \times 10^{-3}/40 \times 10^{-9} = 4000$

The corresponding memory capacity is

$2n^2 = 2(4000)^2$ or 32 M bits

16.22

For leakage current I, the voltage change on C in time T is $V = IT/C$

Correspondingly, $1 = I \times 10 \times 10^{-3}/20 \times 10^{-15}$, and the maximum leakage is $I = 20 \times 10^{-15}/10 \times 10^{-3} = \underline{\underline{2pA}}$

16.23

For leakage current I, the voltage change on c in time T is $V = \dfrac{IT}{c}$

Hence,

$0.2\text{V} = \dfrac{I \times 10^{-3} \times 10}{20 \times 10^{-15}} \Rightarrow I = 0.4 \times 10^{-12}$

$= 0.4$ pA is the maximum leakage current.

16.24

For the bit-line output to reach $0.9\,V_{DD} = 2.7$ V from $V_{DD}/2 = 1.5$ V in 2ns for an initial bit-line signal at $0.1/2 = 0.05$ V:
signal of $0.1/2 = 0.05$ V:

$2.7 = 1.5 + 0.05e^{2/\hat{j}}$

whence $2/\hat{j} = \ln[(2.7 - 1.5)/0.05] = 3.178$

and $\hat{j} = 2/3.178 = 0.629$ ns

Thus $C/G_m = 0.629 \times 10^{-9}$s, and $G_m = 1 \times 10^{-12}/(0.629 \times 10)$
$= 1.589$ mA/V
For matched inverters $g_{mn} = g_{mp} = G_m/2 = 1.589/2$
$= 0.795$ mA/V
Now $g_m = k'(W/L)(v_{GS} - V_t)$
and $0.795 \times 10^{-3} = 100 \times 10^{-6}\,(W/L)_n[3.0/2 - 0.8)$

Thus $(W/L)_n = 0.795 \times 10^{-3}/(100 \times 10^{-6})/0.7$
$= 11.36$
Now, for devices assumed to have length L μm (or, alternatively, for each micron of device length) $W_n = 11.36$ μm and $W_p = 3(11.36) = 34.1$ μm
Now, for a differential input signal of 0.2V (and 0.1V on each bit-line), the response time is t, where $2.7 = 1.5 + 0.1\,e^{t/0.629}$ whence $t = 0.629 \ln (2.7 - 1.5)/0.1 = 1.56$ ns

16.25

Note that for the inverters

$k_n = k_n'\,(W/L)_n = 100(6/1.5) = 400\ \mu\text{A/V}^2$

$k_p = k_p'\,(W/L)_p = (100/2.5)(15/1.5) = 400\ \mu\text{A/V}^2$

Thus we see that the inverters are matched,
Generally, $i_D = 1/2\,k_n\,(v_{GS} - V_t)^2$ and $g_m = 2_{iD}/$
$3V_{os} = kn(v_{gs} - v_t)$
Now, at $v_{GS} = v_o = V_{DD}/2 = 3.3/2 = 1.65$ V.
$g_{mn} = 400\,(1.65 - 0.8) = 340\ \mu$A/V
Thus $G_m = g_{mn} + g_{mp} = (340) = 680\ \mu$A/V

For a bit-line capacitance of 0.8 pF $\hat{j} - C/G_m$

or $\hat{j} = 0.8 \times 10^{-12}/680 \times 10^{-6} = 1.176$ ns
Now, for 0.9 V_{DD} reached in 2ns, for a signal Δv,
$0.9(3.3) = 1.65 + \Delta v e^{2/1.176}$
or $\Delta v = (2.97 - 1.65)/5.478 = 0.241$ V
Thus the initial voltage between B lines must be $2(0.241) = 0.482$ V
If an additional 1ns is allowed : $t = 2 + 1 = 3$ ns
and $\Delta v = (2.97 - 1.65)/e^{3/1.176} = 0.103$ V allowing a signal to be used of $2(0.103) = 0.206$ V
Now, with the original bit-line signal of 0.241 V, and a delay of 3 ns:
$2.97 = 1.65 + 0.241e^{3/\hat{j}}$
and $e^{3/\hat{j}} = (2.97 - 1.65)/0.241 = 5.477$
$3/\hat{j} = \ln(5.477) = 1.7006$
whence $\hat{j} = 3/1.7006 = 1.764$ ns

Thus $C = G_m\,\hat{j} = 680 \times 10^{-6} \times 1.764 \times 10^{-9} =$ 1.20 pF

This is an increase (from 0.8 pF) of $\left(\dfrac{1.2 - 0.8}{0.8}\right)$

$100 \times 50\%$
For the longer line, the initial delay to establish a suitable signal becomes 150% of 5ns = 7.5ns

16.26

(a) For an initial difference between bit lines of ΔV, each bit-line signal is ΔV/2.

for the rising line: $v_B = \frac{V_{DD}}{2} + \frac{\Delta V}{2} e^{t/(C_B/G_m)}$

$$e^{t/(C_B/G_m)} = \frac{2}{\Delta V} \cdot (0.9 - 0.5) V_{DD} = \frac{0.8 V_{DD}}{\Delta V}$$

Taking the natural log of both sides

$$\ln e^{t/(C_B/G_m)} = \ln\left(\frac{0.8 V_{DD}}{\Delta V}\right)$$

$t_d = \left(\frac{C_B}{G_m}\right) \ln\left(\frac{0.8 V_{DD}}{\Delta V}\right)$ as stated

(b) For reduction of one half the original, G_m has to be doubled. $G_m \propto (W/L)$

G_m is doubted by doubling the width of all transistors

(c) $V_{DD} = 1.8$ V, ΔV = 0.2 V

original design:

$$t_d = \left(\frac{C_B}{G_m}\right) \ln\left(\frac{0.8(1.8)}{0.2}\right) = 0.9\left(\frac{C_B}{G_m}\right)$$

Reducing ΔV by 4: $\Delta V = \frac{0.2}{4} = 0.05$ V

$$t_d = \left(\frac{C_B}{G_{m2}}\right) \ln\left(\frac{0.8(1.2)}{0.05}\right) = 1.5\left(\frac{C_B}{G_{m2}}\right)$$

for these to be equal:

$\frac{0.9}{G_m} = \frac{1.5}{G_{m2}}$ and $G_{m2} = \frac{1.5}{0.9} G_m = 1.7\ G_m$

Thus the transistors must be made 70% wider (or increased by a factor of 1.7x)

16.27

For the DRAM arrangement, the signal is applied to only one side; Thus in comparison to the SRAM treatment, the applied signal is only half as large.

Now, the specification must be met for either a '0' a '1' stored. The worst case is a differential signal of 40 mV (corresponding to a single-side signal of 70 mV)

Thus $2.0 = 20 \times 10^{-3} e^{5/\hat{j}}$, and $5 = \hat{j} \ln(2/20 \times 10^{-3})$, or $5 = \hat{j} \ln(100) \times 4.605\ \hat{j}$, whence $\hat{j} = 1.086$ ns (next)

For a 1pF bit-line capacitance, $G_m C/\hat{j}$ or $G_m = 1 \times 10^{-12}/1.086 \times 10^{-9} = 0.921$ mA/V, with 0.921 = 0.46 mA/V from each transistor.

Now, for the n-channel device, $g_m = k'_n(W/L)_n(v_{GS} - V_t)$ or $0.46 \times 10^{-3} = 100 \times 10^{-6}(W/L)_n(2.5-1)$

Thus $(W/L)_n = (0.46/0.1)/1.5 = 3.07$

For matched inverters, $(W/L)_p = 2.5(3.07) = 7.68$

When a '1' is read, the response time will be $t = \hat{j}\ \text{lm}\ (2/20 \times 10^{-3}) = 1.086 \ln 100 = 5$ns

(note: this is as designed!)

When a '0' is read, $t = 1.086 \times 10^{-9} \ln(2/(100/2) \times 10^{-3}) = 4.01$ns

16.28

$$\Delta t = \frac{CV_{DD}}{I}$$

$$I = \frac{CV_{DD}}{\Delta t} = \frac{60 \times 10^{15} \times 1.2}{0.3 \times 10^{-9}} = 240\mu\text{A}$$

$p = V_{DD} I = 1.2 \times 240\mu = 288\mu$W

16.29

Here $2^n = 512$, $n\log_{10}2 = \log_{10} 512$, $n = 2.709/0.301 = 9.00$
Thus the number of bits is 9
The decoder has 512 output line, one of which is active (high). The NOR away requires true and complement input lines for each bit: $2 \times 9 = 18$
Each row uses 9 NMOS for a total of $9 \times 512 = 4608$ NMOS and 512 PMOS, for a total of 5120 transistors.

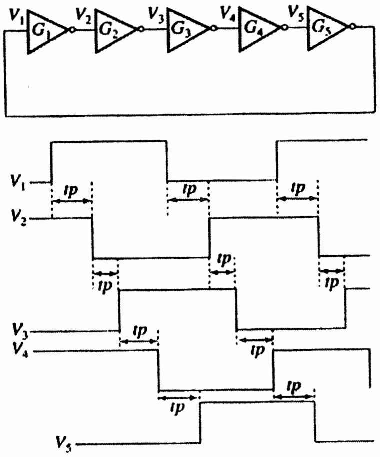
16.30

For a 256 K bit square away, there are $(256 \times 1024)^{1/2} = 512$ rows and columns
Number of column-address bits is $\log_2 512 = 9$
Two multiplexors are needed, since both true and complement bit lines are required. For each multiplexor, there are 512 output lines.
For each (half) multiplexor, 512 NMOS needed for a total of 1024 NMOS pass gates.
For the 512 output NOR decoder itself, $512 \times 9 = 4608$ NMOS and 512 PMOS are needed.
The address-bit inverters need 9 NMOS and 9 PMOS Overall, the need is for $1024 + 4608 + 9 = 5641$ NMOS and $512 + 9 = 521$ PMOS, for a total of 6162 transistors.

16.31

From the solution . above, a square 256 K-bit array has 512 rows and 512 columns for which 9 row and 9 column address bits are needed
Check: $2^{9+9} = 2^{18} = 262144$
For the tree 9 levels of pass gates are needed.
The total number of pass gates is N =
2+4+8+16+32+64+128+256+512
See that N = 2 + 2 (N − 512), or N = 2 + 2N − 1024, whence N = 1022
Thus a tree column decoder for 9 bits needs 1022 pass transistors
For true and complement bit lines, a total of 2(1022) = 2044 pass transistors are needed. Compare this with the number required beyond the input inverters namely 6162 − 18 = 6144

16.32



$$t_P = \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2}(6n + 4n) = 5 \text{ ns.}$$

$$f = \frac{1}{10t_P} = 20 \text{ MHz}$$

16.33

$$N = 11$$

$$f = 20\ \mu\text{Hz} = \frac{1}{2Nt_p} = \frac{1}{2(11)t_p}$$

$$\therefore\ t_p = 2.3 \text{ nsec}$$

16.34

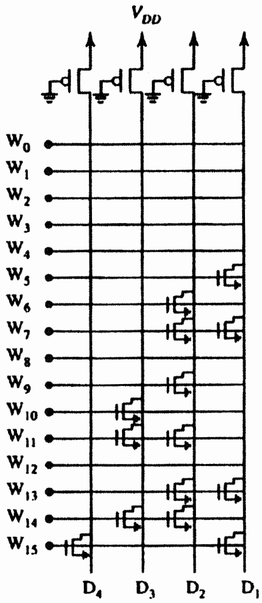
Note that the output is high if no word is selected. Thus, logically, high must correspond to logic 0 (and no transistor, as noted in the text).

Correspondingly, the words stored in are 0100, 0000, 1000, 1001, 0101, 0001, 0110, and 0010.

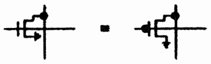
16.35

Need z = x + y

X	Y	Z
00	00	0000
00	01	0000
00	10	0000
00	11	0000
01	00	0000
01	01	0001
01	10	0010
01	11	0011
10	00	0000
10	01	0010
10	10	0100
10	11	0110
11	00	0000
11	01	0011
11	10	0110
11	11	1001



Note that a total of 14NMOS and 4PMOS are used.



16.36

(a) For the PMOS, with $V_B = 2.5$ V

$$L_D = (90/3)10^{-6}(12/1.2)[(5-1)2.5 - 2.5^2/2]$$

$$= 30 \times 10^{-6}(10)[4(2.5) - 2.5^2/2]$$

$$= 2.0625 \text{ mA}$$

Thus the average charging current is 2.06 mA

Time for precharge $t = CV/I$

whence

$$t = 1 \times 10^{-12}(5-0)/(2.06 \times 10^{-3}) = 2.42 \text{ ns}$$

(b) For the word-line rise,

$$T = RC = 5 \times 10^3 \times 2 \times 10^{-12} = 10 \text{ ns}$$

Here, $v_w = 5(1 - e^{-t/10})$

Thus the rise time (10% to 90%) is essentially the time t to 90%, where

$$0.9(5) = 5(1 - e^{-t/10})$$

$$e^{-t/10} = 0.1$$

and $t = -10 \ln(0.1) = 23$ ns

At the end of one time constant, $T =] = 10$ ns and $v_w = 5(1 - e^{-10/10}) = 3.16$ V

For discharge,

$$i_{D_{av}} = 1/2\, k_n' \,(W/L)_n (v_{gs} - V_t)^2$$

$$= 1/2(90)(3/1.2)(3.16 - 1)^2 = 525\ \mu\text{A}$$

Thus, the bit-line voltage will lower by 1V in about $\Delta t = C\Delta V / i_{D_{av}} = 1 \times 10^{-12}, (\times \ldots$

$\ldots) \times 1/(525 \times 10^{-6}) = 1.90$ ns